On Torque Ripple Reduction in Current-Fed Switched Reluctance Motors

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Abstract—This paper addresses a basic control issue in switched reluctance motor (SRM) drives—the production of a ripple-free torque. Simple and largely model-independent conventional supply waveforms are not able to satisfy this requirement. The goal of this paper is to improve SRM dynamical performance by compensating for motor nonlinearities, while maintaining the robustness of conventional methods. The method is based on a complete parameterization of position-dependent voltage and current profiles in ripple-free operation, and on a waveform optimization to minimize power supply requirements. Furthermore, model uncertainties are included to show that the proposed strategy consistently outperforms the conventional policy. Experimental data verifying the analytical approach are included.

Index Terms—Nonlinear systems, reluctance motor drives, torque control.

I. INTRODUCTION

ADVANTAGES of switched reluctance motor (SRM) drives include a simple structure (hence, low cost), the ability to operate in harsh environments (such as at high temperatures) and under partial hardware failures, and a wide speed range. The SRM’s inherent nonlinearity, however, results in difficulties when smooth and quiet operation is required. Industrial acceptance of SRM drives is presently limited by crude, almost model-independent control patterns that are easy to implement, but do not achieve the desirable smooth operation.

Current profiling for torque ripple minimization has been previously pursued; the unique feature of the approach described here is the complete parameterization of current waveforms yielding (nominally) torque ripple-free operation, serving as a basis for waveform optimization. Several optimization criteria can be used, with the general goal of minimizing power supply requirements and reducing sensitivity to model inaccuracies. The design based on one such criterion—peak input voltage minimization—is illustrated in both simulations and hardware experiments, demonstrating the robustness of its performance advantage over conventional designs. This paper builds on previous developments in [1], where bounded-input–bounded-output stability is discussed as well.

Control strategies for ripple-free SRM operation have been studied extensively, and only a sample of approaches of immediate relevance to this paper is listed here. Reference [2] reports a feedback linearizing velocity controller; it uses a single phase at a time and, thus, requires very high flux variations, often leading to voltage saturation and to high sensitivity to inevitable model uncertainties. Difficulties in practical implementations of feedback-linearization-based control are documented in [3]. An interesting cascade control structure is proposed in [4], but it was derived for a motor with a very high number of rotor poles (150), resulting in a simple torque—flux relationship that does not hold in every SRM (such as the one used in the experiments of this paper). Reference [5] presents methods for computing simple reference currents for a current-tracking feedback control to minimize torque ripple, while [6] proposes a solution to the same problem that utilizes contour functions. In [7], nominal currents which result in constant torque are computed for reduced peaks and slopes under the constraint that, at “critical” rotor positions, each of the active phases contributes half of the total torque while excited with the same current. Reference [8] considers adjustments in firing angles for an SRM with nonlinear magnetics, and presents simulations for an “8/6” SRM (this choice of current shapes and the influence of motor geometry is discussed later). Although [9] claims zero torque ripple, it requires a very special (and ideal) motor construction and does not address current actuator limitations. In [10], the goal motivated by energy efficiency is to minimize the peak phase current while requiring linear torque change in the angular range where two phases overlap. Several papers have addressed adaptive control ideas for SRM drives: [11] presents adaptive feedback linearization for an SRM with linear magnetics; [12] describes a “backstepping” control design for an SRM with linear magnetics; [13] presents a model reference adaptive controller based on B-spline functions; and [14] addresses adaptive control under fault conditions.

The organization of this paper is as follows. In Section II, standard models of an SRM under various assumptions are presented. In Section III, the proposed low ripple control policy is described. Section IV contains simulations with accurate and uncertain models. The experimental results are in Section V, followed by conclusions in Section VI.

II. SRM MODEL

SRM dynamics are governed by the following time-domain equations [16]

\[
\frac{dX}{dt} = -Ra + v, \quad \frac{d\theta}{dt} = \omega, \quad J\frac{d\omega}{dt} = \tau_M - D\omega - T_L
\]  

(1)
where \(i, \lambda \in \mathbb{R}_+^n\) and \(v \in \mathbb{R}^m\) are the respective phase current, flux linkage, and voltage vectors and \(n\) is the number of stator phases; \(\theta\) and \(\omega\) are the angular position and velocity; \(J, R,\) and \(B\) are the moment of inertia, the electrical resistance in each phase, and the friction coefficient, respectively; and \(\tau_M\) is the motor torque and \(T_L\) the load torque.

A key component here is the function \(\lambda(\theta, i)\), relating flux linkage to current and angular displacement. The function is periodic in \(\theta\), and monotonically increasing in \(i\). It can be expressed as \(\lambda(\theta, i) = L(\theta, i) i\), in terms of the inductance function \(L\). Models assuming linear magnetics ignore the current dependence of \(L\) (thus, \(L = L(\theta)\)), while in more realistic, nonlinear magnetics models \(L(\theta, i)\) is monotonically decreasing in \(i\) (due to saturation). The analysis and control design approach that are presented here are based on nonlinear magnetics, and it is assumed that \(L(\theta, i)\) is (approximately) known. Fig. 1 shows the flux dependence on the phase current for various positions for the SRM used in the experiments.

The expression for the torque produced by a single phase \(k\) in the case of magnetically independent phases can be evaluated from the coenergy as

\[
\tau_{Mk}(\theta, i) = \frac{\partial}{\partial i} \left[ \int_0^i \lambda_k(\theta, \dot{i}) d\dot{i} \bigg|_{\dot{i} = \text{const.}} \right]
\]

summing over all phases \(k\) for the total torque. This expression is simplified to \(\tau_{Mk}(\theta, i) = \frac{1}{2} \dot{i}^T \frac{dL(\theta)}{d\theta} \dot{i}\) in the linear magnetics context. Using the relations of current and flux, (1) can easily be transformed to a differential equation in the state \((\vec{\theta}^T, \dot{\theta}, \omega)^T\) or \((\lambda^T, \dot{i})^T\).

In analysis of many electric machines it is advantageous to seek a (Blondel–Park type) coordinate transformation in which the flux and speed equations do not depend on \(\theta\). However, it was shown in [17] that, in general, such transformation does not exist for an SRM, even in the case of linear magnetics. It is, thus, natural to base control design directly on (1) and to allow control strategies to explicitly depend on (measured or estimated) \(\theta\).

The control inputs in (1) are voltages. An alternative is to assume current actuation, whereby only the last two equations in (1), corresponding to the mechanical subsystem, are of interest; such drives are denoted as “current fed.” In practice, commanded currents in the latter setting are usually produced with voltage input, employing a form of current-tracking hysteretic control. The current actuation assumption is justified by the fact that, in small- and medium-size drives, the dynamics of the electrical subsystem are much faster than the mechanical subsystem. (This approach can be theoretically formalized using an argument based on singular perturbations [18].) The experimental setup described in this paper uses a current-fed SRM and a hysteretic tracking of the commanded current.

### III. Low Ripple Control

The first step in the control design procedure is to derive a complete and computationally efficient parameterization of all \(n\)-tuples of currents that achieve a fixed torque \(\tau^0\). In this paper, motor operation is considered in the region below the “base speed,” where voltage actuators can produce the required control inputs (phase currents in laboratory experiments reported in Section V). Thus, inputs that will avoid actuator saturation will be investigated. In other words, by optimizing the current waveform used, the “base speed” is maximally extended for given actuator (voltage) limitations.

For conceptual simplicity, SRM’s are considered where, at most, two phases can contribute positive torque at any given time. Many popular designs satisfy this condition, including the four-phase stator and six-pole rotor “8/6” SRM used in the experiments. For the same reason, it is assumed that phases are magnetically decoupled (i.e., \(L(\theta, i)\) is diagonal). This condition is, again, often satisfied to a large degree and, in particular, it approximately holds for the SRM of this paper. An extension of the design suggested here to a general setting where either or both assumptions are removed can be carried out at the price of straightforward, but notationally unpleasant, computations.
The torque developed by an SRM is independent of the polarity of stator currents, but it depends on $\theta$, as its sign equals the sign of $\partial L/\partial \theta$. To maintain constant torque efficiently, nominal current trajectories should contribute only positive components to $\tau_M$. Hence, phase currents will be kept at zero over the half-period arc where the phase inductance is decreasing. The arc corresponding to positive $\partial L/\partial \theta$, where a nonzero $k$th phase current $i_k$ will be produced, is denoted by $\Theta_k = [d_{k-1}, d_k]$. As shown in Fig. 2, there is an overlap between $\Theta_k$ and the arcs $\Theta_{k-1}$ and $\Theta_{k+1}$, for the preceding and following phases (denoted with $k-1$ and $k+1$, respectively). For the rotor positions $\theta \in (d_{k-1}, d_{k+1})$, the only nonzero phase current is $i_k$, and the value of $i_k$ that is needed to produce a desired torque $\tau_M = \tau^0$ is uniquely determined by (2). From the fact that phase currents are periodic and form constant angular shifts in relation to each other, it suffices to consider $i_k(\theta)$ for half of the region $[c_k, d_k]$ to determine $i_k$ for the remaining part of $\Theta_k$. The desired torque production thus allows one degree of freedom, subject to the constraints that $i_{k-1}(d_{k-1}) = 0$ and $i_k(c_k) = 0$ have already been determined. This degree of freedom will be used in optimizing the selected current waveform. For “8/6” SRM’s, the points $d_{k-1}$ and $c_{k+1}$ coincide. Currents are periodic with the spatial angle of 60°, where, in the second half of these 60°, the current is kept zero to avoid negative (breaking) torque. To make current waveform optimization tractable, the original set of time-domain candidate waveforms will be approximated by a finite dimensional parameter set. In experiments, the parameterization was based on specifying the current $i_k$ in $N_p = 9$ points over the spatial domain of 30°; $\dot{i}_k$ was then evaluated over $N_c = 12N_p$ points using cubic splines, and $\ddot{i}_k$ was determined to ensure that the two phases jointly generate the torque $\tau^0$. The choice of angles above is such that the current is expected to change little between two points, so that it can be well approximated with splines, justifying the term “complete.” The choice of $N_p$ and $N_c$ requires a tradeoff between the computational time and the capability to capture the current variation.

Previous studies of control policies for ripple-free operation of an SRM (e.g., [7] and [19]) have repeatedly come to the conclusion that the required input voltages tend to be “spiky.” This phenomenon is due to the nonlinear relationship between phase voltages and phase currents, whereby even relatively smooth current waveforms may require very high input voltages. Another source of control difficulties is in the fixed magnitude (Vdc) of the dc voltage in (hysteretically) switched power inverters. Setting a high value for Vdc, to accommodate a rapidly varying waveform, will result in increased average switching frequency over time intervals where a moderate continuous voltage is needed. This leads to increased switching losses and poor efficiency. Voltage magnitude is, therefore, the main factor constraining control capability, and a useful criterion for the selection of a current waveform is the minimization of the highest voltage required to produce the constant torque $\tau^0$. This criterion relates also to yet another source of control limitations, which is the proportionality between the bandwidth of speed regulation and the difference between $V_{dc}$ and the voltage required at a given time. The optimized current and voltage waveforms that minimize the required voltage magnitude are shown in Fig. 3. Both the optimized current and the traditional current are sketched in Fig. 1. Observe that the proposed policy and the traditional policy are similar in terms of efficiency (as the areas under the current curves are comparable). It can be seen from the figure that the peaks of the optimized current are around the denser regions of flux-current-angle plot, as expected from (2).

The parameterization of the current set can be used for efficient computations with optimization criteria other than voltage minimization, which could prove beneficial in other applications. Some examples are as follows: 1) in high-speed operation (i.e., above the “base speed”) it is of interest to maximize the achievable ripple-free torque subject to actuator constraints; 2) maximization of efficiency at low speed operation; 3) torque ripple minimization with one or more faulted phases (where the parameterization has to be modified to accommodate the broken symmetry between phases); and 4) the minimization of ripple in radial forces which are primarily responsible for acoustic noise of an SRM through the “ovalization” of the stator [20].

IV. SIMULATIONS WITH UNCERTAIN MODELS

The simulation and experimental data were obtained with an “8/6” SRM, the inductance characteristics of which are shown in Fig. 1.

The speed of the SRM is approximately 300 r/min, and the torque is, with current waveforms optimized for minimal voltage requirements, $\tau^0 = 0.8$ N·m (the base speed is $\approx 570$ r/min with a 35-V supply voltage). The performance of the proposed controller will be compared to the “traditional” SRM controller [16]. The traditional control waveform is determined by four parameters: 1) the turn-on angle; 2) the turn-off angle; 3) the constant chopping current level; and 4) the supply voltage. The particular traditional switching pattern considered
in the experiments is optimal for its class, in the sense that it minimizes torque ripple. Because of its limited number of degrees of freedom, the optimal selection of the traditional control parameters depends on both the desired nominal speed and (average) torque (and yields an increasing torque ripple away from the nominal speed). On the other hand, the optimized policy suggested here guarantees (nominal) ripple-free torque at varying speeds (but may be suboptimal in terms of minimizing the required voltage at speeds different from the nominal). Fig. 4 compares the optimal current waveform generated by the proposed policy and by the traditional design.

Torque deviations (under the assumption of a perfect model) are also presented in Fig. 4, where the traditional controller is designed to produce minimal absolute deviations of torque around the nominal value \( \tau_n \). The ripple in the optimized policy is only due to finite computational accuracy, as compared to significant torque ripple (about 30% of the average value) under the traditional switching policy. This result, while encouraging, is not very surprising, as one expects a current waveform optimized over a (much) larger family to outperform the more constrained traditional current shape.

From an implementation standpoint, a critical issue is the robustness of the proposed solution to model uncertainties and differences among phases. To simulate deviations of the model from the “actual” dependence between phase currents and fluxes, the “true” current-dependent inductance curve is perturbed in a probabilistic way (independently in each of the four phases). The numerically simulated variations in the rate of inductance change \( \partial L/\partial \theta \) were significant, and reached 20% for the phase current of 10 A.

Representative current-dependent inductances and the perturbations are shown in Fig. 5 for two neighboring phases. The resulting torque deviations are presented in Fig. 6; the ripple is still approximately 50% smaller with the optimized policy than in the case of traditional control. These simulations suggest that the new switching policy is preferable when a reliable model is available and that it maintains an advantage over the conventional one in cases of inaccurate models and timing in inverter switching. The proposed policy thus retains some of the inherent robustness of the conventional design, while achieving the high accuracy characteristic of feedback-linearization-type controllers.

V. EXPERIMENTAL RESULTS

The experimental results were obtained using the “8/6” SRM, the model of which was the basis for the simulations above, and it is equipped with an optical shaft encoder with 200 pulses per revolution. Mechanical load for the motor was provided by a dynamometer equipped with a hysteretic brake. Torque measurements were obtained with a strain gauge dynamic torquemeter (Himmelstein MCRT 4901V) with the measurement bandwidth of 300 Hz. Double-flex couplings were used to connect the motor, torquemeter, and the load cell, and the test stand was placed on vibration isolation mountings. Given the limitations of the torquemeter, torsional resonant frequencies (estimated to be in the range 300–650 Hz) and the need to observe the improvement in the torque ripple,

![Fig. 4.](image)
Fig. 5. True (solid) and perturbed (dashed) inductance for two neighboring phases.

Fig. 6. Torque deviations for the proposed controller (solid) and traditional controller (dashed), assuming inaccurate model and unequal phases.

Fig. 7. Measured steady-state waveforms for the conventional policy: top trace—actual currents for two neighboring phases; middle trace—torque variations (amplitude is 32% of the mean); bottom trace—reference current waveforms for the same two phases.

The measurements were obtained at low speeds (200 r/min corresponds to the torque ripple fundamental at 80 Hz, since each of the four phases is excited six times per revolution in this “8/6” SRM).

The experimental setup employs a half-bridge inverter with two transistors and two diodes per phase; it operates in a hysteretic current mode with “hard chopping” [16] (i.e., the voltage on an active phase is either positive or negative dc voltage); the hysteresis band is 0.2 A. Optimal currents are implemented rather coarsely—there are 16 quantization levels, and the target waveform for the hysteretic control is updated every 1.8 mechanical degrees. Both policies are implemented using a Motorola MC68332 microcontroller.

Figs. 7 and 8 show the steady-state waveforms for the two switching policies. A remarkable similarity between the simulated torque ripple (for the case of an uncertain SRM
model) and actual recordings can be observed. Thus, a repeated (or on-line) identification of $L(\theta, i)$ is likely to lead to further improvements in ripple minimization in the proposed control policy.

Magnitude spectra (normalized at dc) corresponding to the two control policies are shown in Fig. 9. Fig. 9 also shows results obtained with “soft” chopping inverter operation for optimal control policy in which the voltage on an active phase can be zero, in addition to $\pm V_{dc}$. Under soft chopping, a further 3–6-dB attenuation is observed in the main and higher harmonics. This is expected, because in the case of soft chopping, the current variation and the number of switchings per period are both reduced.

These measurements have been obtained using an HP35665A spectrum analyzer and “flat-top” windowing that offers high amplitude accuracy ($\pm 0.005$ dB), but somewhat lower frequency resolution. The peak at 80 Hz corresponds to the fundamental of the torque ripple (rotational speed is $\approx 200$ r/min). A marked improvement is evident in the range 150–250 Hz; in particular, the second harmonic is attenuated almost 8 dB (i.e., 2.5 times). The measurements at low frequencies tend to rule out shaft misalignment as a major source of torque ripple; nonsymmetries of the phases are a likely cause for spectral components at 20 Hz and its multiples. Together with inevitable quantization and position errors, these findings point toward an on-line self-tuning approach to torque ripple minimization as a promising research direction. One challenge is the development of an efficient and accurate parametrization of optimized current waveforms over a wide speed and torque range. The other challenge is in further reduction of the sensitivity of these waveforms to inaccuracies in the flux model. One strategy, pursued in [15], is to compute the current waveforms using a simplified torque function $T_M(\theta, i)$, then to dynamically estimate the harmonic components of the resulting torque ripple and, finally, to utilize feedback to correct the currents.

VI. CONCLUSION

In this paper, the production of ripple-free torque in SRM’s has been addressed. The control goal has been to minimize torque ripple produced by an SRM using simple position and current feedback structures and standard microcontroller technology. A complete parameterization of position-dependent voltage and current profiles that result in (nominal) ripple-free operation of an SRM has been developed. This parameterization serves as a computational base for optimization over candidate current waveforms. The main concern has been the minimization of the required voltage magnitude, since it is a common constraint in practice. Other useful criteria described in the paper also rely on the same parameterization for efficient calculations. The effects of model uncertainties in an SRM have been explored and showed, via simulations, that the proposed switching strategy consistently outperforms the conventional switching policy. Experimental data presented in the paper verify the analytical approach and point out possible improvements. The proposed switching strategy is relevant for
several areas of current interest in SRM drive control, as it can be naturally included in self-tuning and adaptive control implementations.

REFERENCES


