Qualitative kinematics of planar robots: Intelligent connection

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Abstract

This paper proposes a qualitative representation for robot kinematics in order to close the gap, raised by the perception–action problem, with a focus on intelligent connection of qualitative states to their corresponding numeric data in a robotic system. First, qualitative geometric primitives are introduced by combining a qualitative orientation component and qualitative translation component using normalisation techniques. A position in Cartesian space can be mathematically described by the scalable primitives. Secondly, qualitative robot kinematics of an n-link planar robot is derived in terms of the qualitative geometry primitives. Finally, it shows how to connect quantitativeness and qualitativeness of a robotic system. On the one hand, the integration of normalisation and domain knowledge generates normalised labels to introduce the meaningful parameters into the proposed representation. On the other hand, the normalised labels of this representation can be converted to a quantitative description using aggregation operators, whose numeric outputs can be used to generate desired trajectories based on mature interpolation techniques.

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1. Introduction

There exists an interesting gap between traditional robotics and cognitive robotics, or robot motion and human perception [1]. More specifically, that is the intelligent connection of numeric data used in conventional robotics and symbols in high-level cognitive functions. The reason is twofold. On the one hand, research in robotics has traditionally emphasized low-level sensing and control tasks including sensor processing, path planning and control; on the other hand, research in cognitive robotics is concerned with endowing robots and software agents with higher level cognitive functions that enable them to reason, act and perceive in changing, incompletely known, and unpredictable environments. This gap is one of crucial issues for interdisciplinary research in intelligent robotics among engineering community, robotics community and AI community. It emphasises the goal of robotics research that “robotics is the intelligent connection of perception to action” [2,3].

Research on qualitative reasoning and model-based technology can be found in [4–7]. Generally speaking, there are two type of approaches to qualitative spatial representations [8,9]. One is to explore what aspects do lend themselves to qualitative representation, the other is to use a quantitative representation as a starting point and compute problem-specific qualitative representations to reason with. Cohn and Hazarika [10] gave sufficient overview of qualitative spatial representation and reasoning techniques by investigating the main aspects of the representation of qualitative knowledge including ontological aspects, distance, orientation and shape, and qualitative spatial reasoning including reasoning about spatial change. The representation of qualitative kinematics is the well developed field in qualitative spatial representation. Its history can be covered by the following research work. Firstly, the possible motions of objects are represented by qualitative regions in configuration space representing the legitimate positions of parts of mechanisms [11]. Faltings built upon Nielsen and Forbus’ earlier work on qualitative kinematics [12], and developed a first principles algorithm for analysing planar mechanisms. However, this work suffered from the limitation that certain problems could not be solved without including quantitative information. Secondly, Olivier et al. proposed a qualitative kinematics reasoning method based upon the use of occupancy arrays [13]. This approach worked simply on the constraint that no two objects occupy the same occupancy array position and can be extended to include semi-quantitative information. Thirdly, Kramer [14] proposed ‘The Linkage Assistant’ kinematics simulator which demonstrated that mechanism kinematics analysis did not solely have to rely on exact geometric mechanism information. Fourthly, Liu [15] presented a qualitative representation and reasoning approach based upon the formalism of qualitative trigonometry, qualitative arithmetic, and qualitative spatial inference. In addition, Liu and Coghill [16,17] proposed fuzzy qualitative trigonometry assisting qualitative calculation and reasoning of trigonometry-based systems. However, developing a general approach to the representation of qualitative kinematics is still an open problem. This study aims at developing a general qualitative representation for kinematics planar robots in order to help to solve the perception–action problem in robotics; the approach can also be extended to general mechanisms.

The rest of this paper is organised as follows: Section 2 presents qualitative geometric primitives in Cartesian space. Section 3 derives a qualitative representation for qualitative robot kinematics. Section 4 addresses how the representation connects both qualitative states and numeric robot data. Section 5 concludes this paper.
2. Qualitative geometric primitives

The degrees of freedom of a robotic system can be simply viewed as the number of coordinates that it takes to uniquely specify the position of the system. Consider a rigid planar figure \( \mathcal{A} \) that is free to move in a two-dimensional plane. Its motion along its two degrees of translation and around its one degree of orientation can be described in terms of its degrees of freedom, whatever coordinates are used to describe its position. The position representation of a system consists of two components: a translation component and an orientation component. The position of figure \( \mathcal{A} \) is denoted by \( \mathcal{A}_p(C_t, C_o) \) in general coordinates, where \( C_t \) stands for a translation component, \( C_o \) for an orientation component.

The formula \( \mathcal{A}_p(C_t, C_o) \) can be used to describe the position in both quantitative or qualitative terms. Its quantitative representation is \( \mathcal{A}_p(p_l, p_h) \), while its qualitative representation is given by \( \mathcal{A}_p(qpl, qp_h) \). In order to connect robot motion and perception, we need to define the mathematical description of \( \mathcal{A}_p(C_t, C_o) \) for qualitative analysis.

Further, let us consider the facts presented by Freksa [18–20]. First, qualitative knowledge is relative knowledge where the reference entity is a single value rather than a whole set of categories. Secondly, qualitative knowledge is obtained by comparing features within the object domain. The two facts inspire a solution to the intelligent connection problem. That is that the qualitative position description can be obtained by mapping numeric data into a unit circle using normalisation techniques, where a full orientation \([0, 2\pi)\) can be normalised into a unit range \([0, 1)\). Hence, the range \([0, 1)\) provides a reference entity for comparison of translation and orientation features. Furthermore, the general representation of figure \( \mathcal{A} \) can be given by \( \mathcal{A}_p(C_t(s), C_o(r)) \), where \( s, r \) are defined as the mapping parameters over quantisation. Basically, \( s \) and \( r \) are the numbers of the interval ranges that a translation component and an orientation component have, respectively, in the context of normalisation reference. As \( s \to \infty \) and \( r \to \infty \), the limits of \( C_t(s) \) and \( C_o(r) \) in Eq. (1) are approaching the set of real numbers \( \mathbb{R} \), that is, quantitative description \( \mathcal{A}_p(p_l, p_0) \)

\[
\begin{align*}
\lim_{s \to \infty} C_t(s) &= p_l \\
\lim_{r \to \infty} C_o(r) &= p_0
\end{align*}
\]

On the other hand, as \( s \to \infty \) and \( r \to \infty \), the limits of \( C_t(s) \) and \( C_o(r) \) in Eq. (2) are approaching a set of \( s_0 \) qualitative states for a translation component and a set of \( r_0 \) qualitative states for an orientation component in qualitative terms

\[
\begin{align*}
\lim_{s \to s_0} C_t(s) &= qp_l \\
\lim_{r \to r_0} C_o(r) &= qp_0
\end{align*}
\]

Hence, the intelligent connection herein actually is the problem of how to construct the mapping between the right-side terms of Eqs. (1) and (2). The proposed method of solving the connection problem can be achieved by data normalisation and aggregation. For instance, a set of two-dimensional numeric data \( P(p_x(i), p_y(i)) \), where \( i = 1, \ldots, n \), and assume a reasoning system requires a set of corresponding symbols. The data set can be mapped into a unit circle by data normalisation, i.e., Eq. (3)

\[
\hat{I}_i = \frac{I_i}{\max(I_i)}, \quad \hat{\theta}_i = \frac{\theta_i}{2\pi}
\]

where \( I_i = \sqrt{p_x^2(x) + p_y^2(y)} \), \( \theta_i = \arctg \left( \frac{p_y(x)}{p_x(x)} \right) \).
For simplicity, we separate the orientation component from the translation component, and also evenly divide a full orientation into five interval ranges and a unit length into eight interval ranges. That is to say, \( r = 5 \) and \( s = 8 \) if the interval ranges are described in terms of normalised intervals. Then please see Fig. 1, the normalised numeric data in the dark area can be represented by a qualitative symbol or qualitative state \( QS(2, 3) \) in Eq. (4), in which elements “0.25” and “0.6”, are symbols instead of numeric data. They show qualitative information in terms of the orientation and translation positions with “1” as a reference entity. The combined qualitative description of \( QS(2, 3) \) is given in Fig. 2

\[
QS(2, 3) = \begin{bmatrix} 2/5 \\ 3/8 \end{bmatrix} = \left[ \begin{array}{c} 0.25 \\ 0.6 \end{array} \right]
\]

On the other hand, the connection requires that output symbols from a reasoning system should be able to transfer back to the description in terms of numeric data. For instance, a numeric sampling point has to be extracted from the dark area in Fig. 2 to represent symbol \( QS(2, 3) \). There are many techniques that can be used for this purpose such as hand-coded methods, random selection and aggregation methods [21]. In the paper aggregated values are used for numeric data extraction such as different type of mean functions. The aggregated value \( P(l_a, \theta_a) \) can be obtained by

\[
l_a = f(QS(2)); \quad \theta_a = f(QS(3))
\]

where \( f() \) is an aggregation operator.

For instance, given the trajectory of an ellipse shown in Fig. 3, its qualitative representation (i.e., the inside continuous black area within a unit circle), can be obtained, where \( s \) and \( r \) are set as 20 and 19, respectively. Please note that the number of symbols in either an orientation component or a translation component is decided by the domain
knowledge of a symbol-based system (e.g., a reasoning system). In this sense, the generated normalised symbols can be integrated into the symbol-based system. The set of either orientation symbols or translation symbols is called a quantity space. There is not a well-defined definition for quantity space yet, it can be understood as that a quantity space is utilized to represent continuous values via sets of ordinal relations, it can be thought of as partial information about a set of elements [22]. The elements can be represented by intervals, ratio and fuzzy intervals. Fuzzy intervals have also been used in fuzzy reasoning about mechatronics systems [23]. We use the representation of a four-tuple fuzzy number in paper [23], say, a fuzzy number \([a, b, \tau, \beta]\). Due to the fact that a motion component comprises evenly distributed normalised numeric data, fuzzy numbers should have the same shape; what is more, for simplicity, we have \(b - a = \kappa_0 \tau\) and \(\tau = \beta\), and the membership value of the crossing point of adjacent fuzzy numbers is 0.5. Hence, a function for

![Fig. 2. The combined qualitative description of QS(2,3).](image1)

![Fig. 3. The qualitative representation of a set of numeric data \((s = 20, r = 19)\).](image2)
generating a quantity space including \( n \) normalised qualitative states can be obtained in the following, where \([a_i, b_i, \tau_i, \beta_i]\) denotes the \( i \)th symbol \( QS(i) \)

\[
QS(i) = \begin{cases}
[0, \kappa_0 \tau, 0, \beta], & i = 1 \\
[(\kappa_0 + 1)(i - 1)\tau, (\kappa_0 + 1)i\tau - \tau, 0, \beta], & i = 2, \ldots, n - 1 \\
[1 - \kappa_0 \tau, 1, \tau, 0], & i = n 
\end{cases}
\]  

where \( \tau = \frac{1}{n(\kappa_0 + 1) - 1} \). \( \kappa_0 \) is a threshold parameter to define the shape of the fuzzy numbers, \( \kappa_0 \) and \( n \) are chosen by symbolic systems.

3. Qualitative robot kinematics

This section starts with the introduction of conventional robot kinematics, next qualitative robotic primitives are proposed, and then a qualitative version of robot kinematics is derived based on the robotic primitives. Finally the change of qualitative states is discussed.

3.1. Conventional robot kinematics

A robot can be considered to consist of a series of links connected together with joints. For simplicity, we concern an \( n \)-link planar robot with revolute joints only, we assign a frame of reference to each link, which are named systematically with numbers, for instance, the \( i \)th link is numbered \( i \) from the immovable base part of a robot. The frame of reference joined to the base is named as global reference frame which is the reference frame for a robot; the frame joined to the \( i \)th link is named as \( i \)th local reference frame.

Robot kinematics is the study of motion of robots; it includes forward kinematics and inverse kinematics. The former is to calculate the position of any point in the work volume of a robot given the length of each link and the angle of each joint; the latter is to calculate the angle of each joint given the length of each link and position of the point in work volume. We consider forward kinematics in this paper, the end position of the \( n \)th link of the robot \( P(p_x, p_y) \) can be formalized in Eq. (6) provided its each link length \( l_i \) and its corresponding absolute angle \( \theta_i \). Please note that \( l_i \) and \( \theta_i \) are numeric

\[
P(\Theta) = \begin{bmatrix} p_x(\Theta) \\ p_y(\Theta) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} l_i \cos(\theta_i) \\ \sum_{i=1}^{n} l_i \sin(\theta_i) \end{bmatrix}
\]  

3.2. Qualitative robotic primitives

Roughly speaking, there are two types of robotic primitives: revolute and prismatic joints, see Figs. 4 and 5. They can be used to construct a wide range of robotic systems. For the former, \( l_0 \) denotes the length of its rigid link, \( \theta \) denotes its orientation variable; for the latter, \( l \) denotes its translation variable, \( \theta_0 \) denotes its starting angular state.

The qualitative representation of the end-effector of a revolute primitive in Fig. 4 is given in Eq. (7). The quantity space of a translation component \( Q^d \) is a one-item quantity space \([l_0]\), the quantity space of an orientation component \( Q^o \) is on the closed range \([0 \ 2\pi]\)
The qualitative representation of a prismatic primitive shown in Fig. 5 is given in Eq. (8). The quantity space of its translation component $Q_d$ belongs to closed range $[0, l]$, its $Q_a$ is a one-item quantity space, $[\theta_0]$

$$\begin{align*}
Q^d &= q_p l | q_p l \in [l_0] \\
Q^a &= q_p_0 | q_p_0 \in [0, 2\pi]
\end{align*}$$

The general qualitative representation of robot components including two joints shown in Figs. 6 and 7 is given in Eq. (9). The difference from those single primitives in Figs. 4 and 5 is that a constraint function $C_{\text{dof}}$ is introduced to confine the order of degrees of freedom of a robot from its base. The robotic structures in Figs. 6 and 7 are distinguished by value assignment of their $C_{\text{dof}}$, whose entries, $q_p_0, q_p_1$ in Fig. 6 are assigned to 1 and 2, those in Fig. 7 are assigned in the other way around

$$\begin{align*}
Q^d &= q_p l | q_p l \in [0, l_0] \\
Q^a &= q_p_0 | q_p_0 \in [0, 2\pi] \\
C_{\text{dof}} &= \{q_p_1, q_p_0\}
\end{align*}$$
Furthermore, Eq. (9) can be rewritten with $s$ normalised symbols of the translation component and $r$ normalised symbols for the orientation component in the following:

\[
\begin{align*}
Q_d &= q_{pl} \in \left[ \frac{l_1}{l}, \frac{l_2}{l}, \ldots, \frac{l_s}{l}, 1 \right] \\
Q_a &= \frac{q_{ph}}{2\pi} \in \left[ \frac{q_{\theta_1}}{2\pi}, \frac{q_{\theta_2}}{2\pi}, \ldots, \frac{q_{\theta_{r-1}}}{2\pi}, 1 \right] \\
C_{dof} &= \{ q_{\theta} = 1, q_p = 2 \}
\end{align*}
\]

where

\[
\begin{align*}
0 &\leq \frac{l_1}{l} \leq \frac{l_2}{l} \leq \cdots \leq \frac{l_s}{l} \leq 1 \\
0 &\leq \frac{q_{\theta_1}}{2\pi} \leq \frac{q_{\theta_2}}{2\pi} \leq \cdots \leq \frac{q_{\theta_{r-1}}}{2\pi} \leq 1
\end{align*}
\]

3.3. Qualitative representation of robot kinematics

The components of a robot are described at a coarse but important level by just two attributes: their position and their orientation. The aim of robot qualitative representation is the manner in which we qualitatively represent these quantities and manipulate them mathematically. The quantitative description of an $n$-link serial robot shown in Eq. (6) can be rewritten in qualitative terms

\[
P(QS(\Theta)) = \begin{bmatrix} \sum_{i=1}^{n} p_x'(QS(\theta_i)) \\ \sum_{i=1}^{n} p_y'(QS(\theta_i)) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} l_i \cos(QS(\theta_i)) \\ \sum_{i=1}^{n} l_i \sin(QS(\theta_i)) \end{bmatrix}
\]

Though the above equation is very similar to Eq. (6), it differs from Eq. (6) in two essential aspects. First, Eq. (11) deals with qualitative symbols, $QS(\theta_i)$, instead of numeric input; secondly trigonometric functions in numeric terms are replaced by fuzzy qualitative trigonometric functions [16,17,24]. Fuzzy qualitative trigonometric functions are the conventional trigonometric functions in fuzzy qualitative terms. They allow dealing with normalised fuzzy intervals as parameters; their outputs are usually a set of fuzzy intervals.

Hence, an $n$-link robot can be decomposed into $n$-link based segments, each of which can be described in its local coordinate system by robot primitives, the position and orientation of its end-effector can be qualitatively described in global reference coordinates. The local representation of the $i$th link can be described by $s_i'$($C_l(s_i), C_o(r_i)$) as follows:
\begin{align}
\begin{cases}
    \mathbf{q}_i^p &= qp_i^i | qp_i^i \in [qp_i^1, qp_i^2, \ldots, qp_i^{n_i-1}, l_i] \\
    \mathbf{q}_0^p &= qp_0^i | qp_0^i \in [q_0^1, q_0^2, \ldots, q_0^{n_i-1}, 2\pi]
\end{cases}
\end{align}

where

\begin{align}
    qp_j^i &= \frac{l_{j_i}}{s_i}, \\
    q\theta_j^i &= \frac{2\pi j}{r_i}, \\
    0 \leq qp_1^i &\leq qp_2^i \leq \cdots \leq qp_{n_i-1}^i \leq l_i, \\
    0 \leq q\theta_1^i &\leq q\theta_2^i \leq \cdots \leq q\theta_{n_i-1}^i \leq 2\pi
\end{align}

The qualitative position of the \(i\)th joint component can be described by a pair of its qualitative position and qualitative orientation. Mapping the representation of the \(i\)th segment into a unit circle using normalisation in its local coordinates, Eq. (12) can be rewritten as

\begin{align}
\begin{cases}
    qp_i^i | qp_i^i \in \left[ \frac{qp_i^1}{\sum_{i=1}^{n_i} l_i}, \frac{qp_i^2}{\sum_{i=1}^{n_i} l_i}, \ldots, \frac{qp_i^{n_i-1}}{\sum_{i=1}^{n_i} l_i} \right] \\
    qp_0^i | qp_0^i \in \left[ \frac{q_0^1}{2\pi}, \frac{q_0^2}{2\pi}, \ldots, \frac{q_0^{n_i-1}}{2\pi}, 1 \right]
\end{cases}
\end{align}

The representation of the position of an end-effector in global coordinates is essential in robotics because an end-effector is used to carry out workspace tasks. The qualitative representation of an end-effector can be obtained in the following:

\begin{align}
\begin{cases}
    qp_i = \oplus_{i=1}^{n} qp_i^i | qp_i^i \in UC_{q_i} \\
    qp_0 = \oplus_{i=1}^{n} qp_0^i | qp_0^i \in UC_{q_0}, \\
    C_{dof} = UC_{dof}
\end{cases}
\end{align}

where

\begin{align}
    UC_{qp_i} &= \left[ \frac{qp_i^1}{\sum_{i=1}^{n_i} l_i}, \frac{qp_i^2}{\sum_{i=1}^{n_i} l_i}, \ldots, \frac{qp_i^{n_i-1}}{\sum_{i=1}^{n_i} l_i} \right], \\
    UC_{qp_0} &= \left[ \frac{q_0^1}{2\pi}, \frac{q_0^2}{2\pi}, \ldots, \frac{q_0^{n_i-1}}{2\pi}, 1 \right], \\
    UC_{dof} &= [qp_0^i = i], \quad i \in (0, 1, \ldots, n)
\end{align}

Here \(UC_{qp_i}\) stands for the translation component of the \(i\)th link segment in a unit circle; \(UC_{qp_0}\) for the orientation component. The constraint function \(C_{dof}\) is employed to define the degrees of freedom constraints between components. \(qp_0^i = 0\) stands for the base of the robot when \(i\) is equal to zero. The qualitative representation of the end-effector is a qualitative addition of the translation and orientation components of each link segment based on its constraints of their degrees of freedom. The role of qualitative addition can be carried out by a variety of qualitative techniques. For example, fuzzy arithmetic can be employed given the components are described by fuzzy numbers.
3.4. Description of the change of qualitative states

In terms of the representation of robot qualitative position, $\Delta \alpha (\Delta C_t(s), \Delta C_o(r))$ is used to denote the change of qualitative states, which consists of two components for the change of the translation and orientation. The state change, $\Delta C_t(s_i), \Delta C_o(r_i)$ of the $i$th link segment from time instant $t$ to $t'$ are given as follows:

$$
\Delta C_t(s_i) = \text{sign}(qp^i_0(t') - qp^i(t)) = \begin{cases} 
+ & \Delta qp^i_0 > 0 \\
0 & \Delta qp^i_0 = 0 \\
- & \Delta qp^i_0 < 0 
\end{cases}$$

(15)

$$
\Delta C_o(r_i) = \text{sign}(qp^h_0(t') - qp^h_0(t)) = \begin{cases} 
+ & \Delta qp^h_0 > 0 \\
0 & \Delta qp^h_0 = 0 \\
- & \Delta qp^h_0 < 0 
\end{cases}$$

The state change of an end-effector, $\Delta \alpha_p(\Delta C_t(s), \Delta C_o(r))$, can be derived based on Eq. (14)

$$
\Delta C_t(s) = \text{sign}\left(\bigoplus_{i=1}^n \Delta qp^i_0(t') - \bigoplus_{i=1}^n \Delta qp^i_0(t)\right)
$$

$$
\Delta C_o(r) = \text{sign}\left(\bigoplus_{i=1}^n \Delta qp^h_0(t') - \bigoplus_{i=1}^n \Delta qp^h_0(t)\right)
$$

(16)

Generally speaking, it is impossible to compare two symbolic labels in different representation scales. For instance, if there are two quantity spaces [lowest, lower, medium, faster, fastest] and [lower, medium, fast] for qualitative descriptions of their state change, no one can tell whether or not the label fastest from the former quantity space changes quicker than the label fast in the latter. The reason is that there is no reference standard for the labels that are used to reflect the perception, without which there is no way to carry out the qualitative arithmetic. Sharing labels across subsystems is a crucial problem in AI research. One of the advantages of the proposed approach is the introduction of normalised labels as the reference entity for relationship construction of robot link segments. On the one hand, the items of $UC_{qp}$ and $UC_{qp^h}$ in Eq. (14) correspond to the normalised symbols of the $i$th link segment. On the other hand, they also have relative quantitative description of knowledge features with a unit circle.

4. Intelligent connection

This section presents how the proposed approach connects symbolic qualitative states to their numeric trajectories.

4.1. Connection to qualitative states

Almost all reasoning systems are based on symbols including intervals, fuzzy numbers and pure symbols so it is crucial to generate scalable symbols, which can properly reflect their symbolic meaning. It requires that normalised qualitative states not only serve as atomic symbols for symbols used in symbolic systems, but also can be qualitatively calculated in terms of normalised symbols. Hence, connection to qualitative states should include normalised symbol generation and qualitatively calculation/reasoning.
Eq. (3) presents the data normalisation for a set of two-dimensional numeric data in Section 2, its variants can be used to deal with data in different types of coordinates. The ellipse trajectory in Fig. 3 actually is the end-effector trajectory of a three-link planar robot, from which it can be seen that numeric data is mapped into a unit circle and can be described by normalised symbols.

The qualitative workspace of a link segment in Eq. (13) can be mathematically described in a matrix

$$W^i_{q} = \begin{bmatrix}
\left( \frac{q_{pi} + q_{pi}'}{2\pi} \right) & \cdots & \left( \frac{q_{pi} + q_{pi}'}{2\pi} \right)
\end{bmatrix}
$$

and the qualitative workspace of its end-effector can be qualitatively derived by the union of qualitative workspaces of link segments, the union operation can be taken by a variety of reasoning techniques (e.g., interval computation and fuzzy arithmetic)

$$W^q = \bigcup_{i=1}^{n} W^i_{q}$$

Each entry of the $W^i_{q}$ is comprised of the qualitative states of the orientation and translation of the $i$th link segment with normalised symbols. The dimension of the envisonnement of the end-effector of an $n$-link robot, $E^d$, is given by

$$E^d = \prod_{i=1}^{n} (r_i \times s_i).$$

Qualitative calculation of normalised symbols can be carried out by fuzzy qualitative trigonometric functions [17]. For instance, a robot shown in Fig. 8 is presented to calculate its end-effector’s qualitative position using fuzzy qualitative trigonometric functions. Let the components of links $l_1$ and $l_2$ have the same number of normalised fuzzy numbers and $\kappa_0$ is set as $5$ (i.e., $n_o = 16$ is for their orientation components and $n_t = 21$ is for their translation components), we obtain

$$UC_{q_{l_1}} = \begin{bmatrix}
\frac{1}{21} & \frac{2}{21} & \cdots & \frac{20}{21} & 1
\end{bmatrix}^T$$

$$UC_{q_{l_2}} = \begin{bmatrix}
\frac{1}{16} & \frac{2}{16} & \cdots & \frac{15}{16} & 1
\end{bmatrix}$$

and their qualitative workspace $W^i_{q}$, where $i = 1, 2,$
Given the scenario of link positions $l_1$ and $l_2$ and angle $\theta_2$, their fuzzy numbers can be generated by using Eq. (5)

$$
P_{l_1} \left( QS_d \left( \frac{5}{21} \right) \right) = \left[ 0.1263 \ 0.1789 \ 0.0105 \ 0.0105 \right] \\
P_{\theta_1} \left( QS_a \left( \frac{5}{16} \right) \right) = \left[ 0.4068 \ 0.4915 \ 0.0169 \ 0.0169 \right] \\
P_{\theta_2} \left( QS_a \left( \frac{4}{16} \right) \right) = \left[ 0.5085 \ 0.5932 \ 0.0169 \ 0.0169 \right] \\
P_{l_2} \left( QS_d \left( \frac{6}{21} \right) \right) = \left[ 0.3158 \ 0.3684 \ 0.0105 \ 0.0105 \right]
$$

where $QS_d(\ )$ and $QS_a(\ )$ denote fuzzy numbers in a translation component and those in an orientation component. Applying FQT SAS and AAA theorems and arcsin function [17], the position of the end-effector, $P(QS_a(\theta_1), QS_d(l_{21}))$, are given in Eq. (19) and the qualitative states of the end-effector is given in Table 1

$$
\begin{pmatrix}
QS_d \left( \frac{20}{21} \right) \\
QS_d (1)
\end{pmatrix}
= 
\begin{pmatrix}
0.8136 & 0.8983 & 0.0169 & 0.0169 \\
0.9153 & 1.000 & 0.0169 & 0
\end{pmatrix}
$$

$$
\begin{pmatrix}
QS_a \left( \frac{1}{16} \right) \\
QS_a \left( \frac{9}{16} \right)
\end{pmatrix}
= 
\begin{pmatrix}
0 & 0.0526 & 0 & 0.0105 \\
0.0632 & 0.1158 & 0.0105 & 0.0105
\end{pmatrix}
$$

(19)

Table 1

<table>
<thead>
<tr>
<th>Qualitative position description of the end-effector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$QS_1$</td>
</tr>
<tr>
<td>$QS_d(\theta_1)$</td>
</tr>
<tr>
<td>$QS_d(l_{21})$</td>
</tr>
</tbody>
</table>

Fig. 8. An example of a Kawasaki FA 20N robot.
4.2. Connection to robot motion

Section 4.1 presents the aspect of intelligent connection on how to connect to qualitative symbols. This section demonstrates how to link normalised symbols to numeric data, that is to say, how to generate numeric data by manipulating normalised fuzzy numbers.

Aggregation operators are chosen to extract numeric data from normalised qualitative states. The reason for that is to solve the states explosion produced by fuzzy qualitative techniques. Aggregation operators model operations such as conjunction, disjunction and averaging on intervals and fuzzy sets [21]. One of popular aggregation operator families are the ordered weighted averaging operators and their variants. Their general mathematical description is given in Eq. (20). The \( \text{OWA} \) operators are the extensions to the quasi-arithmetic mean [25] in the aggregation operation of fuzzy sets [26–29], originally studied by Yager [30].

\[
\text{OWA}(P_1, P_2, \ldots, P_n) = \sum_{i=1}^{n} w_i P_{\sigma(i)}
\]

where \( \sigma \) is an ordinal sequence, \( w_i \geq 0 \) and \( \Sigma w_i = 1 \). The \( \text{OWA} \) operators provide a parameterised family of aggregation operators which can be used for many of the well-known operators by choosing suitable weights, some of the \( \text{OWA} \) are provided in the Appendix.

Combing the ordered weighted averaging operators, Eqs. (6) and (11) are connected together as shown in Eq. (21) whose inputs are qualitative states and output are aggregated numeric data

\[
P(\Theta) = \begin{bmatrix} p_x(\Theta) \\ p_y(\Theta) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \text{OWA}(l_i \cos(QS(\theta_i))) \\ \sum_{i=1}^{n} \text{OWA}(l_i \sin(QS(\theta_i))) \end{bmatrix}
\]

The weights of the \( \text{OWA} \) can be selected based on domain knowledge or application context where the aim is to adjust the \( \text{OWA} \) operator to generate suitable numeric values. Aggregated values can be used to generate smooth trajectories using interpolation techniques in robotics.

An example of a four-link planar robot is demonstrated in this section in order to reveal the effectiveness of the proposed approach. The robot is decomposed into two-link components, each of which includes two robotic primitives shown in Fig. 4, and each link component is modelled by Eq. (21). Input qualitative states for its joints are given in Table 2, \( QS(J'_i) \) denotes the \( j \)th qualitative state for the \( i \)th joint. Quantity spaces used in Section 4.1 are employed here. Fuzzy qualitative trigonometry is firstly used for the two-link

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components, respectively; it calculates the end-effectors of the two-link components. Basically it generates the qualitative position states in terms of overlapped intervals, e.g., Fig. 9 shows the overlapped position intervals for qualitative states $QS(x_{13}, y_{13})$. Secondly the median aggregation operator is used to generate aggregated values for the positions. Aggregated values are labelled as ‘o’ in Fig. 10. It shows that the aggregated values are within the intervals of its qualitative states. The intervals of the qualitative states in Fig. 10 are actually the projections of the fuzzy numbers to the real lines of their universe.

![Fig. 9. Propagated position values of the end-effector of the first-link component for qualitative states $QS(x_{13}, y_{13})$.](image)

![Fig. 10. Joint trajectories of a four-link robot with the qualitative states and aggregated values.](image)
Thirdly, the two-link components are combined together, that is, the base of the second-link component mounts on the position of the end-effector of the first-link component. Finally, interpolation techniques and inverse kinematics are applied to the two components to produce respective joint trajectories. The snapshots of the four-link robot’s motion are shown in Fig. 11, the aggregated values are labelled with ‘o’. Qualitative descriptions of robotic sampling position points, e.g., those in Table 2, can be used to produce the generation of the desired trajectories in order to control the motion of a robot.

Fig. 11 has demonstrated that the proposed method is able to qualitatively controlling atomic behaviours of planar robots; fuzzy qualitative trigonometry and aggregation operators are used to implement the control between qualitative descriptions and desired joint trajectories. The proposed approach has pointed out a novel way towards intelligent connection of motion control task and symbolic tasks, e.g., robot qualitative motion planning and control. On the one hand, atomic behaviours of robots described by qualitative states have the capability of being used to construct symbolic functions. On the other hand, desired trajectories are the inputs for motion control modules. The proposed approach works as a middleware which not only supplies atomic behaviours for symbolic sub-systems but also generates the inputs for numerical subsystems. It also indicates that the more accurate scale of measurement reflecting more quantitative termed states, the more precision of controlled robotic behaviours.

5. Concluding remarks

This research has proposed a novel qualitative representation for robotic intelligent connection. This method has first presented qualitative primitives for robotic components, then gradually constructs qualitative representation for a complex robot. This representation works as a converter for in-depth understanding the connection between low-level control and sensing (i.e., robotic control modules), and high-level aggregation operators (i.e., symbolic systems). Aggregation operators are used to describe meaningful functions. The two advantages of the proposed method should be noted: one is the proposed
normalised technique, it allows sharing normalised symbols across multiple robots or sub-

| systems with the condition that they have the same domain knowledge; the other is the scalable normalised qualitative states/symbols for Cartesian motion components. The number of normalised symbols determines the precision of motion description in that the bigger the number of motion component symbols, the higher the precision of a robotic system. It naturally provides a facility for the negotiation of the connection between the qualitative and quantitative descriptions. However, robots are used to manipulate or interact with other objects for instance human robot interaction [31], the proposed method is sensitive to the size and shape of the objects that they handle during behaviour generation. Learning algorithms are needed for study how to dynamically control the number of normalised symbols for robot behaviour generation and robot motion planning in object-existing environment. Our further work also targets the extension of the proposed method to spatial robots and the problem of integrating kinematics parameters, e.g., DH parameters, into the proposed qualitative model.

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Appendix

See Table 3

References