Adaptive Torque-Ripple Minimization in Switched Reluctance Motors

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Abstract—This paper addresses torque-ripple reduction in current-fed switched reluctance motors (SRMs). Ripple-free torque production in SRMs requires an accurate model that is often too complex for practical implementation. The algorithm proposed here combines the use of a simplified model with adaptation. Explicitly, it includes dynamic estimation of low harmonics of the combined unknown load torque and the ripple in the produced torque (due to model simplification), and adds appropriate terms to the commanded current to cancel these harmonics. Several simulations are presented first, suggesting that our method is effective for constant-speed reference commands, even when a very simple model is used in control design. Experimental results are included next to demonstrate that the algorithm performs well in reducing the torque ripple. Finally, limitations of the adaptive algorithm are explored and quantified.

Index Terms—Adaptive control, switched reluctance machines, torque control.

I. INTRODUCTION

SWITCHED reluctance motors (SRMs) are attractive for industrial applications because of their structural simplicity and low cost, ruggedness, and reliability in harsh environments, and the capability to cover a wide speed range and relatively high torque-to-mass ratio. A comprehensive source on SRM structure and on standard control approaches is [1]. The fact that SRMs are driven into magnetic saturation in energy-efficient use causes their main disadvantages: inherent nonlinearities that present a modeling challenge and a substantial torque ripple. The advent of power electronics and fast digital hardware, together with improved nonlinear and adaptive control methods, enables a reevaluation of these issues. This paper presents one such control design method with the dual goal of torque-ripple minimization and simplicity for ease of real-time implementation.

Ripple-free control strategies for SRMs have been studied extensively, and we review papers that have immediate relevance to our work. Reference [2] reports a feedback linearizing velocity controller; it uses a single phase at a time and, thus, requires very high flux variations, often leading to voltage saturation and to high sensitivity to inevitable model uncertainties. Reference [3] presents methods for computing simple reference currents for a current-tracking control to minimize torque ripple, while [4] proposes a solution to the same problem that utilizes contour functions. In [5], nominal currents that result in constant torque are computed for reduced current peaks and slopes, under the constraint that at “critical” rotor positions each of the phases contributes half of the total torque. Although [6] claims zero torque ripple, very special (and ideal) motor construction is required; the paper also does not address actuator limitations. In [7], the control goal, motivated by energy considerations, is to minimize the peak phase current while requiring linear torque change in the angular range where the two phases overlap. In [8], spline functions are used to model the motor torque function, where the parameters are estimated using adaptive rules. With this approach, the problem of inverting the torque function is alleviated considerably. Reference [9] proposes a new commutation strategy along with a proportional plus integral (PI) controller to minimize torque ripple, where an easily invertible flux function is used in calculating reference phase currents. In [10], optimal reference currents are calculated based on a complete model that produces minimal supply voltage under the constraint that the produced torque is constant. Reference [11] addresses the same issue by applying an adaptive fuzzy logic controller whose parameters are adjusted through a least-mean-squares algorithm. Close to our approach are also [12] and [13], where nonlinear adaptive torque-ripple cancellation is applied in permanent-magnet (PM) drives, and harmonic coefficients of the phase inductance (due to structural nonidealities) are dynamically estimated and current harmonics are adaptively injected to cancel the torque ripple. Our control goal is to optimize performance as measured by low torque ripple; later, we demonstrate that other control metrics important for industrial implementation, like voltampere (VA) rating of the converter and average torque per ampere [1], [15], are not substantially degraded in our solution.

When an accurate machine model is used for control design, both feedback linearization techniques and the use of precalculated optimal torque-sharing functions result in good dynamic performance. However, model inaccuracies are known to degrade performance, and are often hard to avoid; adding to possibly inaccurate measurements, the use of an accurate model entails complex online computations, or the use of large lookup tables that may be impractical. In this paper we address this issue by combining the use of a simple, easily computable torque-
sharing functions with adaptation. Our main goal is to reduce the torque ripple. The underlying control idea is simple: assuming a position dependent—and, therefore, periodic—current excitation, model inaccuracy will result in a periodic ripple in the produced torque. Harmonic components of the produced electromagnetic torque combined with the unknown load torque are dynamically estimated from the speed signal, and corresponding correction terms are added to the commanded phase currents until speed ripple is eliminated (or considerably reduced in practice). Simulations and experiments demonstrate that our algorithm reduces the torque ripple considerably.

The organization of the paper is as follows. Section II presents the SRM model. Section III discusses control design, including the commutation strategy and adaptation. Section IV presents and analyzes simulation and experimental results. Section V analyzes the limitations of the algorithm in terms of practical implementation issues. Conclusions are drawn in Section VI.

II. SRM Model

A standard time-domain machine model governs the SRM dynamics

\[
\begin{align*}
\frac{d\lambda}{dt} &= -R_l i + v \\
\frac{d\theta}{dt} &= \omega \\
J \frac{d\omega}{dt} &= \tau_M(\theta, i) - B\omega - \tau_L
\end{align*}
\]

where \(i, \lambda \in \mathbb{R}^l\) and \(v \in \mathbb{R}^l\) are the respective phase current, flux linkage and voltage vectors and \(l\) is the number of stator phases, \(\theta\) and \(\omega\) are the shaft angular displacement and speed, \(J, R_l, B\) are the moment of inertia, the electrical resistance in each phase, and the friction coefficient, respectively, \(\tau_M(\theta, i)\) is the motor torque, and \(\tau_L\) is the load torque.

The flux function \(\lambda(\theta, i)\) and the produced motor torque \(\tau_M(\theta, i)\), for the “8/6” \((N_p = 4\) phase, \(N_r = 6\) rotor pole) SRM used in our simulations are shown in Fig. 1, and are quite typical in their shapes. Flux data were obtained from static electrical measurements of the motor phases for various locked-rotor angles, and the torque function is calculated using standard energy methods [1]. The expression for the torque produced by a single phase \(k\) in the case of magnetically independent phases can be evaluated from the coenergy as

\[
\tau_{M_k}(\theta, i) = \frac{\partial}{\partial \theta_k} \left[ \int_0^\theta \lambda_k(\theta, i') d\theta' \right] \bigg|_{i = \text{const}}
\]

and summation over all phases \(k\) yields the total torque. This expression would simplify to \(\tau_M(\theta, i) = 1/2i^T(dL/d\theta)i\) in the linear magnetics context which is not assumed here. Using the relations of current and flux, (1) can be transformed to a differential equation in the state \((i^T, \theta, \omega)^T\), or \((\lambda^T, \theta, \omega)^T\).

In the analysis of many electric machines it is advantageous to seek a Blondel–Park-type coordinate transformation in which the flux and speed equations do not depend on \(\theta\). However, it was shown in [16] that, in general, such transformation does not exist for an SRM, even in the case of linear magnetics. It is, thus, natural to base control design directly on (1), and to allow control strategies to explicitly depend on measured or estimated \(\theta\).

The control inputs in (1) are voltages; an alternative is to assume current actuation (current-fed machine), in which case only the last two equations in (1), corresponding to the mechanical subsystem, are of interest. In practice, commanded currents in the latter setting are usually produced with voltage input, employing a form of current-tracking hysteretic control. The current actuation assumption is justified by the fact that, in small- and medium-size drives, the electrical subsystem is typically much faster than the dynamics of the mechanical subsystem. The experimental setup used to validate the proposed adaptive algorithm in this paper uses a current-fed SRM, and a hysteretic tracking of commanded (reference) currents.

III. Control Design

The control structure involves two stages: first, a given torque reference is translated to current actuation, as discussed in Section III-A; second, a closed-loop compensator that provides a torque reference is described in Section III-B.
A. Commutation Strategy

Positive torque production by a single phase is restricted by position and available currents, and two consecutive phases are typically excited to produce a smooth torque. In an “8/6” SRM, phase-currents waveforms are periodic, with a 60° period, and currents are kept at zero during the second half of the period to avoid generation of braking torque. To maintain the same control policy in all phases, current waveforms for the various phases (for a given produced torque reference \( \tau_{\text{ref}} \)) are identical, subject to an appropriate angular shift. In particular, the currents of companion torque-producing phases, at any motor position, are shifted by 15° [10]. The condition restricting current waveforms is, thus, that the sum of torques produced by the angular sections \( 0°-15° \) and \( 15°-30° \) should be constant. An implementation of this policy requires precise representations of the nonlinear function \( i \mapsto \tau_M(\theta, i) \) (from Fig. 1), and of its inverse. Feedback linearization controllers require online evaluations of these functions, typically by means of representations involving numerous parameters, or by very large lookup tables. This reduces the practical value of such schemes, or mandates simplifications, resulting in inaccurate torque production and ripple.

One possible solution, termed “torque sharing,” is to precalculate and parameterize current waveforms as functions of position and \( \tau_{\text{ref}} \). For example, in [14], neural networks are used for this purpose. Such parameterizations, however, tend to be computationally intensive when high accuracy is required. In this paper, we explore the use of a very easily computable torque-sharing current waveform, while the resulting torque ripple is reduced by an adaptive controller. For the purpose of demonstrating our point, our selection in the presented experiments and simulations is a very simple two-parameter model

\[
i_{p1} = \sin(6\theta)(p_1 + p_2\tau_{\text{ref}})
\]

where \( p_1 \) and \( p_2 \) are design parameters, and \( 0 \leq \theta \leq \pi/6 \). This parametrization is just one possible selection; our control structure will compensate for its inaccuracy by adaptation. Comparing this parameterization (for a fixed level of \( \tau_{\text{ref}} \)) with an ideal torque-sharing current waveform (Fig. 2), it is evident that (3) will result in torque ripple. The elimination of such ripple will be achieved by the adaptation of \( \tau_{\text{ref}} \), as described in Section III-B. Fig. 2 compares the torques produced for one desired \( \tau_{\text{ref}} \) level when the accurate and the simplified waveforms (3) are used (\( p_1 = 0, p_2 = 1.58 \)). In fact, the use of adaptation eliminates the need for a precise knowledge of constants \( p_1 \) and \( p_2 \) for example, selections \( p_1 = 0 \) and \( p_2 = 1 \) are also acceptable. Note that the torque ripple generated due to the use of simplified torque sharing current waveform is periodic with a 15° period in an 8/6 SRM.

B. Closed Loop and Adaptation Law

The mechanical equation of the motor can be rewritten as follows:

\[
J \frac{d\omega}{dt} = \tau_M - B\omega - (\tau_L + \tau_{M1} - \tau_M)
\]

where \( \tau_{M1} \) stands for the torque predicted by the simplified model, i.e., \( \tau_{M1} = \tau_{\text{ref}} \) in (3). The term \( \tau_L := \tau_{M1} - \tau_M + \tau_L \) is referred to as the “generalized load”. The closed-loop speed control will be based on a dynamic estimate \( \hat{\tau}_L \) of \( \tau_L \) that we introduce next. Assuming a constant \( \tau_L \) and position-periodic current actuation, \( \tau_L \) is periodic and can be expanded in Fourier series

\[
\tau_L = \alpha + \sum_{\kappa=1}^{\infty} c_k \cos(kN\theta) + s_k \sin(kN\theta)
\]

where \( N = 24 \) in an “8/6” SRM. For simplicity, (5) is abbreviated as \( \tau_L = F_{0,\infty}(\theta)C_{0,\infty} \), where \( F_{0,\infty}(\theta) := [1, \cos(N\theta), \sin(N\theta), \ldots, \cos(nN\theta), \sin(nN\theta)] \), and \( C_{0,\infty} := [a, c_1, s_1, \ldots, c_n, s_n] \). The estimate \( \hat{\tau}_L \) is expressed as

\[
\hat{\tau}_L = F_{0,n}(\theta)C_{0,n}^T
\]

where \( n \) designates the highest harmonic used (with the total of \( 2n + 1 \) coefficients). Only a finite number of harmonics is of practical significance, and the estimated vector of coefficients
\( \hat{C}_{\alpha n} \) is obtained using a dynamic adaptation rule that utilizes speed error measurements

\[
\frac{d}{dt} \hat{\alpha}_n = -J^2 F_{0n}(\theta)^T e_\omega \tag{7}
\]

where \( e_\omega = \omega - \omega_d \). \( \omega_d \) is the reference speed \( \Gamma = \text{diag}(\gamma_0, \gamma_1, \gamma_2, \ldots) > 0 \) and the \( \gamma_s \) are design parameters, determining the rate of adaptation.

To close the loop, the reference torque in (3) will be \( \tau_{M1} \), selected as

\[
\tau_{M1} = J \frac{d \omega_d}{dt} + B \omega_d + \mathcal{F}_L - a e_\omega \tag{8}
\]

where the proportional negative feedback \(-ae_\omega\) acts as an incremental friction to accelerate convergence of the speed error; other terms in (8) are chosen to satisfy (4) at the desired speed and subject to the estimated “generalized load.” In particular, the harmonic components of \( \mathcal{F}_L \) translate, via (8) and (3), to harmonic modulation of the commanded currents. These will be used to reduce the actual torque ripple, as shown next.

Using (4) and (8), the dynamics of the speed error \( e_\omega \) becomes

\[
J \frac{de_\omega}{dt} = -(B + \alpha) e_\omega + F_{0n}(\theta) \hat{\alpha}_n - F_{n+1,\infty}(\theta) e^T_{n+1,\infty} \tag{9}
\]

where \( \hat{\alpha}_n = \hat{C}_{\alpha n} - C_{\alpha n} \). To formulate a combined error dynamics equation for speed and parameters, we introduce

\[
\epsilon = [Je_\omega \quad \hat{C}_{\alpha n} \Gamma^{-1}]^T. \tag{10}
\]

Then, (9) and (7) yield

\[
\dot{\epsilon} = \begin{bmatrix}
-(B + \alpha) J e_\omega \\
-\Gamma F_{0n}(\theta) \Gamma \\
\end{bmatrix} e_\omega \\
+ \begin{bmatrix}
F_{n+1,\infty}(\theta) e^T_{n+1,\infty} \\
-\Gamma^{-1} \frac{d}{dt} C^T_{\alpha n} \\
\end{bmatrix}. \tag{11}
\]

The first entry of the inhomogeneous term becomes smaller as \( n \) increases, and the second entry is zero along ideal (periodic) trajectories, where the harmonic coefficients in (5) are time invariant.

The homogeneous part of (11) can be written as

\[
\dot{\epsilon} = (\mathcal{D} + S(\theta)) \epsilon \tag{12}
\]

where \( \mathcal{D} = \text{diag}\{-B + \alpha/J_{2n+1} \} \) and \( S \) is skew symmetric. A candidate quadratic Lyapunov function is selected as \( V = \delta^T \epsilon^2 \) and along trajectories of (12) it satisfies

\[
\dot{V} = -J(B + \alpha) \delta^2. \tag{13}
\]

Since the origin is the only invariant set of (12), where \( \epsilon_\omega = 0 \), asymptotic stability follows from LaSalle’s Theorem [17]. In fact, the stronger property of uniform exponential stability of (12) can be established, using the fact that the periodic linearization of the pair \([\mathcal{D} + S(\theta), \mathcal{D}]\) around the ideal limit trajectory is observable [18]. This stronger property also implies bounded-input–bounded-output (BIBO) stability of the inhomogeneous system (11).

One point we wish to stress here is that the expansion (5) is, in principle, operating-point dependent. The simplicity of our scheme is due to the fact that we do not seek parameterization with respect to both position and torque reference (or current). The penalty that we incur is the possible requirement for readaptation following a major change in the operating point; later, we address this issue experimentally. Note, however, that if between such changes there is enough time for the adaptation to converge, then our algorithm will be able to successfully track the references.

IV. SIMULATIONS AND EXPERIMENTAL RESULTS

Simulation tests of the proposed algorithm are based on data from a 350-W 8/6 SRM (Magna Physics model #124SR), whose flux and torque characteristics are shown in Fig. 1. Simulations of the motor dynamics rely on a detailed and accurate nonlinear model of the motor, while control design is based on the simple family (3) of approximate torque-sharing current waveforms, together with adaptation on the zero and the first two harmonics of \( \mathcal{F}_L \).

Fig. 3 shows the torque generated by the motor when first two harmonics of \( \mathcal{F}_L \) are estimated. The estimates of the five harmonics coefficients converge to constant values, as displayed in Fig. 4. In Figs. 5 and 6, simulations are repeated for the case
when only the dc term is estimated (other control parameters are unchanged). Since the torque ripple introduced due to simplification is periodic, the estimated dc component in this case is also periodic; this fact is demonstrated in Fig. 5. It is worth pointing out that, due to noise, it may be impossible to implement such an aggressive controller in actual experiments. Fig. 6 shows the corresponding torque. The signal power of the torque ripple in this case is now approximately 10 times larger than that achieved when harmonic adaptation is used, demonstrating the benefit of adaptation.

The efficiency of the proposed scheme is not significantly inferior to the one achieved with standard schemes like the trapezoidal current profiling [1], as we illustrate next. One relevant figure of merit is specific peak voltamperes, defined in [1, p. 180] as the number of transistors times the peak current times the supply voltage divided by the power conversion. Thus, for a given hardware setup and a given operating point, a comparison of two current-profiling schemes reduces to the ratio of peak currents. For the torque of 2.2 N-m and speed of 10 rad/s (the operating point considered in our experiments), the ratio of peak currents in the proposed and the trapezoidal current profiling is $3.87/3.50 = 1.11$. The peak current ratio in the range 1.10–1.14 is obtained at torque levels 2.2–3.3 N-m. The rms values of currents corresponding to the two control schemes are even closer (typically within 1%). We thus conclude that the adaptive current profiling achieves the torque-ripple minimization without significantly compromising the drive efficiency.

The algorithm was tested experimentally with our SRM at a constant speed reference. An encoder providing 2000 incremental angular positions per revolution is mounted on the shaft. The speed estimate is obtained by counting the number of high-frequency clock pulses between two encoder lines. A hysteretic dynamometer, Magtrol model HD-705, is used to load the SRM. The algorithm is realized on a Texas Instruments TMS320C31 digital-signal-processor (DSP)-based dSPACE controller (DS1102). The produced torque is measured using a noncontact torque meter (Himmelstein MCRT 4901 V) with a measurement bandwidth of 300 Hz (this torque measurement is not used in control). Double-flex couplings were used to connect the motor, torque meter, and the load cell, and the test stand was placed on vibration isolation mountings.

The spectrum of the produced torque is measured using a dynamic signal analyzer, model Hewlett Packard 35 665A, with “flat-top” windowing which offers high amplitude accuracy ($\pm0.005$ dB), but somewhat lower frequency resolution. The experimental setup employs a half-bridge inverter with two transistors and two diodes per phase; it operates in a hysteretic current mode with “hard chopping” [1] (i.e., the voltage on an active phase is either positive or negative dc voltage); the hysteresis band is 0.2 A. The reference currents are realized through 12–bit D/A converters.

Fig. 7 shows the experimental results when there is no adaptation of harmonics, while Fig. 8 shows experimental results including adaptation (in both cases the torque spectrum is normalized at dc and the frequency of the fundamental is 38 Hz). Note that the adaptation attenuates the main torque harmonic below $-40$ dB (approximately 20-dB attenuation when compared to the nonadapted case). Observe that in Fig. 7 there are two harmonics at approximately 19 and 28.5 Hz that are due to the fact that the phases are not identical (the reasons include the motor construction and nonidentical implementation of the
analog hysteretic control circuit in various phases). We eliminate those subharmonics by including them in the adaptation mechanism. The overall torque ripple spectrum (up to the bandwidth of the torque meter) is below $-33$ dB. In the experiment, the motor is loaded ($\approx 2.47$ N-m), so that it is driven into magnetic saturation. In both simulations and experiments, the reference speed is low (10 rad/s); this avoids filtering of torque harmonics by the mechanical subsystem of the motor (which acts as a low-pass filter). The adaptation procedure is intentionally slow to suppress the effect of noise in the speed measurements that we discuss in more detail in Section V.

Finally, we show experimental results (obtained through a DSpace Trace module) in Fig. 9 for the case when the speed reference undergoes a 20% step change (from 8 to 10 rad/s) with full (nominal) torque load. We note the good behavior of the estimates of harmonic components of the torque. We do stress, however, that our control structure is intended for operation that is close to steady state (i.e., with close to periodic torque ripple), and its performance under other large transients has to be evaluated on a case-by-case basis.

V. LIMITATIONS OF THE ADAPTIVE ALGORITHM

The control algorithm is basically composed of two parts.

- The coarse tuning part is responsible for bringing the speed to the desired speed reference trajectory. This subsystem is fast, and can be manipulated according to the needs of the application as long as the hardware can provide the required actuation.
- The fine-tuning part includes the adaptation of higher (non-dc) harmonic components. The harmonic estimation task needs to be slower than the estimation of the dc component, as the dc component is responsible for producing a stationary speed error spectrum (as it renders the "generalized load" periodic).

In the following sections, we quantify the main limitations of our adaptive algorithm. The two main issues are: 1) to what level can we reduce the torque ripple and 2) what is the frequency cutoff above which the algorithm ceases to be effective? It will turn out that the answer to the first question depends mostly on the current resolution of the inverter, as the currents are realized by a hysteretic switching. The answer to the second question depends on the useful signal content in the measured quantities and on the noise present in the speed measurement. Our analytical and experimental data show that, in our laboratory setup, we can estimate the first two subharmonics and one main harmonic, as we explain next.

A. LIMITATIONS DUE TO HYSERETIC CURRENT CONTROL

To assess the importance of this limitation, we provide simulations of the inverter, and compare the spectra in cases where reference currents can be realized either fully (when the inverter is an ideal current source) or approximately (by a finite hysteretic current band). In the ideal inverter case, reference currents generate a very low torque ripple: the spectrum of the simulated torque ripple is around $-140$ dB for the adapted harmonics. Fig. 10 shows a very good hysteretic approximation of reference currents; the torque ripple spectrum corresponding to the approximation is shown in Fig. 11. Thus, even with a very good hysteretic approximation, and a high switching frequency of up to 10 kHz, the harmonics cannot be eliminated completely. In our simulations, when the switching frequency is reduced
It is thus clear that if the harmonic coefficient of the torque ripple is too small, the inverter may be unable to realize the part of the reference current that eliminates that component.

B. Limitations Due to Quality of Speed Measurement

Our controller critically depends on the quality of speed measurement. The traditional method of obtaining the speed estimate through a band-limited differentiation of the position (encoder) signal may prove suboptimal for all harmonics of interest. The differentiation amplifies noise, and adds delay to the estimate. Moreover, the quality of the speed estimate obtained through differentiation decreases at low speeds. For these reasons, we calculated a speed estimate by counting the number of pulses $n$ of a high-frequency (crystal) clock between two encoder lines, namely, $\omega \approx (\theta_0/n)T_{clk}$, where $T_{clk}$ is the period of the clock signal. This approach still does not resolve the issue of delays that are important for higher harmonics. As a practical rule, in order to have satisfactory closed-loop operation, the delay of the filter, regardless of speed measurement technique, at the frequency of the harmonic to be estimated should be less than one-quarter of the period of the harmonic being estimated.

The torque harmonic to be estimated should be visible in the speed signal, and the noise level due to speed measurement should not corrupt the signal. For this reason, in our experiments we kept the motor speed low, so that several torque-ripple harmonics can be recovered from the speed signal. To assess the amount of noise introduced in the measured quantities, we analyze the driving term of the adaptation rule in (7), $\beta(t) = c_\omega(t) \sin(Nk \theta)$. While the details are presented in the Appendix, here, we summarize the main result—the noise power in $\beta(t)$ is

$$E[\hat{\beta}(t) - \beta(t)]^2 \approx \sin^2(Nk \theta(t_m)) \left( S_m^2 - S_{m+1}^2 + \frac{T_{clk}^2 S_m^4}{\omega_0^2} \right)$$

where $\hat{\beta}(t)$ is the estimate of $\beta(t)$. In our experimental setup, this quantity can be as much as $-49$ dB in the frequency range from dc to several times the fundamental frequency ($f = 38$ Hz); this value will turn out to be comparable with the speed signal power, thus determining the cutoff frequency for adaptation.

Another important parameter is the number of encoder lines, which should be high enough so that the number of encoder readings per period satisfies the Nyquist rate for the estimated harmonic. Moreover, the computational cycle time should also be fast enough (above the Nyquist rate for the highest harmonic of interest) to capture the variation in the speed spectrum.

C. Limitations Due to the Controller Structure

Next, we analyze the signal content of the measured data by assuming that the closed-loop system acts as a filter on the torque ripple. The output of the filter is the corresponding speed ripple, and the filter represents the closed-loop mechanical system. The speed ripple signal power at a specific harmonic is calculated from

$$P_f(\omega_{ripple}) = \left| \frac{1}{Js + (\alpha + B)} \right|^2 P_f(\tau_{ripple})$$

where $P_f$ stands for the signal power in the harmonic of frequency $f$, and $s = j2\pi f$. Observe that, as the frequency increases, the signal power in the speed decreases because of the low-pass filtering action. Note that we have approximate knowledge of torque harmonic power, which can be used above to estimate the signal power in the speed. From the signal power level and the noise power level in the data we can derive conclusions about the highest harmonics that can be estimated. From a practical standpoint, if the noise power is 25% or more of the signal power, then the harmonic cannot be successfully estimated. Our calculations (with details given in the Appendix) show that the noise power in the speed signal is approximately $-49$ dB in the frequency range from zero to several times the fundamental. From the measured torque ripple (Fig. 7) and (15), we calculate the signal power in the speed, and it turns out to be $-50$ dB at $f = 57$ Hz and $-44$ dB at $f = 76$ Hz. These calculations are based on parameter values of our experimental setup where $J = 0.0004$ (kg m$^2$) and the contribution of the term $(B + \alpha)$ can be neglected above dc. The torque-ripple information is obtained from the dynamic spectrum analyzer (data are shown in Fig. 7). Comparing the signal power in the speed and the noise power of $\beta(t)$, (14), we note that they are comparable at $f = 57$ Hz. Thus, we conclude that frequency components above approximately 57 Hz cannot be estimated, as at higher frequencies the signal power decreases, while the noise power does not.

VI. CONCLUSIONS

An adaptive SRM control algorithm for torque-ripple minimization has been proposed in this paper and verified in simulations and experiments. The algorithm alleviates the effects of modeling errors, and considerably reduces the computational burden associated with the use of accurate but complex torque-sharing current waveforms. Our controller combines simple waveform parameterizations with dynamic estimation of low harmonic components of the resulting torque error to reduce the torque ripple. Studies of our controller with a varying number of harmonics reveal the benefits of the
proposed method. Finally, analysis of the limitations of the algorithm were presented, quantifying our algorithm in terms of magnitude of torque-ripple reduction and in terms of the cutoff frequency.

APPENDIX

Fig. 12 schematically describes the encoder signal; $S_m$ is the slope (hence, the average speed) between time instances $t_{m-1}$ and $t_m$. Note that $t_m$ is the time at which a new encoder line is received; it is measured with some random error, $\Delta t_m$. The speed measured as an average in $[t_{m-1}, t_m]$ is used as the speed estimate in $[t_m, t_{m+1}]$.

We calculate the speed estimate from

$$\hat{\omega}_m = \frac{\theta_0}{t_m - t_{m-1} + \Delta t_m - \Delta t_{m-1}} = \frac{S_m}{1 + \xi_m}$$

where $S_m = \theta_0/(t_m - t_{m-1})$, $\xi_m = (\Delta t_m - \Delta t_{m-1})/(t_m - t_{m-1})$, and $\Delta t_m$ is independent identically distributed (i.i.d.) random variables (assumed uniform on $[-T_{\text{clk}}/2, T_{\text{clk}}/2]$) modeling the inaccuracy in the measurement of time due to finite resolution of the speed measuring clock. As a consequence, the expected value $E[\xi_m] = E[\Delta t_m - \Delta t_{m-1}/t_m - t_{m-1}] = 0$. Assuming the frequency of the clock is high, we can approximate (16) by

$$\hat{\omega}_m \approx S_m(1 - \xi_m + \xi_m^2 - \xi_m^3 + \cdots) \approx S_m(1 - \xi_m)$$

where we retain the first two terms in the Taylor series. Now, we define $\beta(t) = c_\omega(t)\sin(Nk\theta(t))$ to be the actual driving term in the adaptation rule (without noise) and $\hat{\beta}(t) = (\hat{\omega}_m - \omega_d)\sin(Nk\theta(t_m))$ to be its estimate, where $t_m \leq t < t_{m+1}$. Here, our objective is to characterize the quantity $E[\beta(t) - \hat{\beta}(t)]^2$ as the noise power, where $E$ is the mathematical expectation operator. Using the property $E[Z]^2 = \text{var}(Z) + |E(Z)|^2$ of expectation operator, we obtain

$$E[\beta(t) - \hat{\beta}(t)]^2 = E[\beta(t) - \hat{\beta}(t)]^2 + |E[\beta(t) - \hat{\beta}(t)]|^2.$$  

We now turn to calculate both terms on the right side of the above equation.

First, we evaluate $E[\beta(t)]$, viz.,

$$E[\beta(t)] = E\{c_\omega(t)\sin(Nk\theta(t_m))\} \approx (S_m - \omega_d)\sin(Nk\theta(t_m))$$

where we use the fact that $E\xi_m = 0$ is used. To assess the noise in the measurement, we still need to have a knowledge of $\beta(t)$. If we assume that the actual speed is constant between two encoder lines (see Fig. 12), then the additional inaccuracies enter because of delay effects and because of the evaluation of the trigonometric quantity. The delay effect is because the speed estimate used is the one calculated in the previous encoder slot. The inaccuracy in the calculation of the trigonometric quantity comes from the fact that the angle is assumed constant in the time interval preceding the arrival of a new encoder pulse. Thus, $\beta(t) = (S_{m+1} - \omega_d)\sin[Nk(\theta(t_m) + (t - t_m)S_{m+1})].$

Using (19) and (20), a conservative upper bound for $|E[\beta(t) - \hat{\beta}(t)]|^2$ is

$$|E[\beta(t) - \hat{\beta}(t)]|^2 \leq (|S_{m+1} - \omega_d| + |S_m - \omega_d|)^2.$$  

A more practical expression can be found if the error in the calculation of the trigonometric quantity is disregarded, which is acceptable for low-order harmonics. In that case, the expression simplifies to

$$|E[\beta(t) - \hat{\beta}(t)]|^2 \approx \sin^2(Nk\theta(t_m))(S_m - S_{m+1})^2.$$  

Observe that, for higher harmonics, this approximation is not valid, and the inaccuracy in the calculation of the trigonometric quantity becomes important.

To calculate the variance $E[\beta(t) - \hat{\beta}(t)]^2$, we write

$$E[\beta(t) - \hat{\beta}(t)]^2 = |S_m(1 - \xi_m)\sin(Nk\theta(t_m)) - S_m\sin(Nk\theta(t_m))|^2$$

$$= \text{var}(\xi_m)S_m^2\sin^2(Nk\theta(t_m)) = \frac{T_{\text{clk}}^2}{6\theta_0^2}S_m^2\sin^2(Nk\theta(t_m)).$$  

The final result (averaged squared error term driving the adaptation) will be the sum of expressions (22) and (23), showing that the quality of the speed estimate that uses (16) decreases at higher speeds. Note that, when the speed gets higher, we can include more terms in the Taylor expansion, which, in turn, adds more noise to the estimate in addition to the fourth power dependence on speed, proportional to $S_m^4$, or use a different speed estimation algorithm. Using the parameter values from our experimental test bed, the quantity contributing to the error sum coming from (23) is small due to the high-frequency (6.25 MHz) clock. Equation (22) thus provides the dominant contribution, and experimental speed data show that this can reach levels around $-49$ dB in the frequency range from dc to several times the fundamental.

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