Planar Motion Steering of Underwater Vehicles by Exploiting Drag Coefficient Modulation

M. Aicardi, G. Casalino, G. Indiveri

Abstract—An underwater planar vehicle, actuated by rear thrusters and equipped with longitudinal control surfaces which allow the drag coefficient modulation in the sway direction, is considered in a dynamic setting. The maneuvering controls for the vehicle, in order to reach the required final position and attitude are devised by exploiting both the rear thrusters actuation and the capability offered by the admitted presence of the longitudinal, modulable, control surfaces.

I. INTRODUCTION

The control of non-holonomic systems has been considerably put under attention in the last years even in connection with the development of the theory of control of non linear systems. Many results are available regarding either the theoretical aspects ([1], [2], [4]) and their application to terrestrial ([3], [5]), space or marine vehicles ([8] [9]). As to marine vehicles, specific results have been provided either regarding the task of following a predefined trajectory ([6] [9]) and for the stabilization of the position and orientation on a target point ([7] [8]).

In this paper, the second of the above problems (let us call it parking problem) is considered.

Following the same reasoning line that has been proven effective in [5] we shall consider a polar representation of the position and orientation of the vehicle. For the unicycle model such an approach has revealed that a smooth and time-invariant strategy exists that stabilizes the vehicle on the target point. Such a result does not contradict the Brockett’s result since there exist a non-smooth transformation between polar and cartesian state variables. Nevertheless, the stabilization in the polar domain does not imply the stabilization in the cartesian domain (by the way a unicycle controlled by the strategy proposed in [5] is asymptotically stable in the polar domain and unstable in the cartesian domain in the sense of Lyapunov). On the other hand, polar coordinates seem to be more natural to represent the ‘posture’ of the vehicle (they mimic the ‘human’ choice).

Starting from this point of view, we shall consider a marine vehicle (therefore with the presence of a sway component of the velocity) and extend the result obtained in the terrestrial case to the new situation. To do this, one may be tempted to assume available lateral thrusters that can compensate the sway velocity.

In this paper we do not consider such a possibility but on the other hand we shall assume that the drag coefficient can be modified by means of a modulation of longitudinal control surfaces. More specifically, the role assigned to such control surfaces results in that of suitably controlling (without any specific compensation) the sway velocity in order to allow the asymptotic convergence toward the desired final position and attitude for the vehicle. Analytical proofs of stability and convergence are provided within the paper. Different simulation experiments are also provided and compared with the purely kinematic case showing the effectiveness of the proposed dynamic control technique. The so obtained satisfactory results actually encourage further studies devoted to their eventual extension to the more general 3D dynamic case.

II. MODEL AND CONTROL PROBLEM

Consider the following figure:

Fig. 1. The local vs. reference frame described by polar coordinates
The use of polar coordinates have already been successfully used in [5] where they have been shown to be suitable to encompass the limitations of the Brock-ett’s theorem [1]. The state equation can be written (following [5] and [9]) as

\begin{align}
\dot{e} &= -u \cos \alpha - v \sin \alpha \quad (1) \\
\dot{\alpha} &= -\omega + u \frac{\sin \alpha}{e} - v \frac{\cos \alpha}{e} \quad (2) \\
\dot{\theta} &= +u \frac{\sin \alpha}{e} - v \frac{\cos \alpha}{e} \quad (3) \\
\dot{v} &= -Dv - mu \omega \quad (4)
\end{align}

having denoted with $u$ and $v$ respectively the surge and sway velocity, with $\omega$ the angular velocity, and with $D, m$ the drag and inertia parameters. In (4) we shall assume to be able to modify the term $D$ modulating the quantity of underwater surface. Before entering into the details of the paper, it is worth observing that the model does not report the presence of the quadratic drag term $H|v|^2$. However, as it is apparent the quadratic drag component can only help in reducing the magnitude of the sway velocity. Then, we omit such a term and will make some remark at the end of the paper.

As to the control problem, we want to determine suitable laws for $u, \omega, \alpha, \theta, v$ in order to park asymptotically the vehicle over the reference frame. In other words we want $e, \alpha, \theta, \omega, v$ to converge to zero in an asymptotic way.

For a planar unicycle the appropriate control laws have already been determined in [5]. We recall them here (using the parameter $h$ in [5] set to 1):

\begin{align}
u &= \gamma e \cos(\alpha) \\
\omega &= k\alpha + (\alpha + \theta) \frac{\gamma \sin 2\alpha}{2\alpha} \\
\end{align}

A key role was played by the structure of $u$ given in (5) that allowed to eliminate the dependence on $e$ in the denominator of the state equations and of (6).

It is now somehow appealing to understand if and when the same laws can be used to control a marine vehicle. Obviously, a-priori, we cannot say anything about the structure of $u$ and then we could face a singularity in the state equations. However, as shown in the following section, a suitable structure for $D$ exists such that also $v$ will help to smooth the behaviour of the system. So, we shall divide the study in two steps:

- determine if there exist a structure for $D$ suitable to make $v = e \, g()$
- show the performance of the system controlled by (5) and (6)

\section{Structure of $v$}

Assume

\begin{equation}
v = eg() \quad (7)
\end{equation}

being $g()$ a suitable function not depending on $e$.

If (7) were true it would satisfy the differential relation given by (4), i.e.,

\begin{equation}
eg e + e \dot{g} = -De - mu \omega \quad (8)
\end{equation}

Now, considering the control laws given by (5) and (6) we have

\begin{equation}
u w = e f() \quad (9)
\end{equation}

with $f()$ not depending on $e$. Then, by using (9) and (5) in (8) we have

\begin{equation}
(-\gamma e \cos^2 \alpha - eg \sin \alpha)g + e \dot{g} = -De - me f \quad (10)
\end{equation}

Equation (10) means that $g$ must satisfy a differential equation independent of $e$ given by

\begin{equation}
\dot{g} = (-D + \gamma \cos^2 \alpha + g \sin \alpha)g - mf \quad (11)
\end{equation}

The differential equation (11) is not so good. In fact, the quadratic dependence on $g$ may generate a finite-escape-time of the solution. In fact, from a physical point of view, it may happen that $e$ can be zero with the vehicle having a non null sway velocity $v$. Hence in this case, the two conditions $e = 0$ and $v \neq 0$ together with (7), would imply an infinite value for $g$. To overcome such a fact, it is simple to note that if $D$ is chosen in the following way:

\begin{equation}
D = D' + \gamma \cos^2 \alpha + g \sin \alpha \quad ; \quad D' > 0 \quad (12)
\end{equation}

the differential equation for $g$ turns out to be a linear one with the term $mf$ acting as a forcing signal, i.e.,

\begin{equation}
\dot{g} = -D'g - mf \quad (13)
\end{equation}

where the forcing term $mf$ does not depend on $e$. Using the control laws given by (5) and (6), and also the structure in (7) the closed-loop equations can be rewritten as:

\begin{align}
\dot{e} &= -\gamma e \cos^2 \alpha - eg \sin \alpha \quad (14) \\
\dot{\alpha} &= -k\alpha - \gamma \sin 2\alpha - g \cos \alpha \quad (15) \\
\dot{\theta} &= \frac{\gamma \sin 2\alpha}{2} - g \cos \alpha \quad (16) \\
\dot{g} &= -D'g + m \gamma \cos \alpha \left( \frac{\kappa \alpha + \gamma \sin 2\alpha}{2} + \gamma \theta \sin 2\alpha \right) \quad (17)
\end{align}

First note that the set of equations (15), (16), (17) form an autonomous subsystem whose solution is independent on $e$. This means that the time evolution
of $\alpha \theta$ and $g$ is invariant with respect to the absolute ‘distance’ of the vehicle from the target. The conjecture about the structure of $v$ is then proven provided $D$ is chosen as in (12).

Obviously, since $D$ has to be always positive we shall take care about the choice of $D'$. This fact will be taken into account at the end of the study.

### B. Use of the ‘terrestrial’ controls: performance analysis

The result given in the previous subsection guarantees that the state equations of the system do not have any singularity.

We can now use the stability theory to analyze the behaviour of the system. In [10] it has been proven that the unicycle system controlled by (5) and (6) shows an exponentially asymptotically stable behaviour. Then, (see [11]) there exist a Lyapunov function $W_u(\alpha, \theta)$ and four strictly positive scalars $z_1, z_2, z_3, z_4$ such that:

$$z_1 \|\alpha, \theta\|^2 \leq W_u(\alpha, \theta) \leq z_2 \|\alpha, \theta\|^2$$

$$W_u \leq -z_3 \|\alpha, \theta\|^2$$

$$\|\partial W_u / \partial \alpha \partial W_u / \partial \theta\| \leq z_4 \|\alpha, \theta\|$$

The important point of the above properties is, as will be apparent in the following, that we rewrite as:

$$W = W_u + \frac{1}{2} g^2$$

Differentiating with respect to time,

$$\dot{W} = \frac{\partial W_u}{\partial \alpha} \dot{\alpha} + \frac{\partial W_u}{\partial \theta} \dot{\theta} + g \ddot{g}$$

Substituting now (15), (16) in the above derivative we get, by virtue of the linear presence of $g$ in such equations:

$$\dot{W} = \dot{W}_u - \left(\frac{\partial W_u}{\partial \alpha} + \frac{\partial W_u}{\partial \theta}\right) g \cos \alpha - D' g^2 - gmf$$

that we rewrite as:

$$\dot{W} = -D' g^2 - \left[(\partial W_u / \partial \alpha + \partial W_u / \partial \theta) \cos \alpha + m f\right] g + \dot{W}_u$$

(23)

Note that, as can be seen by (17)

$$\lim_{\alpha, \theta \to 0} f = 0$$

(24)

Now, consider (23) as a second order equation in $g$. Since the second order coefficient is surely negative, if such an equation would not admit real solutions then the overall function would be always non positive. With a straightforward application of the elementary properties of the second order equations we can compute the $\Delta$ as

$$\Delta^2 = [(\partial W_u / \partial \alpha + \partial W_u / \partial \theta) \cos \alpha + mf]^2 + 4 D' \dot{W}_u$$

(25)

The properties reported in (18) allows to say that $\Delta^2$ is negative if

$$D' > \frac{[(\partial W_u / \partial \alpha + \partial W_u / \partial \theta) \cos \alpha + mf]^2}{4z_2 \|\alpha, \theta\|^2}$$

(26)

To conclude the proof of the result we just have to say something about the structure of the r.h.s of (26). In particular, it is interesting to understand what happens as $\alpha$ and $\theta$ extremely small. It can be easily seen that, when the denominator of $\Delta^2$ becomes small, also the numerator shows the same behaviour with a dependence not slower than the denominator itself. This is due to the third of (18) and from the fact that $f$ is uniformly Lipschitz function in its arguments. Then, the upper bound given by the r.h.s. of (26) is always well defined. Finally, as to the positivity of $D$ we can easily note that, whenever the system parameters has been fixed, the behaviour of $g$ is surely upper-bounded so that we can always choose $D'$ such that both (26) is satisfied and $D = D' + \gamma \cos^2 \alpha + g \sin \alpha > 0$.

At this point we have just shown the convergence of $\alpha, \theta$ and $g$ to zero.

As regards $\epsilon$ we can say that, on the basis of (14)

- $\epsilon$ cannot cross zero
- for small values of $\alpha$ the behaviour is exponentially decreasing

On the basis of such considerations we can say that there exist a time instant $t^*$ (corresponding to the practical convergence of $\alpha, \theta$ and $g$ to zero, beyond which also $\epsilon$ will monotonically and exponentially decrease to zero.

Note that, we cannot say that $\epsilon$ always decreases (as would happen in the unicycle case) but just that it eventually exponentially converges to zero. This is however sufficient to our aim.

**Remark.** Coming back to the absence of the quadratic drag term, it should now be apparent that its presence would automatically increase the value of
\(D'\) and then help us for our objective. Nonetheless, if such a term would be present, we could not say any more that exactly \(v = \varepsilon g\). However, we could always say that \(|v| \leq \varepsilon g\), hence allowing always to exclude singularities in the closed loop state equations.

### III. Simulation results

In this section we report the simulation of the system under consideration compared with the behaviour of the terrestrial unicycle vehicle, in order to point out the effect of the sway velocity and the actions taken by the controller. The system’s parameters are the following: \(k = 8, \gamma = 2, m = 1, D' = 20\).

Figure 2 shows the behaviour if the trajectory, state variables and controls for a vehicle initially put in \(\{0, -1, -\pi\}\) with null initial sway velocity \(v\):

![Figure 2. Case of null initial \(v\)](image)

Figure 3 shows the effect of a non-zero initial \(v\).

![Figure 3. Case of non-null initial \(v\)](image)

To deal with ‘far’ manoeuvring, we report in figure 4 the same simulation as the one in Fig. 2 except for the fact that in the following case \(y(0) = -100\). As it can be seen, the trajectories are the same, and more important function \(g\) has exactly the same pattern as the one in Fig. 2 as was expected from the fact that \(\dot{g}\) does not depend on \(e\) as already noted.

![Figure 4. Case of ‘far’ initial conditions](image)

Figure 5 refers to a null initial absolute angle. As it can be seen the ‘\(S'\) manoeuvring is accomplished quite well.

![Figure 5. ‘\(S'\) manoeuvring with \(D' = 20\)](image)

Let us finally choose \(D' = 6\), and consider the same ‘\(S'\) manoeuvring. Obviously, the trajectory in Figure 6 is very different than the previous one, but the general properties of the closed loop system are preserved. As it can be seen in Figure 7, in the very early phases of the navigation, the vehicle is not tangent to its trajectory showing the sliding due to the sway velocity.
Fig. 6. ‘S’ manoeuvring with \( D' = 6 \)

Fig. 7. Sliding with \( D' = 6 \)

REFERENCES


