Sale of Price Information by Exchanges: Does it Promote Price Discovery? *

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Abstract

Exchanges often sell both trading services and price information. We study the effect of combining the sale of these two products on price discovery, in a rational expectations model. We show that pricing errors and thereby speculators’ trading profits decline when price information becomes available in real-time to a wider number of speculators. As a result, speculators’ willingness to pay for trading declines with the number of speculators with real-time price information. Hence a for-profit exchange optimally restricts access to price information more than if price information was sold by a pure information seller. As a result, pricing errors are not minimized and price discovery is impaired.

Keywords: Sale of Market Data, Transparency, Price Discovery.

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1 Introduction

“No member of this board, nor any partner of a member, shall hereafter give the prices of any kind of Stock, Exchange or Specie, to any printer for publication [...]” Constitution of the NYSE board, 1817 (quoted in Mulherin et al.(1991), p.597).

“Black box traders, direct market access traders and algorithmic traders are all in a race to beat the other guy. And the best way to do that is to get their hands on the market data first [...]” Traders Magazine, October 2009.

An essential function of securities markets is to discover asset values by aggregating investors' private information. Thus, securities prices contain information, which can then be used for trading decisions (Grossman and Stiglitz (1980)) or investment decisions by firms (e.g., Hayek (1945) or Dow and Gorton (1997)). This information is not free however as prices are owned by exchanges and exchanges usually sell information on prices.\(^1\) Selling information represents an increasing share of exchange's revenue. For instance, in 2010, the sale of market data accounted for 9% and 34% of the NYSE and the London Stock Exchange total revenue, respectively (source: annual reports)\(^2\)

Exchanges often supply price information at various speeds, charging a higher fee to traders who receive information more quickly.\(^3\) In the past trading was taking place on the floor of stock or derivatives exchanges. In this case, floor brokers had, thanks to their physical presence on the floor, a much faster access to price information than off-floor traders (see Easley, Hendershott, and Ramadorai (2009)). Today, trading is increasingly electronic, which greatly accelerates the speed at which price information can be delivered to market participants. Yet, there are still fast and slow traders in terms of access to price and trade data (see Hasbrouck and Saar (2011)). Indeed, some market participants (often proprietary trading firms) buy a direct access to trading platforms' data feeds while other traders obtain the same information with a delay (but at a lower cost).\(^4\)

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\(^1\) In the U.S., stock exchanges must make their data available since 1975 according to the so called “Quote Rule.” Yet, they can charge a price for disseminating their market data. See Mulherin et al. (1991) for an historical account of how exchanges established their property rights over market data.

\(^2\) A 2010 report of the Aite Group estimates that market data revenue accounts for 19% of revenue on average for NYSE-Euronext, Nasdaq, Deutsche Börse and the Tokyo Stock Exchange. See “Exchange Data Solutions: Reeling in the Revenue” (Aite Group, 2010).

\(^3\) Information on past trades is generally available for free only after some delay (e.g., twenty minutes on the NYSE, fifteen minutes on Nasdaq and Euronext). See \(\text{http://finance.yahoo.com/exchanges}\) for the delays after which information on transaction prices from major stock exchanges is freely released on yahoo.com. However, real-time price information is not free and given the speed at which trading takes place in electronic securities markets, an information delayed by one second is already stale.

\(^4\) This delay can be very small but is still sufficient to give an informational advantage to traders with direct access to market data. For instance, U.S. trading platforms must also transmit their data to plan processors (the...
firms buying direct access to streaming prices enjoy, in cyberspace, the informational advantage that floor traders used to have.

Does the sale of price information by exchanges hinder or promote price discovery? Why do exchanges offer differential speeds of access to their data? To our knowledge, there is no formal economic analysis of these questions, despite the importance of price discovery for well functioning financial markets.

To fill this gap, we consider the market for a security with risk averse speculators who possess heterogeneous signals about the payoff of the security and investors who trade for exogenous reasons (“liquidity traders”). We view speculators as proprietary trading firms and liquidity traders as long-term investors who occasionally trade to rebalance their portfolios. Speculators absorb the net demand or supply of shares from liquidity traders and thereby provide liquidity to these investors. Some speculators – the “insiders” – observe the entire history of prices (the “real-time ticker”). Other speculators – the “outsiders” – observe past prices with a delay (latency). As transaction prices are informative about the asset payoff, insiders have an informational advantage over outsiders. The market structures in which speculators are either all insiders or all outsiders result in a level playing field since, in each case, speculators have access to price information at the same speed. Otherwise some traders (the insiders) receive price information faster than others.

We measure price discovery by the size of the average pricing error in each trading round (the average squared deviation between the payoff of the security and the transaction price). Price discovery is more efficient when pricing errors are smaller. Our first result is that price discovery becomes more efficient when the fraction of insiders increases or when latency is smaller. Thus, price discovery is most efficient when all speculators are insiders.

We then study whether a for-profit exchange has an incentive to restrict or foster the dissemination of price information.\footnote{Exchanges are not pure information sellers: they derive revenues both from the sale of price information and the sale of trading services. To capture this feature, we assume that the for-profit exchange can charge a fee for real time price information and a market access fee (i.e., a fee for the right to trade). We show that there is a tension between these two sources of revenues: the trading revenue for an exchange declines when price information is sold to Consolidated Tape Association and Consolidated Quote Association) that then consolidate the data and distribute them to the public (the proceeds are then redistributed among contributors). As this process takes time, market participants with a direct access to the individual data feeds can obtain market data faster than participants who obtain the data from plan sponsors. See the SEC concept release on equity market structure, Section IV.B.2 for a discussion.} Exchanges are not pure information sellers: they derive revenues both from the sale of price information and the sale of trading services.\footnote{Major exchanges (e.g., NYSE-Euronext, Nasdaq, London Stock Exchange, Chicago Mercantile Exchange) are now for-profit. See Aggarwal and Dahiya (2006) for a survey of exchanges’ governances around the world.} For instance in 2010, revenues from trading and information sales accounted for respectively 75% and 9% of NYSE revenues (source: annual report). Other revenues stem from the sale of listing services. See Foucault and Parlour (2004) for a model of competition for listings between stock exchanges.

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a greater number of speculators. The reason is that an increase in the number of insiders exerts a negative externality on all speculators, thereby reducing their willingness to pay for trading.

There are two causes for this negative externality. First, an increase in the fraction of insiders raises the exposure of outsiders to adverse selection. Indeed, insiders bid conservatively for the security when they privately observe a decline in recent transaction prices (a bad signal) and aggressively when they observe an increase in recent transaction prices (a good signal). As a result, outsiders are exposed to a form of winner’s curse: they are more likely to end up with relatively large (small) holdings when the true value of the security is low (large). Second, pricing errors become smaller when the fraction of insiders increases, which reduces the expected profits of all speculators since both insiders and outsiders trade against pricing errors. The exposure to adverse selection is not a problem when all speculators are insiders but the second effect is maximal since, in this case, pricing errors are minimized. As a result, we show that speculators’s expected utility is always higher when they are all outsiders than when they are all insiders.

As the exchange derives revenues both from the sale of “trading rights” and the sale of price information, it internalizes the negative effect of the sale of price information on its revenue from trading. For this reason, the exchange never finds optimal to sell price information to all speculators since, in this case, their willingness to pay for both market access and price information is less than their willingness to pay for only market access when price information is delayed for all speculators. For some parameter values, the exchange may even find optimal not to sell price information at all (a solution that a pure data vendor would never find optimal). In this case, the market is quite opaque.

Otherwise, the optimal solution for the exchange is to sell price information to only a fraction of speculators. In this case, the market structure chosen by the exchange features fast and slow traders in terms of access to real-time information, as is observed in reality. In all cases, the fee for price information is too high for price discovery to be maximal: a reduction in the fee would improve price discovery since only a fraction of speculators find optimal to buy real time price information.

To sum up, in our setting, an exchange has an incentive to restrict access to price information because wider dissemination of price information has a negative effect on its revenue from trading. A pure information seller does not face this conflict of interest and thus always prices information more cheaply than an exchange.
2 Related Literature

Our analysis is related to the literature on financial markets transparency (see, e.g., Biais (1993), Madhavan (1995), Pagano and Roëll (1996), Bloomfield and O’Hara (2000), Rindi (2008), or Boehmer, Saar and Yu (2005)) since a swift dissemination of post trade prices is one dimension of market transparency. Post trade transparency is much discussed in regulatory debates but it has not received much attention theoretically. Our model suggests that for profit exchanges do not have natural incentives to widely disseminate post trade price information since an increase in the fraction of speculators with real-time price information can lower their revenue from the sale of trading services. In fact, for some parameter values, it can be optimal for the exchange in our model to provide no price information at all.

Our paper is also linked to the literature on markets for financial information (e.g., Admati and Pfleiderer (1986, 1987, 1990), Fishman and Hagerthy (1995), Veldkamp (2006), Cespa (2007), and Lee (2009)). However, this literature has not considered the case in which the vendor of information is also a vendor of trading services, which is the relevant case for the sale of price information by trading platforms. We show that when the data vendor also sells trading services, it charges a higher fee for price information than a pure data vendor since a wider dissemination of prices reduce speculators’ willingness to pay for trading.

To the best of our knowledge only two papers analyze the decision to acquire price information by investors. In Boulatov and Dierker (2007), speculators buy price information to reduce their uncertainty on execution prices (“execution risk”). This motivation for buying price information is absent from our model since traders submit price contingent orders (thus, they face no execution risk). Moreover, Boulatov and Dierker (2007) do not analyze the optimal pricing of price information and its impact on price discovery. Easley, O’Hara and Yuang (2011) study the effect of selling price information on the cost of capital in a framework similar to our model. They show that the cost of capital is lowest when price information is available for free. Our work is complementary since we focus on the effect of selling price information on price discovery.

Last our paper contributes to the research agenda described in Cantillon and Yin (2011). They call for using a combination of industrial organization and finance to understand market structures as the “microstructure of exchanges is often part of their business models.” (p. 335). This is precisely the approach taken in this paper: the fraction of speculators with access to price information and therefore the transparency of the market in our model and market efficiency are ultimately

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7See Biais et al. (2006) for a review of the literature on transparency in financial markets.
8Caglio and Mayhew (2008) show empirically that the allocation of tape revenues among contributing platforms by plan sponsors affect trade sizes in U.S. equities markets.
determined by optimal pricing decisions of an exchange.

3 Model

We consider the market for a risky asset with payoff \( v \sim N(0, \tau v^{-1}) \). Trade in this market takes place at dates 1, 2, \ldots, N between two types of traders: (i) a continuum of speculators (indexed by \( i \in [0, 1] \)) and (ii) liquidity traders with inelastic demands. Traders leave the market at the end of each trading round, and a new cohort of speculators and liquidity traders enters the market at the beginning of the next trading round. We denote by \( e_n \) the aggregate order imbalance from liquidity traders at date \( n \), with the convention that a positive (negative) order imbalance means that liquidity traders are net sellers (buyers) of the security. This order imbalance has a normal distribution with mean zero and precision \( \tau e^{-1} \).

Each speculator \( i \) arriving at date \( n \) observes a private signal \( s_{in} \) about the payoff of the security:

\[
s_{in} = v + \epsilon_{in},
\]

where \( \epsilon_{in} \sim N(0, \tau_{\epsilon n}^{-1}) \). The precision of private signals \( \tau_{\epsilon n} \) is the same for all speculators in any period \( n \), but can change across periods. We say that “fresh” information is available at date \( n \) if the precision of speculators’ signals at this date is strictly positive, \( \tau_{\epsilon n} > 0 \). Error terms \( \epsilon_{in} \) are independent across speculators, across periods, and from \( v \) and \( e_n \). Moreover, they cancel out on average (i.e., \( \int_0^1 s_{in} di = v \), a.s.).

We denote by \( p_n \) the clearing price at date \( n \) and by \( p^n \) the record of all transaction prices up to date \( n \) (the “ticker”):

\[
p^n = \{p_t\}_{t=0}^n, \text{ with } p_0 = E[v] = 0.
\]

Speculators differ in their speed of access to ticker information. Speculators with type I (the insiders) observe the ticker in real-time while speculators with type O (the outsiders) observe the ticker with a lag equal to \( l \geq 2 \) periods\(^9\). That is, insiders arriving at date \( n \) observe \( p^{n-1} \) before submitting their orders and outsiders arriving at date \( n \) observe \( p^{n-l^*} \) where \( l^* = \min\{n, l\} \). That is,

\[
p^{n-l^*} = \begin{cases} 
\{p_1, p_2, \ldots, p_{n-l}\}, & \text{if } n > l, \\
p_0, & \text{if } n \leq l.
\end{cases}
\]

We refer to \( p^n \) as the “real-time ticker” and to \( p^{n-l^*} \) as the “lagged ticker.” The “delayed ticker” is the set of prices unobserved by outsiders (i.e., \( p^n - p^{n-l^*} \)). The proportion of insiders at date \( n \) is denoted by \( \mu \). In the first period, the distinction between insiders and outsiders is moot since there are no prior transactions. Figure 1 below describes the timing of the model.
Each speculator has a CARA utility function with risk tolerance \( \gamma \). Thus, if speculator \( i \) holds \( x_{in} \) shares of the risky security at date \( n \), her expected utility is

\[
E[U(\pi_{in})|s_{in}, \Omega^k_n] = E[-\exp{-\gamma^{-1}\pi_{in}}|s_{in}, \Omega^k_n],
\]

where \( \pi_{in} = (v - p_n)x_{in} \) and \( \Omega^k_n \) is the price information available at date \( n \) to a speculator with type \( k \in \{I, O\} \).

As usual in rational expectations model, the clearing price in each period aggregates speculators’ private signals and constitutes therefore an additional signal about the asset payoff. As speculators submit price contingent demand functions, they all act as if they were observing the contemporaneous clearing price (whether or not they have information on past transaction prices). Thus, in period \( n \geq 2 \), we have \( \Omega^I_n = \{p^n\} \) and \( \Omega^O_n = \{p^{n-1}, p_n\} \). We denote the demand function of an insider by \( x^I_n(s_{in}, p^n) \) and that of an outsider by \( x^O_n(s_{in}, p^{n-1}, p_n) \). In each period, the clearing price, \( p_n \), is such that speculators’ aggregate demand is equal to the net order imbalance from

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\(^3\)See Dewan and Mendelson (1998) for a model in which investors access financial information at different speeds.
liquidity traders, i.e.,
\[
\int_0^\mu x_n^I(s_{in}, p^n) di + \int_\mu^1 x_n^O(s_{in}, p^{n-l^*}, p_n) di = e_n, \quad \forall n.
\] (5)

We refer to \( l \) as the latency in information dissemination and to \( \mu \) as the scope of information dissemination (in real-time). When \( 0 < \mu < 1 \), some speculators (the insiders) have access to ticker information faster than other speculators.

The structure of the model in each period is similar to other rational expectations model such as Hellwig (1980) for instance. Multi-periods rational expectations models usually assume that all investors have information on past prices, i.e., \( \mu = 1 \) (see, e.g., Grundy and McNichols (1989)). In contrast, we consider the more general case in which \( 0 \leq \mu \leq 1 \), so that some speculators have a faster access to information on past prices than others. The first step in our analysis is to study the equilibrium of the security market in each period (next section). We then analyze the effect of varying \( \mu \) on price discovery and speculators’ welfare. Last, we endogenize the scope of information dissemination, \( \mu \), by introducing a market for price information.

In reality, investors (including proprietary trading firms) need information about past transaction prices because they are not part to all transactions and therefore they do not automatically know the terms of prior transactions. To capture this in the simplest way, we assume that speculators stay in the market for only one period. A more general model could also feature speculators who participate to all trading rounds (“recurrent speculators”) in addition to “episodic” speculators. Recurrent speculators know the price history because they participate to all transactions. We have studied this more general case when \( n = 2 \). The model becomes significantly more difficult to analyze but does not deliver additional insights relative to the case in which all speculators stay in the market for only one trading round.

4 Equilibrium prices with differential access to price information

We focus on rational expectations equilibria in which speculators’ demand functions are linear in their private signals and prices. In this case, the clearing price is itself a linear function of the asset payoff and of liquidity traders’ net trade, in equilibrium. We refer to \( \tau_n(\mu, l) \) as the informativeness of the real-time ticker at date \( n \) and to \( \hat{\tau}_n(\mu, l) \) as the precision of outsiders’ forecast conditional on their price information at date \( n \) – as the informativeness of the “truncated” ticker. The next lemma provides a characterization of the unique linear rational expectations equilibrium in each period.
Lemma 1 In any period $n$, there is a unique linear rational expectations equilibrium. In this equilibrium, the price is given by

$$p_n = A_n v - \sum_{j=0}^{l^*-1} B_{n,j} e_{n-j} + D_n E[v \mid p^{n-l^*}],$$

where $A_n, \{B_{n,j}\}_{j=0}^{l^*-1}, D_n$ are positive constants characterized in the proof of the lemma. Moreover, speculators’ demand functions are given by

$$x^I_n(s_{in}, p^n) = \gamma(\tau_n + \tau_{\epsilon_n})(E[v|s_{in}, p^n] - p_n),$$

$$x^O_n(s_{in}, p^{n-l^*}, p_n) = \gamma(\hat{\tau}_n + \tau_{\epsilon_n})(E[v|s_{in}, p^{n-l^*}, p_n] - p_n),$$

where $\tau_n + \tau_{\epsilon_n} \equiv \text{Var}[v|p^n, s_{in}]^{-1}$ and $\hat{\tau}_n + \tau_{\epsilon_n} \equiv \text{Var}[v|p_n, p^{n-l^*}, s_{in}]^{-1}$.

To interpret the expression for the equilibrium price, consider the case in which $l = 2$ (the same interpretation applies for $l > 2$). In this case, equation (6) becomes

$$p_n = A_n v - B_{n,0} e_n - B_{n,1} e_{n-1} + D_n E[v \mid p^{n-2}], \text{ for } n \geq 2. \quad (9)$$

We now contrast two particular cases: in the first case, speculators do not receive fresh information at dates $n - 1$ and $n$ whereas in the second case, fresh information is available at date $n - 1$ but not at date $n$. For the discussion, we define $z_n \overset{def}= a_n v - e_n$ and $a_n \overset{def}= \gamma \tau_{\epsilon_n}$.

Case 1. No fresh information is available at date $n - 1$ and at date $n$ ($\tau_{\epsilon_{n-1}} = \tau_{\epsilon_n} = 0$, for $n \geq 3$).

In this case, $A_n = 0$, $B_{n,0} = (\gamma \tau_{n-2})^{-1}$, $B_{n,1} = 0$, and $D_n = 1$ (see the expressions for these coefficients in the appendix). Thus, the equilibrium price at date $n$ is

$$p_n = E[v \mid p^{n-2}] - (\gamma \tau_{n-2})^{-1} e_n. \quad (10)$$

As speculators entering the market at dates $n$ and $n - 1$ do not possess fresh information, the clearing price at date $n$ cannot reflect information above and beyond that contained in the lagged ticker, $p^{n-2}$. Thus, the clearing price is equal to the expected value of the security conditional on the lagged ticker adjusted by a risk premium (the compensation required by speculators to absorb liquidity traders’ net supply).

Case 2. Fresh information is available at date $n - 1$ but not at date $n$ ($\tau_{\epsilon_n} = 0$ but $\tau_{\epsilon_{n-1}} > 0$).

In this case, the transaction price at date $n - 1$ contains new information on the asset payoff ($A_{n-1} > 0$). Specifically, we show in the proof of Lemma 1 that the observation of the price at date $n - 1$ conveys a signal $z_{n-1} = a_{n-1} v - e_{n-1}$ on the asset payoff. Moreover, the equilibrium price at date $n$ can be written as follows

$$p_n = E[v \mid p^{n-2}] + A_n a_{n-1}^{-1} \left( z_{n-1} - E \left[ z_{n-1} \mid p^{n-2} \right] \right) - B_{n,0} e_n. \quad (11)$$
If $\mu = 0$, we have $A_n = 0$ and the expression for the equilibrium price at date $n$ is identical to its formulation in Case 1 (equation [10]). Indeed, in this case, no speculator observes the last transaction price. Thus, the information contained in this price ($z_{n-1}$) cannot transpire into the price at date $n$.

In contrast, if $\mu > 0$ some speculators at date $n$ observe the last transaction price and trade on this information. Thus, the information contained in the price at date $n-1$ “percolates” into the price at date $n$ and the latter is informative ($A_n > 0$), even though there is no fresh information at date $n$. Specifically, equation (11) shows that an outsider can extract a signal $\hat{z}_n$, from the clearing price at date $n$:

$$\hat{z}_n = \alpha_1 z_{n-1} - \alpha_0 e_n = \alpha_1 z_{n-1} + \alpha_0 \beta_n,$$

(12)

with $\alpha_0 \equiv A_n^{-1} B_{n,0}$ and $\alpha_1 \equiv a_{n-1}^{-1}$. This signal does not perfectly reveal insiders’ information ($z_{n-1}$) as it also depends on the supply of the risky security at date $n$ ($e_n$). Thus, at date $n$, outsiders obtain information ($\hat{z}_n$) from the clearing price but this information is not as precise as insiders’ information (since $\alpha_0 > 0$). For this reason, being an insider is valuable in our set-up. ■

When no fresh information is available at dates $\{n-1, \ldots, n-\ell^* + 1\}$ (as in Case 1), observing the delayed ticker is useless and there is no difference between insiders and outsiders. Thus, to focus on the interesting case, we assume from now on that, at any date $n$, there is at least one date $j \in \{n-1, \ldots, n-\ell^* + 1\}$ in which fresh information is available (i.e., $\tau_{\epsilon_j} > 0$). This restriction does not exclude the possibility that no fresh information is available at date $n$ (i.e., $\tau_{\epsilon_n} = 0$, as in Case 2).

In general, the price at date $n$ contains information on the asset payoff (i.e., $A_n > 0$) because (a) speculators’ demand depends on their private signals (when $\tau_{\epsilon_n} > 0$) and (b) insiders’ demand depends on the signals $\{z_{n-j}\}_{j=1}^{\ell^* - 1}$ that they extract from the prices yet unobserved by outsiders at date $n$ (as in Case 2). For outsiders, the clearing price at date $n$ conveys the following signal (see the proof of Lemma 1):

$$\hat{z}_n = \sum_{j=0}^{\ell^* - 1} \alpha_j z_{n-j},$$

(13)

where the $\alpha_s$ are positive coefficients. Intuitively, the signal $\hat{z}_n$ is a less precise estimate of the asset payoff than the series of signals $\{z_{n-j}\}_{j=0}^{\ell^* - 1}$ since it is a linear combination of these signals. Hence, the current clearing price is not a sufficient statistic for the entire price history as the latter enables insiders to recover the signals $\{z_{n-j}\}_{j=1}^{\ell^* - 1}$. For this reason, observing past prices has value even though speculators can condition their demand on the contemporaneous clearing price.\(^{10}\)

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\(^{10}\)In Brown and Jennings (1989) or Grundy and McNichols (1989) clearing prices are not a sufficient statistic for past prices as well. In contrast, Brennan and Cao (1996) and Vives (1995) develop multi-period models of trading
analyze the determinants of this value in Section 6.2 below.

5 Price discovery and the scope of information dissemination

We now study the effect of the scope of information dissemination ($\mu$) and of latency ($l$) on price discovery. We first consider how these variables affect (i) the informativeness of the “truncated ticker,” $\hat{\tau}_n(\mu, l) = (\text{Var}[v|p^{n-l^*}, p_n])^{-1}$, and (ii) the informativeness of the real-time ticker, $\tau_n(\mu, l) = (\text{Var}[v|p^n])^{-1}$. The first (resp. second) measure of price informativeness takes the point of view of outsiders (resp. insiders) since it measures the residual uncertainty on the asset payoff conditional on the prices that outsiders (resp. insiders) observe.

Let $\tau^m_n(\mu, l) \equiv (\text{Var}[\hat{z}_n|v])^{-1}$. The next proposition shows that $\tau^m_n$ is the contribution of the $n^{th}$ clearing price to the informativeness of the truncated ticker. For this reason, we refer to $\tau^m_n$ as the informativeness of the $n^{th}$ clearing price for outsiders.

**Proposition 1** At any date $n \geq 2$,

1. The informativeness of the truncated ticker, $\hat{\tau}_n$, is given by

$$\hat{\tau}_n(\mu, l) = \tau_{n-l^*} + \tau^m_n(\mu, l). \quad (14)$$

The informativeness of the truncated ticker increases in the scope of information dissemination ($\mu$), (weakly) decreases with latency ($l$) and is strictly smaller than the informativeness of the real-time ticker, $\tau_n$.

2. The informativeness of the real-time ticker, $\tau_n$, is independent of latency and the scope of information dissemination. It is given by

$$\tau_n(\mu, l) = \tau_v + \tau_e \sum_{t=1}^{n} a^2_t, \text{ with } a_t = \gamma \tau_{e_t}. \quad (15)$$

As explained in the previous section, the $n^{th}$ clearing price is informative about the signals $\{z_{n-j}\}_{j=1}^{l^*-1}$ obtained by insiders from the delayed ticker (the prices yet unobserved by outsiders). For this reason, the precision of an outsider’s forecast at date $n$ is greater than if he could not condition his forecast on the contemporaneous clearing price ($\hat{\tau}_n > \tau_{n-1^*}$). Yet, an insider’s forecast is more precise than an outsider’s forecast ($\hat{\tau}_n < \tau_n$) because the clearing price at date $n$ is not a sufficient statistic for the delayed ticker.

in which the clearing price in each period is a sufficient statistic for the entire price history. In this case, observing past prices has no informational value.
As the proportion of insiders increases, their demand (and therefore their signals) weighs more on the clearing price realized in each period. Hence, the informativeness of the truncated ticker increases in $\mu$. In addition, the informativeness of the truncated ticker decreases with latency for two reasons. First, an increase in latency implies that outsiders have access to a shorter and, therefore less informative, price history. Second, it implies that the number of past prices unobserved by outsiders is higher. As they have only one signal (the current clearing price) about the information contained in these prices, their inference is less precise.

In contrast, the informativeness of the real-time ticker, $p_n$, does not depend on the level of transparency, i.e., $l$ or $\mu$ (second part of the proposition). The intuition for this finding is as follows. In equilibrium, a speculator’s demand can be written as

$$x_n^k(s_{in}, \Omega_n^k) = (\gamma \tau_{\epsilon_n})s_{in} - \varphi_n^k(\Omega_n^k),$$

where $\varphi_n^k$ is a linear function of the prices observed by a speculator with type $k \in \{I, O\}$. Thus, the sensitivity of speculators’ demand to their private signals ($\gamma \tau_{\epsilon_n}$) is identical for outsiders and insiders. Accordingly, the sensitivity of the $n^{th}$ clearing price to the fresh information available in this period (i.e., $\int_0^1 s_{in} \, di$) does not depend on the proportion of insiders. For this reason, the informativeness of the entire price history does not depend on the proportion of insiders.

Price discovery is more efficient when transaction prices are closer to an asset fundamental value. Hence, we measure the efficiency of price discovery by the mean squared deviation between the payoff of the security and the clearing price (the average “pricing error” at date $n$).\footnote{This measure of market efficiency is often used in experimental studies where the asset payoff is known to the econometrician (see Bloomfield and O’Hara (1999) for instance).} Using the law of iterated expectations and the fact that $E[v] = 0$, it is immediate from equation (6) that

$$E[v - p_n] = 0.$$  

Thus, the average pricing error at date $n$ is equal to $\text{Var}[v - p_n]$\footnote{This would also be the case if $E[v] \neq 0$ since even in this case $E[v - p_n] = 0$.}

**Proposition 2** At any date $n \geq 2$, the average pricing error ($\text{Var}[v - p_n]$), decreases with $\mu_n$, the proportion of insiders at date $n$.

The intuition for this result is simple. Outsiders and insiders buy the asset when its price is less than their forecast of the asset payoff and sell it otherwise. As a result the clearing price in each trading round is an average of insiders and outsiders’ forecasts weighted by the proportion of each type of traders and adjusted by a risk premium. When the fraction of insiders increases, the clearing price becomes closer to their forecast. This effect improves price discovery since insiders’s
forecast of the asset value is closer to the true value of the asset than outsiders’ forecast (Proposition 1).

We have not been able to study analytically the effect of an increase in latency on the average pricing error. However, extensive numerical simulations indicate that an increase in latency has a positive impact on the average pricing error at each date \( n \geq 2 \), as illustrated in Figure 2 (compare for instance the pricing error for \( l = 10 \) and \( l = 20 \)).

Figure 2: Variance of the pricing error \( \text{Var}[v - p_n] \) as a function of latency. Parameter values: \( \tau_v = 2, \tau_e = 1, \tau_{\epsilon_n} = 1, \) for \( n = 1, 2, \ldots, N, \gamma = 1, \mu = 0.01, N = 50, \) and \( l \in \{10, 20, 30, 40, 50\} \).

In Figure 2, we assume that fresh information arrives at each date \( (\tau_{\epsilon_n} > 0, \forall n) \). This information is reflected into subsequent prices through trades by insiders and outsiders. For this reason, the pricing error decreases over time (i.e., \( n \)). Interestingly, Figure 2 shows that the speed at which the pricing error decays with \( n \) increases sharply when outsiders start obtaining information on past prices, that is, when \( l < n \). Intuitively, in this case, the information contained in past prices is better reflected into current prices because all speculators (insiders plus outsiders) trade on this information. This effect dramatically accelerates the speed of learning about the asset payoff compared to the case in which outsiders trade in the “dark” \( (n \leq l) \). This observation suggests that the time at which ticker information becomes available for free in the trading day should coincide with a positive jump in the speed of price discovery in financial markets (see Biais, Hillion and

To sum up, we find that restricting the dissemination of ticker information unambiguously impairs price discovery. Indeed, an increase in the proportion of insiders improves the informativeness of the truncated ticker and reduces the dispersion of pricing errors.

6 For profit exchanges and price discovery

We now endogenize $\mu$, the fraction of insiders, by introducing a market for real-time price information. We assume that speculators and liquidity traders meet on a trading platform operated by a for-profit exchange, which charges a fee for market access and a fee for access to real-time ticker information. Our main result is that the exchange always chooses a fee for real-time ticker information such that not all investors choose to be insiders. As a result, price discovery is not as efficient as it would if price information was sold by a pure data vendor.

6.1 Speculators’ welfare and the dissemination of price information

To understand this result, it is first useful to analyze how a change in the fraction of insiders affect speculators’ expected utilities. In particular, it is useful to compare speculators’ welfare in three market structures: (i) the market in which all speculators are outsiders ($\mu = 0$), (ii) the market in which all speculators are insiders ($\mu = 1$), and (iii) a two-tiered market in which only a fraction of speculators observe prices in real time ($0 < \mu < 1$).

We measure speculators’ welfare by the certainty equivalent of their ex-ante expected utility, i.e., before they learn their signals. Findings are identical if we work directly with speculators’ expected utilities but using certainty equivalents facilitate the exposition (the same approach is used, for instance, in Dow and Rahi (2003)). By definition, the certainty equivalent is the maximal fee that a speculator is willing to pay to participate to the market. We denote this fee by $C_n^k(\mu, l)$ for a speculator with type $k$ entering the market at date $n$ and we call it the speculator’s payoff.

Speculators’ payoffs can be written as follows\footnote{The calculation for investors’ certainty equivalent in the CARA-Gaussian framework is standard (see for instance Admati and Pfleiderer (1987)). A derivation of equation (17) is available from the authors upon request.}

$$C_n^k(\mu, l) = \frac{\gamma}{2} \ln \left( \frac{\text{Var}[v - p_n]}{\text{Var}[v | s_{in}, \Omega_{in}^k]} \right) = \frac{\gamma}{2} \ln(1 + \gamma^{-1} \text{Cov}[x_{in}^k, v - p_n]).$$

(17)

Thus, a speculator’s payoff increases in the covariance between the true return on the security ($v - p_n$) and its position in the security ($x_{in}^k$). This covariance is simply the expected dollar profit of a speculator since: $E[(v - p_n)x_{in}^k] = \text{Cov}[x_{in}^I, v - p_n]$ as $E[v - p_n] = 0$. The precision of insiders’ forecast is higher. Thus, insiders are more likely to buy the asset when it is undervalued ($p_n < v$)
and sell it when it is overvalued \((p_n > v)\). As a result, the covariance between their position and the return on their position \((v - p_n)\) is higher and they obtain a higher expected payoff than outsiders, as shown in the next proposition.

**Proposition 3** At any date \(n\), other things equal, an insider’s ex-ante expected utility is strictly greater than an outsider’s expected utility. Specifically:

\[
C^I_n(\mu, l) - C^O_n(\mu, l) = \frac{\gamma}{2} \ln \left( \frac{\tau_{e_n} + \tau_n(\mu, l)}{\hat{\tau}_{e_n} + \hat{\tau}_n(\mu, l)} \right) > 0. \tag{18}
\]

Now we analyze the effect of a change in the fraction of insiders on speculators’ expected utilities. When \(\mu\) increases, some speculators shift from being outsiders to being insiders. If the increase in \(\mu\) is small, these “switching” speculators are always better off (Proposition 3). But for the moment, we are interested in the welfare of other speculators. Thus, for clarity, we refer to these other speculators, i.e., those who do not shift groups, as “incumbent insiders” and “remaining outsiders”.

**Proposition 4** At any date \(n \geq 2\), incumbent insiders’ expected utility and remaining outsiders’ expected utility decline when the proportion of insiders increases.

In sum, acquisition of ticker information by one speculator exerts a negative externality on other speculators. Figure 3 illustrates Propositions 3 and 4 for specific parameter values and \(n = l = 2\).

This negative externality arises for two reasons. First, speculators benefit from large pricing errors in the model. Indeed, their expected payoff is positively related to the covariance between the size of their position and the difference between the asset payoff and its price (equation \(17\)). This covariance is positive (speculators tend to buy the asset when it is undervalued and sell it otherwise) and larger when pricing errors are larger. An increase in the number of insiders reduces pricing errors (Proposition 2) and therefore it harms all speculators. Second, as explained previously, insiders are more likely than outsiders to buy the security when its price is low compared to its payoff and sell it when its price is high relative to its payoff. Thus, insiders expose outsiders to adverse selection, which is another reason for which outsiders’ expected payoff declines with the fraction of insiders.

Now, we compare speculators’ payoff when \(\mu = 0\) and when \(\mu = 1\). In both market structures, all speculators are equal in terms of their access to price information. But when \(\mu = 1\) price information is available in real-time for all speculators while when \(\mu = 0\), access to price information is delayed for all speculators. Here, we are not comparing the expected utilities of a given group of speculators

\[^{14}\text{Take the extreme case in which the pricing error vanishes (}p_n = v\text{). Then, the covariance between a speculator’s position and the true return on the security would be zero.}\]
for two different values of \( \mu \) as in Proposition 4. Rather we are comparing the expected utility of speculators when they are all insiders \( (\mu = 1) \) with their expected utility when they are all outsiders \( (\mu = 0) \). When \( \mu = 0 \), the average pricing error is higher than when \( \mu = 1 \) but the precision of speculators’ forecast is smaller as they have access to less price information. The first effect works to make speculators’ payoff higher when \( \mu = 0 \) while the second effect works to make it smaller when \( \mu = 0 \). The next proposition shows that the first effect always dominates.

**Proposition 5** At any date \( n \geq 2 \), speculators’ welfare is higher when \( \mu = 0 \) than when \( \mu = 1 \), i.e., \( C^I_n(1,l) < C^O_n(0,l) \).

In other words, speculators prefer a market in which price information is delayed for all speculators to a market in which price information is available in real-time to all speculators (Proposition 5). As we shall see below, this result is important to understand why selling price information to all speculators is never optimal for a for-profit exchange. As an illustration, consider the example in Figure 3. Speculators have a payoff equal to \( C^O_2(0,2) = 0.206 \) when they are all outsiders and a payoff equal to \( C^I_2(1,2) = 0.147 \) when they are all insiders.

Figure 3: Speculators’ welfare is higher in a fully opaque market \( (\mu = 0) \) compared to a fully transparent one \( (\mu = 1) \): \( C^O_2(0,2) > C^I_2(1,2) \). Parameters’ values: \( \tau_v = 2 \), \( \tau_e = 1 \), \( \tau_{\epsilon_1} = 1 \), \( \tau_{\epsilon_2} = 0.05 \), \( \gamma = 1 \), and \( n = N = 2 \).

Now, we compare the market structure in which information is delayed for all speculators \( (\mu = 0) \) with a two-tiered market structure in which only some investors have access to information in real
time ($\mu > 0$). To this end, let $\overline{p}_n$ be the fraction of insiders at date $n$ such that $C^I_n(\overline{p}_n, l) = C^O_n(0, l)$. That is, when $\mu = \overline{p}_n$, insiders have exactly the same payoff in a two-tiered market structure with $\mu = \overline{p}_n$ or a market structure in which information is delayed for all speculators. Proposition 3 implies that $C^I_n(0, l) > C^O_n(0, l)$ while Proposition 5 implies that $C^I_n(1, l) < C^O_n(0, l)$. As $C^I_n(\mu, l)$ decreases with $\mu$, we deduce that $\overline{p}_n$ is strictly greater than zero and strictly smaller than one. Moreover:

$$C^I_n(\mu, l) < C^O_n(0, l) \text{ if } \overline{p}_n < \mu \leq 1,$$

(19) $$C^I_n(\mu, l) > C^O_n(0, l) \text{ if } 0 < \mu < \overline{p}_n.$$ (20)

Hence, we obtain the following proposition.

**Proposition 6**

1. If $\overline{p}_n < \mu_n \leq 1$ then, at date $n$, insiders and outsiders would be better off if information was delayed for all investors.

2. If $\mu_n < \overline{p}_n$ then, at date $n$, insiders would be worse off if information was delayed for all investors and outsiders would be better off.

As an illustration, consider Figure 3 again. In this example, $\overline{p}_2 = 0.696$. Moreover, if information is delayed for all speculators, their payoff is equal to $C^O_2(0, 2) = 0.206$. Now suppose that $\mu$ switches from zero to 10%. The speculators who become insiders are better off since their payoff becomes $C^I_2(0.1, 2) = 0.381$ while those who remain outsiders are worse off since their payoff becomes $C^O_2(0, 2) = 0.187$.

In this case, the aggregate increase in the payoff of speculators who become insiders is $0.1 \times (C^I_2(0.1, 2) - C^O_2(0, 2)) = 0.0175$ whereas the aggregate decrease in outsiders’ payoffs is $0.9 \times (C^O_2(0.1, 2) - C^O_2(0, 2)) = -0.0171$. Thus, in aggregate, insiders’ utility gain more than offsets outsiders’ utility loss. That is, the segmentation of the market in terms of speed of access to ticker information yields a net average welfare gain for speculators.

The reason is as follows. As previously explained, insiders’s expected profit is higher because they are more likely to buy the asset when its true return is positive and sell it when its true return is negative. Thus, speculators switching from outsiders to insiders work to increase the expected profit on speculators’ average position ($\mu x^I_n + (1 - \mu) x^O_n$), at the expense of liquidity traders since speculators’ net position must be the opposite of liquidity traders’ net trade. However, there is a countervailing effect: as the fraction of insiders increases, the average pricing error decreases which tends to lower the expected profit of all speculators. This countervailing effect always dominates for $\mu$ large enough but may not for small values of $\mu$. 

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This observation is key to understand why it can sometimes be optimal for a for-profit exchange to create a two-tier market for price information, rather than simply shutting down access to this information for all speculators (by charging a very high fee for price information). To prepare the ground for this result, let \( W_n(\mu, l) \) be the average payoff of speculators at date \( n \) when the fraction of insiders is \( \mu \). Observe that if \( \mu = 0 \), \( W_n(0, l) \) is equal to speculators’ payoff when price information is delayed for all speculators \((C^n_O(0, l))\). Let \( \mu_{\text{max}}^n \) be the value of \( \mu \) that maximizes \( W_n(\mu, l) \). Proposition 6 implies that \( \mu_{\text{max}}^n < 1 \) and there exist parameter values such that \( \mu_{\text{max}}^n > 0 \). For instance, for the parameter values considered in Figure 3, \( W_2(0, 2) = 0.206 \) and \( W_2(0.1, 2) = 0.207 \). Thus, in this example, \( \mu_2^{\text{max}} > 0 \).

It is difficult to characterize analytically the set of parameters for which \( \mu_{\text{max}}^n > 0 \). However, when price information is never disclosed to outsiders \((l = \infty)\) and fresh information arrives only at date 1 \((\tau_{e_1} > 0 \text{ but } \tau_{e_n} = 0 \text{ for } n \geq 2)\), we can obtain a simple sufficient condition on the parameters for \( \mu_{\text{max}}^n > 0 \) at all dates, as shown in the next proposition.

**Proposition 7** If \( l = \infty \), \( \tau_{e_n} = 0 \) for \( n \geq 2 \), \( \gamma^2 \tau_v \tau_\epsilon > 2 \text{ and } 0 < \tau_{e_1} < \left( \frac{\tau_v (\gamma^2 \tau_v \tau_\epsilon - 2)}{\gamma^2 \tau_\epsilon} \right)^{1/2} \) then speculators’ average welfare, \( W_n(\mu, l) \), is maximal in a two-tier market structure with a fraction \( \mu_{\text{max}}^n > 0 \) of insiders at any date \( n \geq 2 \).

The proposition considers the case in which fresh information arrives only at date 1. Thus, in this case, speculators who arrive at dates 2, 3, \ldots, do not have private signals. Yet, even in this case, creating a two-tiered market, and therefore introducing asymmetric information among speculators, can raise speculators’ average payoff. The reason is that price information raises the expected profit of insiders. This increase occurs primarily at the expense of liquidity traders when \( \mu_{\text{max}}^n > 0 \). Thus, it more than compensates the loss in welfare for outsiders and as a consequence, speculators’ average welfare increases.

### 6.2 Optimal sale of price information by for profit exchanges

We now show that a for-profit exchange chooses its fee for price information so that the fraction of speculators who decide to buy price information is \( \mu_{\text{max}}^n \), i.e., the fraction of speculators for which speculators’ average welfare is maximal. Hence, a for-profit exchange either chooses to delay access to price information for all market participants or sets a two-tier market structure. As a result, price discovery is never as high as it could be.

\[ 15 \] When \( \tau_{e_n} > 0 \), dissemination of ticker information has an ambiguous effect on the level of asymmetric information. Indeed, it exacerbates informational asymmetries between insiders and outsiders but it homogenizes information among investors who become insiders (as their private signals become less important).
We first describe how the market for price information operates in the model. At the beginning of each period, before receiving their private signals, speculators decide (i) whether to participate to the market and (ii) whether to purchase ticker information. We denote the price of ticker information at date $n$ by $\phi_n$. A speculator entering the market at date $n$ is an insider if she pays this fee. We denote the proportion of speculators buying ticker information by $\mu(\phi_n, l)$.

Let $\overline{\phi}_n(\mu, l)$ be the maximum fee that a speculator is willing to pay to observe the real-time ticker at date $n$. This fee is:

$$\overline{\phi}_n(\mu, l) = C^I_n(\mu, l) - C^O_n(\mu, l),$$

and we call it the value of the ticker. Using equation (18), we obtain that

$$\overline{\phi}_n(\mu, l) = \frac{\gamma}{2} \ln \left( \frac{\tau_{\epsilon_n} + \tau_n(\mu, l)}{\tau_{\epsilon_n} + \hat{\tau}_n(\mu, l)} \right) > 0. \quad (22)$$

The value of the ticker increases with the difference between the informativeness of the real-time ticker and the informativeness of the truncated ticker. Proposition 8 implies that this difference is reduced when the proportion of insiders increases or when latency is reduced. Thus, we obtain the following result.

**Proposition 8** For a fixed latency, the value of the real-time ticker at any date $n \geq 2$ decreases with the proportion of insiders. Moreover, for a fixed proportion of insiders, the value of the real-time ticker weakly increases with the latency in information dissemination, $l$. More precisely:

$$\overline{\phi}_n(\mu, l) < \overline{\phi}_n(\mu, l + 1) \text{ for } n > l,$$

$$\overline{\phi}_n(\mu, l) = \overline{\phi}_n(\mu, l + 1) \text{ for } n \leq l.$$

At date $n$, a speculator buys ticker information if the price of the ticker is strictly less than the value of the ticker ($\phi_n < \overline{\phi}_n(\mu, l)$). She does not buy information if $\phi_n > \overline{\phi}_n(\mu, l)$. Finally, she is indifferent between buying ticker information or not if $\phi_n = \overline{\phi}_n(\mu, l)$. Thus, the equilibrium proportion of insiders is

$$\mu^e(\phi_n, l) = \begin{cases} 
1 & \text{if } \phi_n \leq \overline{\phi}_n(1, l), \\
\mu & \text{if } \phi_n = \overline{\phi}_n(\mu, l), \\
0 & \text{if } \phi_n \geq \overline{\phi}_n(0, l). 
\end{cases} \quad (23)$$

Figure 4 shows how the equilibrium proportion of insiders is determined. As the value of the ticker declines with $\mu$ (Proposition 8), for each price of the ticker, there is a unique equilibrium proportion of insiders $\mu^e(\phi_n, l)$. Moreover, the equilibrium proportion of insiders decreases with the price of ticker information (to see this, consider an upward shift in $\phi_{10}$ in Figure 4).
In Section 5, we have shown that measures of informational efficiency improves when the fraction of insiders increases (see Propositions 1 and 2). As this fraction is inversely related to the fee for price information, we obtain the following testable implication.

**Corollary 1** A decrease in the fee for price information improves price discovery.

We now study how a for-profit exchange would choose its fee for ticker information. In each period, the for-profit exchange charges two fees: a fee for real-time ticker information, $\phi_n$, and a market access fee, $E_n$ (a “trading right”). In this way, we account for the fact that exchanges also derive revenues from market participation. All speculators pay the access fee and only speculators who receive ticker information in real time pay the information fee. We refer to $(\phi_n, E_n)$ as being the exchange’s tariff. In each period, the for-profit exchange chooses its tariff to maximize its profit.

Given this tariff, the proportion of insiders is $\mu^e(\phi_n, l)$ as explained previously and the (per capita) expected profit of the exchange is:

$$\Pi_n(\mu^e(\phi_n, l), l) = \mu^e(\phi_n, l)\phi_n + E_n$$

\[\text{16}\] The choice of a tariff in a given period does not influence the exchange’s expected profits in subsequent periods because the speculators’ payoffs in each period only depend on the fraction of insiders in this period. Hence, the tariffs that maximize per period expected profits of the exchange also maximize the total expected profit of the exchange over all periods.
As the exchange is a monopolist, it optimally chooses its tariff to extract all gains from trade from speculators. Thus, the optimal tariff of the exchange is such that speculators’ payoff net of the fees they pay is zero. This implies:

\[ E_n = C_O^e(\mu^e(\phi_n, l), l) \quad \text{(outsiders’ net payoff is zero),} \tag{24} \]
\[ \phi_n + E_n = C_I^e(\mu^e(\phi_n, l), l) \quad \text{(insiders’ net payoff is zero).} \tag{25} \]

Observe that the access fee is determined by the fee for ticker information since this fee determines the equilibrium proportion of insiders. Hence, ultimately, \( \phi_n \) is the only decision variable of the for-profit exchange. Equations (24) and (25) imply:

\[ \phi_n = C_I^e(\mu^e(\phi_n, l), l) - C_O^e(\mu^e(\phi_n, l), l) = \bar{\phi}_n(\mu^e(\phi_n, l), l), \]

where the last equality follows from the definition of \( \bar{\phi}_n(\mu^e(\phi_n, l), l) \). Thus, the objective function of the for-profit exchange is:

\[ \max_{\phi_n} \Pi_n(\mu^e(\phi_n, l), l) = \mu^e(\phi_n, l)\bar{\phi}_n(\mu^e(\phi_n, l), l) + C_O^e(\mu^e(\phi_n, l), l). \tag{26} \]

The solution to this optimization problem can be found by solving

\[ \max_{\mu} \Pi_n(\mu, l) = \mu\bar{\phi}_n(\mu, l) + C_O^e(\mu, l), \tag{27} \]

since there is a one-to-one relationship between the equilibrium proportion of insiders and the fee charged for ticker information. Consequently, if \( \mu_n^* \) is the solution of equation (27) then \( \phi_n^* = \bar{\phi}_n(\mu_n^*, l) \) solves equation (26).

The for-profit exchange faces the following trade-off. On the one hand, by increasing the proportion of insiders, it gets a larger revenue from information sale (\( \mu\bar{\phi}_n(\mu, l) \)). However, to achieve such an increase, the exchange must lower (i) the price for ticker information (since \( \partial \bar{\phi}_n(\mu, l)/\partial \mu < 0 \)) and (ii) the access fee since speculators’ gain from market participation decreases with the proportion of insiders (\( \partial C_O^e(\mu, l)/\partial \mu < 0 \)). Using the definition of \( \bar{\phi}_n(\mu, l) \), we can rewrite equation (27) as:

\[ \max_{\mu} \Pi_n(\mu, l) = \mu C_I^e(\mu, l) + (1 - \mu)C_O^e(\mu, l) = W_n(\mu, l). \tag{28} \]

Hence, the exchange’s expected profit is equal to the average payoff of all speculators. This is intuitive since the exchange’s fees are set to extract all speculators’ surplus. The following result is then immediate.

**Proposition 9** At any date \( n \geq 2 \) and for all values of the parameters, the for-profit exchange chooses its tariff so that the proportion of speculators buying ticker information maximizes speculators’ average payoff. That is, \( \mu_n^* = \mu_n^{\max} \) and \( \phi_n^* = \bar{\phi}_n(\mu_n^{\max}, l) \). Thus, rationing access to ticker information is always optimal for a for-profit exchange.
As $\mu_n^{\text{max}} < 1$ (see previous section), Proposition 9 implies that the for-profit exchange either delays access to price information for all speculators ($\mu_n^{\text{max}} = 0$) or gives access to price information only to a subset of speculators ($0 < \mu_n^{\text{max}} < 1$). Figure 5 illustrates Proposition 9 by plotting the exchange’s profit ($\Pi_2$) as a function of the proportion of insiders, for specific parameter values (the same as those in the numerical example of the previous section, see Figure 3). For these parameters, the exchange’s expected profit peaks at a relatively low proportion of insiders ($\mu^*_2 = \mu_2^{\text{max}} \simeq 6\%$ which implies $\overline{\phi}(\mu_2^*, 2) \simeq 0.2$).

Figure 5: Profit from ticker sale and entry fee: The Exchange optimally segments the market ($\mu^* = 0.062$). Parameters’ values: $\tau_v = 2$, $\tau_e = 1$, $\tau_{\epsilon_1} = 1$, $\tau_{\epsilon_2} = 0.05$, $\gamma = 1$, $l = 2$, and $n = N = 2$.

Thus there are cases in which a two-tier market structure in terms of access to price information is optimal for the exchange. It is a way to optimally strike a balance between two opposite goals: (i) maximize speculators’ willingness to pay for trading, which requires to restrict access to price information and (ii) make profits on the sale of information, which requires to sell information to at least some speculators.

Corollary 1 implies that informational efficiency is maximal when the fraction of insiders is one. The next corollary immediately follows from this result and Proposition 9.

**Corollary 2** The fee for price information set by a for-profit exchange is too large for price discovery to be maximal.
One way to alleviate this problem is to separate the sale of trading services from the sale of price information. Indeed, a pure data vendor earns revenues only from the sale of price information (i.e., its expected profit is $\mu \tilde{\phi}_n(\mu, l)$). Hence it has no incentive to internalize the negative effect of disseminating price information on speculators’ willingness to pay for market participation. As a result the fee for price information charged by a pure data vendor is always higher than that charged by a for-profit exchange and price discovery is enhanced. For instance, Figure 6 plots the expected expected from the sale of price information for a pure data vendor in our model for the same parameters as those used in Figure 5. This expected profit for $n = 2$ is maximal for $\mu^{**}_2 \simeq 90\%$ and a price for price information at this date equal to $\tilde{\phi}_2(0.9, 2) \simeq 0.08$ (i.e., less than half the price charged by a for-profit exchange in the same conditions).

Figure 6: Profit from ticker sale only: The pure Data Vendor sets a lower fee compared to the Exchange allowing a larger number of insiders ($\mu_{DV}^{**} = 0.907$). Parameters’ values: $\tau_v = 2$, $\tau_e = 1$, $\tau_{\epsilon_1} = 1$, $\tau_{\epsilon_2} = 0.05$, $\gamma = 1$, $l = 2$, and $n = N = 2$.

In the recent years, competition among stock exchanges has increased. In particular new trading platforms (such as BATS in the U.S. or Chi-X in Europe) started trading shares listed on incumbent markets such as NYSE-Euronext and Nasdaq. As a result, the same security is often traded on multiple trading platforms. However prices in these trading platforms are not perfect substitutes as they are not equally informative.\footnote{For instance, Hasbrouck (1995) finds that the contribution of NYSE prices to price discovery is higher than the contribution of regional exchanges. Thus, the price of ticker information for the same security should vary across}
than incumbent exchanges for direct access to their real-time data, maybe because their prices are less informative.\footnote{For instance, the NYSE and BATS charge, respectively, $60,000/month and $5,000 for real-time last sale price in securities traded on their platforms. See http://www.batstrading.com/market_data/products/#last-sale and http://www.nyxdata.com/Data-Products/NYSE-Real-Time-Reference-Prices.} This evolution however may have forced however incumbent markets to cut their fee for price information. This reduction could be one mechanism by which price discovery improves when markets become more fragmented. A full analysis of this conjecture is left for future research.

7 Conclusions

Exchanges play an important role by providing market places to share risk and discover asset values. They play this role with an eye to making profits from various activities, in particular the sale of trading services and the sale of information on prices. In this paper, we show that there is a conflict between the efficiency of price discovery and profit maximization by exchanges.

This conflict arises because selling real-time price information to more speculators enhances price discovery but reduces speculators’ expected trading profits. Now, an exchange’s revenue from the sale of trading services is positively related to speculators’ expected profit since speculators must cover their cost of market participation by trading profits. Hence, a for-profit exchange faces a trade-off between its expected revenue from trading and its expected revenue from the sale of price information. We show that it always resolves this trade-off by choosing a fee for price information such that no speculator or only a fraction of all speculators buy access to price information in real-time. This pricing policy is detrimental to price discovery since price discovery is most efficient when all speculators observe prices in real-time.

In reality, exchanges sell price information directly to proprietary trading firms, as explained in the introduction. They also sell market data (including prices) to pure data vendors (e.g., Bloomberg or Reuters) who then resell these data to other investors. Moreover, as mentioned in the last section, a given security often trades on multiple platforms and prices on these platforms are imperfect substitutes. Hence, exchanges face competition both from data vendors (to whom they sell information) and other exchanges. Studying this competition and its effects on market efficiency is an interesting venue for future research.

References


Appendix

Proof of Lemma 1

Step 1. Informational content of equilibrium prices.

In a symmetric linear equilibrium, speculators’ demand functions in period \( n \geq 1 \) can be written as follows:

\[
\begin{align*}
    x_n^I(s, p^n) &= a_n^I s - \varphi_n^I(p^n), \\
    x_n^O(s, p^{n-2}, p^n) &= a_n^O s - \varphi_n^O(p^{n-2}, p^n),
\end{align*}
\]

where \( \varphi_n^k(.) \) is a linear function of the clearing price at date \( n \) and the past prices observed by a speculator with type \( k \in \{I, O\} \) (\( \varphi_n^I(.) = \varphi_n^O(.) \) since price information is identical for insiders and outsiders at date 1). In any period \( n \), the clearing condition is

\[
\int_0^\mu x_{in}^I di + \int_\mu^1 x_{in}^O di = e_n.
\]

Thus, using equations (29) and (30), we deduce that at date \( n \)

\[
a_n v - \mu \varphi_n^I(p^n) - (1 - \mu) \varphi_n^O(p^{n-2}, p^n) = e_n, \quad \forall n \geq 2,
\]

with \( a_n \equiv \mu a_n^I + (1 - \mu) a_n^O \). We deduce that \( p^n \) is observationally equivalent to \( z^n = \{z_1, z_2, \ldots, z_n\} \) with \( z_n = a_n v - e_n \).

Step 2. Equilibrium in period \( n \).

Insiders. An insider’s demand function in period \( n \), \( x_n^I(s, p^n) \), maximizes

\[
E[-\exp\left\{\frac{(v - p_n) x_n^I}{\gamma}\right\} | s, p^n].
\]
We deduce that
\[ x_n^I(s_{in}, p^n) = \gamma \frac{E[v - p_n | s_{in}, p^n]}{\text{Var}[v - p_n | s_{in}, p^n]} = \gamma \frac{E[v | s_{in}, p^n] - p_n}{\text{Var}[v | s_{in}, p^n]} . \]

As \( p^n \) is observationally equivalent to \( z^n \), we deduce (using well-known properties of normal random variables)
\[
\begin{align*}
E[v | s_{in}, p^n] &= E[v | s_{in}, z^n] = (\tau_n(\mu, l) + \tau_{\epsilon_n})^{-1}(\tau_n E[v | z^n] + \tau_{\epsilon_n} s_{in}), \\
\text{Var}[v | s_{in}, p^n] &= \text{Var}[v | s_{in}, z^n] = (\tau_n(\mu, l) + \tau_{\epsilon_n})^{-1},
\end{align*}
\]

where
\[
\tau_n(\mu, l) \overset{\text{def}}{=} (\text{Var}[v | p^n])^{-1} = (\text{Var}[v | z^n])^{-1} = \tau_v + \tau_e \sum_{t=1}^n \sigma_t^2 .
\]

Thus,
\[
x_n^I(s_{in}, p^n) = \gamma (\tau_v + \tau_{\epsilon_n})(E[v | s_{in}, p^n] - p_n) \\
= a_n^I(s_{in} - p_n) + \gamma \tau_n(E[v | p^n] - p_n),
\]

where \( a_n^I = \gamma \tau_{\epsilon_n} \).

**Outsiders.** An outsider’s demand function in period \( n \), \( x_n^O(s_{in}, p^{n-l^*}, p_n) \), maximizes:
\[
E \left[ -\exp \left\{ \frac{(v - p_n)x_n^O}{\gamma} \right\} | s_{in}, p^{n-l^*}, p_n \right].
\]

We deduce that
\[
x_n^O(s_{in}, p^{n-l^*}, p_n) = \gamma \frac{E[v - p_n | s_{in}, p^{n-l^*}, p_n]}{\text{Var}[v - p_n | s_{in}, p^{n-l^*}, p_n]} = \gamma \frac{E[v | s_{in}, p^{n-l^*}, p_n] - p_n}{\text{Var}[v - p_n | s_{in}, p^{n-l^*}, p_n]} .
\]

In equilibrium, outsiders correctly anticipate that the clearing price at each date is given by
\[
p_n = A_n v - \sum_{\tau = 0}^{l^*-1} B_{n,\tau} e_{n-\tau} + D_n E[v | p^{n-l^*}] \text{, for } n \geq 1 .
\]

When \( A_n \neq 0 \), the price at date \( n \) is informative about \( v \). This happens iff (i) \( \mu > 0 \) and \( \tau_{\epsilon_n-j} > 0 \) for some \( j \in \{1, ..., l^* - 1\} \) or (ii) \( \tau_{\epsilon_n} > 0 \). In this case, let \( z_n \) be the signal on \( v \) that an outsider can obtain from the equilibrium price \( p_n \), given that he observes \( p^{n-l^*} \). Using equation (34), we obtain that
\[
\hat{z}_n = \frac{p_n - D_n E[v | p^{n-l^*}]}{A_n} \\
= v - \sum_{j=0}^{l^*-1} \frac{B_{n,\tau} e_{n-j}}{A_n} \text{, for } n \geq 1
\]

(35)
Thus, \(\{s_{in}, p^{n-l^*}, p_n\}\) is observationally equivalent to \(\{s_{in}, p^{n-l^*}, \hat{z}_n\}\). Moreover, equation (35) implies

\[
\hat{z}_n \mid v \sim N\left(v, A_n^{-2}\left(\sum_{j=0}^{l^*-1} B_{n,j}^2\right)^{-1}\right).
\]

Let \(\Gamma \overset{\text{def}}{=} A_n^{-2}\left(\sum_{j=0}^{l^*-1} B_{n,j}^2\right)^{-1}\). Using well known properties of normal random variables, we obtain

\[
E[v|s_{in}, p^{n-l^*}, p_n] = (\hat{\tau}_n(\mu, l) + \tau_{\epsilon_n})^{-1}(\hat{\tau}_n(\mu, l)E[v|p^{n-l^*}, p_n] + \tau_{\epsilon_n}s_{in}),
\]

\[
\text{Var}[v|s_{in}, p^{n-l^*}, p_n] = (\hat{\tau}_n(\mu, l) + \tau_{\epsilon_n})^{-1},
\]

where

\[
\hat{\tau}_n(\mu, l) \overset{\text{def}}{=} \left(\text{Var}[v|p^{n-l^*}, p_n]\right)^{-1} = (\text{Var}[v|z^{n-l^*}, \hat{z}_n])^{-1} = \tau_{n-l^*}(\mu, l) + \Gamma_{\epsilon}.
\]

When \(A_n = 0\) (which happens when (i) \(\mu = 0\) or \(\tau_{\epsilon_n-j} = 0\) for all \(j \in \{1, \ldots, l^*-1\}\), and (ii) \(\tau_{\epsilon_n} = 0\)), the price at date \(n\) does not provide information about \(v\). This is as if the precision of the signal inferred from the price at date \(n\) by insiders were nil, that is \(\Gamma = 0\). This is indeed what we obtain for \(\Gamma\) when \(A_n = 0\).

Thus, for all values of \(\mu\) and \(\tau_{\epsilon_n}\), we have:

\[
x_n^O(s_{in}, p^{n-l^*}, p_n) = \gamma (\hat{\tau}_n(\mu, l) + \tau_{\epsilon_n}) (E[v|s_{in}, p^{n-l^*}, p_n] - p_n)
\]

\[
= a_n^O(s_{in} - p_n) + \gamma \hat{\tau}_n(\mu, l)(E[v|p^{n-l^*}, p_n] - p_n).
\]

(37)

with \(a_n^O = a_n^I = \gamma \tau_{\epsilon_n}\). In the rest of the proofs, we sometimes omit arguments \(\mu\) and \(l\) in \(\hat{\tau}_n(\mu, l)\) and \(\tau_{n-l^*}(\mu, l)\) for brevity.

**Clearing price in period** \(n\). The clearing condition in period \(n\) imposes

\[
\int_{0}^{\mu} x_{in}^I d\epsilon + \int_{\mu}^{1} x_{in}^O d\epsilon = e_n.
\]

Using equations (33) and (37), we solve for the equilibrium price and we obtain

\[
p_n = \frac{1}{K_n} \left(z_n + \mu \gamma \tau_n E[v|p^n] + (1 - \mu) \gamma \hat{\tau}_n E[v|p^{n-l^*}, p_n]\right),
\]

(38)

where

\[
K_n = a_n + \gamma (\mu \tau_n + (1 - \mu) \hat{\tau}_n),
\]

(39)

with \(a_n = \mu a_n^I + (1 - \mu) a_n^O = \gamma \tau_{\epsilon_n}\). Observe that

\[
E[v|p^{n-l^*}, p_n] = E[v|p^{n-l^*}, \hat{z}_n] = \hat{\tau}_n^{-1}\left(\tau_{n-l^*} E[v|p^{n-l^*}] + \Gamma_{\epsilon} \hat{z}_n\right),
\]

and

\[
E[v|p^n] = E[v|p^{n-l^*}, \{z_{n-j}\}_{j=0}^{l^*-1}] = \tau_n^{-1}(\tau_{n-l^*} E[v|p^{n-l^*}] + \gamma \sum_{j=0}^{j-l^*-1} a_{n-j}z_{n-j}).
\]
Substituting \( E[v|p^{n-l^*}, p^n] \) and \( E[v|p^n] \) by these expressions in equation \((38)\), we can express \( p_n \) as a function of \( v, \{e_{n-j}\}_{j=0}^{l^*-1} \), and \( E[v|p^{n-l^*}] \). In equilibrium, the coefficients on these variables must be identical to those in equation \((34)\). This condition imposes

\[
A_n = \frac{a_n + \mu \gamma \tau_e \sum_{t=n-l^*-1}^{n-1} a^2_t}{K_n},
\]

\[
B_{n,0} = \frac{1 + \mu \gamma a_n \tau_e + (1 - \mu) \gamma \tau_e B_{n,0} A_n^{-1}}{K_n},
\]

\[
B_{n,j} = \frac{\mu \gamma a_{n-j} \tau_e + (1 - \mu) \gamma \tau_e B_{n,j} A_n^{-1}}{K_n}, \quad \forall j \in \{1, \ldots, l^* - 1\},
\]

\[
D_n = \frac{\gamma \tau_{n-l^*}}{K_n}.
\]

Equations \((40), (41), (42)\) form a system with \( l^* + 1 \) unknowns: \( A_n \) and \( \{B_{n,j}\}_{j=0}^{l^*-1} \). Solving this system of equations, we obtain

\[
A_n = \frac{a_n + \mu \gamma (\tau_n - \tau_{n-l^*})}{K_n} \left( 1 + \frac{(1 - \mu) \gamma \tau_e (a_n + \mu \gamma (\tau_n - \tau_{n-l^*}))}{(1 + \mu \gamma a_n \tau_e)^2 + \sum_{j=1}^{l^*-1} (\mu \gamma a_{n-j} \tau_e)^2} \right),
\]

\[
B_{n,0} = \frac{A_n (1 + \mu \gamma a_n \tau_e)}{a_n + \mu \gamma (\tau_n - \tau_{n-l^*})},
\]

\[
B_{n,j} = \frac{A_n (\mu \gamma a_{n-j} \tau_e)}{a_n + \mu \gamma (\tau_n - \tau_{n-l^*})}, \quad \text{for} \ 1 \leq j \leq l^* - 1,
\]

\[
D_n = \frac{\gamma \tau_{n-l^*}}{K_n}.
\]

Observe that \( A_n \) is positive. Moreover, it is equal to zero iff (i) \( \mu = 0 \) or \( \tau_{\epsilon_{n-j}} = 0 \) for all \( j \in \{1, \ldots, l^* - 1\} \), and (ii) \( \tau_{\epsilon_n} = 0 \), as conjectured. Last, we deduce from these expressions and equation \((36)\) that

\[
\hat{\tau}_n(\mu, l) = \tau_{n-l^*} + \frac{(a_n + \mu \gamma (\tau_n - \tau_{n-l^*}))^2}{(1 + \mu \gamma a_n \tau_e)^2 + \sum_{j=1}^{l^*-1} (\mu \gamma a_{n-j} \tau_e)^2} \tau_e.
\]

This achieves the characterization of the equilibrium in each period in closed-form.

**Remark.** Remember that when \( A_n > 0 \), the observation of the \( n^{th} \) transaction price at date \( n \) conveys the following signal to outsiders:

\[
\hat{z}_n = v - \sum_{j=0}^{l^*-1} \frac{B_{n,j}}{A_n} e_{n-j}
\]

Using equations \((41), (44)\) and \((46)\), we deduce that:

\[
\hat{z}_n = \sum_{j=0}^{l^*-1} \alpha_j \hat{z}_{n-j},
\]

30
with
\[
\alpha_0 = (1 + \mu \gamma \tau_e) \left( a_n (1 + \mu \gamma \tau_e) + \mu \gamma \tau_e \left( \sum_{j=1}^{\tau-1} a_{n-j} \right) \right)^{-1},
\]
and
\[
\alpha_j = (\mu \gamma \tau_e) \left( a_n (1 + \mu \gamma \tau_e) + \mu \gamma \tau_e \left( \sum_{j=1}^{\tau-1} a_{n-j} \right) \right)^{-1}.
\]

\[
\Box
\]

**Proof of Proposition 1**

**Step 1.** We have shown in the proof of Lemma 1 that

\[
\tau_n(\mu, l) = \tau_v + \tau_e \sum_{t=1}^{n} a_t^2.
\]

where \(a_t = \gamma \tau_e \) (see equation (52)). As \(a_t\) does not depend on \(\mu\) and \(l\), the second part of the proposition follows immediately.

**Step 2.** From equation (48) in the proof of Lemma 1, we deduce that

\[
\hat{\tau}_n(\mu, l) = \tau_{n-l'} + \tau_m^{\mu}(\mu, l),
\]

with

\[
\tau_m^{\mu}(\mu, l) \overset{\text{def}}{=} \frac{(a_n + \mu \gamma (\tau_n - \tau_{n-l'}))}{(1 + \mu \gamma a_n \tau_e)^2 + \sum_{j=1}^{\tau-1}(\mu \gamma a_{n-j} \tau_e)^2 \tau_e}. \tag{52}
\]

As

\[
\frac{\partial \tau_m^{\mu}(\mu, l)}{\partial \mu} = \frac{2 \gamma \tau_e (\tau_{n-1} - \tau_{n-l'}) (a_n (1 + \gamma \mu \tau_e a_n) + \mu \gamma (\tau_{n-1} - \tau_{n-l'}))}{((1 + \gamma \mu \tau_e a_n)^2 + (\mu \gamma \tau_e a_n)^2 (\tau_{n-1} - \tau_{n-l'}))^2} \geq 0,
\]

we deduce that \(\hat{\tau}_n(\mu, l)\) increases with \(\mu\). We now show that \(\hat{\tau}_n(\mu, l)\) decreases in \(l\). Using equation (48), we have that \(\hat{\tau}_n(\mu, l) = \hat{\tau}_n(\mu, l+1)\), for \(n \leq l\). For \(n > l\), we deduce from equation (48) that

\[
\hat{\tau}_n(\mu, l) - \hat{\tau}_n(\mu, l+1) = \left( a_{n-l}^2 + \frac{G_n^2(\mu, l)}{Q_n(\mu, l)} - \frac{G_n^2(\mu, l+1)}{Q_n(\mu, l+1)} \right) \tau_e \tag{53}
\]

where \(Q_n(\mu, l) \overset{\text{def}}{=} (1 + \mu \gamma \tau_e)^2 + \sum_{j=1}^{l-1}(\mu \gamma a_{n-j} \tau_e)^2\) and \(G_n(\mu, l) \overset{\text{def}}{=} a_n + \sum_{j=0}^{l-1}(\mu \gamma \tau_e a_{n-j})^2\). We therefore have that \(\hat{\tau}_n(\mu, l) > \hat{\tau}_n(\mu, l+1)\) for \(n > l\).

Last, \(\hat{\tau}_n(\mu, l)\) can be written as follows

\[
\hat{\tau}_n(\mu, l) = \tau_{n-l'} + \tau_e \left( \frac{\sum_{j=0}^{l-1} \rho_{n-j} a_{n-j}^2}{\sum_{j=0}^{l-1} \rho_{n-j}} \right),
\]

31
with \( \rho_n = (1 + \mu\gamma a_n\tau_e) \) and \( \rho_{n-j} = (\mu\gamma a_{n-j}\tau_e) \) for \( j \geq 1 \). Using equation \( (50) \), it is then direct to show that \( \hat{\tau}_n < \tau_n \) since \( a_{n-j} > 0 \).

Proof of Proposition 2

As \( v \) and \( p_n \) are normally distributed for all \( n \), we have

\[
\text{Var}[v - p_n] = \text{Var}[v - p_n | p^{n-I_i^*}] + \text{Var} \left[ E \left[ v - p_n | p^{n-I_i^*} \right] \right].
\]

Using the expression of the clearing price given in Lemma 1, we obtain

\[
E \left[ v - p_n | p^{n-I_i^*} \right] = E \left[ v | p^{n-I_i^*} \right] - (A_n + D_n)E \left[ v | p^{n-I_i^*} \right].
\]

Now, equations \( (40) \), \( (43) \) and the definition of \( K_n \) in the proof of Lemma 1 yields

\[
A_n = \frac{(a_n + \mu\gamma \tau_e \sum_{t=n-(l^*-1)}^{n} a_t^2) + (1 - \mu)\gamma(\hat{\tau}_n - \tau_n - l^*)}{K_n} = \frac{K_n - K_nD_n}{K_n} = 1 - D_n.
\]

Thus, \( A_n + D_n = 1 \). We deduce that

\[
E \left[ v - p_n | p^{n-I_i^*} \right] = 0,
\]

which yields

\[
\text{Var}[v - p_n] = \text{Var} \left[ v - p_n | p^{n-I_i^*} \right] = (1 - A_n)^2\tau_n^{-1} - \left( \sum_{j=0}^{l^*-1} B_{n,j}^2 \right) \tau_e^{-1} = (1 - A_n)^2\tau_n^{-1} - A_n^2(\hat{\tau}_n - \tau_n - l^*)^{-1},
\]

where the last equation follows from equation \( (36) \) in the proof of Lemma 1. We can now differentiate the R.H.S of equation \( (55) \) with respect to \( \mu \) to show that \( \text{Var}[v - p_n] \) decreases with \( \mu \). We omit details of the calculations for brevity. They are available from the authors upon request.

Proof of Proposition 3

Using equation \( (17) \), we deduce that:

\[
C_n^I(\mu, l) - C_n^O(\mu, l) = \frac{\gamma}{2} \ln \left( \frac{\text{Var}[v | s_{in}, \Omega_{in}^I]}{\text{Var}[v | s_{in}, \Omega_{in}^O]} \right).
\]

Proposition 3 follows from the fact that \( \text{Var}[v | s_{in}, \Omega_{in}^I] = (\tau_{\epsilon_n} + \tau_n(\mu, l))^{-1} \) and \( \text{Var}[v | s_{in}, \Omega_{in}^O] = (\tau_{\epsilon_n} + \hat{\tau}_n(\mu, l))^{-1} \).
Proof of Proposition 4

The payoff of a speculator with type $k$ is:

$$C_n^k(\mu, l) = \frac{\gamma}{2} \left[ \ln \left( \frac{\text{Var}[v - p_n]}{\text{Var}[v|s_n, \Omega_n^k]} \right) \right].$$

(56)

Consider the effect of a change in $\mu$ on the payoff of an insider, $C_n^I(\mu, l)$. For insiders, $\text{Var}[v|s_n, \Omega_n^I]$ does not depend on $\mu$ since $\tau_n$ does not depend on $\mu$. Moreover, we know from Proposition 2 that $\text{Var}[v - p_n]$ decreases with $\mu$. We deduce from these observations and equation (56) that

$$\frac{\partial C_n^I(\mu, l)}{\partial \mu} < 0.$$  

For outsiders, the argument is more complex because $\text{Var}[v|s_n, \Omega_n^O]$ decreases with $\mu$ (the precision of outsiders’ forecasts improves when the proportion of insiders enlarges). However, tedious calculations show that

$$\frac{\partial C_n^O(\mu, l)}{\partial \mu} < 0.$$  

For brevity, the proof of this result is omitted and is available from the authors upon request. □

Proof of Proposition 5

We prove that $C_n^I(1, l) < C_n^O(0, l)$. Using equation (55) in the proof of Proposition 2 and the expressions for the coefficients $A_n$ and $B_{n,j}$ in the proof of Lemma 1, we obtain after some algebra

$$C_n^I(1, l) = \frac{\gamma}{2} \ln \left( \frac{\gamma^2 \tau_n - l^* + \tau_{e-1}((1 + \gamma a_n \tau_e)^2 + \sum_{j=1}^{l^*-1}(\gamma a_{n-j} \tau_e)^2)}{\gamma^2(\tau_{e_n} + \tau_n(1, l))3/2} \right),$$

$$C_n^O(0, l) = \frac{\gamma}{2} \ln \left( \frac{\gamma^2 \tau_n - l^* + \tau_e^{-1}(1 + \gamma a_n \tau_e)^2}{\gamma^2(\tau_{e_n} + \tau_n(0, l))3/2} \right).$$

Observe that $C_n^I(1, l) = C_n^O(0, l)$ iff $\tau_{e_n-j} = 0$, $\forall j \in \{1, \ldots, l^*-1\}$ since in this case $a_{n-j} = 0$, $\forall j \in \{1, \ldots, l^*-1\}$ and $\tau_n(1, l) = \tau_{e_n}(0, l)$. Now, it is easily checked that $C_n^I(1, l)$ decreases in $a_{n-j}$, for $j \in \{1, \ldots, l^*-1\}$. This implies that it decreases with $\tau_{e_n-j}$ since $a_{n-j} = \gamma \tau_{e_n-j}$. Moreover, $C_n^I(0, l)$ does not depend on $a_{n-j}$, $\forall j \in \{1, \ldots, l^*-1\}$. As $\tau_{e_n-j} > 0$, we deduce that $C_n^I(1, l) < C_n^O(0, l)$.

□

Proof of Proposition 6, 8 and 9

Immediate from the arguments in the text. □

Proof of Proposition 7

For brevity, we omit the proof of this lemma (the complete proof is available from the authors upon request). □