Abstract— Optimal operation of hydrothermal systems is very complex because it corresponds to a multiperiod, stochastic, large scale and characterized by a nonseparable objective function optimization problem. As a result it is traditionally carried out without taking into account transmission constraints or considering them in a very simplified way. This approach is not adequate to address certain kind of problems such as cost-benefit studies of transmission interconnections in hydro systems or the study of spatial distributions of spot prices (“zones” identification) through the electric network, due to hydro and power flow constraints. In this paper we address the problem of stochastic optimization of large scale transmission constrained hydrothermal systems with applications to the Brazilian case.

Index Terms— Hydroelectric power generation, Power Transmission Economics, Power Generation Dispatch, Optimization Methods

I. INTRODUCTION

The objective of the optimal operation of a hydrothermal system is to determine an operation strategy that for each stage of the planning period and given the system state, produces generation targets for each plant. This strategy should minimize the expected value of the operation cost along the period, composed of fuel cost and penalties for failure of load supply [1].

As discussed in [2,6], the availability of limited amount of hydroelectric energy in the form of stored water in the system reservoirs, creates a link between decision in a given stage and its future consequences. In other words, if we deplete the stock of hydroelectric energy, and low inflow volumes occur, it may be necessary to use very expensive thermal generation and low inflow volumes occur, it may be necessary to use very expensive thermal generation in the future or even fail to supply the load. On the other hand, if we keep the reservoir levels high through a more intensive use thermal generation and high inflow volumes occur, there may be a spillage in the system, which means a waste of energy and, consequently, higher operating costs.

Because it is impossible to have perfect forecasts of the future inflow scenarios, the hydrothermal operation problem is essentially stochastic. As a consequence of existence of multiple and large reservoir as observed in the Brazilian system, it is necessary to consider a large number of time periods to have a correct evaluation of a decision made today in the terms future costs. This implies in a high problem dimension. Finally, because the worth of energy generated in a hydro plant cannot be measure directly as a function of the plant state alone, but rather in terms of expected fuel savings from avoiding thermal generation, the objective function is nonseparable [3].

The above complexities implied that traditionally, large scale hydro thermal operation optimizations are carried out without taking into account transmission constraints or considering them in a very simplified way. Other traditional approach is to consider the transmission system but with a very simplified representation of the hydro plants by specifying a water value for them, neglecting time related constraints or water inflows uncertainties, and conducting operation optimization at each snapshot (classical OPF problems). Both approaches are not adequate to address certain important issues, such as cost-benefit studies for transmission interconnections in hydro systems or the study of spatial distributions of spot prices through the electric network (that can lead to the identification of possible “zones” with the same spot price and a locational marginal pricing scheme can then be applied), due to hydro and power flow constraints.

In this paper we address the problem of stochastic optimization of large-scale hydrothermal systems with transmission constraints. The solution algorithm is based on stochastic dual dynamic programming (SDDP) [4-6], which approximates the expected future cost function of stochastic dynamic programming by piecewise linear functions. These approximate functions are obtained from the dual solutions of one-stage optimizations problem at each stage. No state discretization is necessary and the combinatorial “explosion” with the number of states – the well known “curse of dimensionality” of dynamic programming - is avoided.

The transmission constraints are represented through a linearized power flow model. In this case each stage in the SDDP algorithms corresponds to a linearized optimal power flow model (OPF) with additional variables and constraints.

This work is organized as follows: section II presents problem formulation and in Section III the optimization model is briefly described. Section IV describes a case study of transmission reinforcement problem in the Brazilian system. Section V illustrates the effect of hydro and transmission constraints in the spatial distribution of Brazilian spot prices throughout the electric network and its consequences in terms of submarkets (or zones) definition. Section V concludes.
II. PROBLEM FORMULATION

If the inflows volumes are known at the beginning of each stage, the operation planning problem can be represented as a stochastic dynamic programming (SDP) recursion [4] as follows:

\[
\alpha_t(X_t) = \min_{A_t} \{ E_t \{ \sum_{k \in K} \sum_{j \in T_k} C_{ij}(G_{ij}) + \beta \alpha_{t+1}(X_{t+1}) \} \}
\]

subject to

\[
\sum_{i \in H_k} (V_{i+1} - V_i) + \sum_{j \in T_k} G_{ij} - \sum_{l \in \Omega_k} F_{kli} = D_{tk} \quad (2)
\]

for all \( k \in K \)

\[
V_{i+1} = V_i + A_t + M(Q_{i+1} - S_{i}) \quad (3)
\]

\[
L_i(Q_{i+1}, S_i) \geq 0 \quad (4)
\]

\[
F_{kli} = \gamma_{ki} (\theta_{ki} - \theta_{kl}) \quad (5)
\]

for all \( k \in K, l \in \Omega_k \)

\[
\sum_{l \in \Delta} \phi_{kl} F_{kli} \leq \Pi_i \quad (6)
\]

\[
G \leq G_i \leq \bar{G} \quad (7)
\]

\[
\bar{V} \leq V_{i+1} \leq \bar{V} \quad (8)
\]

\[
|F_i| \leq \bar{F} \quad (9)
\]

for all \( t = T, T-1, \ldots, 1 \) for all \( X_t \)

where:

- \( t \) indexes the stages (T planning horizon)
- \( X_t \) state vector at the beginning of stage \( t \). It includes reservoir storage levels and hydologic trend [7]. The latter information can be given, for example, by the lateral inflow volumes during previous stages: \( X_t = [V_t, A_{t+1}, A_{t+2}, \ldots] \)
- \( \alpha_t(X_t) \) expected value of operation cost from state \( X_t \)
- \( A_t, A_{t+1}, \ldots \) probability distribution of inflow vector \( A_t \) conditioned to state \( X_t \)
- \( E(\{\}) \) expected value
- \( \beta \) discount factor
- \( k \) indexes system buses (K is the set of buses)
- \( j \) indexes thermal plants (\( T_k \) is the set of thermal plants in bus \( k \))
- \( i \) indexes hydro plants (\( H_k \) is the set of hydro plants in bus \( k \))
- \( G_{ij} \) generation of thermal plant \( j \) in stage \( t \). Failure in load supply are represented as dummy thermal plants
- \( C_j \) generation cost of plant \( j \)
- \( \rho_i \) production coefficient of hydro plant \( i \)
- \( Q_{ii} \) turbined outflow of hydro plant \( i \) in stage \( t \)
- \( S_{ii} \) spilled volume of hydro plant \( i \) in stage \( t \)
- \( V_{ii} \) volume of reservoir of hydro plant \( i \) in stage \( t \)
- \( M \) hydro plant incidence matrix
- \( D_{ki} \) active load in bus \( k \) in stage \( t \)
- \( \bar{G} \) vector of thermal generation capacity
- \( F_{kli} \) active flow in circuit \( k-l \) in stage \( t \)
- \( \bar{F} \) vector of circuit flow limits
- \( \Delta \) any subset of circuits
- \( \Omega_k \) set of buses connected to bus \( k \)
- \( \theta_{ki} \) voltage angle at bus \( k \) in stage \( t \)
- \( \gamma_{kl} \) susceptance of circuit \( k-l \)
- \( \phi_{kl} \) coefficient of circuit flow \( (k,l) \) in additional flow constraints
- \( \Pi_i \) right hand side of additional flow constraints in stage \( t \)

For a given \( A_t \) Problem (1) corresponds a linearized optimal power flow problem (OPF) with additional variables and constraints. The new variables are the turbined outflow, spilled volume and reservoir volume. The new constraints are the reservoir water balance equations (3), constraints involving turbined outflow and spilled volume (4) and on reservoir volumes (8). Constraints (2) and (5) represent Kirchhoff’s first law (flow conservation at each bus) and second law (energy conservation), respectively. Constraints (7) represent maximum and minimum thermal generation in each bus. Constraints (6) and (9) correspond to limits on total flows in sets of circuits and on individual circuit flow. The objective function is the sum of thermal generation and deficit cost plus the discounted future cost function (\( \alpha_t(X_t) \)).

III. THE OPTIMIZATION MODEL

The solution approach, known as stochastic dual dynamic programming (SDDP) [see [4-7]], consists in working with piecewise linear approximation of the future cost function (FCF). This approximation is built from the dual solution of the LP problem (1), associated to each hydrological scenario and time stage.

The SDDP algorithm was implemented in a transmission-constrained probabilistic production-costing model, which determines the optimal stochastic operation policy of a multi-reservoir hydrothermal system without aggregating hydro plants or using other approximation techniques. The following aspects are represented in detail in the model:

(i) reservoir operation (turbining, spillage, filtration, head change effect etc. along complex cascades), inflow uncertainty through multivariate stochastic inflow models which represent both spatial and time dependence
(ii) thermal plant operation (non linear efficiency curves, fuel limits, startup costs, multiple fuels etc.)
(iii) transmission network, including Kirchhoff’s laws, quadratic losses, power flow limits, area exports, security constraints etc.

Besides the optimal operating policy, the model calculates several economic indices such as bus marginal costs, network congestion revenues, water values, marginal value of gas reserves, etc. Applications of this model will be illustrated next with real case studies from the Brazilian system.

IV. TRANSMISSION REINFORCEMENT PROBLEM

The Brazilian power system has an installed capacity around 74 GW (90 % hydro – 2001 data) and total production of around 40 GWavg. Hydro system generation is characterized by large reservoirs (multiyear inflow regulation capability) organized in a complex topology over several river basins (Fig 1 shows the Brazilian Southeast hydropower system). The system is divided in 4 zones (also called “submarkets”) - South, Southeast (also includes the Center West), North and Northeast - determined by major structural transmission constraints. An overview of the system is shown
in Fig. 2. The transmission system is highly meshed and composed by more than 45,000 miles of transmission lines (Fig. 3).

In this section, the previous model was used to evaluate the attractiveness of a transmission reinforcement between the South and Southeast regions in the Brazilian system. Power transfer capability between these regions is a very important resource to explore the negative correlation between the streamflows in the Brazilian Southern and Southeastern basins. The transmission reinforcement under analysis corresponds to the construction of a new circuit connecting buses Londrinas 500 kV (located in the South) and Campinas 500 kV (located in the Southeast) in the year 2004 (Fig. 4). Fig. 3 also presents a closer look in the area where the transmission reinforcement to be evaluated is located. Traditionally, in this kind of study power transfer capabilities are derived from load flow simulations considering the existing network and typical plant dispatches, and the transmission system is represented through a very simplified transportation model in the hydrothermal optimization. The error brought by this kind of approach is large because in meshed networks like the Brazilian (as shown in Fig. 3), the second Kirchhoff’s law is essential for the transmission system representation in the hydrothermal optimization.

For the evaluation of the interconnection reinforcement, the following results were obtained: individual production, turbined outflow and spillage for each hydroplant, circuit power flow and load marginal cost for each bus in the system.

Figures 5 and 6 show annual mean marginal costs for the South and Southeast submarkets with and without the transmission reinforcement. In this case, the submarket marginal cost is defined as the load weighted average of its buses marginal costs.

1 Even though there is a large amount of thermal generators in the system, they comprise a small share (7 GW) of the overall installed capacity (74 GW).
It can be seen that even without reinforcement South and Southeast annual mean marginal costs are very close indicating that there were no transmission congestion between the two submarkets. Therefore, from this point of view the reinforcement benefit is negligible. Fig. 7 shows the empirical probability distribution curve of annual marginal cost for the year 2004 without the reinforcement, for two buses – one in the Southeast (TIJPRETO-750 kV) and the other in the South (AREIA-525 kV) region. Here again, it can be seen, that the curves are almost identical indicating there were no transmission congestion between the two zones.

Another important result is the amount of spilled volume that could have been turbinated if there were no transmission bottlenecks. These results are shown in Figs 8 and 9 for the Itaipu hydro plant (12.6 GW of installed capacity) located in the South region and also connected to Southeast through a AC network and a direct DC line. The figures present the probability distribution curve of the spilled volume that could be turbinated and the correspondent spot prices in both situations. As a result of the reinforcement the amount of spilled volume that could not be turbinated due to transmission bottlenecks decreases but the overall impact is negligible because this reduction occurs in the scenarios were spot prices are low due to the system high storage level in those scenarios.

The above results showed that based on system data, the construction of this new circuit Londrinas – Campinas in 2004 would not bring any additional economic benefit and the final decision was to not go ahead with the project. This result contradicted the general perception of system experts at the time the study was carried out, since during the Brazilian energy rationing in the year 2001 Southern reservoirs were full but it was not possible to transfer the desired amount of energy to the Southeast submarket. This difference in perception was probably because the perspective of supply and demand balance evolution for Southeast were different from the time of rationing (as confirmed latter on) and due to the representation of region interchange limits in the reservoir equivalent model currently used for the Brazilian system dispatch.

V. SPATIAL DISTRIBUTION OF SPOT PRICES AND SUBMARKETS (OR ZONES) IDENTIFICATION

In this section we turn to the problem of submarkets, or zones, definition. In the Brazilian regulatory framework there are two system dispatches: a physical dispatch, that takes into account the observable equipment availability, bus demand and network constraints, and a commercial dispatch, that takes into account the same availability and characteristics of generators but uses a simplified representation of the demand and transmission network. In the latter, the demand is aggregated by submarkets and transmission constrains within each submarket are ignored; only network constrains between
submarkets are taken into account.

All financial settlement in the Wholesale Market is based on the commercial dispatch. The differences between energy production in the commercial and physical dispatches are compensated as follows: if a generator produced more in the physical dispatch than in the commercial dispatch its additional operating costs are paid by the ISO. Otherwise, if the generator produced less in the physical dispatch than in the commercial dispatch it has to compensate the ISO for the corresponding operating cost. The difference between generation cost in the physical and commercial dispatch is always greater zero and are charged to the consumers as the System Services Charge.

The main motivation for the use of submarkets was to characterize the system in terms of its major structural transmission constraints, in order to facilitate spot prices computation and investment analysis in new plants. Furthermore the aggregation of network buses in submarkets may bring benefits in terms of lower energy prices to consumers due to the increasing of competition among generators located in different buses. On the other hand, it is important that no structural transmission constraint is missed in this aggregation so that the System Service Charge does not get to high or other significant congestion appears.

As within submarkets there are no structural transmission constraint one possible approach to identify them is to carry out a transmission constrained hydrothermal optimization and identify the circuits that are close to be overloaded. This task is not as easy as it may seem because in meshed systems the consequences of an overloaded circuit may spread to buses far from its location. Fig. 10 illustrates this effect.

Fig. 10 – Brazilian System – 2005 – Circuits Close to be Overloaded

Based on a full transmission constrained hydrothermal optimization of the Brazilian system, according to the previous methodology and same system configuration as of Section IV, it shows the circuits which are close to be overloaded (identified in bold and pointed by arrows) for the year 2005 and for a given dispatch scenario. Observe that from the picture it is not easy to identify the regions in which there are no internal transmission constraints.

Another approach to define submarkets is to identify the buses in the network that have similar marginal costs, since differences in marginal costs across buses are due to transmission bottlenecks or losses. Fig. 11 shows, for the same year 2005 and a given inflow scenario, the spatial distribution of bus marginal costs (R$/MWh)² throughout the network (buses with similar marginal costs are shown with the same color pattern).

Fig. 11 – Brazilian System – 2005 – Buses with Close Marginal Costs

It can be seen that buses located in the South and Southeast zones have close marginal costs and in principle could be part of a same zone. The other potential similar “zone” consists of the set of buses located in the North and most of the Northeast regions (light gray in the Fig.11) where marginal costs are, however, much higher. The third set comprises buses located between the Southeast and North-Northeast zones that have an intermediary level of marginal cost. Fig. 11 also shows some circuits that are closer to be overloaded (white circuits, pointed by arrows). Note that they are not at the frontier of the regions that have similar marginal costs. This confirms what was observed before that it is not easy to identify submarkets by observing the circuits that are close to be overloaded. Based on this approach a clustering analysis technique may be used to identify the submarkets. In such technique, N buses would be partitioned in M buses-set (zones) in such a way that each set is spatially connected (considering the neighborhood buses topology) and the corresponding buses in each set have similar spot prices (within a pre-specified range).

A third approach in submarket definition is to evaluate alternative submarkets arrangements through the associated System Service Charge. We will illustrate this approach in the evaluation two alternative submarkets arrangements in Brazil: the current (2002) arrangements of four submarkets (North, Northeast, South and Southeast) and a proposed arrangement of two submarkets: North-Northeast and South-Southeast.

Fig. 12 shows the probability distribution curve of System Service Charge for the year 2005 corresponding to the four submarket case, whereas Fig. 13 presents the one corresponding to the two submarket case.

As expected the mean System Service Charge for four submarkets (R$0.8 Millions) is smaller than the corresponding one for two submarkets (R$2.0 Millions), since it leads to less out-of-merit-order dispatch due to transmission constraints. The difference in the mean System Service Charges would

² R$/US$ exchange rate equals 3.00 (August 2002)
correspond to 0.13 cents/MWh and to less than 0.11 cents per MWh at a 95 % confidence level that was found to be acceptable by the regulator as reasonable figures. This kind of result was the base to recommend the changing of zones definition in Brazil – from the original four submarkets configuration to a two submarkets configuration, to be implemented by the regulator in 2003.

![Fig. 12. System Service Charge Year 2005 – Four Submarkets](image)

![Fig. 13. System Service Charge, Year 2005 – Two submarkets](image)

VI. CONCLUSIONS

Optimal operation of hydrothermal systems is a very complex optimization problem and as a result it is traditionally carried out without taking into account transmission constraints or, when they are considered, it is in a very simplified way. In this paper we presented two types of problems in which transmission network representation, in terms of first and second Kirchoff’s law, is essential in hydrothermal optimization. By having solved problems with more than 100 large plants, 90 thermal plants and a network of 3300 buses and 4900 circuits, the results presented showed how flexible and efficient can be an implementation of the SDDP algorithm to address the stochastic transmission constrained and large scale hydrothermal optimization problem.

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VIII. REFERENCES


IX. BIOGRAPHIES

Sérgio Granville has BSc. degree in Mathematics from PUC/RJ and a PhD degree in Operations Research from Stanford University. He joined PSR in 2000 and is currently engaged in risk management for energy markets and software development for power systems. He has authored and co-authored several technical papers in PSPO and is a member of the IEEE PES.

Gerson Oliveira has a BSc degree in EE from PUC/Rio and a DSc degree in Systems Engineering from COPPE/UFRJ. He joined PSR as an associate consultant in 1999, where he is currently involved with transmission planning models. Previously Mr. Oliveira worked at CEPEL, the Brazilian power research center, where he was a project manager and senior researcher in the areas of optimization and statistical techniques applied to PSPO.

Luiz M. Thomé has a BSc in EE from UFRJ, and was awarded post-graduate credits in power system engineering from COPPE/UFRJ. He joined PSR in 1997, where he directs network analysis software development and research on transmission tariffs and transmission planning issues. Previously Mr. Thomé managed the development of Cepel's network analysis software, used by most Brazilian utilities (AC power flow, contingency analysis, equivalent network, voltage sensitivity, electromechanical transient analysis).

Nora Campodónico has a BSc degree in Statistics from the Catholic University of Lima and a DSc degree in Systems Engineering (optimization) from COPPE/UFRJ. She joined PSR in 1992, where she coordinates the development of software products in the areas of stochastic hydrothermal scheduling, power system planning under uncertainty and competition, reliability evaluation and optimal pricing.

Maria de Luján Latorre has a BSc degree in Mathematics from UNR, Argentina and a PhD degree in Systems Engineering (optimization) from COPPE/UFRJ. She joined PSR in 2001, where she is working on the development and study in Power Systems Optimization area. Previously, she worked at CEPEL (the Brazilian electrical research center), where she developed computational tools for optimal power flow.

Mario Veiga Pereira has a BSc degree in EE from PUC/Rio and DSc degree in Optimization from COPPE/UFRJ. He is currently engaged in regulatory studies and the development of new methodologies and tools for risk management in competitive markets. Previously he was a project manager at EPRI's PSPO program and research coordinator at Cepel, where he developed methodologies and software for expansion planning, reliability evaluation and hydrothermal scheduling tools. He was one of the recipients of the Franz Edelman Award for Management Science Achievement, granted by ORSA/TIMS for his work on stochastic optimization applied to hydro scheduling.

Luiz Augusto Barroso has a BSc in Mathematics and an MSc degree in Operations Research, both from UFRJ. He joined PSR in 1999, where he has been working in project economics evaluation, system planning studies and market power in energy markets. He is a member of IEEE-PES and has been a speaker on deregulated energy markets issues in several countries.