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Statistics of Turbulence between Two Counterrotating Disks in Low-Temperature Helium Gas.

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Abstract. – Turbulent flows, driven between two counterrotating disks in low-temperature helium gas, are investigated experimentally. The statistical characteristics of the flows are studied for various values of R_λ, ranging between 100 and 2100. The inertial range is investigated, and various quantities, such as the probability density function and the exponents of the structure functions, are determined. We find that the characteristics of the inertial range are essentially the same as those obtained in grid and jet turbulence.

Introduction. – High-Reynolds-number turbulence is traditionally investigated in open geometries such as wind tunnels, jets, wakes and duct flows [1]. Similar studies are usually not carried out in confined systems for various reasons: experimentally, high Reynolds numbers are more difficult to produce in such geometries, and hot-wire anemometry is more delicate to use, because of the absence of a strong mean flow; also, the turbulence generated in confined systems is expected to be less isotropic and homogeneous—and therefore with less general properties—than in open flows.

Confined geometries have actually received much attention in the last two years; it has been shown that the direct observation of vortex filaments is possible in a turbulent flow driven between counterrotating disks [2]. The technique relies on the fact that vortex filaments induce low pressures, and therefore a cavitation effect can take place. The method allows to investigate the dynamics of such objects, from their nucleation to their final burst. These observations have been compared with numerical experiments performed on the Taylor-Green vortex [3], for which there are also strong indications of the existence of vortex filaments. Thus, confined systems turn out to be attractive because visualisation studies are easier to perform than in open systems, and a close link with numerical simulations can be established. Indeed, the role of the filaments in the determination of the statistical properties of turbulence is still unclear and questions like their connection with the Kolmogorov cascade are open; one certainly needs further information on the statistics of turbulence in such systems. Results on the pressure field have been obtained recently [4], but, to the best of our knowledge, the statistics of the flow field itself is poorly known in this system, and questions like the small-scale intermittency and the existence of an inertial range have not been addressed experimentally until now.

The aim of this paper is to present a first study of the statistics of the flow field in the turbulent flow generated between two counterrotating disks, *i.e.* the configuration mentioned above, where vortex filaments have been observed. In order to achieve high Reynolds numbers, so as to obtain well-defined scaling properties, we use low-temperature helium gas as the working fluid [5]. In this study, we determine several quantities, such as the spectrum of the velocity fluctuations, the probability density function (p.d.f) of the velocity differences and the structure functions; such quantities will be further compared to those obtained in other configurations, such as jets or wakes [1].

Description of the experiment. – The experimental set-up is shown in fig. 1. The flow is produced in a cylinder, 6.4 cm in diameter and 4.8 cm high, limited axially by two counterrotating disks. Six radial blades, forming an angle of 60° between each other, are fixed on each disk so as to increase the efficiency of the stirring. Each blade is an aluminium plate, 1 mm thick, 1 cm wide and 3 cm long, mounted normally to the plane of the disk. The two disks are driven by two d.c. motors, which have been prepared to run at low temperature. The rotation rates of each motor range from 1 to 30 Hz and the motors can run at distinct speeds, either counterrotating or corotating. The whole system is enclosed in a cylindrical vessel, 23 cm high and 7.4 cm in diameter, in thermal contact with a liquid-helium bath. The temperature of the experiment is regulated at a constant value, comprised between 4.2 and 10 K. The vessel is filled with helium gas, at a controlled pressure, ranging from 0 up to 6 atm. The cryostat, which contains the liquid helium, is built in glass and has a nitrogen envelope.

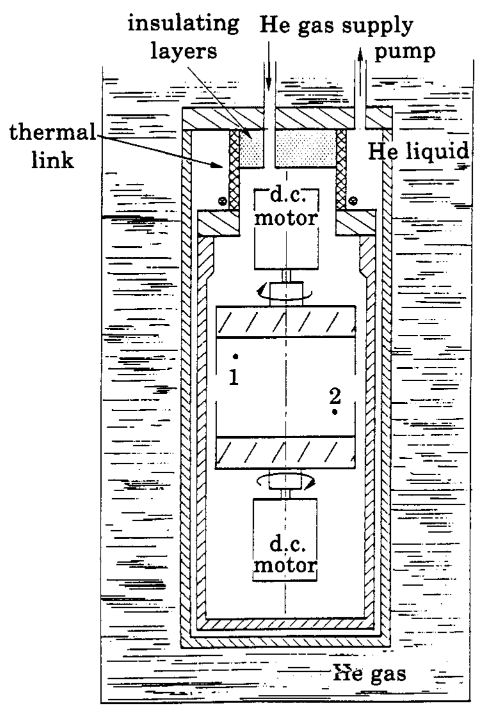

Fig. 1. – Sketch of the experiments. The numbers 1 and 2 indicate the position of the sensors.

The local velocity is measured by «hot»-wire anemometry. The detector is made of a 7 μm thick carbon fibre, whose resistivity increases with decreasing temperature (from 300 K to 4 K), in a way similar to Allen Bradley carbon resistors. This fibre is stretched across a 6 mm wide, 12 mm long frame made of 250 μm insulated steel wire. We then perform a gold evaporation which covers the fibre with a 4000 Å thick layer, everywhere except on a 25 μm long spot at the centre. This is the sensing element, which is thus a cylinder, 7 μm in diameter and 25 μm long. Like ordinary hot wires, the system can work at constant current or constant temperature. For the measurements of the velocity, a constant overheating of typically a few K is imposed. The resulting signal-to-noise ratio is about 60 dB. Two similar probes have been used for the present experiment: they are placed at a distance of 1 cm from the disk and from the outer cylinder, respectively in the upper and lower parts of the cell (see the positions 1 and 2 shown in fig. 1).

The signal is analysed on a HP3562A spectrum analyser, and digitized on a 16 bit converter, controlled by a DSP. The records are of various sizes, from a few to about 30 millions of points.

By changing the pressure and the temperature of the cell, we can investigate a large range of kinematic viscosities. The minimum value, which is reached close to the critical point, is approximately $2.5\cdot 10^{-4}\,\mathrm{cm}^2/\mathrm{s}$; values as large as $10^{-1}\,\mathrm{cm}^2/\mathrm{s}$ can be reached at small pressure, under conditions where local measurements are still possible. The calibration of the probes is achieved by driving the two disks in the corotating mode, at the same rotation rate. Figure 2 shows the evolution of the output voltage as a function of the rotating rate, for an overheating of 1 K. As shown in fig. 2, the experimental data can be reasonably well represented by King's law [6], *i.e.* by an expression of the form

$$V = \sqrt{A + B\sqrt{\Omega}}\,,$$

where Ω is the rotation frequency, and A and B are two constants. These constants depend on various parameters, such as the overheating, the temperature of the fluid, the pressure, the probe itself. We thus perform systematic calibrations of the probes.

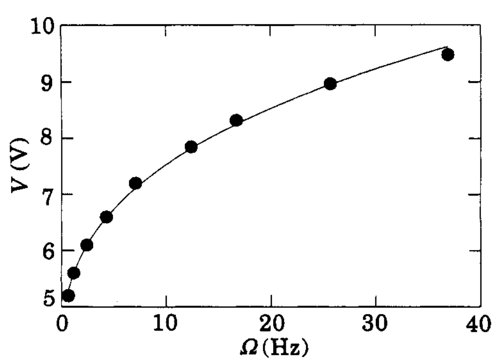

Fig. 2.

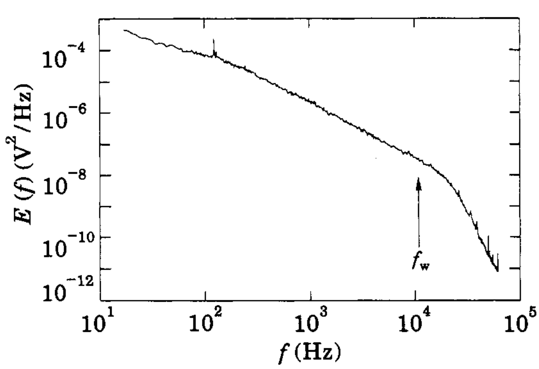

Fig. 3.

Fig. 2. – Calibration curve for a carbon fibre sensor, 7 μm in diameter and 25 μm of active length, obtained in a constant-temperature operating mode (the resistance is maintained to 250 Ω in the present case). The full line is a best fit using King's law with $A = 19\ \mathrm{V}^2$ and $B = 12\ \mathrm{V}^2/\sqrt{\mathrm{Hz}}$ (see text).

Fig. 3. – Power density spectrum of the voltage fluctuation, at 908 mbar, and with a rotation frequency of 20 Hz. The frequency characterizing the effect of the probe size is shown. The inertial range extends over two decades of variation of the frequency; the corresponding exponent is approximately – 1.72.

Results.

a) Energy spectrum. – Figure 3 shows the power density spectrum of the voltage fluctuations, obtained with one of the two probes, in the case where the disks rotate in two opposite directions at a frequency of 20 Hz. In this case, the pressure is 908 mbar, and the temperature 4.9 K, so that the kinematic viscosity is $1.2 \cdot 10^{-3}\,\mathrm{cm}^2/\mathrm{s}$, *i.e.* close to that of mercury. In the conditions where the measurement has been performed, the mean velocity is 250 cm/s, the fluctuation rate is 25%, and the value of the Reynolds number, based on the angular velocity and the disk radius, is 10^6. The corresponding velocity spectrum (obtained after using the calibration curve discussed above) has a form very close to that of fig. 3.

One can see that in the lower-frequency range, *i.e.* from 10 to 120 Hz, the spectrum is weakly dependent on the frequency; there is a well-defined oscillation at 120 Hz, corresponding to the periodic passage of the 6 blades at the rotation velocity of the disks. Above this peak, one observes a well-defined power law dependence of the spectrum, over more than 2 decades of variation of the frequency. By isolating the entire region where the power law holds, and using the least-mean-square method, one gets an exponent of 1.72, which is slightly larger than the Kolmogorov exponent.

At larger frequencies, the spectrum drops into the dissipative range. In this region, the spectrum is affected by the finite size of the probe, whose characteristic frequency f_w is defined by

$$f_w = \frac{U}{2\pi l_w}\,,$$

where $l_w = 25\ \mu\mathrm{m}$ is the length of the active part of the probe and U the local mean velocity. One finds $f_w = 16$ kHz (see the arrow on the spectrum), which indicates that the dissipative range cannot be fully resolved by the probe in this case.

Similar spectra are obtained for other values of the rotation speed, pressure and temperature; they differ from the one of fig. 3 by the extension of the domain of validity of the power law, and also their form in the higher-frequency range, depending on the value of f_w.

In conclusion, we find spectrum characteristics of «à la Kolmogorov» turbulence, with a measurable correction to the $-5/3$ exponent. This correction is usually attributed to intermittency.

b) Histograms and structure functions. – Figure 4 shows the p.d.f of the voltage difference, *i.e.* of the quantity

$$\Delta V(\tau) = \langle V(t) - V(t+\tau)\rangle$$

for the particular value $\tau = 0.8$ ms, obtained in the same conditions as above. Here, the brackets represent time averages; the histogram of fig. 4 is obtained by averaging over 34 millions of points sampled at 125 kHz. Here again, we do not represent the histogram of the velocity increments, but we have checked that its form is similar to that of fig. 4.

As previously noted [7], the p.d.f are not Gaussian for small τ; actually, in general, they are neither exponential; for example, in the case of fig. 4, the tails of the distribution can be fitted by an expression of the form $\exp[-\Delta V^\alpha]$, where $\alpha \sim 0.6$. As τ is increased, as expected, the histograms tend to evolve towards a shape closer to a Gaussian distribution.

Once the histograms are plotted, we are in a position to calculate the structure functions,

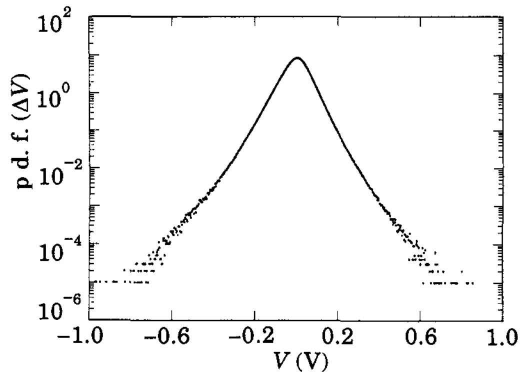

Fig. 4.

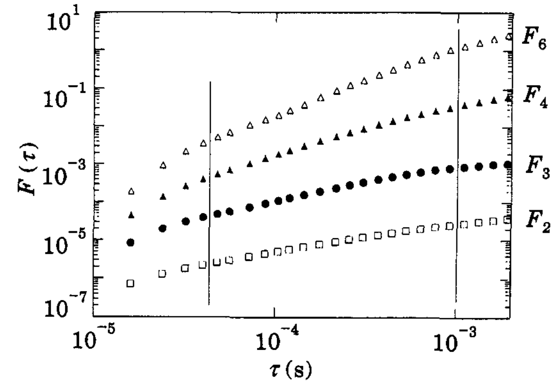

Fig. 5.

Fig. 4. – Probability density function of the voltage difference, in the same conditions as fig. 3, with a time difference τ of 0.80 ms.

Fig. 5. – Structure functions of the voltage difference, of various orders. The vertical lines indicate (somewhat arbitrarily) the boundaries of a domain where a power law may be roughly defined. The units for $F_3(\tau)$ are V^3, and are arbitrary for the other functions.

i.e. the quantities $F_p(t)$ defined by

$$F_p(\tau) = \langle (V(t) - V(t+\tau))^p \rangle ,$$

where p is the order of the structure function.

Figure 5 represents $F_p(\tau)$ for several values of p, ranging from 2 to 6, in the same conditions as above. Of particular interest is the third-order structure function $F_3(t)$, which is traditionally used to estimate the dissipation ε and the boundaries of the inertial range. According to the Kolmogorov relation [8], and using Taylor hypothesis, one has, for homogeneous isotropic turbulence,

$$F_3(\tau) = \frac{4}{5}\,\varepsilon U \tau .$$

Figure 5 shows the existence of a range for which $F_3(\tau)$ is close to be proportional to τ. Note that the limits of such a range are much narrower than that for which a power law is observed on the spectrum. Another information which can be extracted from the third-order structure function is the dissipation ε, the Taylor microscale λ and the corresponding Reynolds number R_λ. The last two quantities are defined by

$$\lambda = \sqrt{\frac{15\nu u^2}{\varepsilon}} \quad \text{and} \quad R_\lambda = \frac{u\lambda}{\nu} ,$$

in which u is the root mean square of the velocity fluctuations. For the experiment corresponding to fig. 3, one finds a value of 1200 for R_λ. For the experiments which we have performed, the range of values of R_λ extends from 100 to 2100. The maximum value is thus only 30% below that currently achieved in the largest wind tunnels [1].

Figure 5 shows that the structure functions roughly follow power laws in the same range of values of τ, with a small (but visible) curvature. If we neglect it, and assimilate such curves to straight lines, one can define a set of exponents $\zeta(p)$, by the relation

$$F_p(\tau) \sim \tau^{\zeta(p)}$$

in the inertial range. Typical values are listed below, for $R_\lambda \approx 1200$, with p varying from 1 to 8. For larger values of p, convergence is questionable.

p	1	2	3	4	6	8
$\zeta(p)$	0.4	0.7	1	1.3	1.8	2 ($\pm$ 0.2)

Similar results are obtained for different values of R_λ. We find agreement with the values currently accepted for jets and wakes [1].

Conclusion. – The main result of this study is that we recover, at small scales, the characteristics of turbulence which have already been established for open systems, such as jets, wakes, and duct flows. This reinforces the idea that, on small scales, turbulence has universal properties, independent of the forcing. In our case, the existence of such characteristics was not obvious, because of the presence of a mean rotation at the position where we measure, the strong anisotropy of the forcing, and the proximity of the walls. All that does not seem to affect strongly the small scales characteristics of the flow. One can thus say that filaments live in an ordinary turbulent world. Indeed, one certainly must perform measurements at various positions in the cell to draw out more accurate conclusions.

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