THE UNCERTAINTY OF DELAYS AT OPTIMAL PRE-TIMED AND VEHICLE ACTUATED SIGNALS

Francesco VITI, Henk J.Van ZUYLEN

Abstract. Although extensive research has been carried out in the past to assess the performance of control signals under different service mechanisms, little interest has been given to the way these control methods deal with the variability of traffic. This paper proposes a probabilistic approach to calculate delay and queue length distributions in time. This method has been used to compare the effects of simple pre-phased controls - based on delay minimization - and of vehicle actuated controls, which look at the actual headway distribution of vehicles without looking explicitly at vehicle delays. This study confirms the superiority of gap-based mechanisms over delay-based controls also in terms of delay variability, given their property to adapt green and cycle lengths according to the real amount of traffic arriving during each cycle.

1. Introduction and problem description

Dynamic traffic signal control systems are designed with the scope of increasing network efficiency and safety. On the other hand, signal controls strongly affect the capacity of an intersection and, on a larger scale, to a part of a network. The importance of this performance measure is confirmed by its use in the definition of Level of Service of the Highway Capacity Manual [1].

One of the determinants of delays is the traffic demand. The dependency of delays on the number of arrivals has been analyzed and several models have been developed in the past, mostly based on heuristics (e.g. [2], [3], [4], [5]). The estimation of such delays is, in all models, based on average demand conditions, and it is simply an expectation value. In spite of the road capacity, which has relatively little fluctuations (for example due to adverse weather conditions, lane closures, accidents etc.), the arrivals at one intersection can be very different from one day to another. The uncertainty reflected to the delay that a traveler may experience can be therefore very large. Figure 1 shows travel time observations collected at an arterial road in the city of Delft, the Netherlands. The stretch of road is interrupted by two intersections with vehicle actuated signals. Data was processed

1 Delft University of Technology, faculty of Transport and Planning, Stevinweg 1 2628CN Delft, f.viti@citg.tudelft.nl and h.j.vanzuylen@ct.tudelft.nl
using camera detection. When the demand is low travel times do not increase considerably. During the morning peak several observations resulted in very large travel times, although the average demand of the total evaluation period is just close to capacity. As one can confirm by looking at the average travel time and the standard deviation during this period a traveler may drive at free flow during one day and experience the double of the expected travel time the day after.

![Figure 1: travel times observed at an urban road in Delft, the Netherlands](image)

Recently, transport policies are more and more asking attention to the analysis of the causes and the effects of travel time uncertainty and they call for solutions to reduce it. Very little consideration has been given in the past to the estimation of the queue and delay variability. Newell [6] formulates mathematically this problem using renewal theory. This approach inspired the work of Olszewski [7], who investigated the queue length distribution in time using a Markov Chain process.

In this paper we examine how different traffic control mechanisms cope with the variability of delays. To do so, we formulate the overflow queuing process as a Markov Chain for optimal pre-timed controls (section 2), which are based on delay minimization, and for vehicle actuated controls (section 3), which are gap-based. Comparison between these two control mechanisms is made in section 4 in terms of expectation value and of standard deviation. Finally conclusions are given in section 5.

2. The Markov model for optimal pre-timed controls

The signalized intersection system is governed by a cyclic mechanism, which allows the use of a discrete time analytical process instead of the continuous approach. Van Zuylen [8] described a Markov model for queues at isolated intersections assuming Poisson arrivals and normally distributed saturation flows. Olszewski [7] independently developed the idea of applying the Markov chain technique to signal control problems. The model described in this section is not different from the ones developed by these authors.

The stochastic queuing process is defined in its discrete time case to be a sequence of stochastic variables in time, where a state $Q_{t,i}$ is described by the previous state $Q_{t,i}$ and the number of arrivals $a_t$ and departures $d_t$ during the interval $[t, t+1]$, according to the
simple relationship $Q_{t+1} = \max\{Q_t + a_i - d_i, 0\}$. Assumed the probability distribution of the arrivals known and deterministic service rate, one can compute the probability of the queue length in time by first computing the transition probability from one cycle to another:

$$q_i(t) = \Pr(i = j + a_i - d_i) \quad \forall \quad j \geq i - d_i, \quad a_i \in [0, a_{\text{max}}]$$

(1)

which represents the probability that the queue length moves from a state $j$ at time $t-1$ to state $i$ at time $t$. The probability of a zero overflow queue comprises all cases where $j + a_i - d_i \leq 0$, while if a maximum value for the queue $Q_{\text{max}}$ is assumed (which can represent the maximum number of vehicles that can buffer at the road section without creating spillback effects), this value will comprise all values for which $j + a_i - d_i \geq Q_{\text{max}}$.

The queue length probability at time $t$ is therefore given by the following formula (2):

$$\Pr(j, t) = \sum_{i=0}^{Q_{\text{max}}} \Pr(i) \cdot q_i(t)$$

(2)

Figure 2: The distribution of overflow queue lengths in time

Figure 2 shows an example of overflow queue length distribution in time for $x=0.95$ and with zero initial queue. Even if the probability of having a zero overflow queue remains the largest chance, it reduces to only 50%.

Figure 3: Individual delay and optimal cycle length
The computation of delays is done by using the probabilistic model proposed by Olszewski [9] and it is left out in this paper. The presented model enables one to correctly evaluate the dynamic and the stochastic character of overflow queues at any loading condition. A direct consequence can be seen in the computation of the optimal cycle length for each demand condition. For example, the optimal cycle computed by Webster [2] is derived by assuming the overflow to be in equilibrium. This state can be reached in conditions of demand very close to capacity only after several cycles (in the example above after around 50 cycles). This means that the Webster delay formula overestimates the individual delay. As it can be seen in figure 3, this error reflects in the computation of the optimal cycle length, which is with the Markov model smaller than the one computed with the Webster’s optimal cycle formula. This error increases the closer the demand is to the capacity.

3. Markov model for vehicle actuated controls

Actuated control phase plans are in general determined by the headway distribution of the arrivals at the intersection. The basic mechanism is to extend the guaranteed green time (say, a minimum value) until the distance between two vehicles is larger than a certain threshold. This green time is usually constrained to be smaller than a maximum value, which is mainly determined by the intersection geometry. Therefore, the assigned green times and the delay incurred are stochastic variables too. The computation of green times is then subdivided into three terms: the green time given to serve the vehicles queuing up during the red phase, the one given to the vehicles queuing while the green phase is started and the green time extension given to vehicles arriving in sequence with short headways.

If $\lambda_i(\tau)$ is the arrival rate (in vehicles per second), and $r_i(\tau)$ is the red time at the previous cycle for stream $i$, one can compute the probability of a certain number of vehicles $k$ queuing up during the red phase (of length $\rho$) as:

$$P[Q_i(\tau) = k] = \int_{\rho_{min}}^{\rho} \left( P(\lambda_i(\tau) \cdot \rho = k) \cdot P(r_i(\tau) = \rho) \right) d\rho$$

(3)

The probability of a green time $g^i(t)$ needed to clear the queue at the end of the red phase to be a value $l$ is therefore given by:

$$P(g_i^i(\tau) = l) = \sum_{k=1}^{\infty} P(Q_i(\tau) = k)$$

(4)

While clearing the queue formed during the red phase, other vehicles may join the queue. These vehicles are computed by replacing the probability of red time in formula (3) with the green time of formula (4). The probability of green time due to all vehicles in queue $g^i$ is thus given by computing the joint probability of green due to vehicles arriving during the red phase and the ones arriving during the green phase (see also [10]).

The probability of green time extension is computed by computing all sequences of vehicles with headway shorter than the unit extension $\tau$. If one computes the probability distribution of a sequence of $n$ vehicles at times $0 < t_1 < t_2 < ... < t_n = t$ the probability of observing this sequence with $t_2 - t_1 < \tau$, $t_3 - t_2 < \tau$, etc. is given by the following formula:
\[ P(t_{\text{ext}} = t) = \sum_{n=0}^{\infty} P(t_1 < t_2 < \ldots < t_n = t) \cdot P(n, t) \quad \text{s.t.} \quad t_1 < \tau, t_2 - t_1 < \tau, \ldots, t_n - t_{n-1} < \tau \quad (5) \]

The probability of having an extension of exactly \( t \) seconds is then given by:

\[ P_i^t(g^i (\tau) = t) = \sum P(t_{\text{ext}} = t) \cdot P(g^i_{\text{max}} - g^i_0 \geq t) \quad (6) \]

The probability of a total green time \( G^\text{tot} \) is finally given by computing the joint probability of green given to clear the queue and the green time extension.

Overflow queues are likely to occur only when the intersection is oversaturated and the maximum green extension is met. The corresponding probability is computed by the following formula:

\[ P(Q_i^\text{ext}(t) = q) = \sum_{k - g^i_{\text{max}} \geq q} P(Q_i(t) = k) \quad (7) \]

Since an eventual overflow queue should be cleared in the next green phase, formula (3) should also consider that, apart from the arrivals, also the eventual overflow queue should be served (see [10] for details). Last step is to derive the probability distribution of the red times at the previous cycle. The corresponding probability of a red time to be a certain value \( r(t) = s \) is thus computed with the following formula:

\[ P(r(t) = \rho) = P(\sum_{j=1}^{n} g_j^\text{tot} (t-1) + TL = \rho) \quad (8) \]

Figure 4: expected green time and overflow queue for different demand conditions

4. Comparison of delay estimates

The application of the Markov model to these two control mechanisms enables one to analyze their efficiency also in terms of variability, according to the scope of this paper. Table 1 presents the results of the Markov models in terms of expectation value and standard deviation of the individual delay for different demand conditions. The corresponding degrees of saturations range from 0.7 to 0.99 with pre-phased controls.
Table 1: expected delay and standard deviation for $s=1800$ veh/s

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<th>500</th>
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<th>570</th>
<th>605</th>
<th>640</th>
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<td>22.6</td>
<td>26.6</td>
<td>32.1</td>
</tr>
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<td>6.5</td>
<td>8.1</td>
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</table>

It should be said that if a larger demand is loaded the optimal cycle method will result in too large cycles and the delay increases more steeply, since overflow queues will appear systematically. The vehicle actuated control can still give low delays and standard deviations for a much larger demand (as one can see from figure 4). This is easy to understand: low demands for pre-timed controls result in an unused part of the green time, while this does not occur at vehicle actuated controls because green are variable too.

5. Conclusions

This paper analyzed the behavior of optimal pre-phased controls and vehicle actuated controls in terms of expected delay and standard deviation. To do so, the individual delay at these traffic controls is modeled using Markov renewal theory, which enables one to compute a full probability distribution of the overflow queue and delay in time.

Conclusions from the study point at the superiority of vehicle actuated controls, during conditions of demand near capacity, for two reasons: 1) it outperforms the optimal pre-phased control in terms of both expectation value and standard deviation and 2) it enables one to serve a higher total demand at the intersection. This conclusion can be helpful information for practitioners and policy makers; for an intersection, which does not have to serve a large demand, the simple fixed control can suffice, while it is recommended to use a responsive control like the vehicle actuated control if this demand is close to the capacity.

References


