A vibro-acoustic signature in the context of power transformer tap-changers comprises a burst of transients generated by a sequence of electromechanical events. Comparison of vibro-acoustic signatures lacks precision unless the events in the sequence are synchronised to make juxtaposition of the signatures possible. Time warping offers a solution of this problem in that it temporally reorganises the position of the elements of one of the signatures so that it matches the time frame of the other. Furthermore, the scale of the time warp has a useful diagnostic function. However, the dynamic time-warping (DTW) algorithm used in speech processing is inapplicable in the context of electrical switchgear since some discontinuities are present in the time frame. To overcome this DTW limitation, we propose a multi-scale correlation algorithm which processes these discontinuities adequately and allows us to find the time relationship between the events of two signatures. This paper presents the operating principle of both these algorithms along with results that bear witness to the sharp time-warping correction obtained with multi-scale correlation. This correlation can even correct the order of appearance of events so that the time sequences are juxtaposed and comparison of signatures can be undertaken.

© 1999 Academic Press

1. INTRODUCTION

Specific knowledge about the health of distribution substation equipment is becoming more and more crucial for the proper operation of an electric power grid. Power equipment used for switching electrical apparatus in substations is exposed to strong operating stresses and consequently merits special attention when maintenance is performed. On-load tap changers (OLTC), for example, cause about 50% of the faults that occur in the transformers on which they are installed. The OLTC is a switching device specifically designed to regulate the distribution voltage by selecting the number of turns required in the secondary transformer.

One source of information about the health of a system is the vibro-acoustic signature from the switching operation of a particular unit of apparatus. When one such signature is recorded by an accelerometer and sampled at an appropriately selected rate, it has a useful spectral content which can reach over 30 kHz. This signature can be either reduced to a single envelope or split into several frequency bands. In the case of an OLTC, the signature comprises various, more of less superposed transients corresponding to electromechanical sequences that occur in parallel and it is therefore a complex one to handle. The relative position of each transient is not exactly identical from one signature to the other; the difference may be barely perceptible or, on the other hand, quite substantial. The amplitude, the frequency content and even the time position of the transients provide useful data for establishing a diagnostic.
The so-called absolute diagnostic method consists in comparing the signature with a set of tolerable amplitudes, referred to here as the ‘amplitude chart’, together with the time positions of the events, or timing chart. This method is used when the equipment is put into service or for an unscheduled verification. The so-called relative method, for its part, consists in comparing the signature recorded at different points in time for the same piece of equipment. On-line monitoring is based on the latter approach, which is more sensitive for detecting the appearance of a fault. However, it is difficult to compare signatures when the transients are not located precisely at the same points in time and do not have the same duration. In this case, deviation or ‘warping’ of the time frame occurs. Incidentally, note that the time warp contains diagnostic information that may be just as pertinent as the amplitude of the transients.

Figure 1 shows a slight difference in the time frames between two signatures. In this example, it is the viscosity of the oil, affected by the temperature, which has caused the variation in the time frame: the signature recorded at 23 h has a 3.5% lead over that recorded at 18 h, resulting in the appearance of a peak at the point where the amplitude was low on the previous signature. Moreover, fault tests performed on equipment at the plant [1, 2] have revealed that a loose spring can also affect the time frame. Lastly, a significant difference can lead to a change in the order of occurrence of the events: the time sequence of the appearance of transients is no longer the same. This situation is better described as time folding, a more severe type of time warping.

In order to compare signatures, it is therefore essential to ‘unwarp’ them in time. This implies that each transient must be superimposed on its counterpart and, also, that any transient with time warping must be straightened out so that it matches its equivalent in the reference signature as closely as possible.

Two solutions can be envisaged when it comes to signature unwarping. The first consists in comparing only the signatures recorded under identical operating conditions, i.e. at the same load and same temperature: unless there is a fault, these two signatures will have a similar time frame. The disadvantages of this solution are, first, that it is costly in memory space allocation and, second, that it raises a number of different problems including incursion into zones where no reference signature is recorded for some time after the monitoring system has gone into operation. The second solution involves unwarping the different time events or transients in order to obtain a comparison that will not be affected, or not much, by the time warp. It is this second solution, illustrated in Fig. 2, which will be studied in this paper. The unwarping begins with a pattern comparison between a signature.
and a reference signature. The latter, more often than not, is a signature average. The result of time warping is a ‘mapping’ between the time indices of the two signatures. This mapping is subsequently used to unwarp the new signature and is also sent to the diagnostic module. The unwarped signature can also be added to the signature average.

We then describe a time-warping algorithm in wide use together with a new one, which we find more suitable for comparing vibro-acoustic signatures, and give a few details about the module providing the ‘unwarped’ signature. The following sections present three application examples of signatures affected, respectively, by a small time warping, a sudden significant time warping and a time folding.

2. CONVENTIONAL DTW ALGORITHM AND ITS LIMITATIONS

For over 25 years, speech processing has been using an algorithm known as dynamic time warping (DTW) based on dynamic programming [3–5]. This technique is currently used in a commercial product for the diagnosis of high-voltage circuit breakers [6, 7] and in various applications such as ultrasound image processing [8]. It will be seen that, although the DTW algorithm is suitable for speech processing, it is not at all appropriate for comparing vibro-acoustic signatures. Moreover, the computational complexity characterising speech processing has been the focus of several publications dealing with the possibility of reducing it [9–11]. Yet the monitoring of electrical switching equipment does not call for real-time analysis because the switching operations are usually performed at intervals of a few minutes and the fact that one is not recorded from time to time makes no difference: the computation time is not such a great constraint, whereas the accuracy of the time unwarping is important.

Let us take two discrete time-frequency signatures $A = \{a_1, a_2, \ldots, a_n, \ldots, a_N\}$, and $B = \{b_1, b_2, \ldots, b_m, \ldots, b_M\}$. The indices $n$ and $m$ refer to a constant time-step scaling. The numbers of samples $N$ and $M$ are usually similar. When the signatures are spectrograms, $a_n$ and $b_n$ are vectors containing the short-time energy spectrum of a measured vibro-acoustic signal. We also call these vectors the time slice of a spectrogram. The desired solution is a dynamic time-warping function

$$m = w(n)$$

which will relate these two signatures is such a way as to minimize the time-normalised distance

$$D(A, B) = \frac{1}{N} \sum_{n=1}^{N} d(a_n, b_{w(n)})$$
between them where
\[
d(a_n, b_m) = \|a_n - b_m\|
\] (3)
is the local distance. The boundary conditions are
\[
w(1) = 1
\] (4)
for the initial endpoint and
\[
w(N) = M
\] (5)
for the final endpoint. It is easy to write the time-warping function when there is a linear discrepancy in the time scale between the two signatures. In this case, the function takes the form
\[
w(n) = \left[\frac{(M - 1)(n - 1)}{N - 1}\right] + 1
\] (6)
with the boundary conditions expressed in equations (4) and (5). In more general terms, the difference in the time frame should be corrected using a time-warping function of \(n\), which can be obtained locally by starting from the origin \(n = m = 1\) and progressing, time slice by time slice. At each step, minimisation of the time-normalised cumulative distance determines whether the progress of a signature should be accelerated or held back by one slice in relation to the other signature. Translated into equations, we have the rule
\[
w(n + 1) - w(n) = \begin{cases} 
0, 1, 2 & \text{if } w(n) \neq w(n - 1) \\
1, 2 & \text{if } w(n) = w(n - 1) 
\end{cases}
\] (7a, b)
which defines the rate of progression of the signature \(B\). In other words, equation (7) specifies that signature \(B\) be delayed by one time slice with respect to \(A\), that is \(w(n + 1) - w(n) = 0\), insofar as it was not delayed by one time slice the previous time. When \(w(n + 1) - w(n) = 2\), signature \(B\) is advanced by one time slice with respect to \(A\), and, lastly, the two signatures progress at the same rate when \(w(n + 1) - w(n) = 1\). It should be mentioned that, at each step on \(A\), we evaluate few possibilities of pathways with corresponding cumulative distance and retain only the lowest distance for each pathway. To accomplish the latter task, our algorithm resembles other optimised algorithms [11] in that it uses a circular buffer containing a limited number of pathway possibilities.

Figure 3 shows a particularly illustrative example of an unwarping trace. The first four steps are shown with arrows. Steps 1 and 3 correspond to a zero increment in the time-warping function, step 2 to an increment of 1 and step 4 to an increment of 2. The shaded area represents the region of possible paths.

The rule defined in equation (7) commonly used for speech processing [12] but it has the disadvantage of being unable to superimpose a transient on its counterpart when the two transients are separated by a number of steps more than the half-width of these transients. It is only in the presence of the first transient to be detected that this rule accelerates the rate of progression on the other signature in order to catch up with the corresponding transient. The maximum and minimum unwarping slope allows progress to be anywhere between twice as fast and twice as slow. This restriction is useful for speech processing where there is no pause in the middle of a word and no change in the order in time of arrival of the phonemes. Moreover, progress is gradual from one phoneme to another without any possibility of superposition (i.e. the sounds ‘a’ and ‘u’ being uttered at the same time). In the case of tap-changers, as with circuit breakers, however, we have a sequence of transients where there is no continuity between the end of one transient and the beginning of a new
one: a spectral sequence generated by one event is suddenly replaced by another, which tries to superpose itself on or replace the first. The algorithm cannot keep track of this sudden jump because there is nothing to tell it whether a new transient is present before the other appears. This algorithm is only effective in the presence of a time warp that has a marked limitation on the slope. One solution might be to modify the rule announced in equation (7). For example, a variant of this algorithm exists in which the constraint governing the boundary conditions given in equations (4) and (5) is less stringent [13]. At a stretch, jumps over a larger number of steps could be tolerated at each stage of the comparison, which boils down to increasing the maximum slope and reducing the minimum slope (see Fig. 3). This produces another problem however: whenever an object correlates too well with a series of consecutive objects present in the second signal, a blockage occurs in the progression. This is how the rule expressed in equation (7b) manages to avoid by forcing the progression forward on both signals. The real failing of this algorithm, despite its popularity in speech-processing applications, is the highly local decision rule acting on the smallest scale, i.e. by $\pm 1$ time slice, as opposed to a multi-scale decision and action. We will compare this algorithm’s performance with that of the proposed algorithm later.

3. TIME UNWARPING BY DYADIC MULTI-SCALE CORRELATION (DMSC)

3.1. OPERATING PRINCIPLE

If we try to adjust two signatures by hand, intuitively we start by superimposing the entire trace of each and then take what seem to be the objects comprising each signature and superimpose them among themselves. Since some of these objects can be split into sub-objects, we can also carefully superimpose each sub-object of the objects concerned. We will also take care to set aside from the comparison process any objects that do not have a corresponding object in the other signature.

It is not easy to automate a process like this because the human eye carries out several very complex image-processing processes without us being even aware of it and which for
the most part remain unknown to us. However, the algorithm we have developed is based on the mental image that we would form of the task if it were entrusted to a human being equipped with a pair of scissors. The first stage consists in adjusting the two signatures in their entirety by performing a cross-correlation in which the position of the maximum determines the overall time warp. Rather than the latter value, however, we take the result of the cross-correlation between the signatures and duplicate it in order to fill in a matrix like the one shown in Fig. 4. The symbol $\otimes$ represents the cross-correlation. Subsequently, we cut one of the signatures into two and perform two cross-correlations to fill in the two corresponding portions of the matrices. The results of each cross-correlation are added to the result of the first cross-correlation. Then each half-signature is again cut into two and we perform four cross-correlations, add the results to the previous results, and so on. Thus, the view that we have of the signal becomes multi-scale: we correlate the entire signature, the constituent parts of objects, the sub-objects comprising each object, etc. The trick is to select the right way of arranging the cross-correlation results for doing the summation. Each result is placed in such a way that the correlation maximum occurs in the middle of the matrix, in this case of a vertical line, if we have two identical signals to compare. Any blank spaces are filled with zeros. The summation result is therefore a matrix comprising elements created by the sum of cross-correlations performed on different scales. From this matrix, referred to as the warp matrix, it will be seen that we can extract the warp trace which, to the nearest slope, matches the dynamic time-warping function $w(n)$. The warp trace provides information on the time differences between the signatures and allows us to adjust them for the purpose of comparison.
One element $x_n$ of the vector $X = \{x_1, x_2, \ldots, x_N\}$ shown in Fig. 4 can be a time sample or a vector containing a portion of the signature envelope. If the raw signal is split into a multi-band signal, one element $x_n$ can also be a matrix containing the values of one or several time samples recorded over a few frequency bands. Lastly, one element $x_n$ may be a vector containing a time slice of the time-frequency distribution of the signature \[14, 15\].

A spectrogram \[15\] or a wavelet transform \[16, 17\] may be used to generate this distribution. For a spectrogram estimated by the power of a windowed short-time fast Fourier transform (WSTFFT), one time slice corresponds to one fast Fourier transform (FFT).

Lastly, the algorithm described here was developed for $X$ and $Y$ vectors of identical dimension.

3.2. DYADIC MULTI-SCALE CORRELATION (DMSC) AND WARP MATRIX DISPLAY

If we use the term ‘multi-scale correlation’, it is because the same part of the signature is subjected to successive correlations on increasingly reduced scales. We cut one of the vectors at systematic locations distributed in such a way as to optimize the computation time, hoping that the number of cuts and the resolution obtained with the smallest increment will offset the choice of a cut not based on the shape of the signature. DMSC is based on this approach. The term ‘dyadic’ refers to the way the portions signatures are cut: the objects of the computation are split into two at each step. The term ‘multi-scale correlation’ is also used by Yu Fan et al. \[18\] for ultrasonic strain reconstruction. In addition, multi-scale approach is largely used in image processing \[19–22\]. However, the latter algorithms are different from the one proposed here.

The number of cuts determines the size of the warp matrix and the time resolution of the process. In the specific context of tap-changer monitoring, multi-scale correlations are performed on time-frequency signatures with 64 to 256 cuts. Thus, we obtain matrix dimensions ranging from $64 \times 127$ to $256 \times 511$: the memory space required for handling the warp matrix is considerable.

Expressed more formally for a general case, we define a matrix scalar product

$$x_n \otimes y_m = \sum_i \sum_j x_{n,i,j} \cdot y_{m,i,j}$$

(8)

with the matrix norms expressed as

$$\| x_n \|^2 = x_n \otimes x_n \quad \text{and} \quad \| y_m \|^2 = y_m \otimes y_m.$$  

(9)

We use a normalised correlation function

$$\text{corr}_m \{X, Y\} = \frac{\sum_{n=1}^{N} x_n \otimes y_{n+m}}{\sqrt{\sum_{m=1}^{N} \| x_n \|^2 \Psi_x \{n + m\} \sum_{m=1}^{N} \| y_{n+m} \|^2 \Psi_y \{n\}}}$$

(10)

with $\Psi_x \{i\}$ defined as the existence function where $\Psi_x \{i\}|_{x \text{ exist}} = 1$ and $\Psi_x \{i\}|_{x \text{ do not exist}} = 0$. Without normalisation, the element $y_{m+j}$ with the greatest amplitude will always predominate and will draw the correlation peak towards it. From the energy point of view, a normalised correlation gives the proportion of energy that has been correlated while for the vectors, it gives the square of the cosine of the angle between $X$ and $Y$. It will also be noticed that the correlation is ‘non-centered’, in other words, the mean value has not been removed before the computation. However, it could happen that certain types of signal call for the mean value to be eliminated first.
Based on the normalised correlation defined in equation (10), the DMSC can be written

$$\text{corr}_{m,n}(X, Y) = \frac{\sum_{k=0}^{\lfloor N/I \rfloor} P^k \cdot \text{corr}_{m}(X, Y) |_{X_k = \lfloor I(n-1)/2^{k}\rfloor/N/2^k, \int \lfloor I(n-1)/2^{k}\rfloor/N/2^k |_{X_k = 0} \text{ elsewhere}}} {\sum_{k=0}^{\lfloor N/I \rfloor} P^k}$$

(11)

for $n = 1, 2, \ldots, \lfloor N/I \rfloor$ and $m = 1, 2, \ldots, 2N - 1$, with the weighting coefficient $P$. The warp matrix dimension is $(2N - 1) \times \lfloor N/I \rfloor$. The weighting factor $P$ is fixed around unity and serves to increase or reduce the weight of the correlation with the scale change of the interval correlated. The variable $I$ which determines the number of consecutive elements $y_n$ of the smallest portion of signal is to be correlated. Usually, we fix $I = 1$ for time-frequency signatures and $I > 2$ for envelope signatures. It should be noted that Fig. 4 is easier to understand than equation (11): in fact the idea of multi-scale correlation is based on a mental image rather than an abstract equation.

The warp matrix or its transpose can be displayed in different ways including pseudo-3D with the surfaces in relief, for example, or in 2D with variation in the light intensity (Fig. 5). The display of the warp matrix reveals immediately the presence of the warp trace near a line passing through the centre of the matrix. The warp trace represents the location where...
the multi-scale correlation is maximum. The position of this trace is referenced so that a null position corresponds to an identical time frame for the two signatures. Signatures \( X \) and \( Y \) are related by the warp trace such that

\[
x_{n + T(n)} \approx y_n.
\]

Since the DTW warping function links the signatures in a similar manner, i.e.

\[
b_{w(n)} \approx a_n,
\]

the warp trace \( T(n) \) and the warping function \( w(n) \) are two functions that apply a similar transformation and are related by the equation

\[
T(n) \approx w(n) - n.
\]

However, in contrast to the DTW function \( w(n) \) which can only develop by \( \pm 1 \) element per increment, the warp trace can contain marked discontinuities covering dozens of matrix elements.

3.3. EXTRACTION OF THE TIME WARP TRACE

Let us define the auto-warp trace as follows:

\[
T_0(n) = \text{indice}(\max_l \{\text{corrm}_{n,l}(Y, Y)\})
\]

as the trace resulting from multi-scale correlation of a signature with respect to itself. The auto-warp trace is located in the column at the centre of the warp matrix and ideally has a value independent of the signature \( s_1 \) which is constant for any value of \( n \). If there is no interpolation of the warp trace, we usually have \( T_0(n) = N \). However, the position of the auto-warp trace may be slightly different, by less than one element of the column index, depending on the type of numerical interpolation used. For example, for some 1D or 2D interpolation algorithms, the warp trace contains deviations which may be amplified near the edges of the warp matrix. These deviations represent an artifact of numerical manipulations and shift the warp trace. However, this can be compensated and the value of the warp trace position can be shifted by \( N \) if we define the value of the warp trace

\[
T(n) = \text{indice}(\max_l \{\text{corrm}_{n,l}(X, Y)\}) - T_0(n)
\]

as the index value of the position of the correlation maximum subtracted from the position of the auto-warp trace. The latter function yields the magnitude of the time deviation directly, according to the index \( n \) between the signatures.

The time warp trace is readily visible to the naked eye and all we have to do is to put the indices corresponding to the maximum correlation values into \( T(n) \), as indicated by equation (16). However, jump in an index value corresponds to an increase of several time samples on the original signature. For a discrete spectrogram, for example, we usually have several dozen such samples between two consecutive FFTs, each corresponding to an increment in the index \( n \). It is for this reason that we use a method of interpolating the warp trace which allows us to better define the position of the correlation peak. This interpolation significantly increases the time resolution of the extraction of the warp trace. In practical terms, we fix the interpolation resolution in such a way that it corresponds to the sample period of the raw signal.

Figure 5 shows an example of warp trace extraction with interpolation where a time lag of 0.65 ms (43 samples) can be seen between two elements adjacent to the matrix. There is a difference of 4 matrix elements between the first and the fifth transient, i.e. 2.6 ms, which corresponds approximately to the time lag observed between the envelopes shown in Fig. 1.
3.4. CORRECTION OF WARPED SIGNATURE BASED ON THE WARP TRACE

Generally speaking, it seems impossible to correct the raw signal without dissociating the different transients that overlap each other. In fact, a time re-sampling called for by the warp trace would change not only the position of the transients but also their frequency content. Ideally, we should be able to extract each transient separately, modify its position and add them together to reconstitute the signal. In practice, the newly developed algorithm corrects the time flow of the envelopes or spectrograms of the signatures but not the raw signal.

In order to correct an envelope, we simply interpolate it to the positions $I(n + T(n))$ where $I$ is the number of time samples per element index of the warp matrix. To correct a spectrogram, we recalculate it by sliding a spectral window over the raw data using an increment of $d(n + T(n))$ rather than a constant increment $d$ corresponding to the distance between two consecutive FFTs.

3.5. APPLICATION OF DMSC TO ENVELOPES OF VIBRO-ACOUSTIC SIGNATURES

The envelope can be computed in different ways, by decimating the mean quadratic value of the signal, by using the Hilbert transform or taking the peak value of the signal in the decimation interval. A vibro-acoustic signature of a tap-changing operation sampled at 60 k samples/s thus gives a smooth, decimated envelope with a sampling rate of 2–5 k samples/s.

Figure 6 shows an analysis similar to that in Fig. 5, but this time it is an analysis of the envelope of the signal rather than the spectrogram. With a decimation rate of 16, and 12 000 raw data yield $N = 750$ such envelope samples. The vertical scale of Figure 6 represents envelope samples and shows the warp trace close to the $m = N = 750$ value: a vector element $x_n$ contains only one envelope time sample. For the horizontal scale, we fix $I = 2.9$ in such a way as to limit the number of cuts to 256. The result of the multi-scale correlation on the signal envelope is slightly different from that obtained from spectrograms. Comparison of the envelopes seems to be less accurate. Actually, in the case of envelopes, correlation between the portions of two different transients for a same signature is too close to the correlation value obtained for a portion of the same transient observed for two signatures.

We can also distinguish the presence of strong cross-correlations between non-related transients generating lighter horizontal bands. It is mainly the large-scale shapes of the transients and their layout that reveal the warp trace from the envelopes. It is possible to display this trace despite the meagreness of the data provided by the envelope.

3.6. APPLICATION OF DMSC TO TIME-FREQUENCY SIGNATURES

The comparison is always between signatures with identical spectral processing parameters: the type of spectral window, the window width, the number of frequency bands and the way the spectral mean in a band is computed. On the other hand, the time duration may
vary between two signatures. In the correlations that we undertook in this work, the log of the spectral density gives the best result as far as the amplitude or the spectral density is concerned. The warp matrices presented in this paper are therefore from signatures formed from the log of the spectral density.

Normalisation of the warp matrix by the denominator of equation (11) is such that the value is unity when the correlation is perfect (white in the illustrations of the warp matrix). Application of time warping to time-frequency signatures usually yields a warp trace over 0.8 for signatures from the same phenomenon. Figure 5 presents a comparison of signatures for a same tap-changing operation and unit weighting. For an isolated transient, the intensity below the warp trace warp and the width of the supporting lobe are, respectively, function of the relative amplitude of the transient and of the wealth of detailed information that is contains. Figure 7 presents the same analysis with a fixed weighting $P$ of 0.8. It may be seen that the warp trace is smooth in appearance, for the reason that any weighting less than unity reduces the importance of correlations on finer details. It is therefore possible to limit or encourage large shifts in the time warp of the signature. The weighting parameter $P$ is fixed by the user to adjust the smoothness of the warp trace to the context in which it is used.

4. BEHAVIOUR OF THE DTW AND DMSC ALGORITHMS IN THE PRESENCE OF DISCONTINUITIES IN THE TIME FRAME

Tap-changing signatures contain discontinuities on a small scale, namely a few increments on the time index. This is why the DTW algorithm does not work very well here and becomes noisy as soon as a discrepancy occurs between two transients on the smallest scale. Figure 8 shows time deviations estimated by the DTW and DMSC algorithms. In this example, as in many others, the DTW algorithm shows time deviation oscillations between signatures on a small scale. On the other hand, the DMSC algorithm has a lower oscillation amplitude and a better resolution provided by the interpolation. Moreover, it yields similar results for signatures that have been interchanged, i.e. when the trace is extracted from
Figure 8. Time deviation observed by the DMSC and DTW algorithms in the comparison of tap-changing signatures shown at the top of Fig. 5: - - - - - , DTW; ** , DMSC \( p = 0.8 \).

corr_{m,n} \{ Y, X \} instead of corr_{m,n} \{ X, Y \}. Naturally, the deviation has the opposite sign when the signatures are interchanged. Similarly, the time discrepancy is similar when we compare the estimated values obtained for the spectral density with those obtained for the log of the spectrum of the signal. If we smooth the details of the deviations by setting a weighting factor \( P \) of around 0.7–0.8, the DMSC has a stable performance.

In order to glean more detailed information about the behaviour of these algorithms, we altered one of the signatures and compared it with the original. Tests [23] were performed which revealed that a simple delay of several time steps introduced into the middle of a signature is not processed properly by the DTW algorithm if the circular buffer provided for the calculation of best possible paths is not wide enough. Figure 9 illustrates an attempt to prove the efficiency of the other algorithm, the DMSC, by cutting out a portion of raw signal and pasting it back 300 samples further on. The behaviour of this algorithm is very close to the ideal, whereas the DTW with a wide buffer has a poor response to the appearance of the discontinuity. In this example, the discontinuity moves the dynamic time-warping function \( w(n) \) within the region of possible paths but also outside the buffer width containing the expected paths for the DTW with a narrow buffer. This explains why the latter algorithm fails and the time-unwarping function \( w(n) \) progresses in a random fashion and cannot keep on a straight course in order to make up for the warp generated by the discontinuity. In other examples, when the discontinuity is more than a few steps but less than the buffer width, the mapping \( w(n) \) in 0, 1 and 2 step sizes limits the possible paths to a set of non-optimal solutions.

Figure 10 presents an example of inversion in the order of occurrence of two transients. In this simulation, we also cut the vector of time samples prior to performing the spectrogram. The DTW algorithm with a very wide buffer (filled with over 200 paths) failed: it is clearly unable to unwarp time folding where the time flows in the reverse direction at certain points. The DMSC algorithm, on the other hand, with the appropriately adjusted inter-scale weighting parameter \( P \), passes this test. It should be noted that both inverted transients stand out clearly on the spectrogram. Transients that are too similar could not have been put into the right order by the DMSC algorithm. The notion of dissimilarity between the time-frequency objects on different scales therefore comes up. The weighting \( P \) allows us only to choose between small or entire scales to give the time cut command. Introduction of the weighting function \( P(k) \) as being related to size could prove useful when objects show dissimilarities over a small number of intermediate scales. In this case, we can adjust the focus on the right scale to reverse the transient. Inversely, if knowledge of the
physical phenomenon demands non-inversion of these transients, $P$ or $P(k)$ should be adjusted to prevent inversion from taking place.

The last two tests represent difficult cases, proving how the DMSC algorithm performs well. Yet its conventional counterpart, the DTW algorithm, has an advantage in that it offers less computational complexity. Although this advantage is essential for continuous speech processing, it is not required for our application where we can allow ourselves to process the information quite some time after the recording. We should specify that the computation time needed with the DTW algorithm is less than few seconds whereas the DMSC takes more than 5 min with the latter example (CPU: M604 at 200 MHz). The greater computation time required by the optimised DTW is a linear function of the dimension of the signature, whereas in the case of the CMED, the computational complexity grows by $N^3 \ln(N)$ without optimisation and by $N^3$ with partial optimisation, where $N$ is the number of time increments in the signature. The increase in memory space requirements increases, respectively, by $N$ and by $N^2$ for these two algorithms. Among other things, 43 M-bytes are needed by LabVIEW™ to process 256-step signatures. It should be pointed out that LabVIEW™ is slow and inefficient for this type of processing. Implementation under Fortran, Pascal or C++ would have significantly reduce both the memory and the computational requirements.
5. CONCLUSION

A new algorithm has been developed for comparing vibro-acoustic signatures emitted by power switching equipment. The dyadic multi-scale correlation (DMSC) algorithm estimates the time warp for cases where the DTW is ineffective. In a monitoring system used with this type of equipment, multi-scale correlation makes it possible to identify differences resulting from time warping, as distinct from differences due to the amplitude associated with each transient. The warp trace extracted from the multi-scale correlation matrix, the time-mapping function, is used to correct the time warping on one of the signatures. The
signatures are then aligned making a detailed comparison of transients having the same origin, two by two. Moreover, differences observed in the warp trace contain useful diagnostic information. Simulations based on measured signatures reveal the great flexibility and high degree of robustness of the proposed algorithm.

In this paper, we have partially explored multi-scale correlation for the time warp of vibro-acoustic signatures. It is not difficult to imagine a non-dyadic multi-scale correlation algorithm with an inter-scale ratio of less than two, and correlation weighting by spectral window to smooth any rough parts, which reveal different advantages of the multi-scale correlation algorithm used in our demonstration. On the other hand, this algorithm could be used for image processing or for speech processing although in differed time only, since real-time processing involves a considerable amount of computation time for the DMSC algorithm.

ACKNOWLEDGEMENTS

The contribution of other members of the team dedicated to the development of a tap-changer monitoring system, namely Marc Foata, researcher and project leader, Mohamed Farhat, researcher, Réal Beauchemin and Jean-Yves Paqyin, technicians, is gratefully acknowledged. Without them, there would be no monitoring prototype, no data to be processed, and no signatures to be corrected. Mention should also be made of the three years collaboration between ABB Corp. and Hydro-Québec. In particular, we would like to thank Tord Bengtsson and his team at ABB Corp. for the valuable technical exchanges we have had. Lasty, we are grateful to Claude Rajotte of TransÊnergie, Hydro-Québec, for his support for the system development.

REFERENCES

14. 1996 Proceedings of the IEEE 84, Special Issue on time frequency analysis.