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Analysis of the message propagation on the highway in VANET

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Abstract Intelligent transportation systems (ITS) are becoming more and more important nowadays. In these systems, vehicles and possibly the infrastructure communicate with each other by vehicular ad-hoc networks (VANETs). VANETs are being deployed and widely used in urban as well as in highway applications. Several standard use cases have been identified over the last decade (i.e., alert messages, car following support, data exchange between vehicles, etc.). In this paper, we focus on the alert message propagation on the highway. We derive the stationary and the transient solution of the message propagation distance by constant vehicle speed. Since these messages frequently indicate an accident on the road leading to a traffic jam, we extend the model to take the queueing system due to the traffic jam also into consideration. Our analytical results are compared with SUMO-/Veins-based simulations.

1 Introduction

The vehicular ad-hoc network (VANET) technology supports several types of communication. In the vehicle-to-vehicle (V2V) case vehicles are allowed to communicate only with each other. In vehicle-to-infrastructure (V2I) systems the vehicles can also communicate with the road-side infrastructure if it is present. The most flexible solution, the vehicle-to-everything (V2X) communication does not pose any restriction on the communicating entities.

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Since nowadays road-side infrastructure is scarce, and V2V communication is always possible between vehicles equipped with the necessary capabilities, we focus on the V2V communication in this paper.

Many common V2V use cases have been identified by the IETF in [1], including

- Context-aware navigation for driving safety and collision avoidance;
- Cooperative adaptive cruise control in an urban roadway;
- Platooning on the highway;
- Cooperative environment sensing.

In the V2V setting, VANET supports several message types to serve the requirements of these use cases. There are periodic messages – called beacons – for status and position updates, event-driven messages [2], safety messages, etc., for a taxonomy see [3].

To provide sufficient quality of service, it is important to develop analytical models to calculate various performance measures related to the message propagation. For instance, by computing how far the message propagates, and how long it takes to deliver a message to the vehicles, we can help the drivers to take appropriate decisions on time. Having the message received, the driver can leave the road before reaching the accident and the corresponding traffic jam, or vehicles can maintain a suitable distance between each other to avoid accidents.

There are many factors affecting the efficiency of the message delivery. In this paper, we assume that the communication is based on the IEEE 802.11p WAVE protocol. In this protocol, the information is carried by various channels defined for different purposes ([4]). In our work, we do not analyze the details of the radio transmission, but assume an idealistic behavior, namely that all messages are successfully transmitted immediately to all vehicles in the transmission range of the sender. We also assume that vehicles traveling in the opposite direction do not contribute to the message delivery ([5], [6]). The message transmission considered in this paper is one-dimensional, which is a proper model for highways, but not for urban scenarios. While these assumptions seem restrictive, there are surprisingly few results available in the literature for even such a simple model. There are some papers on the stationary analysis of the message propagation distance ([7], [8]), however, according to our best knowledge, there are no results available on the analysis of the transient behavior.

Nevertheless, the analysis of the transient behavior is crucial in many safety scenarios. When the speed of a vehicle changes suddenly, or when an accident occurs, it is essential to know how fast the related message reaches a given distance. Answering this question needs transient analysis, which is the main topic of this paper.

The rest of the paper is organized as follows. The related work is summarized in Section 2. The description of the system with the detailed discussion of the assumptions is provided by Section 3. The main results on the stationary and the transient properties of the message propagation distance are presented by Section 4. In Section 5, the results are extended to take the queueing system due to the traffic jam also into consideration. Finally, Section 6 concludes the paper.

2 Related work

In the recent decades, a large number of research papers appeared related to VANETs, proposing new protocols, studying the properties of the radio channel, analyzing various performance measures, etc. In this section we summarize only those which are closely related to the topic of our paper, that is the analysis of the message propagation.

The message propagation delay and the number of vehicles receiving the messages depend on many factors. When road-side units (RSUs) are present, the message delivery is more efficient, since RSUs can collaborate to increase the throughput of the network. A cooperative load balancing solution is proposed in [9] that prioritizes requests based on their urgency. However, in this paper, we assume that no RSUs are present, hence the messages can be passed only between vehicles.

Several papers investigate the effect of the radio channel and the media access control (MAC) on the message transmission delay. The survey [10] lists the most widely used propagation models (including deterministic and probabilistic models) to analyze the probability of successful message delivery given the parameters of the channel. Like our paper, [11] also focuses on the message delivery delays, but it assumes a dense-enough (urban) network such that there is always at least one vehicle present in the transmission range. Our paper does not have this restriction. Furthermore, [11] considers the properties of the physical layer and the MAC layer (fading, interference, packet loss, etc.) as the primary source of delays, while our paper builds on a higher level model with idealistic physical and MAC layers. The MAC layer is analyzed in [12], too, where the authors consider a Markov model for the contention window to obtain the packet reception rate taking the potential collisions due to concurrent transmissions and queue overflow into consideration.

Protocol layers above the MAC have been studied in the literature as well. A new broadcast protocol has been introduced in [13] with a new forwarding scheme, that is capable of achieving very high message propagation speed. In ad-hoc networks, the broadcast communication sometimes leads to a so-called storm problem. In paper [14] the authors proposed a new protocol, called DRIVE, based on local one-hop neighbor information that achieves short delays and low overhead.

In this paper, we investigate the same performance measures as the aforementioned papers, namely the message propagation distance and the message propagation speed (or the delay). However, we ignore all the effects of the underlying physical/MAC/network layers, which were analyzed in details in the papers listed above. We assume idealistic behavior. Thus, the messages inside a given range are always delivered successfully, and with zero transmission delay. Even with these restrictive assumptions, the exact mathematical analysis is challenging. The following couple of papers pose the same assumptions and analyze the dynamics of the message passing in VANETs with purely mathematical tools.

In [15] a slightly different problem is solved. That paper aims to determine the optimal lifetime of the messages, to ensure that all the vehicles in a given zone receive the information. The authors assume a fixed transmission range, Poisson arrival process for the vehicles, and normal distributed speed, but the results are not explicit and are approximate when there are vehicles with different speeds. In our paper, the message lifetime is considered to be infinite. Hence, messages are never dropped even if they have not been forwarded for a long time.

The message propagation distance in one-dimension is studied in [8]. Making use of an interesting approach, based on the workload process of a G/D/∞ queue, the probability density function (pdf) of the message propagation distance is derived in Laplace transform domain. From the Laplace transform the mean value is determined, and the pdf in time domain is obtained explicitly by inverse transformation. We study the same problem in our paper, by a different approach: we develop a differential equation for the pdf of the message propagation distance. With this approach, we were able to derive a differential equation for the transient behavior as well, which has never been published before.

In [7] the one-dimensional scenario of [8] has been extended to a two-dimensional lattice, for urban environment. This paper still considers the stationary behavior only and presents no results on the transient.

3 Description of the system

We assume that the arrival process of the vehicles is a Poisson process (with intensity parameter λ), which is a common assumption made by the waist majority of the related publications, including [15], [7].

All vehicles are assumed to have the same, constant speed, denoted by v. Taking multiple speeds into consideration makes the analysis very difficult. However, according to our simulation results (to be shown later), the results for the multi-speed case are almost the same as the ones we get by assuming the same speed for all vehicles.

For the radio communication, we assume that all vehicles communicate according to the IEEE 802.11p standard. Several papers have been published in the past on the analysis of the message reception probabilities as the function of the inter-vehicle distance at radio level. The models developed can be classified into deterministic or probabilistic models [10]. We use the deterministic propagation model in this paper (free space propagation model, also referred to as Friis model, [16]), that defines a coverage radius R, and assumes that all messages are successfully transmitted to all vehicles inside the coverage radius. This model is used in many VANET simulation frameworks, including the Simulation of Urban MObility (SUMO, [17]) and Vehicles in Network Simulation (Veins, [18]).

Table 1 summarizes the notation used in the sequel.

4 Analysis of the message propagation

Assume that an event occurs on the highway at position A. In the rest of the paper, this position is assumed to be fixed, and messages advertising the event are generated continuously for a long time. Here we study the right-continuous stochastic process $\{\mathcal{D}(t), t > 0\}$, where $\mathcal{D}(t)$ is the information propagation distance, that is the position of the last car measured from A having the message received at time t, plus R (the radius of its radio coverage). We provide the differential equations governing the evolution of $\mathcal{D}(t)$, and characterize the stationary behavior by deriving the distribution function and the mean value of $\mathcal{D} = \lim_{t\to\infty} \mathcal{D}(t)$.

In this section we assume that the arrival process of the vehicles is a Poisson process with parameter denoted by λ (measured in vehicles/second) and the speed of all vehicles is constant, given by v (measured in meters/second). With these parameters, we have that the distance between the vehicles is exponentially distributed with parameter $\vartheta = \lambda/v$ and the direction of the vehicles is constant, and it is one-directional as shown in Figure 1.

For a similar problem, the stationary message propagation distance has been derived both in [7], [8]. We arrive at the same results in Sections 4.1 and 4.2. Our new contribution, the transient analysis is described in Section 4.3.

4.1 Clusters of informed vehicles

A set of vehicles, where the distance between subsequent vehicles in the set is less than R, is called a *cluster* in this paper. When the head (the first car) of the cluster receives a message, all the vehicles in the cluster will receive it eventually. In our model, we assume that all vehicles will be informed immediately, although in reality, the speed of the information propagation depends on the beacon periods, too. (Later, Section 4.4 studies the consequences of this assumption.)

Let us introduce random variable $\mathcal{G}$, referred to as the *cluster length*, that plays an important role in the analysis. $\mathcal{G}$ represents the distance between the position of the first car in the cluster and the last position where the alert information is available, that is, the position of the last car plus R (see Figure 2). In the rest of the section the complementary cumulative distribution function (ccdf) of $\mathcal{G}$, denoted by $G(x) = P(\mathcal{G} > x)$, is derived.

From the stochastic interpretation of the system, $G(x)$ satisfies the recursive expression

$$G(x) = \begin{cases} 1, & \text{if } x \le R, \\ \int_{y=0}^{R} \vartheta e^{-\vartheta y} G(x-y)\, dy, & \text{if } x > R. \end{cases} \quad (1)$$

The first term corresponds to the case when x is close enough to the message source to receive the message directly. In the second case the vehicle that is the closest to the message source falls into the coverage area (in distance y); it receives the message and starts broadcasting it, hence the message has to take only the remaining $x - y$ distance to reach the target.

The next theorem provides the mean of $\mathcal{G}$ based on this recursion.

Theorem 1 *The mean cluster length $E(\mathcal{G})$ can be expressed by*

$$E(\mathcal{G}) = \frac{1}{\vartheta}(e^{\vartheta R} - 1). \quad (2)$$

Proof To obtain the mean value the integral of the ccdf is calculated:

$$\begin{aligned} E(\mathcal{G}) &= \int_{x=0}^{\infty} G(x)\, dx \\ &= \int_{0}^{R} 1\, dx + \int_{R}^{\infty} \int_{0}^{R} \vartheta e^{-\vartheta y} G(x-y)\, dy\, dx \\ &= R + \int_{0}^{R} \vartheta e^{-\vartheta y} \left[\int_{y}^{\infty} G(x-y)\, dx - \int_{y}^{R} G(x-y)\, dx \right] dy \\ &= R + (1 - e^{-\vartheta R}) E(\mathcal{G}) - R(1 - e^{-\vartheta R}) \\ &\quad + (1 - e^{-\vartheta R})/\vartheta - R e^{-\vartheta R}, \end{aligned}$$

from which (2) follows. □

The differential equation providing the ccdf $G(x)$ itself is given by the following theorem.

Theorem 2 *The ccdf of $\mathcal{G}$ is the solution of the delayed differential equation (DDE)*

$$\frac{d}{dx} G(x) = -\vartheta e^{-\vartheta R} G(x - R), \quad x > R, \quad (3)$$

with boundary condition $G(x) = 1, x \le R$.

Proof For $G(x+R)$ the second case of (1) is valid (since $x > 0$), that, by swapping the variables in the integral, can be transformed to

$$\begin{aligned} G(x+R) &= \int_{y=0}^{R} \vartheta e^{-\vartheta y} G(x+R-y)\, dy \\ &= \int_{u=x}^{x+R} \vartheta e^{-\vartheta (x+R-u)} G(u)\, du, \end{aligned} \quad (4)$$

which, after taking the derivative and making use of the Leibniz integral rule, leads to

$$\begin{aligned} \frac{d}{dx} G(x+R) &= \vartheta G(x+R) - \vartheta e^{-\vartheta R} G(x) \\ &\quad - \vartheta \underbrace{\int_{x}^{x+R} \vartheta e^{-\vartheta (x+R-u)} G(u) du}_{G(x+R)} = -\vartheta e^{-\vartheta R} G(x), \end{aligned}$$

which establishes the theorem. □

4.2 The stationary solution of $\mathcal{D}(t)$

The stationary solution $\mathcal{D} = \lim_{t \to t} \mathcal{D}(t)$ is the distance between the event, A, and the position where the information is available, as seen by an external observer at a random point of time. With other words, when an external observer takes a snapshot of the system, $\mathcal{D}$ represents the distance the message can travel through a chain of vehicles being closer than R to each other, starting in A.

Observe that this quantity is almost the same as the cluster length $\mathcal{G}$ defined in Section 4.1. The only slight difference is that in this case, the head of the cluster is the event at A instead of a vehicle, which does not need different mathematical treatment. Hence we have that $\mathcal{D}$ is in fact equal to $\mathcal{G}$, thus $E(\mathcal{D}) = E(\mathcal{G})$ and $F(x) = P(\mathcal{D} > x) = G(x)$ hold.

4.3 The transient analysis of $\mathcal{D}(t)$

After the analysis of the stationary solution we also investigate the transient behavior of the process $\mathcal{D}(t)$, that is, we characterize the ccdf $F(t, x) = P(\mathcal{D}(t) > x)$ for $x \ge R$. Note that this distribution has probability mass at $x = R$, $\mathcal{D}(t) = R$ occurs when there are no vehicles on the highway having the message received.

Theorem 3 *The transient ccdf $F(t, x)$ satisfies the partial differential equation (PDE) for $x > R$*

$$\frac{\partial}{\partial t} F(t, x) - v \frac{\partial}{\partial x} F(t, x) = \lambda (1 - F(t, R_+)) G(x - R), \quad (5)$$

where $F(t, R_+)$ denotes $\lim_{x \searrow R} F(t, x)$. For $x \le R$ we have $F(t, x) = 1$.

Proof To prove the theorem we describe the evolution of $\mathcal{D}(t)$ in the infinitesimally small time period $(t, t+\Delta)$. There are two possibilities leading to $\mathcal{D}(t+\Delta) > x, x > R$:

– either the position of the last informed vehicle was beyond $x + \Delta \cdot v$ at time t,
– or there was no informed vehicle present at time t (hence $\mathcal{D}(t) = R$, having probability $1 - F(t, R_+)$), a new vehicle entered the range R of the event (with probability $\lambda\Delta$), received the message, and through a chain of vehicles the information eventually reached beyond distance $x - R$ (the corresponding probability is $G(x - R)$).

All the remaining events have probability $o(\Delta)$, for which $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$ holds. More formally,

$$\begin{aligned} F(t + \Delta, x) = {} & F(t, x + \Delta \cdot v) \\ & + (1 - F(t, R_+)) \cdot \lambda\Delta \cdot G(x - R) \\ & + o(\Delta), \end{aligned} \tag{6}$$

which, after dividing by Δ, some transformation, and taking the limit $\Delta \to 0$ leads to (5). □

To show that $G(x)$ (and Theorem 2) is the stationary solution of $F(t, x)$ indeed, we take the limit $t \to \infty$ and substitute $G(x) = \lim_{t\to\infty} F(t, x)$, leading to

$$\frac{d}{dx} G(x) = -\vartheta(1 - G(R_+))G(x - R), \tag{7}$$

since $\lambda/v = \vartheta$. Taking the limit $x \to R$ in (3) and exploiting that $G(x-R)$ becomes 1 in this range yields $G(R_+) = 1 - e^{-\vartheta R}$, that, together with (7), gives the same DDE as shown in Theorem 2.

To actually compute the transient probabilities the differential equation (5) has to be evaluated numerically. (6) provides one possibility for this purpose, with an appropriately small Δ it enables to obtain the probabilities progressively.

An other, particularly interesting quantity to study is the evolution of the mean value $E(\mathcal{D}(t))$ in time. The mean value is given by the integral of the ccdf, hence

$$E(\mathcal{D}(t)) = \int_0^\infty F(t, x)\, dx = R + \int_R^\infty F(t, x)\, dx.$$

Integrating both sides of (5) with regards to x from R to ∞ gives

$$\frac{d}{dt}(E(\mathcal{D}(t)) - R) - v[F(t, x)]_R^\infty = \lambda(1 - F(t, R_+))E(\mathcal{D}),$$

that, making use of (2), simplifies to

$$\frac{d}{dt} E(\mathcal{D}(t)) = v(e^{\vartheta R}(1 - F(t, R_+)) - 1). \tag{8}$$

Similar to the mean value, the variance $Var(\mathcal{D}(t))$ can be calculated similarly.

4.4 Numerical examples

In the first example the mean message propagation distance $E(\mathcal{D})$ is depicted as the function of the car arrival rate λ and radio transmission range R (see Figure 3). The speed of the vehicles is set to $v = 36$ m/s throughout this section. The results are as expected: the higher the car density and the wider the transmission range is, the longer is the message propagation distance. Figure 3 confirms that the radio coverage plays a crucial role in the message propagation, increasing R to twice the value increases the message propagation distance by several orders of magnitudes.

Next, we plot the ccdf $F(x)$ by $\lambda = 0.65$ and various R parameters in Figure 4.

To investigate the effect of the simplifying assumptions making the scenario idealistic (including the infinitely large acceleration and deceleration, infinitesimally small bacon period), we developed various simulation tools in the SUMO/Veins/OMNeT++ framework [17–19]). These simulation tools are

– An idealized simulation tool respecting the same assumptions as our analytical models. The only purpose of this simulator is to verify our analytical results. Since the performance measures obtained by this simulator matched our formulas up to a very small error, we do not include the corresponding results on our plots.
– The realistic simulation where the radio communications are realistic. The message is transferred by beacons, with the beacon period set to 50ms. The behavior of the vehicles (including the acceleration, deceleration, car-following model, etc.) is also realistic, for the parameters see Table 2.
– A realistic simulation where the vehicles have different speed, according to a normal distribution with variance $\sigma^2 = 8$.

In Figure 4 the results obtained by the detailed simulations and the analytical results are compared, by parameters $R = 150, \lambda = 0.65$. As reflected by the plots, assuming immediate message transmission between the vehicles and assuming constant vehicle speed have little impact on the shape of the ccdf.

Finally, we demonstrate some studies that can be carried out using the results for the transient behavior. We note that the transient behavior is especially difficult to investigate efficiently by simulation: to compute the message propagation probabilities at a certain time t a huge number of independent simulations must be performed up to time t with different random seeds. Our analytical formulas, on the other hand, returned the results quickly without any numerical issues. Figure

5 depicts the transient ccdf $F(t,x)$ at $t = \{1,2,4\}$ seconds together with the stationary solution $F(x)$, while Figure 6 shows the mean of the message propagation distance $E(\mathcal{D}(t))$ and its squared coefficient of variation (SCV) defined by $Var(\mathcal{D}(t))/E^2(\mathcal{D}(t))$. According to the figures, by such high vehicle density, the stationary solution is achieved very fast, in just 4 seconds.

5 Queueing model for the traffic jam

Based on the results of the previous section it is possible to calculate the distribution of the message propagation distance when there is an accident on the highway at position A. However, there is the other factor that has to be taken into consideration. Typically, the accident can be passed by at a very low speed, leading to a traffic jam. To take the effect of the traffic jam into account, we introduce a queueing model. The arrival process to this queue is a Poisson process with rate λ, and the service time is given by the time taken by a vehicle to leave the accident. If the vehicle length is L and the speed in the accident region is v_{slow}, the service time is deterministic, $\tau = L/v_{slow}$, leading to an M/D/1 (infinite capacity) queue.

5.1 Stationary solution of the queue

To obtain the stationary solution of the queue length $\mathcal{X}$, the queue length process embedded at vehicle service instants is studied. Since the arrival process is Poisson, the stationary solution of the embedded process is equal to the stationary solution seen by a random observer due to the PASTA property [20].

The transition probability matrix P of the embedded Markov chain has an upper-Hessenberg structure ([20], see also Figure 7). Hence we have

$$P = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \cdots \\ a_0 & a_1 & a_2 & a_3 & \cdots \\ & a_0 & a_1 & a_2 & \cdots \\ & & a_0 & a_1 & \cdots \\ & & & \ddots & \vdots \end{bmatrix} \quad (9)$$

In matrix P, a_i gives the probability that i vehicle arrive to the accident region while one vehicle leaves it. It can be computed as

$$a_i = \frac{(\lambda D)^i}{i!} e^{-\lambda D}, \quad i \geq 0. \quad (10)$$

If the queue is stable ($\lambda D < 1$ holds), the stationary queue length probabilities $\pi_i = P(\mathcal{X} = i), i \geq 0$ can be obtained by the solution of

$$\pi P = \pi, \qquad \sum_{n=1}^{\infty} \pi_n = 1, \quad (11)$$

where π is the (infinitely long) row vector consisting of probabilities π_i. To compute the queue length distribution numerically, we have $\pi_0 = 1 - \lambda D$ and for $i > 0$ the π_i is given by the recursive formula

$$\pi_{i+1} = \frac{\pi_i - \pi_0 a_i - \sum_{j=1}^{i} \pi_j a_{i-j+1}}{a_0}. \quad (12)$$

For the mean queue length the Pollaczek-Khinchine formula provides a simple explicit solution

$$E(\mathcal{X}) = \rho + \frac{1}{2}\frac{\rho^2}{1-\rho}. \quad (13)$$

5.2 Message propagation distance

The message propagation distance in the presence of the queue, denoted by $\mathcal{B}(t)$, has two components: the queue length at time t, given by $\mathcal{X}(t)$, and the message propagation distance counted from the end of the queue given by $\mathcal{D}(t)$. These two random variables are not independent, but in this section, we approximate $\mathcal{B}(t)$ as if the two components were independent. Hence, in steady state, for the mean values we have

$$E(\mathcal{B}) = E(\mathcal{X}) + E(\mathcal{D}), \quad (14)$$

where $E(\mathcal{X})$ is given by (13) and $E(\mathcal{D}) = E(\mathcal{G})$ is given by (2).

For the ccdf of $\mathcal{B}$, defined by $B(x) = P(\mathcal{B} > x)$, our approximation is given by

$$B(x) = \sum_{i=0}^{\infty} \pi_i \cdot F(x - i \cdot L), \quad (15)$$

since the length of i cars in the traffic jam is $i \cdot L$.

5.3 Simulation results

In this section we prepare a simulation study to evaluate the accuracy of the results. Our SUMO/Veins/OMNeT++-based simulations are detailed, in the sense that the messages are transferred through beacons, and the behavior of the radio transmission, as well as the motion model of the vehicles, is taken into account in full details (for the parameters see Table 2).

We assume that the vehicles slow down from $v = 36$m/s to $v_{slow} = 3$m/s when they reach the accident. Since the car length is set to $L = 4.5$m and the car arrival rate is $\lambda = 0.65$cars/s, this means that the utilization of the queue representing the traffic jam is $\rho = 0.975$. All other simulation results are shown in Figure 8.

The comparison between the analytical and the simulation results is depicted in Figure 8. According to the results, the simple model proposed to approximate the effect of the traffic jam turned out to be reasonably accurate.

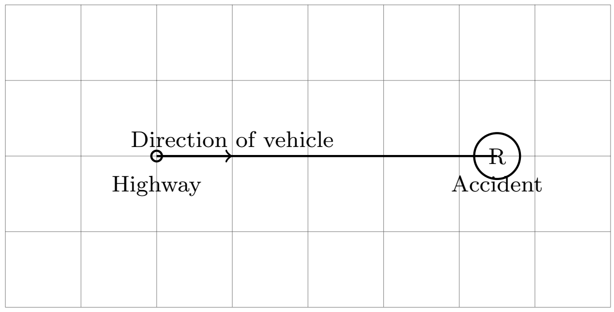

Fig. 1 Propagation alert meaage in the Highway

6 Conclusion

In this paper, we have introduced new contributions regarding the message propagation in VANET systems. We have derived the stationary and transient solutions of the message propagation distance. We validated our analytical results with simulation (Veins and SUMO within OMNET++). We have proposed an accurate approximation to take into account the length of the traffic jam caused by an accident as well.

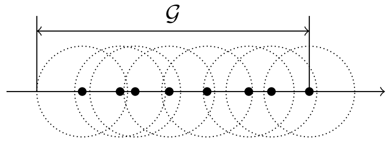

Fig. 2 A cluster of informed vehicles

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Table 1 Notation and parameters used

Parameter	Definition
R	Range of transmission, 150m
L	Length of a car, 4.5m
v	Speed in the highway, 36 m/s
λ	Vehicle arrival rate, in vehicles/second
ϑ	Parameter of the exponentially distributed vehicle density, in car/meters
$\mathcal{D}(t)$	Message propagation distance at time t
$E(\mathcal{D})$, $SCV(\mathcal{D})$, $Var(\mathcal{D})$	Mean, squared coefficient of variation and variance or random variable $\mathcal{D}$

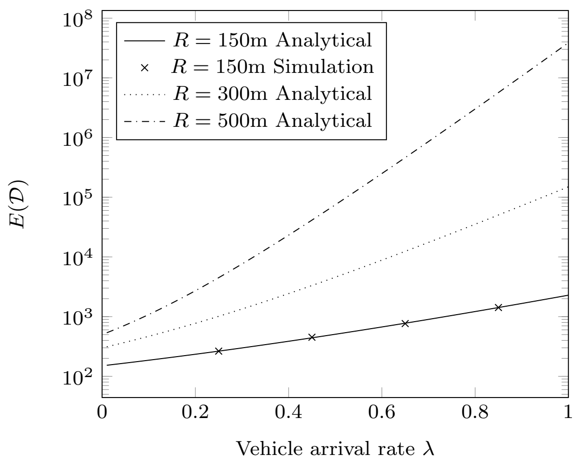

Fig. 3 The mean message propagation distance

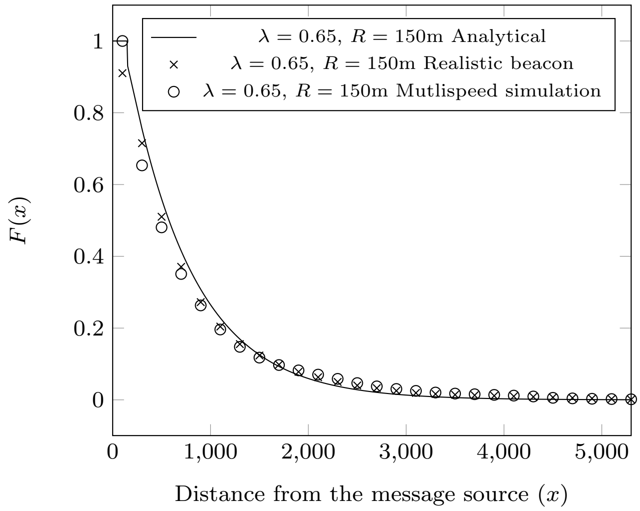

Fig. 4 The comparison of the analytical and simulation-based results

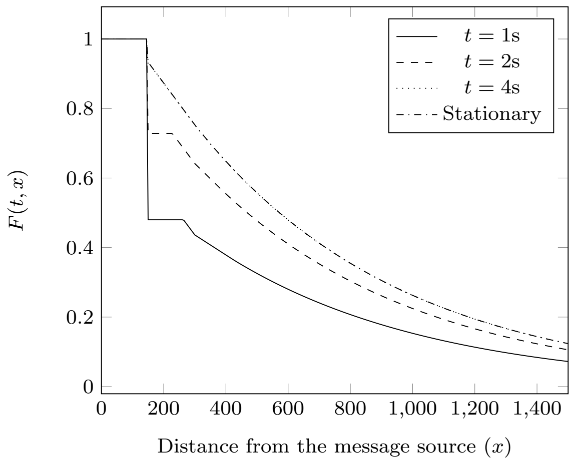

Fig. 5 The transient distribution $F(t, x)$ at some time points, $\vartheta = 0.65$, $R = 150$m

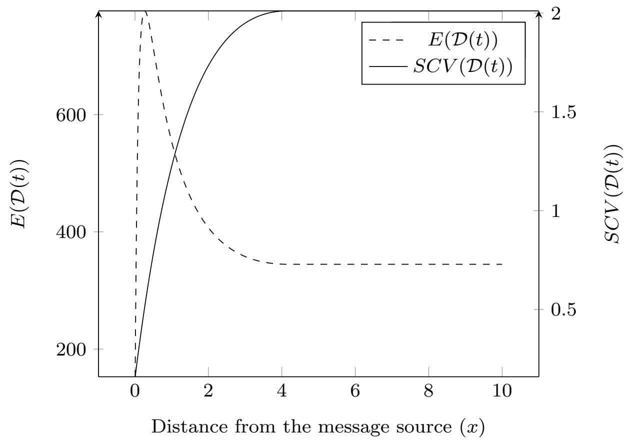

Fig. 6 The mean and the SCV of the message propagation distance as the function of time

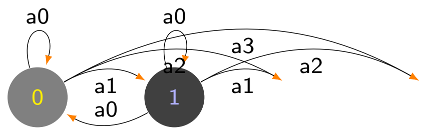

Fig. 7 state-transition diagram for M/D/1 Markov chain

Table 2 Parameters used in simulation

Parameter	Definition
R	Range of transmission, R = 150m
L	Length of the vehicles, $L = 4.5$m
v	Normal vehicle speed, $v = 36$m/s
v_{slow}	Slow vehicle speed at accident, $v_{slow} = 3$m/s
λ	Car arrival rate, $\lambda = 0.65$
Center frequency	5.890 GHz
Analogue Model	Simple path loss model
Safety message duplicate period	50ms
Channel bandwidth	10MHz
Channel data rate	3Mbps
Safety message packet size	100 bytes
Simulation time	8000s

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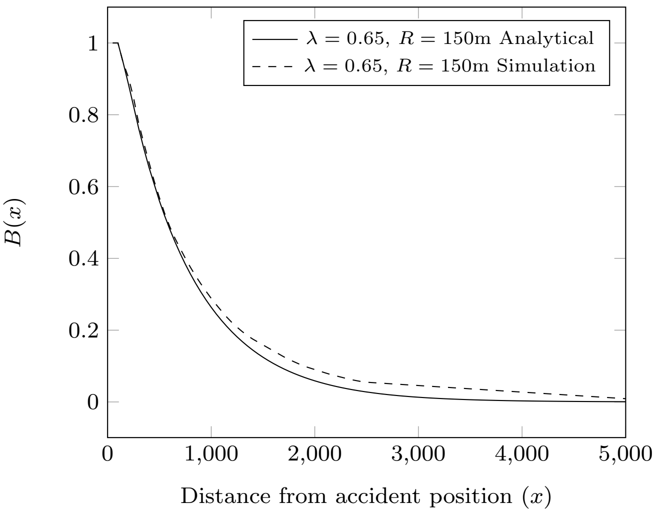

Fig. 8 The comparison of the analytical and simulation-based results

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