Tail dependence structure of the foreign exchange market: A network view

Gang-Jin Wang\textsuperscript{a,b,c,*}, Chi Xie\textsuperscript{a,b}

\textsuperscript{a}College of Business Administration, Hunan University, Changsha 410082, China
\textsuperscript{b}Center of Finance and Investment Management, Hunan University, Changsha 410082, China
\textsuperscript{c}Center for Polymer Studies and Department of Physics, Boston University, Boston, Massachusetts 02215, USA

Abstract

Tail dependence of financial entities describes when the price of one financial asset has an extreme fluctuation (e.g., price sharply rises or falls), the degree of its effect on the price fluctuation of another asset. Under the background of the global financial crisis, tail dependence structure of financial entities plays an important role in financial risk management, portfolio selection, and asset pricing. In this paper, we propose a concept of tail dependence networks to investigate the tail dependence structure of the foreign exchange (FX) market. Lower- and upper-tail dependence networks for 42 major currencies in the FX market from 2005 to 2012 are constructed by combing the symmetrized Joe-Clayton copula model and two filtered graph algorithms, i.e., the minimum spanning tree (MST) and the planar maximally filtered graph (PMFG). We also construct the tail dependence hierarchical trees (HTs) associated with the MSTs to analyze the currency clusters. We find that (1) the two series of lower- and upper-tail dependence coefficients present different statistical properties; (2) the upper-tail dependence networks are tighter than the lower-tail dependence networks; and (3) different currency clusters, cliques and communities are respectively found in the two tail dependence networks. The key empirical results indicate that market participants should consider the different topological features at dif-

\*Corresponding author at: College of Business Administration, Hunan University, Changsha 410082, China. Tel.: +86 731 88823890; fax: +86 731 88823670.

Email addresses: wanggangjin@hnu.edu.cn (Gang-Jin Wang), xiechi@hnu.edu.cn (Chi Xie)

Preprint submitted to Elsevier
ferent market situations (e.g., a booming market or a recession market) to make decisions on the investing or hedging strategies. Overall, our obtained results based on the tail dependence networks are new insights in financial management and supply a novel analytical tool for market participants.

**Keywords:** Dependence structure, Foreign exchange market, Network, Copula, Cluster analysis, Community structure

1. Introduction

The dependence structure of financial markets is a key and debatable issue for financial economists, regulators and investors. It is the useful and important information for some financial activities, such as international portfolio’s diversification and risk management, derivative pricing, and market integration (Buccheri et al., 2013). For example, how to select financial assets in the international portfolio diversification is the crucial step, which needs to examine the dependence of financial asset returns and check whether they have the dependence hierarchical or clustering structure. As such, a growing number of methods are developed to capture the dependence structure among different financial asset returns, while the dependence (or correlation) network analytical tool is one of the most popular and widely used approaches (see, e.g., Mantegna (1999); Brida et al. (2009); Kwapien et al. (2009); Brida and Risso (2010); Tumminello et al. (2010); Fenn et al. (2012); Kwapien and Drozdz (2012); Wang et al. (2012, 2013a)). Different approaches can be used to build the dependence network, such as the minimum spanning tree (MST) approach (Mantegna, 1999), the planar maximally filtered graph (PMFG) method (Tumminello et al., 2005), and the correlation threshold method (Onnela et al., 2004; Boginski et al., 2005), which are designed to select or filter the information presented in the dependence (or correlation) matrix. In other words, before constructing the financial networks, one should build the dependence matrix of the financial asset returns by the dependence measure, while the common practice of the dependence measure is chosen as the Pearson’s correlation coefficient (PCC). Although PCC is easy to calculate and widely used, some questions are posed on the practice of using PCC as a universal dependence measure (see, e.g., Rachev et al. (2005); Zebende (2011); Wang et al. (2012); Zhou and Gao (2012)). There are at least two drawbacks for PCC as follows. (1) The theoretical assumption of PCC is that the joint distributions of time series obey Gaussian distribution. How-
ever, a large deal of evidence shows that the dependence between different financial asset returns is non-Gaussian (see, e.g., Bae (2003); Durante et al. (2014); Wang et al. (2014)). A good instance is that the dependence of financial asset returns during a recession is larger than during a boom (Zhou and Gao, 2012). Besides, many studies report that the dependence of financial markets comes to a remarkable peak during the largest market shocks and financial crises (see, e.g., Aste et al. (2010); Podobnik et al. (2010); Wang et al. (2013b)).

(2) PCC is defined to quantify the linear correlation for the whole range of sample. It ignores the fact that the real world data are characterized by a high level of heterogeneity (Wang et al., 2013b). Namely, it neglects the difference between extreme and commonplace observations. It is a fact that as the frequency of financial crisis doubled in recent years, the extreme returns of financial time series occurred increasingly. Therefore, PCC may lose effectiveness and be misleading if the investigated asset returns are heterogeneous and the extreme returns show different patterns of dependence from the remaining returns (Zhou and Gao, 2012). In a word, the previous dependence network methods cannot accurately detect the dependence structure of financial markets, especially during the financial crises (e.g., US sub-prime crisis, 2008 financial crisis, and European debt crisis).

To overcome the shortcomings of linear correlation functions (e.g., PCC), scholars resort to a powerfully and widely used tool—copulas proposed by Sklar (1959). Copulas are flexible and effective tools to measure dependence structure between two or more variables and model any type of multivariate distributions, which go beyond the linear correlation. Specifically, they allow for the tail dependence that represents the level of dependence among the tails of financial asset distributions (Zhou and Gao, 2012; Sun, 2013). In detail, tail dependence refers to the level of dependence in the lower and upper quadrant tails of a bivariate distribution, so it is a suitable measure of the dependence of extreme events. That is to say, tail dependence can be divided into two measures, namely the lower-tail dependence and upper-tail dependence, which are used respectively to investigate the joint extreme events in financial asset returns during the market downturns and market upturns (Hu, 2010). However, in the common practice of measuring dependence, people usually ignore the extreme returns that hide in the tails and the tail dependence among financial asset returns, which is dangerous for investment portfolios and other financial activities especially during the market downturns. So, more attentions should be paid to the tail dependence of financial markets. Unfortunately, to the best of our knowledge, very little of the ex-
isting research considers the tail dependence of the foreign exchange (FX) market from a network point of view.

Therefore, the aim of this paper is to propose a concept of tail dependence network by combing copulas and two network methods (MST and PMFG) for investigating the tail dependence structure of the FX market. We focus our study on the topology and clustering of tail dependence networks (i.e., lower-tail dependence network and upper-tail dependence network) of the FX market. We choose the FX market as the research object because it is the largest and most liquid financial market that directly or indirectly influences all other financial markets (Wang et al., 2014), and thus is a good representative for financial markets. In practical terms, we select the daily FX rates of the set of 42 major currencies in the FX market during the period 2005–2012 as the empirical data. The procedure of constructing the lower- and upper-tail dependence networks mainly consists of three steps.

Firstly, we compute the lower- and upper-tail dependence coefficients among 42 major currencies by a copula method. In this step, we first use the AR(1)-GARCH(1,1)-t model to characterize the marginal distributions of FX rate returns and then employ the symmetrized Joe-Clayton (SJC) copula proposed by Patton (2006) to calculate the lower- and upper-tail dependences. The motivations that lead us to choose the SJC copula to analyze the tail dependence structure of the FX market can be summarized as follows (Zhou and Gao, 2012; Sun, 2013). (1) SJC is a flexible copula approach to modeling the tail dependence because it can accommodate different types of dependence patterns that range from tail dependence to tail independence for both the lower and upper tails. (2) It allows for the asymmetric tail dependence and nests symmetry as a special case. More recently, the SJC copula has been widely applied in finance and economics (Basher et al., 2014; Hammoudeh et al., 2014; Yang and Hamori, 2014a; Hoesli and Reka, 2015; Huang and Wu, 2015; Jammazi et al., 2015; Ning et al., 2015). For example, Basher et al. (2014) study the tail dependences across Gulf Arab stock markets by the SJC copula and find that the lower-tail dependence is larger than the upper-tail dependence. Yang and Hamori (2014a) use the SJC copula to examine the tail dependence structure between gold prices and FX rates and observe that the upper-tail dependences of GBP/gold and JPY/gold are greater than the lower-tail dependences. Hoesli and Reka (2015) investigate the tail dependences between REIT and stock markets by the SJC copula and further research their contagion channels. Jammazi et al. (2015) analyze the tail dependences between stock and government bond returns for various
countries and find that the tail dependences for most countries are asymmetric. From the aforementioned literature, we conclude that the JSC copula is a powerful tool to investigate the tail dependence structure of financial entities, which motives our choice for the SJC copula.

Secondly, on basis of the lower- and upper-tail dependence coefficients of the set of 42 currencies in the FX market, we construct the lower- and upper-tail dependence matrices, which are denoted as $T_L$ and $T_U$, respectively. Then, we transform the two tail dependence matrices into lower- and upper-tail distance (or dissimilarity) matrices, which are labeled as $D_L$ and $D_U$, respectively.

Finally, we transform the two tail distance matrices into lower- and upper-tail dependence networks by the MST and PMFG approaches, which are respectively called as lower- and upper-tail dependence MSTs and PMFGs. The choice of the MST and PMFG methods is motivated by the following reasons. On the one hand, the MST is a simple and robust method, which connects $N$ nodes with $N-1$ stronger edges such that no loops are produced (Onnela et al., 2003; Wang et al., 2013a). On the other hand, the PMFG maintains the hierarchical structure of the MST but contains more information in comparison to the MST, which links $N$ nodes with $3(N-2)$ edges (Tumminello et al., 2005). At the same time, we also present the lower- and upper-tail dependence hierarchical trees (HTs) associated with the MSTs to analyze the hierarchical structure of the FX market. Since Mantegna (1999) proposes the MST and Tumminello et al. (2005) develop the PMFG to study the dependence structure of financial entities, these two approaches are frequently chosen for construing financial networks (see, e.g., Matesanz and Ortega (2014); Ji and Fan (2014); Matesanz et al. (2014); Mai et al. (2014); Wang et al. (2014); Birch et al. (2015); Wang and Xie (2015); Yan et al. (2015)). For instance, Matesanz and Ortega (2014) investigate the co-movements of 28 major currencies in the period of 1992–2002 in the FX market via combining the phase synchronous coefficient and MST. Ji and Fan (2014) study the topological network of 24 major crude oil markets from 2000 to 2011 by using the MST and HT tools and find that the global crude oil markets are clustered by geographical and organizational features. Matesanz et al. (2014) construct the HT and MST network for a sample of 32 commodity prices during the period 1993–2010 to examine the co-movement of commodity markets. Mai et al. (2014) analyze the constituent stocks of China Securities Index 300 (CSI 300) from 28 September 2009 to 30 March 2012 and build the PMFG network for the CSI 300 market, in which the CSI
300 market is found as a scale-free network. Wang and Xie (2015) construct the HT, MST and PMFG networks of 20 country indices in international real estate securities markets from 2006 to 2012 to study the corresponding hierarchical structure, clustering structure and community structure. Yan et al. (2015) examine the topological stability of stock market networks by identifying the MST and PMFG networks. Although many new developments and outcomes are obtained in the existing literature about the MST and PMFG networks, somewhat surprisingly, the concept of tail dependence networks has never been proposed, which motives us for this research.

Overall, compared to previous works on the dependence networks, the proposed lower- and upper-tail dependence networks have several outstanding advantages. (1) The proposed networks take into account the tail dependence of financial agents and capture the heterogeneous and nonlinear features and tail risks of asset returns. (2) Our primary empirical results show that despite the lower- and upper-tail dependence networks look similar, different topological features (e.g., clusters and communities) exist in the two tail dependence networks. However, the existing literature on the dependence networks did not uncover these differences. (3) Our empirical outcomes suggest that market participants in the FX markets should adjust their investing or hedging strategies at different market situations under the guide of the tail dependence networks, which is a new contribution in the literature of financial networks.

In addition, our proposed tail dependence networks at least have three potential applications in the field of expert and intelligent systems. The first potential application is that the proposed tail dependence networks tool can be integrated into the FX trading systems. The FX trading systems with the embedded tail dependence networks is useful for market participants to construct FX portfolio and forecast the FX rates. Designing and constructing a financial decision-making platform based on the tail dependence networks is the second possible application. This proposed platform maps the complex financial systems and permits investors or hedgers to make network- and data-based decisions. The final conceivable application is to design and build an early warning system (ESW) on the FX markets from a macro perspective. Based on the ESW, one may predict the extreme events and tail risks by investigating the changes of topological properties of tail dependence networks.

The rest of this paper is organized as follows. In Section 2, we present lower- and upper-tail dependence networks by combing copulas and two net-
work methods (MST and PMFG). In Section 3, we show the empirical data and main empirical results. Finally, we draw conclusions and present some future works in Section 4.

2. Methodology

In this section, we first describe the calculation of tail dependence coefficients, which consists of the copula functions, the model for marginal distributions, the SJC copula model, and the estimation of parameters. Then, we present the basics of construction of lower- and upper-tail dependence networks based on the MST and PMFG methods. For the first applications of MST and PMFG to financial markets and a more detailed description, see Mantegna (1999) and Tumminello et al. (2005).

2.1. Copula functions

Copula functions are flexible and useful tools to investigate the dependence structure of a multivariate distribution. A $d$-dimensional copula is a multivariate cumulative distribution function (CDF) with standard uniform marginal distributions (Alouïet al., 2013). Assume $X=(X_1, X_2, \ldots, X_d)$ is a random vector, with a joint CDF $F$ and marginal CDFs $F_1, F_2, \ldots, F_d$. According to Sklar’s (1959) theorem, there exists a copula function $C$: $[0,1]^d \rightarrow [0,1]$ such that:

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)).$$  

(1)

Copula functions supply a convenient way to create a multivariate joint distribution by first modeling the univariate marginal distributions and then examining the dependence structure of a multivariate random variable (Yang and Hamori, 2014b). To be simple enough, in this paper, only the bivariate case are considered. Tail dependence, a useful dependence measure described by copulas, quantifies the probability that both variables are in their lower or upper joint tails (Ning, 2010). In general, two measurements of tail dependence are used to examine the tail dependence, i.e., lower- and upper-tail dependence coefficients, which are very helpful to detect the trend of financial markets to recession or brisk.

Let $X$ and $Y$ be two random variables with marginal CDFs $F_X$ and $F_Y$, respectively, the lower- and upper-tail dependence coefficients are defined as

$$\tau^L = \lim_{\varepsilon \rightarrow 0} \Pr[Y \leq F_Y^{-1}(\varepsilon) | X \leq F_X^{-1}(\varepsilon)] = \lim_{\varepsilon \rightarrow 0} \frac{C(\varepsilon, \varepsilon)}{\varepsilon},$$  

(2)

$$\tau^U = \lim_{\varepsilon \rightarrow 0} \Pr[Y \geq F_Y^{-1}(\varepsilon) | X \geq F_X^{-1}(\varepsilon)] = \lim_{\varepsilon \rightarrow 0} \frac{1 - C(\varepsilon, \varepsilon)}{\varepsilon}.$$  

(3)

7
\[
\tau^U = \lim_{\varepsilon \to 1} \Pr[Y > F_Y^{-1}(\varepsilon) | X > F_X^{-1}(\varepsilon)] = \lim_{\varepsilon \to 1} \frac{1 - 2\varepsilon + C(\varepsilon, \varepsilon)}{1 - \varepsilon}, \tag{3}
\]
where \(\varepsilon\) is the probability, \(\tau^L\) and \(\tau^U\in[0,1]\). If \(\tau^L\) or \(\tau^U\) is positive, the two random variables \(X\) and \(Y\) are lower-tail dependence or upper-tail dependence; otherwise, there are lower-tail independence or upper-tail independence. Further detailed descriptions on copula functions can be seen in Joe (1997) and Nelsen (2006).

2.2. Model for marginal distributions

Some stylized facts of financial asset returns, such as fat-tails, autocorrelations, and volatility clustering, are well documented in previous literature. The GARCH model is the most popular method to model financial time series and capture their stylized features like volatility clustering. In addition, Bollerslev (1987) and Ning (2010) find that the univariate distribution of the daily FX rate returns can be well fitted by the Student’s \(t\)-distribution. Therefore, following Patton (2006); Dias and Embrechts (2010); Wang et al. (2014), we employ the AR(1)-GARCH(1,1)-\(t\) model to characterize the marginal distributions of the FX rate returns. The model is expressed as follows:

\[
\begin{align*}
    r_{i,t} &= \mu_i + \varphi_{1,i} r_{i,t-1} + \varepsilon_{i,t}, \tag{4} \\
    z_{i,t} | I_{t-1} &= \sqrt{\frac{v_i}{\sigma^2_{i,t}(v_i - 2)}} \cdot \varepsilon_{i,t}, z_{i,t} \sim iid \ t(v_i), \tag{5} \\
    \sigma^2_{i,t} &= \omega_i + \alpha_i \varepsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1}, \tag{6}
\end{align*}
\]
where \(r_{i,t}\) is the FX rate returns of currency \(i\) at time \(t\), \(\mu_i\) and \(\varepsilon_{i,t}\) are the mean and error term in the conditional mean equation respectively, \(\varphi_{1,i}\) is the autoregressive (AR) coefficient with the order of one, \(z_{i,t}\) follows a Student’s \(t\)-distribution, \(\sigma^2_{i,t}\) is the conditional variance of the error term \(\varepsilon_{i,t}\) with the following conditions: \(\omega_i>0\), \(\alpha_i>0\), \(\beta_i>0\), and \(\alpha_i + \beta_i<1\), \(v_i\) is the degree of freedom of the Student’s \(t\)-distribution, and \(I_{t-1}\) is the information set at time \(t-1\). In this study, we set the order of AR and the lag orders of the GARCH model all to be one, which is in accordance with the Brooks’s (2008) statement that the volatility clustering in financial asset returns can be sufficiently described by a GARCH-family model with the lag order of one.
2.3. Symmetrized Joe-Clayton (SJC) copula model

After obtaining the marginal distribution for each FX rate returns, we use a copula function to compute the tail dependence. According to Joe (1997) and Nelsen (2006), many copulas can be chosen to model the dependence of financial asset returns. Two frequently used models are the Gaussian and Student’s $t$ copulas. However, the Gaussian copula shows tail independence (i.e., $\tau^L$ and $\tau^U$ are both equal to zero), while the Student’s $t$ copula exhibits symmetric tail dependence (i.e., lower- and upper-tail dependences are the same). Another two copulas are the Gumbel and Clayton copulas, which can be used to analyze the tail dependence. These two copulas are also not chosen in this paper because the Gumbel copula can be only used to measure the upper-tail dependence while the Clayton copula can be only used to calculate the lower-tail dependence. A good choice is the Symmetrized Joe-Clayton (SJC) copula because it has the ability to model asymmetric tail dependence (i.e., lower- and upper-tail dependences).

Given two standard uniform variables $u$ and $v$ (which stand for two different currencies), the JC copula (Joe, 1997) is defined as

$$C_{JC}(u, v|\tau^L, \tau^U) = 1 - \left(1 - [(1 - (1-u)^\kappa)^{-\gamma} + (1 - (1-v)^\kappa)^{-\gamma} - 1]^{-1/\gamma}\right)^{1/\kappa}, \quad (7)$$

where $\kappa = 1/ \log_2(2 - \tau^U)$, $\gamma = -1/ \log_2(\tau^L)$. $\tau^L$ and $\tau^U \in (0, 1)$ are the lower- and upper-tail dependence coefficients respectively. The SJC copula (Patton, 2006) is expressed as

$$C_{SJC}(u, v|\tau^L, \tau^U) = 0.5 \left[ C_{JC}(u, v|\tau^L, \tau^U) + C_{JC}(1 - u, 1 - v|\tau^L, \tau^U) + u + v - 1 \right]. \quad (8)$$

As presented by Sun (2013), the density of SJC copula can be derived as

$$c_{SJC}(u, v|\tau^L, \tau^U) = \frac{\partial^2 C_{SJC}(u,v|\tau^L,\tau^U)}{\partial u \partial v} = 0.5 \left[ \frac{\partial^2 C_{JC}(u,v|\tau^L,\tau^U)}{\partial u \partial v} + \frac{\partial^2 C_{JC}(1-u,1-v|\tau^L,\tau^U)}{\partial (1-u) \partial (1-v)} \right], \quad (9)$$

which allows the lower- and upper tail dependences to be estimated, following the inference-function-for margins (IFM) approach proposed by Joe and Xu (1996) which is presented in the next subsection.
2.4. Estimation of copula parameters

The copula parameters can be estimated by two steps, the first for the marginal distributions and the second for the copula functions. Two widely used methods are the IFM and the exact maximum likelihood (EML). Following Lai et al. (2009); Wang, Chen and Huang (2011); Wang et al. (2014), we employ the IFM to estimate the copula parameters rather than the EML, because the later needs more computation than the former. The two-step estimations of the IFM are presented as follows:

**Step 1.** The parameters in the marginal distributions are estimated by the maximum likelihood as

\[
\hat{\theta}_i = \arg \max_{\theta_i} \sum_{t=1}^{T} \ln f_i(z_{i,t}; \theta_i),
\]

where \( f_i(\cdot) \) refers to the marginal density of the FX rate returns of currency \( i \), and \( \theta_i \) is the parameter set of returns of currency \( i \).

**Step 2.** Given \( \theta_u \) and \( \theta_v \) for two different currencies, the copula parameters can be estimated as

\[
\hat{\theta}_c = \arg \max_{\theta_c} \sum_{t=1}^{T} \ln c_{\text{SJC}}(F_u(z_{u,t}; \theta_u), F_v(z_{v,t}; \theta_v); \theta_c).
\]

2.5. Lower- and upper-tail dependence networks

Once we obtain the lower- and upper-tail dependence coefficients between any two currencies in the FX market, we then build \( N \times N \) lower- and upper-tail dependence matrices \( T^L \) and \( T^U \) with elements \( \tau^L_{ij} \) and \( \tau^U_{ij} \) for currencies \( i \) and \( j \), respectively, where \( N \) is the number of currencies (\( N=42 \), in our case), and \( 1 \leq i, j \leq 42 \). According to the original application of the MST introduced by Mantegna (1999), the dependence coefficient should be transformed into a distance (or dissimilarity) measure. Following Durante et al. (2014), we transform the lower- and upper-tail dependence matrices \( T^L \) and \( T^U \) into two tail distance matrices \( D^L \) and \( D^U \) by the distance measures \( d^L_{ij} = \sqrt{1 - \tau^L_{ij}} \) and \( d^U_{ij} = \sqrt{1 - \tau^U_{ij}} \), respectively. Based on the two distance matrices \( D^L \) and \( D^U \), we can obtain the lower- and upper-tail dependence networks of the FX market filtered by the MST and PMFG methods, which are respectively called as lower- and upper-tail dependence MSTs and PMFGs. That is to say, four tail dependence networks are gained in our study.
The tail dependence MST network filters the tail distance matrix and links all $N$ nodes (currencies) with $N - 1$ stronger edges such that the sum of all currency distances is the minimum. In other words, the construction idea behind MST is to reduce the number of $N(N - 1)/2$ elements originally contained in the tail distance matrix to only the $N - 1$ most important links. In this study, we use the Kruskal’s (1956) algorithm to construct the MST network. Here we show a brief introduction of the procedure of the MST as follows (Aste et al., 2010):

Step 1. Create an order list of edges $e_{ij}$ (which contains $N(N - 1)/2$ edges, i.e., $N(N - 1)/2$ elements in the tail distance matrix), ranking them by non-decreasing tail distance $d_{L_{ij}}$ (or $d_{U_{ij}}$).

Step 2. Take the first element (i.e., the edge with the smallest tail distance) in the ranked list and add the edge into a graph $G$.

Step 3. Take the next element and add the edge if and only if the graph acquired after the edge insertion is still a forest or a tree; otherwise discard the element.

Step 4. Repeat Step 3 until all elements in the ordered list are used up.

The tail dependence PMFG network keeps the same powerful filtering characters of the tail MST network but contains extra links, loops, and cliques, and thus more complex than the tail MST network. That is, the PMFG is a network with loops whereas the MST is a tree without loops. Notable differences between the MST and PMFG networks are summed up as follows. (1) The topological constraint (i.e., the condition at Step 3 in the procedure) for the new inserted edge is different. The topological constraint of the MST network is that the resulting graph is still a forest or a tree (i.e., no loops are produced in the resulting graph), while the topological constraint of the PMFG network is that the resulting graph can still be embedded on a plane or a sphere (Tumminello et al., 2010), i.e., topological surfaces with fixed genus $g=0$.1 (2) The number of edges is different. The MST connects $N$ nodes with $N - 1$ edges, while the PMFG links $N$ nodes with $3(N - 2)$ edges. Namely, the PMFG has 3 times the number of edges as the MST.

---

1 Genus is a topologically invariant property of a surface defined as the largest number of nonintersecting simple closed curves that can be drawn on the surface without separating it. Roughly speaking, a genus of a surface is the number of holes in a surface. A plane or sphere has genus 0, and a torus has genus 1, etc. For a more detailed explication, see Tumminello et al. (2005, 2010); Aste et al. (2010).
Another difference is that the PMFG contains 3-cliques and 4-cliques. In graph theory, an \( n \)-clique is a complete sub-graph on \( n \) vertices such that every two vertices in the sub-graph are linked by an edge. A clique in the financial network refers to a set of the financial assets whose price fluctuations show a similar behavior, i.e., a price change of one asset in the clique can possibly influence the price behavior of all other assets in this clique (Boginski et al., 2005; Wang and Xie, 2015).

3. Empirical data and results

3.1. Empirical data

We analyze the FX rates of the set of 42 major currencies in the FX market from January 4, 2005 to December 31, 2012. The empirical data consist of the daily FX rates obtained from the website of the Pacific Exchange Rate Service (http://fx.sauder.ubc.ca/data.html). The choice of the numeraire (or the base currency) is a difficult and common problem in the FX market research because currencies in the FX market are priced against each other and thus there exists no independent numeraire (Keskin et al., 2011). People once considered the metals (e.g., the gold and silver) but dismissed them due to their high volatility. According to the recent works, there are two main ways to choose a suitable numeraire: one is to choose a minor currency that has little impact on the world economy, e.g., Keskin et al. (2011) take the Turkish New Lira (TRY) as the numeraire; and the other is to consider a currency basket that can reflect the whole changes of the world economy, such as the special drawing right (SDR). The SDR designed by the International Monetary Fund (IMF) is a reserve asset and a unit of account, which is also known as the “paper gold”. Therefore, like in Frankel and Xie (2010); Jang et al. (2011); Wang and Xie (2013), we choose the SDR as the numeraire. The 42 currencies come from seven different continents or regions, and their corresponding symbols are organized in Table 1. As usual, the FX rate returns of currency \( i \) are calculated on a continuous compounding as \( r_{i,t}=100\times(\ln P_{i,t}−\ln P_{i,t−1}) \), where \( P_{i,t} \) and \( P_{i,t−1} \) are the daily FX rates of currency \( i \) on days \( t \) and \( t−1 \), respectively. After supplementing the missing data of several currencies in a few transaction days, we finally obtain 2003 log-returns for each currency.

We make a summary statistics of returns of 42 currencies during the an-
Table 1: 42 currencies and respective symbols.

<table>
<thead>
<tr>
<th>Continent</th>
<th>Currency</th>
<th>Symbol</th>
<th>Continent</th>
<th>Currency</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>Egyptian Pound</td>
<td>EGP</td>
<td>Europe</td>
<td>Romanian New Leo</td>
<td>RON</td>
</tr>
<tr>
<td></td>
<td>South Africa Rand</td>
<td>ZAR</td>
<td></td>
<td>Russian Rubles</td>
<td>RUB</td>
</tr>
<tr>
<td>Asia</td>
<td>Chinese Renminbi</td>
<td>CNY</td>
<td>Latin</td>
<td>Argentine Peso</td>
<td>ARS</td>
</tr>
<tr>
<td></td>
<td>Indian Rupee</td>
<td>INR</td>
<td>America</td>
<td>Brazilian Real</td>
<td>BRL</td>
</tr>
<tr>
<td></td>
<td>Indonesian Rupiah</td>
<td>IDR</td>
<td></td>
<td>Colombian Peso</td>
<td>COP</td>
</tr>
<tr>
<td></td>
<td>Japanese Yen</td>
<td>JPY</td>
<td></td>
<td>Peruvian New Sole</td>
<td>PEN</td>
</tr>
<tr>
<td></td>
<td>Malaysian Ringgit</td>
<td>MYR</td>
<td>South Korea</td>
<td>South Korean Won</td>
<td>KRW</td>
</tr>
<tr>
<td></td>
<td>Pakistani Rupee</td>
<td>PKR</td>
<td>East</td>
<td>Bahrain Dinar</td>
<td>BHD</td>
</tr>
<tr>
<td></td>
<td>Phillipines Peso</td>
<td>PHP</td>
<td>Middle</td>
<td>Israeli New Shekel</td>
<td>ILS</td>
</tr>
<tr>
<td></td>
<td>Singapore Dollar</td>
<td>SGD</td>
<td></td>
<td>Russian Rubles</td>
<td>RUB</td>
</tr>
<tr>
<td></td>
<td>South Korean Won</td>
<td>KRW</td>
<td>North</td>
<td>United Arab Emirates Dirham</td>
<td>AED</td>
</tr>
<tr>
<td></td>
<td>Taiwan Dollar</td>
<td>TWD</td>
<td></td>
<td>Danish Krone</td>
<td>DKK</td>
</tr>
<tr>
<td></td>
<td>Thai Baht</td>
<td>THB</td>
<td>American</td>
<td>Canadian Dollar</td>
<td>CAD</td>
</tr>
<tr>
<td></td>
<td>Vietnamese Dong</td>
<td>VND</td>
<td>Asia</td>
<td>US Dollar</td>
<td>USD</td>
</tr>
<tr>
<td></td>
<td>British Pound</td>
<td>GBP</td>
<td></td>
<td>Australian Dollar</td>
<td>AUD</td>
</tr>
<tr>
<td></td>
<td>Czech Koruna</td>
<td>CZK</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>European Euros</td>
<td>EUR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hungarian Forint</td>
<td>HUF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Icelandic Krona</td>
<td>ISK</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Norwegian Krone</td>
<td>NOK</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Polish Zloty</td>
<td>PLN</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We find that the means of returns of 42 currencies are very small in relation to their standard deviations, presenting relatively high risks. Besides, we also find that skewness, leptokurtosis, and significant Jarque-Bera statistics (at 1% level) exist in the FX rate returns, which suggest that the FX rate returns' unconditional distributions are asymmetric, fat-tailed, and non-Gaussian. From the results of the ARCH-LM test for conditional heteroscedasticity on the returns of 42 currencies, we observe that there is significant volatility clustering in each return series, implying that the GARCH model is suitable for modeling the FX rate returns.

3.2. Statistics of tail dependence coefficients

In this subsection, we analyze the statistics of the non-diagonal and upper (or lower) triangular elements (i.e., tail dependence coefficients between any two currencies) in the tail dependence matrix. For both the lower- and upper-tail dependence coefficient series, there are 861 (i.e., \(N(N-1)/2\)) observations. By comparing the two series of tail dependence coefficients, we find that 278 pairs of observations are the same. Namely, about 32% of

---

2 Due to space limitations, the detailed summary statistics are not presented in this paper, which can be available from the authors upon request.
the lower-tail dependence coefficients are equal to the upper-tail dependence coefficients, while the rest (68%) of observations determine the difference of the dependence structure in the FX market. Therefore, to find the similarities and differences between the lower-tail and upper-tail dependence structures is an interesting and important work.

In Fig. 1, we present the probability density functions (PDFs) $P(\tau^L)$ and $P(\tau^U)$ of the elements $\{\tau^L_{ij}; i<j\}$ and $\{\tau^U_{ij}; i<j\}$ of the lower- and upper-tail dependence matrices $T^L$ and $T^U$. One can observe that the two distributions are different except for the overlapped right-tails. This observation suggests that (1) the strong tail dependence structure of the FX market is stable; and (2) the difference between the two tail dependence structures is mainly reflected in the weak tail dependence structure that is unstable. That is to say, several strong dependences among some currencies do not change along with the market conditions ranging from the depression to the booming or vice versa, while majorities of weak dependences in the FX market are varying as the change of the market. Besides, the number (or the probability) of weak upper-tail dependences is greater than that of weak lower-tail dependences.
Table 2: Descriptive statistics of lower- and upper-tail dependence coefficients $\{\tau_{ij}^L; i<j\}$ and $\{\tau_{ij}^U; i<j\}$.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower-tail</td>
<td>0.1624</td>
<td>8.2718e-025</td>
<td>0.8478</td>
<td>0.1841</td>
</tr>
<tr>
<td>Upper-tail</td>
<td>0.1416</td>
<td>5.1295e-026</td>
<td>0.8411</td>
<td>0.1735</td>
</tr>
</tbody>
</table>

In Table 2, we show the basic descriptive statistics of lower- and upper-tail dependence coefficients $\{\tau_{ij}^L; i<j\}$ and $\{\tau_{ij}^U; i<j\}$. From Table 2, we can find that the values of mean and standard deviation of upper-tail dependence coefficients are both smaller than that of the lower-tail dependence coefficients, which implies that the whole dependences among currencies in the FX market during market downturns is stronger and more volatile than that during market upturns. Both the minimums of lower- and upper-tail dependence coefficients are close to zero, meaning that tail independence exists in the FX market. The coefficients between USD and CNY, and between JOD and AED are maximums of the lower- and upper-tail dependence coefficients respectively, which indicates that the above two pairs of currencies possess the strongest dependence in the FX market under different market situations. A possible interpretation of USD-CNY having the largest lower-tail dependence coefficient is that CNY is re-pegged to USD during 2008 financial crisis (i.e., market downturns).

3.3. Topological properties of tail dependence networks

Networks’ topological properties determine the function of networks and affect their dynamic behavior. In the analysis of networks, two common topological features are the average length path and average clustering coefficient. We show a brief introduction of the two measures as follows.

**Definition 1** Consider an un-weighted graph $G(V, E)$, where $V$ is the set of vertexes and $E$ is the set of edges. If vertexes $v_i$ and $v_j$ are directly connected, the path length $l_{ij}$ between vertexes $v_i$ and $v_j$ is equal to 1; otherwise, it is defined as the length (or steps) of the shortest path between two vertexes $v_i$ and $v_j$ (Wang et al., 2014). The average path length (APL) is defined as the average length along the shortest paths for all possible pairs of network vertexes, i.e.,

$$L_{APL} = \frac{2}{N(N-1)} \sum_{i<j} l_{ij}, \quad (12)$$
where $N$ is the number of vertexes.

**Definition 2** In an un-weighted graph $G(V, E)$, let $k_i$ be the degree of vertex $v_i$ (which means that vertex $v_i$ has $k_i$ neighbors in the set of vertexes $\Gamma_i$), $\langle k \rangle$ be the average degree of the graph $G$, and $e_{ij}$ be an edge connecting vertexes $v_i$ and $v_j$. If there are $E_i$ edges among $k_i$ vertexes, i.e., $E_i = \# \{e_{jk} | v_j, v_k \in \Gamma_i, e_{jk} \in E \}$, the clustering coefficient $C_i$ of vertex $v_i$ is defined as

$$ C_i = \frac{2E_i}{k_i(k_i - 1)}, $$

(13)

where $k_i(k_i - 1)/2$ is the number of edges that could exist among the vertices within the set $\Gamma_i$. The average clustering coefficient (ACC) proposed by Watts and Strogatz (1998) is defined as the average of clustering coefficients of all vertexes $N$, i.e.,

$$ L_{ACC} = \frac{1}{N} \sum_{i=1}^{N} C_i. $$

(14)

As for the economic interpretations of the aforementioned topological features, we show a short presentation as follows. In the financial network, the path length between two assets (e.g., currencies) means that one asset’s price fluctuations need the number of other assets acted as the intermediaries, which can be transferred to another asset. Therefore, the average length refers to the average number of intermediates between any two assets, and characters the size of the network. Each asset and its directly connected assets make up a neighborhood group, while an asset’s clustering coefficient in the financial network represents the level of cluster of the neighborhood group. So, the average clustering coefficient reflects the whole compactness of the network.

In Table 3, we present the computed results of the average degree $\langle k \rangle$, the average path length $L_{APL}$, and the average clustering coefficient $L_{ACC}$ of lower- and upper-tail dependence MSTs and PMFGs. From the results of the average degree in Table 3, it can be found that on average, each vertex has only one or two neighbors in the tail dependence MST network, while in the tail dependence PMFG network it has five or six neighbors. Interestingly, the average path length of the upper-tail dependence networks is larger than that of the lower-tail dependence networks, indicating that the size of the former is greater than that of the later. The average clustering coefficient of the tail dependence MST network is equal to zero because the MST network has no loops. About the compactness of the network measured by the average
clustering coefficient, the lower-tail dependence PMFG network is a little tighter than the upper-tail dependence PMFG network. As reported by Watts and Strogatz (1998), small-world properties are common found in the real-world networks, such as the collaboration network of actors, the electrical power grid network, and the neuron network of the nematode worm. The so-called small-world network is a kind of graph in which majorities of vertexes are not neighbors of one another, but majorities of vertexes can be reachable from all the rest vertexes by a small handful of hops or steps. To examine whether the tail dependence PMFGs are small-world networks, we create a random graph with the same number of vertexes ($N$) and the average degree ($\langle k \rangle$), and present its results of average path length and average clustering coefficient in Table 3. Watts and Strogatz (1998) state that in a small-world network, its average path length is close to that of a random graph which has the same network parameters (i.e., vertexes and the average degree) as the investigated network, while the average clustering coefficient of a small-world network is far larger than a random graph. From this, we can come to a conclusion that the two tail dependence PMFGs are small-world networks.

Table 3: The average degree $\langle k \rangle$, the average path length $L_{\text{APL}}$, and the average clustering coefficient $L_{\text{ACC}}$ of lower- and upper-tail dependence MSTs and PMFGs, and a random graph.

<table>
<thead>
<tr>
<th></th>
<th>$\langle k \rangle$</th>
<th>$L_{\text{APL}}$</th>
<th>$L_{\text{ACC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower-tail MST</td>
<td>1.9524</td>
<td>5.5633</td>
<td>0</td>
</tr>
<tr>
<td>Upper-tail MST</td>
<td>1.9524</td>
<td>6.0128</td>
<td>0</td>
</tr>
<tr>
<td>Lower-tail PMFG</td>
<td>5.7143</td>
<td>3.1417</td>
<td>0.7062</td>
</tr>
<tr>
<td>Upper-tail PMFG</td>
<td>5.7143</td>
<td>3.5436</td>
<td>0.6985</td>
</tr>
<tr>
<td>Random graph</td>
<td>5.7143</td>
<td>2.2451</td>
<td>0.0814</td>
</tr>
</tbody>
</table>

In financial markets, the price fluctuations of one asset usually have an influence on other asset’s price behavior, while the strength of influence may be different for different assets. To quantify how strongly a given vertex (asset) influences others in the financial network, we introduce a physical quantity proposed by Kim et al. (2002), called the influence-strength (IS).

**Definition 3** In a weighted graph $G(V, W)$, let $\Gamma_i$ be a set of vertexes connected to the vertex $v_i$. The influence-strength is defined as the sum of the weights of all edges incident upon a given vertex $v_i$ (Kim et al., 2002), i.e.,

$$s_i = \sum_{v_j \in \Gamma_i} \tau_{ij} = \sum_{v_j \in \Gamma_i} 1 - d_{ij}^2,$$  \hspace{1cm} (15)
where $\tau_{ij}$ stands for the lower- or upper-tail dependence coefficient between currencies $i$ and $j$, while $d_{ij}$ is the corresponding distance.

The quantity of influence-strength not only considers the number of assets that directly link to an asset $i$, but also takes into account their strength of the dependence. The larger influence-strength that an asset (vertex) has, the greater influence of it on other assets (vertexes) is.

As reported by Kim et al. (2002), the distribution of influence-strength usually follows a power-law, i.e.,

$$P(s) \sim s^{-\eta},$$

(16)

where $s$ is the influence-strength, and $\eta$ is the power-law exponent.

In Fig. 2, we draw the log-log plots of the distribution of influence-strength versus the influence-strength for the four tail dependence networks. To detect the power-laws of the influence-strength, we adopt a powerful tool proposed by Clauset et al. (2009), which is a combination of maximum-likelihood fitting approaches and goodness-of-fit tests on basis of the Kolmogorov-Smirnov statistic and likelihood rates. Based on the method developed by Clauset et al. (2009), we compute power-law exponents and the corresponding $p$-values, and show them in Fig. 2. Clauset et al. (2009) suggest that if the estimated $p$-value is larger than 0.1, we can accept the power-law hypothesis for the empirical data. From Fig. 2, we can see that all the four $p$-values are greater than 0.1, which indicates that the influence-strength distributions of the four tail dependence networks for the FX market obey the power-law. The power-law distribution of influence-strength means that a few assets (vertexes) always have the larger influence-strength that significantly affect other assets (vertexes), whereas most assets’ (vertexes’) influence-strength is too weak to influence other assets (vertexes). Besides, in Fig. 2, one can find that the power-law exponents of lower-tail dependence MST and PMFG networks are respectively smaller than that of upper-tail dependence MST and PMFG networks. This finding implies that the influence-strength distribution of upper-tail dependence networks is more uniform than that of lower-tail dependence networks; on the other hand, it suggests lower-tail dependence networks have more assets (vertexes) with a large influence-strength than upper-tail dependence networks.

In Table 4, we list the top and the bottom five currencies ranked by
the influence-strength in the four tail dependence networks.\textsuperscript{3} From Table 4, one can find that USD has the greatest influence-strength in the four networks, suggesting that USD is the well-deserved dominant currency in the world. It is a surprise that JOD has the second ranked influence-strength in the two tail dependence MST networks. It would make sense that the monetary policy of Jordan is a fixed exchange rate pegged to USD, so the influence of JOD may be ascribed to the influence of USD. In the two tail dependence PMFG networks, CNY has the second ranked influence-strength, which reveals that CNY has a powerful influence in the FX market. This result may be attributed to not only the strong correlation between CNY and

\textsuperscript{3} For the full list of the ranked currencies by the influence-strength, it can be obtained from the authors upon request.
Table 4: The top and the bottom five currencies ranked by the influence-strength (IS) in the lower- and upper-tail dependence MST and PMFG networks.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Lower-tail MST</th>
<th>Upper-tail MST</th>
<th>Lower-tail PMFG</th>
<th>Upper-tail PMFG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Currency IS</td>
<td>Currency IS</td>
<td>Currency IS</td>
<td>Currency IS</td>
</tr>
<tr>
<td>1</td>
<td>USD 5.4840 USD</td>
<td>4.967 USD USD</td>
<td>7.969 USD USD</td>
<td>6.159 USD USD</td>
</tr>
<tr>
<td>2</td>
<td>JOD 3.0480 JOD</td>
<td>2.523 CNY 2.523</td>
<td>5.136 CNY 5.136</td>
<td>5.394 CNY 5.394</td>
</tr>
<tr>
<td>4</td>
<td>MXN 2.4262 EUR</td>
<td>2.397 HUF 2.397</td>
<td>4.768 TWD 4.768</td>
<td>4.175 TWD 4.175</td>
</tr>
<tr>
<td>38</td>
<td>CAD 0.3666 COP</td>
<td>0.279 CAD 0.279</td>
<td>0.972 VND 0.972</td>
<td>0.897 VND 0.897</td>
</tr>
<tr>
<td>39</td>
<td>GBP 0.3147 RUB</td>
<td>0.248 GBP 0.248</td>
<td>0.771 RUB 0.771</td>
<td>0.652 RUB 0.652</td>
</tr>
<tr>
<td>40</td>
<td>RUB 0.2880 GBP</td>
<td>0.204 RUB 0.204</td>
<td>0.768 ILS 0.768</td>
<td>0.498 ILS 0.498</td>
</tr>
<tr>
<td>41</td>
<td>ILS 0.2835 ILS</td>
<td>0.191 ILS 0.191</td>
<td>0.753 GBP 0.753</td>
<td>0.475 GBP 0.475</td>
</tr>
<tr>
<td>42</td>
<td>JPY 0.0391 JPY</td>
<td>0.063 JPY 0.063</td>
<td>0.048 JPY 0.048</td>
<td>0.063 JPY 0.063</td>
</tr>
</tbody>
</table>

Notes: Due to space limitations, we omit the “dependence” in the terms of lower- and upper-tail dependence MST and PMFG.

USD but also the increasing power and economic status of China. For the rest of the top five currencies, there are different in different tail dependence networks. As presented in Table 4, the bottom five ranked currencies show two interesting phenomena as follows. (1) Two important currencies GBP and JPY appear as the low influence vertex (asset) in the four tail dependence networks. The following reasons may be accounted for the positions of the two currencies. As for GBP, although Britain is a member of the European Union, its monetary policy and FX regime are independent; while for JPY, the FX regime of Japan, which is a special case in the developed countries, is to keep JPY as a weak currency. (2) Another two currencies RUB and ILS are members of the bottom five currencies ranked by the influence-strength in the four tail dependence networks. The positions of the two currencies in the network are just like their counties’ (i.e., Russia and Israel) special political role and geographic location in the world.

3.4. Results of lower- and upper-tail dependence MSTs and HTs

In Figs. 3 and 4, we show the lower- and upper-tail dependence MSTs of the set of 42 currencies in the FX market obtained from the corresponding tail dependence matrices estimated by using 2003 daily returns during the analyzed period (2005–2012), respectively. In each figure, we mark currencies from the same continent or region as the same color and shape.

From Figs. 3 and 4, one can observe that majorities of currencies are
clustered together according to their geographical distributions. For instance, the European cluster, Asian cluster, Latin American, Middle Eastern cluster with EUR (HUF), MYR, MXN, and JOD at their centers are formed in the tail dependence MSTs. Besides, it can be found that an important cluster—the international cluster with the USD at its hub—is directly or indirectly linked with currencies from Asia, Middle East, Latin America, and Africa.
Another interesting cluster is the Commonwealth cluster that consists of AUD and NZD from Oceania, GBP from Europe, ZAR from Africa, and (or) CAD from North America. The name of the Commonwealth cluster is reported by Wang et al. (2013a, 2014) who consider that countries of the five or four currencies are from the Commonwealth of Nations.

As a whole, the two tail dependence MSTs are similar. But some differences can be found between the two MSTs such as the network’s structure and currencies’ position. For example, the European cluster in the lower-tail...
dependence MST is tighter than in the upper-tail dependence MST. Positions of three currencies (JPY, ILS, and CAD), which are ranked in the last five currencies by the influence-strength, are changed. In detail, JPY, ILS, and CAD connected in the international cluster, the Latin American cluster, and the Commonwealth cluster change to link with the European cluster, the Asian cluster, and the Latin American cluster, respectively. Therefore, participants in the FX market should take care that the currency clusters change in different tail dependence MSTs, and the special currencies like JPY and ILS with a peripheral position in the network have a low influence-strength.

As usual, in Fig. 5, we draw the lower- and upper-tail dependence hierarchical trees (HTs) associated with the MSTs. Currencies from the same continent or region are marked the same color. In a HT, if a horizontal line is plotted between two (or more) vertical lines, the two (or more) currencies (i.e., vertical lines) are connected or clustered together. From Fig. 5(a), we can find that the distance between USD and CNY is the smallest of the empirical data, suggesting a strong lower-tail dependence between the two currencies that forms a cluster. We call this cluster as the international cluster because it also links other currencies like EGP, PEN, PKR, ARS, the Middle Eastern cluster that is composed of BHD, JOD, AED, SAR, and KWD, etc. Except for CNY, PKR, and JPY, currencies from Asia are linked and lined together to form the Asian cluster. Similar to the Asian cluster, except for TRY, GBP, and RUB, currencies from Europe are connected to form the European cluster that consists of two sub-groups, one with EUR and SEK and the other with HUF and PLN. Besides, AUD and NZD make up the Oceanian cluster in the HT. An inhomogeneous cluster, which consists of ZAR, TRY, and MXN from different continents, are formed in the HT.

From Fig. 5(b), we can see that the whole structure observed in the upper-tail dependence HT is in line with Fig. 5(a), but some related changes can be found as follows. (1) The Middle Eastern cluster becomes the first cluster that has the smallest distance among its currencies. (2) Although Asian currencies are lined together, the centers of cluster (i.e., MYR and SGD) appeared in Fig. 5(a) is gone. (3) The sub-group with EUR and SEK in the European cluster is replaced by the sub-group with NOK and SEK. (4) The South American cluster is formed with BRL and MXN in the upper-tail dependence HT. It is interesting to note that the final four currencies lined in the two tail dependence HTs are RUB, GBP, ILS, and JPY, which is consistent with the results ranked by the influence-strength in Table 4.
3.5. Results of lower- and upper-tail dependence PMFGs

In this subsection, we consider the lower- and upper-tail dependence PMFGs of the set of 42 currencies in the FX market obtained from the corresponding tail dependence matrices estimated by using 2003 daily returns during the analyzed period (2005–2012), and present the two tail dependence PMFGs in Figs. 6 and 7. Similar to Figs. 3 and 4, currencies from the same geographical location are marked the same color and shape in Figs. 6 and 7.
As the outcomes observed in the two tail dependence MSTs, the tail dependence structure between currencies in the FX market drawn by the tail dependence PMFGs is primarily of geographical origin. Concretely, expect for some currencies from the international cluster and the Commonwealth cluster, most currencies from the same geographical location (i.e., continent or region) in the FX market are grouped to form the geographical clusters. In comparison to the tail dependence MSTs, the tail dependence PMFGs include more edges and thus generate more information. In Section 2.5, we introduce that one of differences between the MST and PMFG is the later contains 3-cliques and 4-cliques. By using the \( n \)-clique algorithm proposed by Palla et al. (2005), we calculate that the lower-tail dependence PMFG contains 17 and 12 3-cliques and 4-cliques respectively. While in the upper-tail dependence PMFG, we count that the number of 3-cliques and 4-cliques are 20 and 15 respectively. The number of 3-cliques or 4-cliques of the two tail dependence PMFGs is far smaller than the number of possible cliques of three or four elements in a fully linked graph, which is \( C^3_{42} = 11,480 \) or \( C^4_{42} = 111,930 \). The complete lists of 3-cliques and 4-cliques showed in the

Figure 6: (Color online) Lower-tail dependence PMFG of the set of 42 currencies in the FX market during the period 2005–2012. Currencies from the same geographical location (i.e., continent or region) are marked the same the color and shape. Coding is: Africa, orange ellipses; Asia, cyan diamonds; Europe, yellow squares; Latin America, blue triangles; Middle East, green diamonds; North America, red ellipses; and Oceania, magenta squares.
two tail dependence PMFSs are described in Tables 5 and 6, respectively. The lists in Tables 5 and 6 are ranked by the average tail dependence coefficients of three or four elements. From Table 5, one can find that the 3-cliques between the lower- and upper-tail dependence PMFGs is almost completely different, except for one case, namely the 3-cliques with elements of CLP, MXN and SGD. As for the 4-cliques in Table 6, most of them are different between the two PMFGs, except for three cases, i.e., 4-cliques with elements of HUF, PLN, ZAR and TRY, CAD, AUD, NZD and TRY, and CLP, MXN, RUB and TRY. Therefore, market participants should pay attention to differences of the 3-cliques and 4-cliques between the two tail dependence PMFGs, and immediately adjust the decision-makings once the market situation changes. Besides, one can see that most of elements of 3-cliques and 4-cliques are from the same currency cluster observed in the tail dependence MSTs or HTs. From this, portfolio holders should choose currencies (assets) from different 3-cliques and 4-cliques or clusters to build the portfolios, which is helpful for investment diversifications and reducing the risk of portfolios.
Table 5: 3-cliques ranked by the average tail dependence coefficients ($\langle \tau_{ij}^L \rangle$ or $\langle \tau_{ij}^U \rangle$) of lower- and upper-tail dependence PMFGs of the set of 42 currencies in the FX market.

<table>
<thead>
<tr>
<th>Lower-tail dependence PMFG</th>
<th>Upper-tail dependence PMFG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cy. 1</td>
<td>Cy. 2</td>
</tr>
<tr>
<td>BHD</td>
<td>JOD</td>
</tr>
<tr>
<td>USD</td>
<td>PEN</td>
</tr>
<tr>
<td>EUR</td>
<td>NOK</td>
</tr>
<tr>
<td>CNY</td>
<td>PEN</td>
</tr>
<tr>
<td>BRL</td>
<td>MXN</td>
</tr>
<tr>
<td>ARS</td>
<td>EGP</td>
</tr>
<tr>
<td>MXN</td>
<td>ZAR</td>
</tr>
<tr>
<td>CZK</td>
<td>HUF</td>
</tr>
<tr>
<td>COP</td>
<td>MXN</td>
</tr>
<tr>
<td>HUF</td>
<td>NOK</td>
</tr>
<tr>
<td>NOK</td>
<td>PLN</td>
</tr>
<tr>
<td>CHF</td>
<td>NOK</td>
</tr>
<tr>
<td>AUD</td>
<td>HUF</td>
</tr>
<tr>
<td>ISK</td>
<td>PLN</td>
</tr>
<tr>
<td>CLP</td>
<td>MXN</td>
</tr>
<tr>
<td>INR</td>
<td>MXN</td>
</tr>
<tr>
<td>JPY</td>
<td>EGP</td>
</tr>
<tr>
<td>JPY</td>
<td>CHF</td>
</tr>
</tbody>
</table>

Notes: The abbreviation “Cy.” stands for “Currency”.

The community structure is another important topological feature in the complex network, which represents the emergence of clusters of vertexes in a complex network that are more compactly linked internally than the rest of the complex network. In the existing literature on financial networks, the community structure is widely found and reported in the PMFG network. For instance, Song et al. (2011) investigate the PMFG network of the set of 57 market indices from 1 January 1996 to 31 July 2009 and detect four communities in the PMFG. Similar to Song et al. (2011), four communities are detected in the PMFG of industrial indices of U.S. equity markets by Buccheri et al. (2013) who examine the 49 industry index time series during the period 1969–2011. Wang and Xie (2015) uncover three communities in the PMFG network of 20 national real estate securities markets in the period 2006–2012. In previous works, scholars develop many useful methods to detect the community structure in the complex networks (see, e.g., Palla et al. (2005); Blondel et al. (2008); Fortunato (2010)). To identify the community
Table 6: 4-cliques ranked by the average tail dependence coefficients (⟨\tau_{ij}^L⟩ or ⟨\tau_{ij}^U⟩) of lower- and upper-tail dependence PMFGs of the set of 42 currencies in the FX market.

<table>
<thead>
<tr>
<th></th>
<th>Lower-tail dependence PMFG</th>
<th>Upper-tail dependence PMFG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cy. 1</td>
<td>Cy. 2</td>
</tr>
<tr>
<td>BHD</td>
<td>KWD</td>
<td>SAR</td>
</tr>
<tr>
<td>USD</td>
<td>JOD</td>
<td>SAR</td>
</tr>
<tr>
<td>EUR</td>
<td>HUF</td>
<td>NOK</td>
</tr>
<tr>
<td>AUD</td>
<td>NZD</td>
<td>ZAR</td>
</tr>
<tr>
<td>HUF</td>
<td>PLN</td>
<td>ZAR</td>
</tr>
<tr>
<td>BRL</td>
<td>NZD</td>
<td>ZAR</td>
</tr>
<tr>
<td>CAD</td>
<td>AUD</td>
<td>NZD</td>
</tr>
<tr>
<td>INR</td>
<td>MYR</td>
<td>SGD</td>
</tr>
<tr>
<td>GBP</td>
<td>AUD</td>
<td>NZD</td>
</tr>
<tr>
<td>IDR</td>
<td>MYR</td>
<td>KRW</td>
</tr>
<tr>
<td>COP</td>
<td>MYR</td>
<td>MXN</td>
</tr>
<tr>
<td>CLP</td>
<td>MXN</td>
<td>RUB</td>
</tr>
</tbody>
</table>

Notes: The abbreviation “Cy.” stands for “Currency”.

structure in the tail dependence PMFGs, we employ a simple, efficient and easy-to-implement detecting tool proposed by Blondel et al. (2008), which is called as the Louvain approach.\(^4\)

In Figs. 8 and 9, we present the communities of the lower- and upper-tail dependence PMFGs respectively. Like in Song et al. (2011) and Buccheri et al. (2013), four communities are identified in each tail dependence PMFG. From Fig. 8, one can see that the four detected communities correspond primarily to different clusters observed in the tail dependence MSTs: top left, the international cluster and the Middle Eastern cluster; top right, the Asian cluster and Latin American cluster; bottom left, the European cluster; and bottom right, the Commonwealth cluster. We denote the four communities as C1, C2, C3, and C4 from top left to bottom right. Compared with the four detected communities in Fig. 8, we can find some changes in the upper-tail dependence PMFG in Fig. 9 as follows. (1) The community C1 has a small change, namely THB joins the community instead of JPY. (2) The community C2 reduces to the Asian cluster, and the ILS from the (lower-

\(^4\) The cause of such a name of the Louvain approach is that the method was developed when the authors in Blondel et al. (2008) all were at the Université catholique de Louvain.
Figure 8: (Color online) Four communities detected into the lower-tail dependence PMFG (computed by using the daily returns of 42 currencies in the FX market during the period 2005–2012) by the Louvain approach (Blondel et al., 2008). Currencies from the same geographical location (i.e., continent or region) are marked the same color and shape. Coding is: Africa, orange ellipses; Asia, cyan diamonds; Europe, yellow squares; Latin America, blue triangles; Middle East, green diamonds; North America, red ellipses; and Oceania, magenta squares.

tail) community C4 adds in this community. (3) The community structure of C3 is restructured and consists of most European currencies and JPY. (4) The Latin American cluster and the Commonwealth cluster are merged to form the new community C4. By comparing communities with 3-cliques and 4-cliques, one can see that the size of community is larger than the 3-cliques and 4-cliques, and some 3-cliques and 4-cliques are covered in the same community. Therefore, when portfolio holders choose the suitable currencies in
Figure 9: (Color online) Four communities detected into the upper-tail dependence PMFG (computed by using the daily returns of 42 currencies in the FX market during the period 2005–2012) by the Louvain approach (Blondel et al., 2008). Currencies from the same geographical location (i.e., continent or region) are marked the same color and shape. Coding is: Africa, orange ellipses; Asia, cyan diamonds; Europe, yellow squares; Latin America, blue triangles; Middle East, green diamonds; North America, red ellipses; and Oceania, magenta squares.

the portfolios, they should not only consider 3-cliques and 4-cliques, but also take into account the community structure that is different during different market situations (e.g., a booming market or a recession market).

4. Conclusions and future works

In this paper, we examine the tail dependence structure of a set of 42 currencies in the FX market in the range from the beginning of 2005 to the
end of 2012. Based on the SJC copula model and the MST and PMFG approaches, we construct lower- and upper-tail dependence networks to analyze the tail dependence structure of the FX market. In practice, we first employ the AR(1)-GARCH(1,1)-t model to characterize marginal distributions of FX rates returns. Then, we adopt the SJC copula model to compute the lower- and upper-tail dependence coefficients between each pair of FX rates. Next, we build the lower- and upper-tail dependence matrices and transform the two tail dependence matrices to two distance matrices. Finally, we use the MST and PMFG method to construct the tail dependence networks and study their topological properties, the cluster and community structure. Meanwhile, we also investigate the lower- and upper-tail dependence HTs associated with the MSTs. Some basic findings for investigating tail dependence structure of the FX market are summarized as follows.

1. By analyzing the statistics of lower- and upper-tail dependence coefficients, we find that the two series of tail dependence coefficients present different distributions.

2. Based on the analysis of topological properties of the tail dependence networks, we come to conclusions that the two PMFG networks are small-world networks, the upper-tail dependence networks are tighter than the lower-tail dependence networks, and the influence-strength distributions of four tail dependence networks follow a power-law.

3. Some clusters are observed in the tail dependence MSTs and HTs, such as the international cluster, the European cluster, the Asian cluster, the Middle Eastern cluster, the Latin American cluster, and the Commonwealth cluster with USD, EUR, MYR, JOD, MXN, and AUD at their centers, and most clusters are formed according to their currencies’ geographical origin. Currencies like JPY, GBP, ILS, and RUB with a peripheral position in the MST networks have a weak influence-strength.

4. 3-cliques and 4-cliques are found in the lower- and upper-tail dependence PMFGs, and the number of cliques with three and four elements of the later is more than that of the former. Four communities are respectively detected in the two tail dependence PMFGs, and some differences of the four identified communities between the two tail dependence PMFGs are found.

As a whole, the tail dependence structure between the lower- and upper-tail dependence networks (MSTs, HTs, and PMFGs) is similar, but remarked and subtle differences are coexistence. Therefore, market participants should pay attention to the differences between the two tail dependence networks and be ready to adjust the decision-makings once the market status is varying
from a depression period to a booming period and vice versa.

One of the notable contributions of this paper is that we propose the concept of lower- and upper-tail dependence networks, which combines the SJC copula model with the traditional dependence network methods (i.e., the MST and PMFG). The tail dependence networks can provide a new angle of view for analyzing the dependence structure of financial markets, and a novel tool for market participants. In practical terms, our contributions to the field of expert and intelligent systems are three-fold as follows.

(1) The tail dependence networks can be integrated into the foreign exchange trading systems. Analyzing the links and clusters of the tail dependence networks in the FX trading systems is helpful in the construction of FX portfolio and also important in predicting foreign exchange rates and economic decision-making. On another note, our proposed tail dependence network tool is a methodological contribution to the modern portfolio theory.

(2) In the industry, the financial network methods have been designed to relevant expert and intelligent platforms for visual decision-making in financial management. For instance, one of the famous platforms is the Financial Network Analytics (FNA) platform that maps complex financial systems allowing its users to make better data-driven decisions (Birch et al., 2015). Therefore, our proposed tail dependence networks also can be applied and designed to a similar decision-making platform.

(3) From a macro perspective, by identifying the changes and differences of links, clusters and communities in the lower- and upper-tail dependence networks, we can design and construct an early warning system (EWS) to forecast the extreme events and estimate relevant tail risks in the FX market.

Despite the significant contributions of our proposed tail dependence networks to the fields of expert and intelligent systems and financial management, some open topics are not examined in this study but can be extended for the future works as follows.

(1) As noted, a useful application of the tail dependence networks is in the portfolio management. When we construct an asset portfolio, the key problem is how to select the assets in the optional basket based on the tail dependence networks. Therefore, it is urgent to design an intelligent selecting algorithm to achieve the optimal portfolio according to the topological features of the tail dependence networks.

(2) One of the major limitations in our work is that the dynamic properties of the FX market are not considered. It is well-known that financial markets are complex dynamic systems, so the tail dependence structure be-
tween financial agents is time-varying. Thus, another interesting future work is to investigate dynamics of the tail dependence networks, which can be conducive to understand the dynamic mechanism of financial markets from a new perspective and construct visual and self-adapting decision-making system in financial management.

(3) In recent years, many factors such as the frequent bursting of financial crises, rapid shock of oil prices, wild speculation of investors, and unstable political situation of several countries lead to the dramatic fluctuation of the asset prices and then the instability of financial markets. The study of the stability of financial markets becomes a hot topic both in academic and industrial fields. So it would be an insight future research on the stability of financial markets by examining the stability of dynamic tail dependence networks and building relevant early warning systems.

(4) Another potential and insight future study is to construct a crisis simulation model of tail dependence networks for financial markets. According to topological properties (e.g., the degree distribution, average path length, average clustering coefficient, clusters, and community structure) of empirical financial networks, this crisis simulation model can be used to generate simulated tail dependence networks for predicting extreme events and crises.

Acknowledgments

We are grateful to the main editor and two anonymous referees for their helpful suggestions that significantly improved the quality of the paper. The research is supported by the National Natural Science Foundation of China (Grant Nos. 71501066 and 71373072), the China Scholarship Council (Grant No. 201506135022), the Specialized Research Fund for the Doctoral Program of Higher Education (Grant No. 20130161110031), and the Foundation for Innovative Research Groups of the National Natural Science Foundation of China (Grant No. 71221001).

References


Wang, G.-J., Xie, C., Han, F., Sun, B. (2012). Similarity measure and topology evolution of foreign exchange markets using dynamic time warping


