Revenue Maximizing Auctions
with Market Interaction and Signaling∗

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Abstract

This paper studies the effect of post-auction market interaction and asymmetric information on the design of a revenue maximizing mechanism. In this situation, bidders can have incentives to signal their types to influence the outcome of the post-auction market game. Thus a revenue-maximizing mechanism not only specifies who wins the object but also describes the amount of information that is revealed about the bidders. The form of the bidders’ utility functions determines the format of a revenue maximizing mechanism. This paper shows that if bidders have additively separable utility functions in their true and signaled types, then revealing information is revenue maximizing if the utility function is convex in the signaled type, while hiding information is revenue maximizing if the utility function is concave. Previous literature suggested that the key factor is the sign of the first derivative of the utility function with respect to the perceived type, which is contradicted by our findings.

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Keywords: auction theory, mechanism design, externalities, signaling, information disclosure

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1 Introduction

This paper studies the design of the revenue maximizing mechanisms if the winning bidder engages in a post-auction market interaction under asymmetric information. In these situations, the bidding behavior of a player conveys information about his type and thus influences his competitors’ or his customers’ beliefs and behavior in the aftermarket, which in turn affects the profit of the winning bidder. Therefore, the seller can influence the bidding behavior of the players by controlling the amount of information revealed in the auction process. Some examples include treasury auctions with resale\(^1\), and bidding for cost reducing patents, for sequential contracts or for licenses to operate in new markets, e.g. spectrum auctions. In general, all signaling games where players compete to signal through a controlled mechanism can fit into this framework. This way many non-auction transactions can fit in this framework, examples of which are takeovers where bidders compete on a product market after the takeover occurred or Spence’s (1973) educational signaling model with entry examinations.

The main question of this paper is how to design the revenue maximizing mechanism that specifies a winner, a price and an information disclosure policy to achieve its objective. The optimal (revenue-maximizing) amount of information revealed depends on the form of the bidders’ utility (or profit) functions. In this paper, we consider a special type of utility functions, which are additively separable in the true and the signaled types of the bidders, where the term "signaled type" refers to the beliefs of the parties affected (e.g. aftermarket competitors or customers). As an illustration, consider an auction for a

\(^1\)See Bikhchandani and Huang (1990) for an analysis of treasury auctions.
license where the winning bidder enters a market that operates for multiple periods. Each bidder’s type is his probability of success on the market. In period 1, the seller commits to a revenue maximizing mechanism that first asks the bidders to report their types secretly (i.e. to the seller only) and then based on these reports it chooses who wins the license and what information (if any) is released about the type of the winner. In period 2, the winning bidder enters in an employment contract: based on the information the seller reveals in period 1, potential future employers form their beliefs about the type of the winner and they offer competing employment contracts to the entrant. This example can also be considered as a model of startup companies and venture capitalists. In this case the announcement of the seller clearly affects the future utility of the entrant, because it influences the beliefs of future employers. The main novelty, compared to the design of a revenue-maximizing auction in the standard case, is that the seller also controls the amount of information transmitted about the type of the winner to the uninformed public.

In this setting, if the bidder’s utility function is convex in the signaled type of the bidder, then the revenue maximizing policy is to reveal all the information about the type of the winner. If the bidder’s utility function is concave in the signaled type then the revenue maximizing policy is not to reveal any of the information. In the case of symmetric bidders this policy can be implemented through a first-price auction using the appropriate announcement policy. To provide intuition, suppose that the utility function is concave in the signaled type. Then the winner is "risk averse" in the type that is signaled to the market, so ex-ante he has a higher expected utility if no information is revealed about his type. This extra utility can be entirely captured by the seller by charging a
higher price for the license required to enter the industry. As a result the seller wants to maximize this extra utility of the winner and consequently, the revenue-maximizing mechanism reveals no information about the type of the winner. Consider the case where the winner is allowed to operate in a monopoly market where the consumers care about the inferred quality of the product. An example where the utility function is convex in the signaled type may be the case of a cable-tv provider where the consumers might value the extra quality a lot, even if the quality of the service is already high. The opposite case could be a public utility service where all the consumers value is a reliable service and they do not put high value on improved quality.

It is interesting to note that even if the signals are correlated and a full rent-extraction mechanism is available like in Cremer and McLean (1988) the revenue maximizing announcement policy is still the same. In this case the seller is interested in maximizing the ex-post profits of the bidders, because the seller can extract this profit by designing an appropriate payment scheme.\(^2\)

The most similar papers are Das Varma (2003), Goeree (2003), and Katzman and Rhodes-Kropf (2002). These papers analyze an auction game with aftermarket competition where the bidders can signal their types through their bids. Goeree (2003) compares the revenues of the separating equilibria of the first- and second-price auctions. Katzman and Rhodes-Kropf (2002) consider first-price, second-price and English auctions and examines revenue and efficiency implications of announcement policies. They also discuss the effects of the different auction formats on participation and revenues. Das Varma

\(^2\)Moreover, in the case of non additively separable utility functions a result with similar flavor holds: A revenue maximizing announcement policy is such that it maximizes the ex-post profits of the winner.
(2003) analyzes the question of efficiency in the same model. Our paper considers a model where the post auction interaction takes a different form; instead of studying a product market competition, the winner participates in the labor market or the contracts with future clients. We characterize the revenue maximizing selling mechanism allowing any selling mechanism to be employed, the first work that follows a mechanism design approach. Katzman and Rhodes-Kropf (2002) conclude that releasing more information is revenue enhancing when appearing a high is beneficial (Cournot-competition), but not if it hurts the winner (Bertrand-competition). While the intuition behind this results seems appealing our results show that what is important is not the sign of the first derivative with respect to the signalled type, but the second derivative. This intuition fails if $V_2$ is concave (and increasing) for the following reason: there is indeed a higher marginal incentive to bid high when the bids are announced\(^3\), but the bid associated with the lowest type is low with full announcement, since upon winning, the profits are smaller.

Another related paper is Lizzeri (1999) who considers a model of certification institutes that can certify the product quality of a firm after a costless investigation. There are two major differences between his model and ours. First, our model does not allow costless observation of the type of the applicants, so the seller has to rely on the reports of the bidders to extract information. Second, in our model the agent has to go through an examination if it is to enter the market, while in Lizzeri (1999) using the certification service is not required for entering the market.

Our paper is also related to the literature of auctions with externalities initiated by

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\(^3\)In other words, the slope of the bid function tends to be higher when more information is revealed.
Jehiel and Moldovanu. These papers, similarly to ours, allow for the possibility that other market participants are influenced by who wins the object and at what price. The seminal paper of Jehiel-Moldovanu (2000a) makes the assumption that the private information of the bidders is automatically revealed after the auction process, an assumption that excludes the possibility of signaling, which is the focus of this paper.

The layout of the paper is as follows: The setup and a motivating example are presented in Section 2. Section 3 characterizes the optimal auction. Section 4 concludes. The Appendix contains some of the proofs.

2 The Model Setup and some Applications

Consider an auction where \( n \) bidders are competing for a single indivisible license to enter into a monopoly market. The winning bidder receives the license, can enter the market and sell its product. Let \( T_i \) denote the random variable for the type of bidder \( i \in \{1, 2, ..., n\} \) and \( t_i \in [\underline{t}_i, \overline{t}_i] \) be its realization. Denote the joint type space by \( S \), where

\[
S = [\underline{t}_1, \overline{t}_1] \times [\underline{t}_2, \overline{t}_2] \times ... [\underline{t}_n, \overline{t}_n].
\]

Let \( T = (T_1, ..., T_n) \) and \( t = (t_1, ..., t_n) \) and vectors \( T_{-i} \) and \( t_{-i} \) be the type vectors excluding the type of bidder \( i \). Firm \( i \)'s type is distributed according to a strictly positive density function \( f_i \) and the corresponding distribution function \( F_i \). The types of the bidders are drawn independently of each other. The profit of the winner depends on his type \( (t_i) \) and the perception of his type by the market, \( h_i \) in the following way:

\[
V(t_i, h_i) = t_i + V_2(h_i),
\]

where $V_2$ is strictly increasing. The perceived type, $h_i$ is assumed to be the expected value of the type of the winner conditional on the information the market observes. Note, that any function where the first component is more general, i.e. $V = V_1(t_i) + V_2(h_i)$ is captured as long as $V_1$ is strictly increasing; one just needs to change the definition of $t_i$ to transform the profit function into the form of (1).

Two features of this profit function are worth mentioning. First, the profit from entering this market ($V(t_i, h_i)$) depends on the beliefs of the market only through the expected value of the type of the winner. Second, the profit function is additively separable in $t_i$ and $h_i$ in the sense that there is no interacting term of $h_i$ and $t_i$. This means that a firm with a high type ($t_i$) obtains the same increase in profits if the perception of the market ($h_i$) improves as a firm that has a low type.

If firm $i$ with type $t_i$ wins the license and has to pay $b_i$ then his profit is

$$V(t_i, h_i) - b_i.$$ 

Since the profit from losing is normalized to 0, so the expected profit of firm $i$ from an allocation can be written as

$$E[1_i(V(t_i, h_i) - b_i)],$$

where $1_i$ equals to 1 if firm $i$ wins the auction and equals 0 otherwise.

The following example provides an economic application where the profit of the winner takes the form of equation (1).

**Example 1** Consider the situation in which entrepreneurs compete for the possibility of entering a market that operates for two periods. The entrant is self-employed in period
1, but is employed for a fixed wage that depends on the information available to the labor market in period 2. Entering the market requires the purchase of an input from the auctioneer in period 1.\(^5\) Only one unit of this essential input is available and thus only one entrepreneur can enter the market. This input can be the list of available clients, some production equipment or merely a license from the government that allows entry. Upon entering the market, entrepreneur \(i\) is successful with probability \(t_i\) and unsuccessful with probability \(1-t_i\) in each of the two periods, and the outcomes are not serially correlated. In case of success (failure) the revenue is 1 (0). Setting the production costs to zero the only period 1 cost entrepreneur \(i\) incurs is the expected payment to the seller, \(b_i\). Then the total period 1 expected profit (excluding the payment made to the auctioneer) of entrepreneur \(i\) upon entering is

\[
V_1(t_i) = t_i.
\]

The (period 1) auctioneer also announces a signal \(s_i\) based on the bid of the winner of the auction. Without observing the revenue in period 1 but based on the signal \(s_i\) potential future employers form a belief about the type of the entrant \(h_i = E[t_i|s_i]\), which is also the probability of success in period 2 given signal \(s_i\). In period 2 the entrepreneur becomes an employee of one of these employers who bid for the right of employing the entrepreneur for a fixed wage \(w_i\) and obtaining the product this entrepreneur might create in period 2. Since all employers are identical, they bid up to their valuation for the entrepreneur, which depends only on his probability of success and thus can be written as \(g(h_i)\).\(^6\) Then

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\(^5\)For simplicity, we assume that a firm that did not enter in period 1 cannot produce in period 2 either, because it has not acquired the experience for operation.

\(^6\)If it is not possible for the (potentially) risk averse employers to obtain insurance then function \(g\) is linear. Otherwise, depending on the structure of the premium, it can take a different functional form.
the second period utility (wage) of the entrepreneur is

\[ V_2(h_i) = g(h_i). \]

Finally, the total utility of the winning entrepreneur can be written as

\[ V(t_i, h_i) = t_i + V_2(h_i) \]

as it is required in formula (1).

Such a profit function may also arise under procurement contracting. Suppose that a contractor first bids for an initial contract with the government or another big procurer. Then this first procurer may reveal information about the contractor and future procurers care about the quality level this contractor can provide. If the type of the winning contractor \((t_i)\) matters in the first contract, but it is only the market’s perception \((h_i)\) that influences future profits then the profit function takes the form of (1). A third application is the case when some consumers observe the quality of the product of the winner, but some others do not and they try to infer it from the announcement of the mechanism designer. Then there is some revenue that depends on the real type, which comes from the demand of the "informed" consumers; and there is some revenue that depends on the "signaled" type, which comes from the demand of the "uninformed" consumers. The sum of these revenues is then additively separable as required. Finally, consider the case of a start up company that sells a proportion of its company’s share to a venture capitalist where the price of the share depends on the beliefs of the financial market.
3 The revenue maximizing auction

The revenue-maximizing mechanism must specify the variables that are standard in the theory of optimal auctions (the probability of winning for each type and expected payment functions) and in addition it must describe the announcement policy of the seller. By the revelation principle it follows that the seller can restrict attention to truth-telling equilibria of direct mechanisms. In such a mechanism each bidder \( i \in \{1, \ldots, n\} \) makes a report \( \bar{t}_i \) about his type to the seller, and let \( \bar{t} = (\bar{t}_1, \ldots, \bar{t}_n) \). In a truth-telling equilibrium the bidders report their types truthfully, \( \bar{t}_i = t_i \) for \( i \in \{1, 2, \ldots, n\} \). Then depending on these reports, the seller chooses \( p_i(\bar{t}) \), the probability that \( i \) wins, and \( i \)’s expected payment \( b_i(\bar{t}_i) \). Let us denote the probability that bidder \( i \) obtains the object if all the other bidders are truthful as \( P_i(\bar{t}_i) \), where

\[
P_i(\bar{t}_i) = E[p_i(\bar{t}_i, T_{-i})].
\]

In addition the seller sends an announcement to the market. The announcement policy may depend on the reports of the other bidders and on a randomizing device as well. Let \( Z \) be the random variable for this announcement and \( z \) be its realization.\(^8\) By assumption, the market always observes the identity of the winner.\(^9\)

Let random variable \( \eta \in \{0, 1, 2, \ldots, n\} \) denote the winning bidder, where \( \eta = 0 \) means that the object is retained by the seller. This variable may depend on all the announce-

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\(^7\)Since the utility of bidder \( i \) is linear in his payments we can assume without loss of generality that the payment does not depend on the announcement of the other bidders.

\(^8\)Whether \( z \) is observed by the winning bidder is not important, since the post-auction market interaction does not involve any strategic element, so only the beliefs of the market is important and not the beliefs of the winning bidder about the beliefs of the market.

\(^9\)Relaxing this assumption would not cause any complication in the analysis, but it would not be natural in most conceivable applications.
ments plus potentially on a randomizing device. The market cares only about the expected value of the type of the winner, so the belief of the market if firm \( i \in \{1, 2, ..., n\} \) won can be interpreted as the expected value of the types that induce announcement \( z \):

\[
h_i(z) = E[T_i|\eta = i, Z = z].
\]

Consider three examples to illustrate how the announcement policy determines the beliefs of the market. First, suppose that there is full signaling in the sense that the type of the winner is known surely to the market. Then \( h_i(z) \) equals the reported type of the winner, which is \( \tilde{t}_i \). Second, if no information other than the identity of the winner is transmitted by the auction process then for all \( z \)

\[
h_i(z) = E[T_i|\eta = i].
\]

Third, consider the intermediate case where the bidder with highest (reported) type wins, but only the second highest type is announced.\(^{10}\) Denote this second highest type by \( z \). Then \( h_i(z) \) is the expected value of the highest type if the second highest type is \( z \).

If player \( i \) with type \( t_i \) reports \( \tilde{t}_i \), and the other reports are \( \tilde{t}_{-i} \), and signal \( z \) is sent to the market, then the expected profit achieved by \( i \) is

\[
N_i(t_i, \tilde{t}_i, \tilde{t}_{-i}, z) = p_t(\tilde{t})[t_i + V_2(h_i(z))] - b_i(\tilde{t}_i)
\]

His expected profit if the other players report truthfully is the following:

\[
U_i(t_i, \tilde{t}_i) = E[N_i(t_i, \tilde{t}_i, T_{-i}, Z)|\tilde{t}_i],
\]

\(^{10}\)This case becomes relevant when one studies the separating equilibrium of a second price auction. See the discussion after Proposition 1.
where one needs to condition on $\tilde{t}_i$, since the distribution of $Z$ depends on $\tilde{t}_i$. An incentive compatible mechanism is such that type $t_i$ cannot improve his profit by misrepresenting his type:

$$U_i(t_i, t_i) \geq U_i(t_i, \tilde{t}_i) \text{ holds for all } i, t_i, \tilde{t}_i.$$ 

Then the problem of the seller is to maximize the revenue in the class of all incentive compatible mechanisms where each bidder reaches a utility level not lower than his reservation utility, which is normalized to 0.

Before turning to our main result, let us introduce a benchmark setting where the signaling component is absent, i.e. $V_2$ is constant zero, but the type distribution of the bidders is unchanged. In this setting we refer to the revenue maximizing mechanism as the standard Myerson auction, since that mechanism was first characterized by Myerson (1981). A key concept here is the virtual utility of bidder $i$, which is $m_i = t_i - \frac{1-F_i(t_i)}{F_i(t_i)}$.

Define the allocation rule of an auction as the probability component of the mechanism, $p_i(t_i, t_{-i})$. The standard Myerson auction allocates the object based on the virtual utilities of the bidders.\(^\text{11}\) Define the modified virtual utility of $i$ as $M_i = m_i + V_2(t_i)$ and let the modified Myerson auction be the Myerson auction in an environment where the virtual utilities of the bidders are equal to the defined modified virtual utilities.

Then we obtain our main result:

**Theorem 1** If $V_2$ is concave (convex) then the revenue maximizing mechanism is such that no (all) information is announced about the type of the winner. Moreover, if $V_2$ is

\(^{11}\text{If the virtual utilities are increasing then the standard Myerson auction allocates the object to the bidder with the highest virtual utility, or chooses to retain the good if } m_i < 0 \text{ for all } i. \text{ If function } m_i \text{ is not strictly increasing for some } i \text{ then a method called ironing may be needed to calculate the allocation.}
convex then the allocation rule \( \{p_1(t), p_2(t), ..., p_n(t)\} \) is the same as that of the modified Myerson auction.

**Proof.** See Appendix. ■

The intuition behind this first result is the following. If \( V_2 \) is concave then ex-ante (before types are drawn) bidder \( i \) is “risk averse” in \( h_i \). This means that the bidders do not like the risk that the revelation of the types produces from an ex ante point of view. If \( V_2 \) is convex then bidders are "risk lovers", which leads to the opposite conclusion; now they would like the type of the winner to be revealed.

The first part of the Theorem states that in a situation where the *marginal* profit from being perceived as a higher type is increasing (decreasing) in the signaled type it is optimal to announce (hide) the type of the winner completely. In markets where a minimum level of quality is all that is required (an example may be public utilities) it is better to hide the type of the licensee, since there is not much to gain from increasing the signaled type above the average belief. Conversely, in markets where being perceived as a high quality type confers higher and higher benefits the type of the licensee should be announced. An example here could be cable TV companies or other legal monopolies in the telecommunication sector.

The proof of the Theorem follows Myerson (1981) with the exception that now there is an extra component of the revenue that is determined by the announcement policy of the seller. The proof shows that the seller chooses the announcement policy so that it maximizes the total profit of the bidders in the subsequent market game. The reason for this is that the seller is able to extract all the extra profits the bidders enjoy if the
announcement policy is changed in a way that increases the valuations (profits) of the
bidders.\textsuperscript{12}

The revenue maximizing allocation rule when $V_2$ is convex is the modified Myerson
auction. One can show that if for a given distribution function the standard Myerson
auction always allocates the object to the highest type, then the modified Myerson auction
does it as well, but not necessarily the other way around. This follows, because if $m_i$ is
strictly increasing in $t_i$ then $M_i$ is strictly increasing as well, but not the other way around.

If $V_2$ is concave then the seller chooses the allocation function to maximize

$$
\int_{t \in S} f_1(t_1) \ldots f_n(t_n)p_i(t) \{m_i(t) + V_2(\mathbb{E}[T_i|\eta = i])\} \, dt.
$$

Consequently, one cannot pursue a pointwise optimization approach, since changing the
allocation probability $p_i(t)$ at a given point $t$, affects the value $\mathbb{E}[T_i|\eta = i]$. In this case,
we content ourself with characterizing the allocation policy only for the symmetrical case
considered in the Corollary below.

Because of its importance in standard auction theory let us summarize the findings
below in the case when the bidders are symmetric ($F_i$ is the same for all $i$) and the virtual
utility function is strictly increasing:

**Corollary 1** Suppose that $F_i = F$ for all $i$ and $m = x - \frac{1-F(x)}{f(x)} \geq 0$ is strictly increasing
for all $x$ and $V_2$ is positive for all $h$. If $V_2$ is concave or convex then the revenue maximizing
auction always allocates the good to the highest type and never retains it.

**Proof.** The result is proved in the Appendix. \hfill \blacksquare

\textsuperscript{12}This feature is the consequence of the assumption of additive separability, which implies that a change
in the signaled type ($h_i$) changes the profits of all types ($t_i$) in the same way, independent of the type.
It is an interesting question whether such an allocation-announcement policy pair can be implemented as an equilibrium allocation-announcement policy pair of a standard auction format. If $V_2$ is convex, then the revenue maximizing mechanism calls for the full revelation of the type of the winner. A natural candidate is then to conduct a first price auction with announcing the price. However, such an auction is a signaling game with multiple equilibria and we need to select the equilibrium that delivers the maximal revenue, which turns out to be the *separating equilibrium*, where the bidding function $b(t_i)$ is strictly increasing.

Under the assumptions of Corollary 1, the following results hold:

**Proposition 1** In the case where $V_2$ is convex, the separating equilibrium of a first price auction with price announcement is revenue maximizing. In the case where $V_2$ is concave, the first price auction with hidden bids is revenue maximizing.

**Proof.** When $V_2$ is concave, then in the proposed first price auction the bids are not revealed and thus the signaling aspect of the auction disappears. Then this first price auction becomes a standard private value auction. The (unique) equilibrium outcome of this standard auction has the same allocation policy and the same amount of information revealed as in the revenue maximizing mechanism in Corollary 1, thus the revenues of the two mechanisms are equal.

When $V_2$ is convex, the separating equilibrium of the first price auction yields the same allocation and the same (perfect) information revelation about the type of the winner as the revenue maximizing mechanism of Corollary 1. Then it follows that the first price auction provides the same (maximum) revenue as well.
To show that such a separating equilibrium of the first price auction exists, consider the utility of a type $t_i$ who bids $b(\tilde{t}_i)$:

$$U_i(t_i, \tilde{t}_i) = F^{n-1}(\tilde{t}_i)(t_i + V_2(\tilde{t}_i) - b(\tilde{t}_i)).$$

Let us define $B_i(\tilde{t}_i) = V_2(\tilde{t}_i) - b_i(\tilde{t}_i)$ and then rewrite the above equation as

$$U_i(t_i, \tilde{t}_i) = F^{n-1}(\tilde{t}_i)(t_i - B_i(\tilde{t}_i)).$$

Then the incentive constraints can be written in the same form as in the case of a standard first price auction with symmetric bidders and independent valuations, which implies that a strictly monotone equilibrium exists. ■

In this symmetric setup, our findings suggest a way to rank the revenues offered by the first price and the second price auction formats if only the price is revealed.\(^{13}\) The first-price auction reveals more information, since the bid of the winner is revealed while in the second price auction only the bids of the losers are revealed. So, one would think that a second price auction yields higher revenues under concavity while a first-price auction yields higher revenues under convexity of $V_2$. One can show that this is indeed the case if we compare the revenues derived in the separating equilibria of the two auctions. The starting point is that the equilibrium allocation is the same in the two auctions, since always the highest type wins and thus one only needs to maximize the profits that arise from the $V_2$ component of the profit function. Take the case when $V_2$ is concave and consider two announcement policies: In one all the bids are revealed, in the other all, but the highest. The second policy effectively pools some types of the winner, which

\(^{13}\)Katzman and Rhodes-Kropf (2002) and the working paper version of Goeree (2003) compared these auction formats using different functional form assumptions from ours.
increases the expected value of the $V_2$ component by Jensen’s inequality. In the case when $V_2$ is convex, (the separating equilibrium outcome of) the first price auction with price announcement is revenue maximizing, so it must yield a higher revenue than the second price auction with price announcement.

Since a higher signal is beneficial for the winner in this model, one might think that announcing the type of the winner induces the bidders to bid more aggressively to signal their type to the market. Therefore one may expect that the expected revenue is higher than under a policy that hides some information. This intuition fails if $V_2$ is concave for the following reason: there is indeed a higher marginal incentive to bid high when the bids are announced\textsuperscript{14}, but the bid associated with the lowest type is low with full announcement, since upon winning the profits are smaller. Katzman and Rhodes-Kropf (2002) compare first- and second-price auctions with price announcement and conclude that a first-price auction (that reveals more information) yields higher revenue under Cournot-, while a second-price auction is beneficial under Bertrand-competition. The above insight shows this result holds not because in the Cournot-case it is beneficial to appear as a high type, while in the Bertrand-case it is beneficial to appear as a high type. Rather, as Molnar and Virag (2005) shows, it depends on the second derivative of the profit functions.\textsuperscript{15}

\textsuperscript{14}In other words, the slope of the bid function tends to be higher when more information is revealed.\textsuperscript{15}More precisely, two key factors are identified. First, the second derivative of the profit function with respect to the signalled type as in this paper. Second, the cross partial derivative with respect to the true and the signalled types. A positive cross-partial derivative favors price announcement, while a negative cross-partial favors hiding information.
4 Conclusion

This paper characterized the revenue maximizing auction when bidders of an auction care about the perception of their type by a post auction market. We assume that the seller can control the amount of information transmitted through the bidding process in order to maximize revenues. If the bidders’ utility functions are additively separable in their true and signaled type, then the revenue maximizing announcement policy depends on whether the utility functions are concave or convex in the signaled typed. If the utility function is convex (concave) then the revenue maximizing announcement policy is to reveal all (none of) the information that the bids contain. In the case of symmetric bidders and increasing virtual valuations this mechanism can be implemented through a first-price auction where all (none of) the bids are announced after the auction.

We assumed that the seller can both credibly transmit information and can prevent information transmission if it is in his best interest. The implementation of a mechanism that prescribes to hide any information signalled through the bids relies on the second assumption. Upon relaxing this assumption, pooling would be more appealing for the seller, because he can credibly commit not to release information only if he does not acquire it. This commitment issue is absent if there is full information revelation according to the revenue maximizing auction. First, hiding information is not necessary. Second, if we organize this revenue maximizing mechanism as a first-price auction without hiding the winner’s bid then the auction process itself will transmit the information.

In a work in progress, Molnar and Virag (2005) consider a setup where the bidder’s utility function is not additively separable. More precisely, they consider the case of a
takeover competition with post-auction market competition. They show that in the class of mechanisms considered full information revelation is optimal when the post auction competition is Cournot-, while the opposite is true in the case of Bertrand-competition. These results are similar in flavor to the one found by Katzman and Rhodes-Kropf (2002) who show that a first price auction yields higher revenue than a second price auction in the case of Cournot-competition, since it reveals more information about the type of the winner than the second price format.

Appendix

Lemma 1 For any incentive compatible mechanism the probability that bidder $i$ with type $t_i$ wins, $p_i(t_i)$ is weakly increasing in $t_i$ for all $i$. Moreover, any mechanism in which this probability is increasing in $t_i$ is incentive compatible with an appropriate payment schedule.

Proof. The probability of winning for bidder $i$ if $\tilde{t}_i$ was the announcement of the bidder to the seller, as follows:

$$p_i(\tilde{t}_i) = E_{t_{-i}} p_i(\tilde{t}_i, t_{-i})$$

The expected utility of type $t_i$, if he pretends that his type is $\tilde{t}_i$, is the following:

$$U_i(t_i, \tilde{t}_i) = p_i(\tilde{t}_i)\{V_1(t_i) + E[V_2(h_i(z))|\eta = i]\} - b_i(\tilde{t}_i)$$

We may write the incentive compatibility constraints as follows:

$$U_i(t_i, t_i) \geq U_i(t_i, \tilde{t}_i) \quad \text{(IC1)}$$

$$U_i(\tilde{t}_i, t_i) \geq U_i(\tilde{t}_i, t_i) \quad \text{(IC2)}$$
Adding (IC1) to (IC2), using formula (3), and canceling terms that appear on both sides yields:

\[ [p_i(t_i) - p_i(\tilde{t}_i)][V_1(t_i) - V_1(\tilde{t}_i)] \geq 0 \]

Using the monotonicity of \( V_1 \) yields that \( p_i(t_i) \) is weakly increasing. A converse argument proves the second part of the Lemma. □

**Proof of Theorem 1:**

**Proof.** The only difference between our analysis and the standard Myerson setup is that in our case there is an extra term in the social welfare function, which originates from the second component of the profit functions of the firms. After conducting similar analysis to Myerson (1981) one may write the expected revenue of the auctioneer as follows:

\[
R = \sum_{i=1}^{n} \left( E \left[ p_i(T) \left( T_i - \frac{1 - F_i(T_i)}{f_i(T_i)} \right) \right] \right) + E \left[ V_2(h(Z)) \right],
\]

(4)

where expression \( E [V_2(h(Z))] \) denotes the expectation of the \( V_2 \) component according to the auction the seller employs. Formally,

\[
E [V_2(h(Z))] = \sum_{i=1}^{n} \Pr(\eta = i)E[V_2(h_i(Z)|\eta = i].
\]

(5)

Formula (4) implies that, given an allocation rule \((p_1, p_2, ..., p_n)\) the seller needs to design an announcement policy that maximizes the expected value of the second component, \( E [V_2(h(Z))] \), since the first component of (4) does not depend on \( Z \) in any way.

First, suppose that \( V_2 \) is concave. Then Jensen’s inequality implies that

\[
E[V_2(h_i(Z)|\eta = i] \leq V_2(E[h_i(Z)|\eta = i]).
\]

(6)
By the law of iterated expectations

\[ E[h_i(Z)|\eta = i] = E[T_i|\eta = i]. \]

Then the expression on the right hand side of inequality (6) is the expected value of the \( V_2 \) component if \( i \) wins and no extra information is announced by the seller. So, inequality (6) means that if the seller does not disclose any information if \( i \) wins then the \( V_2 \) component is maximized. But since it holds for all \( i \) separately, given any probability functions \( (p_1,p_2,...,p_n) \) the value of \( E[V_2(h(Z))] \) is maximized when no information is announced about the winner. Together with a previous observation it implies that a revenue maximizing auction does not announce any information when \( V_2 \) is concave.

Now, assume that \( V_2 \) is convex. Take again the equation

\[ R = \sum_{i=1}^{n} \left( E \left[ p_i(T) \left( T_i - \frac{1 - F_i(T_i)}{f_i(T_i)} \right) \right] + E[V_2(h(Z))] \right). \]

First, prove that for any allocation policy \( (p_1,p_2,...,p_n) \) the second term on the right hand side achieves its maximum value under the announcement policy that announces the type of the winner. Assume that a type \( T_i \) has the same signal, \( z \) as some other types with positive probability. Denote the corresponding beliefs of the market by \( h_i(z) \). Switch now to a mechanism that reports the type of the winner in the case when \( z \) was reported before. Then by convexity of \( V_2 \) this step will increase the expectation of the term in question, because Jensen’s inequality applies for this convex function. Formally for all \( z \)

\[ E[V_2(T_i)|\eta = i, Z = z] \geq V_2(E[T_i|\eta = i, Z = z]) = V_2(h_i(z)). \]

Let \( G_i \) denote the distribution function of \( Z \) if \( i \) wins and let \( \tilde{S} \) be the corresponding
support. Then
\[
E[V_2(h_i(Z)|\eta = i] = \int_S V_2(h_i(z))dG_i(z)
\]
\[
\leq \int_{\tilde{S}} E[V_2(T_i)|\eta = i, Z = z]dG_i(z) = E[V_2(T_i)|\eta = i],
\]
where the last equality follows from the law of iterated expectations. Since this holds for all \(i\) it means that the second component of the revenue is maximal when the type of the winner is announced, while the first component does not depend on the announcement policy. Thus the revenue maximizing policy announces the type of the winner. Using this last result implies that the optimal revenue is such that
\[
R = \sum_{i=1}^{n} \left( E \left[ p_i(T) \left( T_i + V_2(T_i) - \frac{1 - F(T_i)}{f(T_i)} \right) \right] \right).
\]
One can then follow Myerson (1981) to prove the last part of the Theorem. ■

**Proof of Corollary 1:**

**Proof.** Let \(V_2\) be concave and consider the mechanism that does not announce any information about the winner, allocates the object to the highest type and never retains it. The first sum in (4) is maximized by this mechanism as it follows from the analysis of Myerson (1981). We only need to prove that the second term of (4) is maximized as well. Using the results of Theorem 1 one can restrict attention to mechanisms where no information is revealed about the winner. In Step 1 we show that the mechanism that maximizes the second component of (4) retains the good with probability 0. Step 2 describe this second component, while Step 3 concludes the proof.

Step 1: By a standard argument, in this symmetric model one can restrict attention to symmetric mechanisms when looking for the revenue maximizing mechanism.\(^{16}\) Take

\(^{16}\)If an asymmetric mechanism is optimal then so is the mechanism that would just flip over the alloc-
a mechanism that retains the good with positive probability. Consider then a modified mechanism that uses the same allocation rule except that when the first mechanism retained the object, this mechanism allocates it to the bidder with the highest type, \( T_H = \max \{ T_1, ..., T_n \} \). Let \( \eta \) and \( \lambda \) denote the random variables for the winner in the two mechanisms. Because of symmetry it is true that for all \( i \in \{1, 2, ..., n\} \)

\[
E[T_{\eta}|\eta \neq 0] = E[T_{\eta}|\eta = i].
\]

The fact that no information is revealed in the revenue maximizing mechanism yields

\[
\sum_{i=1}^{n} \Pr(\eta = i)E[V_2(h_i(Z)|\eta = i] = \Pr(\eta \neq 0)V_2(E[T_\eta|\eta \neq 0]).
\]

Also,

\[
E[T_\lambda] = \Pr(\eta \neq 0)E[T_{\eta}|\eta \neq 0] + (1 - \Pr(\eta \neq 0))E[T_H|\eta = 0]
\]

and

\[
V_2(E[T_\lambda]) = V_2\{\Pr(\eta \neq 0)E[T_{\eta}|\eta \neq 0] + (1 - \Pr(\eta \neq 0))E[T_H|\eta = 0]\} \geq \Pr(\eta \neq 0)V_2(E[T_{\eta}|\eta \neq 0]) + (1 - \Pr(\eta \neq 0))V_2(E[T_H|\eta = 0]) \geq \Pr(\eta \neq 0)V_2(E[T_{\eta}|\eta \neq 0])
\]

by Jensen’s inequality and the positivity of \( V_2 \). But the value of the second component of (4) is exactly \( \Pr(\eta \neq 0)V_2(E[T_{\eta}|\eta \neq 0]) \) for the first mechanism and \( V_2(E[T_\lambda]) \) for the modified mechanism, which concludes Step 1.

Step 2: Consider any symmetric mechanism that sells the object with probability 1 and denote the random variable for the winner as \( \eta \). The second component of (4) may
be written as

\[ E[V_2(h(Z))] = V_2(E[T_i|\eta = i]) = V_2(E[T_\eta]). \]  

(7)

Step 3: Thus one needs to maximize \( E[T_\eta] \) in the class of symmetric mechanisms that sell the object with probability 1. Let

\[ T_H = \max\{T_1, \ldots, T_n\}. \]

It is obvious that

\[ T_\eta \leq T_H \]

and thus

\[ E[T_\eta] \leq E[T_H], \]

which concludes the proof. Thus a mechanism that always sells the object to the highest type and does not disclose any further information is revenue maximizing. Note, that by Lemma 1 such a mechanism satisfies the global incentive compatibility constraints.

When \( V_2 \) is convex then the revenue maximizing auction allocates the object to the bidder with the highest modified virtual utility, \( M = x + V_2(x) - \frac{1-F(x)}{f(x)} \), which is a positive and strictly increasing function of \( x \). Then Theorem 1 implies that a mechanism in which always the highest type wins and his type is announced is revenue maximizing. Again, global incentive conditions for the bidders are satisfied by Lemma 1. ■

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