Cooperative Spectrum Sensing over Correlated Log–Normal Sensing and Reporting Channels

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Abstract—In this paper, we investigate the performance of a general multi–layer decentralized system setup for cooperative spectrum sensing, which includes realistic sensing/reporting channels and correlated Log–Normal shadow–fading in all wireless links of the cooperative network. Our investigations will show that, even though often overlooked in theoretical arguments and Monte Carlo simulations, and, in particular, novel approximation methods to account for correlated Log–Normal shadowing in cooperative spectrum sensing problems will be introduced.

I. INTRODUCTION

Cognitive Radio (CR) is commonly considered a key enabling technology to provide high bandwidth to mobile users via heterogeneous wireless architectures and Dynamic Spectrum Access (DSA) capabilities (see, e.g., [1], [2]). In particular, the inspirational principle of DSA relies on the broadly recognized experimental evidence of low spatial and temporal usages of the frequency spectrum [3]. The basic idea of DSA is to allow some users (which are often called Secondary Users, SUs) to dynamically borrow, for data transmission, unused frequency bands, provided that no harmful interference is generated towards those users either having high transmit priorities (i.e., gold users with a high quality of service) or owning the exclusive rights to transmit over a specific spatial correlation on the reporting channel are accounted for to design optimal decision strategies at the fusion center, but the reporting channel is still neglected in the analysis, and iv) in [23], approximation methods to analyze correlated Log–Normal shadowing on the sensing channel are introduced, but the reporting channel is still neglected. In

As recently noticed by [12], [13], cooperative spectrum sensing for CRs belongs to the general research area of decentralized and distributed data/signal fusion problems (see, e.g., [14], [15] and references therein). However, the vast majority of published results available in the literature so far have investigated the cooperative spectrum sensing problem under the typical assumptions of the so–called classical decentralized data fusion framework [14]. For example, i) in, e.g., [16], [17] the wireless channel is not considered in the analysis, and ii) in, e.g., [18]–[23] the effect of the wireless channel on the reliability and performance of the spectrum sensing process is taken into account only on the sensing channel and is overlooked on the reporting channel. To the best of the authors’ knowledge, only in a few contributions the authors have applied the guidelines of the emerging decentralized data fusion framework [14] to the analysis of cooperative spectrum sensing problems. For example, i) in [13], the authors consider correlated observations on the sensing channel, but an error–free reporting channel is still retained, and ii) in [7], [24] a noisy reporting channel is analyzed, but simplified fading channel models are still considered.

In particular, as far as the fading channel is considered, recent measurement campaigns, as well as extensive simulation and experimental activities, have pointed out the inconsistency of several assumptions that are typically used for analysis and simulation of mobile ad–hoc and multi–hop networks [25]–[27]. It has been shown that when the system to be analyzed gets distributed, one of the most important factors to be taken into account to get reliable performance predictions and system designs is Log–Normal shadow–fading, as well correlation effects that characterize this statistical phenomenon over short/medium distances [27]. Actually, the analysis of correlated Log–Normal shadowing for cooperative spectrum sensing has already been investigated by some authors, even though several limitations are retained in these analyzes (see, e.g., [7], [8], [11], [13], [18], [19]). For example, i) in [18], the effect of correlated shadow–fading on the system performance is analyzed, but only for a very large number of cooperative users and for a specific spatial correlation function; moreover, the reporting channel is assumed to be ideal and error–free, ii) in [19], Log–Normal shadowing is analyzed only via Monte Carlo simulations, and still neglecting the existence of a realistic reporting channel, iii) in [13], correlated Log–Normal shadowing on the sensing channel is accounted for to design optimal decision strategies at the fusion center, but the reporting channel is still overlooked in the analysis, and iv) in [23], approximation methods to analyze correlated Log–Normal shadowing on the sensing channel are introduced, but the reporting channel is still neglected. In
summary, the few contributions that try to take into account correlated Log–Normal shadowing for analysis and design of cooperative spectrum sensing only focus on one side of the communication system, i.e., the sensing channel, while the other side, i.e., the reporting channel, is often overlooked. However, recent results in [7], [24] have clearly illustrated (even though without considering shadowing propagation) that errors on the reporting channel yield fundamental performance limits for cooperative spectrum sensing, which are similar to Signal–to–Noise Ratio (SNR) walls in [28].

Motivated by the above considerations, the aim of the this contribution is to propose an advanced framework for analyzing the performance of cooperative spectrum sensing problems under the assumptions of the emerging decentralized data fusion framework, and to take into account realistic propagation conditions on both sensing and reporting channels. In particular, the contributions of the paper are as follows.

- **Realistic Wireless Channel Assumptions.** Differently from typical system setups where correlated Log–Normal shadowing is just considered on the sensing channel, we consider the effect of correlated Log–Normal shadowing on both sensing and reporting channels. In fact, regardless of the cooperative protocol used for data reporting, shadowing correlation is expected to exist in all wireless links (see, e.g., [7, Fig. 4.3] and [27, Fig. 2]).

- **Simple Cooperative Protocol for Data Reporting.** Moving from the good performance/complexity trade–off offered by the Amplify–and–Forward (AF) relying mechanism [29], we propose an AF–inspired cooperative setup for data reporting from local sensors to a fusion center. This allows to keep the complexity of sensors at a very low level, since each of them needs only to forward the sensed signal to the remote fusion center (see, e.g., [7], [30]).

- **Improved Framework for Modeling Correlated Log–Normal Shadowing.** It is well–known that no closed–form expressions to model the power–sum of generically correlated Log–Normal Random Variables (RVs) exist in the literature (see, e.g., [31], [32] and references therein). In this contribution, we move on and generalize recent results on modeling Log–Normal power–sums to take into account the particular needs of cooperative spectrum sensing over realistic sensing and reporting channels.

The remainder of the manuscript is organized as follows. In Section II, the multi–layer system model that accounts for realistic sensing/reporting channels is introduced. In Section III, the problem of cooperative spectrum sensing is formalized from the analytical point of view. In Section IV, the analytical framework for modeling correlated Log–Normal shadowing in the cooperative spectrum sensing scenario is introduced. In Section V, numerical and simulation results are shown, and the accuracy of the proposed method substantiated. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

A. Distributed Model for Cooperative Spectrum Sensing

Let us consider the general multi–layer decentralized detection problem shown in Fig. 1. In general, there is an event, which is often called ‘Phenomenon of Interest’ (PoI) [15], we are interested in detecting by using a cooperative network of dispersed local nodes. In this contribution, we assume PoI is the determination of either presence ($H_1$) or absence ($H_0$) of a Primary User (PU) transmitting over a given frequency band. In this latter scenario, we assume that some local nodes (SUs in Fig. 1) sense/observe a given frequency band independently and in a decentralized fashion. As shown in Fig. 1, the quality of the observations depend on the wireless channel between PoI and SUs (i.e., the “sensing channel”), which affects the sensing capabilities of each local sensor. As described in Section I, most literature on either generic data fusion (see, e.g., [14], [15]) or cooperative spectrum sensing (see, e.g., [7], [24]) problems makes the unrealistic assumption that the decisions of local sensors can be sent with high reliability (i.e., error–free) over a dedicated channel (i.e., the “reporting channel”) to a fusion center, which, upon reception of local data, produces an estimate of PoI by choosing either $H_0$ or $H_1$. Following the principles of the emerging data fusion framework, we remove this limitation and consider an unreliable wireless reporting channel, as shown in Fig. 1.

B. AF–Inspired Relay Protocol for Data Reporting

In particular, to consider realistic operations of local sensors when reporting their own local estimates to the fusion center, we consider the following protocol for data reporting. After local sensing, every local SU is assumed to act as a relay node, which forwards the sensed signal to the fusion center through a faded (reporting) channel. In particular, moving from recent advances in the field of cooperative wireless communications, we suggest to use the simple AF relay protocol for data forwarding/reporting [29]. Among the various possibilities for cooperative relying [29], the reason for choosing AF is twofold: i) it allows to keep the complexity of local sensors at a low level since only analog forwarding is required by them, and ii) it offers an implicit and simple way to report raw data to the fusion center, which it has been shown to improve spectrum sensing performance [17]. We also note that similar spectrum sensing relay protocols have been recently suggested in [7], [30]. However, their operational principle and motivation are different from those proposed in the present paper: we do not use relay for local messages exchange among SUs to avoid bad shadowed channels, but simply exploit the similarity of data fusion problems and cooperative relaying [33] to report raw data to the fusion center without assuming ideal reporting channels and to keep the complexity of local
sensors at a moderate level (no local detection, but simple analog forwarding). Finally, for the sake of simplicity, we assume every SU in Fig. 1 forwards its data over orthogonal sub-channels [29], so that we can neglect interference among SUs during data reporting.

### III. Problem Statement

#### A. Notation

According to Fig. 1 and Section II, we adopt the following notation, which is similar to dual-hop cooperative systems [33], [34]. i) PU is denoted by $S$ (i.e., source), ii) the fusion center is denoted by $D$ (i.e., destination), iii) $\{ R_l \}_{l=1}^L$ are the $L$ active SUs (i.e., relays), which report their local sensed signals to $D$, and iv) the dual-hop wireless link $S$--$R_l$--$D$ is the $l$-th branch of the network. Moreover, i) $G_{l,1}^L$ is the relay gain associated to relay $\{ R_l \}_{l=1}^L$, ii) $\{ \alpha_{l,SR} \}_{l=1}^L$ and $\{ \alpha_{l,RD} \}_{l=1}^L$ are the fading gains of sensing and reporting channels, respectively, iii) $N_0$ is the one-sided power spectral density of the Additive White Gaussian Noise (AWGN) at the input of $\{ R_l \}_{l=1}^L$ and $D$, iv) $\gamma_{SR} = \alpha_{l,SR}^2 E_s / N_0$ and $\gamma_{RD} = \alpha_{l,RD}^2 E_s / N_0$ are the SNRs of sensing and reporting links on the $l$-th branch, respectively, and v) $E_s$ is the average radiated power in every transmission.

Finally, i) $\alpha_{l,SR}^2$, $\alpha_{l,RD}^2$, i.e., the channel power gains on sensing and reporting channels, respectively, are assumed to be Log--Normal distributed and generically correlated RVs, as a consequence of shadow--fading propagation [27], and ii) motivated by [35], we consider the fusion center to be equipped with a simple Energy Detection (ED) for spectrum sensing, and to use a Square--Law Combining (SLC) [9] mechanism to combine the signals received from the SUs (this choice leads to combine in a Maximal Ratio Combining (MRC) fashion the received SNRs, see Section III-B).

#### B. Analytical Formulation

According to [9], the decision statistic of a SLC decentralized detector is $y_{SLC} = \sum_{l=1}^L y_l$. In particular, when the fusion center uses square--and integrate operations for every received signal (i.e., ED), we have $y_l = \frac{2}{\Psi_l} \int_0^{t_l} r_l^2(t) \, dt$, where $t_l$ (.) is the signal received by the fusion center from $R_l$. $\Psi_l$ is one-sided power spectral density of the noise component of $t_l$ (.), and $T$ is the observation (i.e., integration) window. By relying on the AF--inspired data reporting mechanism, the received signal from each SU is $r_l(t) = (\alpha_{l,SR} \alpha_{l,RD} G_l) s(t) + (\alpha_{l,RD} G_l) n_{RD}(t) + n_{l}(t)$, where $s(\cdot)$ denotes the presence or absence of the PU, i.e., $s(t) = 0$ if $H_0$, and $n_{l}(\cdot)$, $n_{RD}(\cdot)$ are the AWGNs at the input of terminals $R_l$ and $D$, respectively [33], [34]. Accordingly, we also have $\Psi_l = (\alpha^2_{l,SR} G_l + 1) N_0$.

Some important comments deserve to be made with regard to $y_{SLC}$. i) Only the fusion center needs to compute $\Psi_l$ for every received local observation, and this can be directly done by its own without any message exchange with the SUs ($\Psi_l$ is independent from $\alpha_{l,SR}$). In particular, the estimation of $\Psi_l$ can be done directly from $t_l$ (.) when PU is not transmitting, but SUs are still relaying during the PU’s silence periods. ii) In this contribution we will not analyze the effect of estimation errors on $\Psi_l$ [36]. This is left to a future contribution.

Given that the main aim of this paper is to analyze the performance of the AF--inspired reporting method for general settings, we will consider several options for choosing $\{ G_{l,1}^L \}_{l=1}^L$. In particular, two basic AF relay mechanisms will be considered: i) Channel State Information (CSI) [33], and ii) Fixed--Gain (FG) [34] relaying. In the former case, every SU is assumed to have full CSI about the sensing channel, and the relay gain is $G_l = G^CS_l = 1/\alpha_{l,SR}$ [33]. On the contrary, in the latter case the relays are not required to have instantaneous CSI. In particular, the relay gain is $G_l = G^FG_l = \sqrt{E_s/(C_l N_0)}$, where $C_l$ is a constant factor for blind relays [34], and it is a function of the Average CSI (A--CSI) on the sensing channel for semi--blind relays (see [37] for analytical details for this latter case).

Following similar analytical steps as described in [9], it is possible to show that, for both relaying mechanisms, $y_{SLC}$ follows either a central or non--central chi–square distribution depending on PU is not $(H_0)$ or is $(H_1)$ transmitting, respectively. Accordingly, the performance of the distributed spectrum sensing problem in Fig. 1 can be characterized by two performance measures, i.e., False Alarm Probability ($P_{fa}$) and Detection Probability ($P_d$). These metrics can be computed as follows, when removing the conditioning upon fading channel statistics on both sensing and reporting channels:

$$
\begin{align*}
\text{P}_{fa} & = \frac{\Gamma\left( LN/2, \lambda/(2\sigma^2) \right)}{\Gamma\left( LN/2 \right)} \\
\text{P}_d & = \frac{1}{\lambda} Q_{LN/2} \left( \sqrt{a\xi p/\sigma^2}, \sqrt{\lambda/\sigma^2} \right) \, d\xi \tag{1}
\end{align*}
$$

where the following definitions have been used: i) $\Gamma(\cdot)$ is the Gamma function [38, pp. 255, Eq. (6.1.1)], ii) $\Gamma(\cdot)$ denotes the incomplete Gamma function [38, pp. 260, Eq. (6.5.3)], iii) $Q_{LN}(\cdot)$ is the generalized Marcum Q–function [19, pp. 73], iv) $N$ is the number of degrees of freedom of the system [9], v) $\lambda$ is the detection/decision threshold used by the fusion center in the binary hypothesis testing problem to discriminate between presence and absence of a PU, and vi) $\sigma^2 = 1/a = 2$. Moreover, $f_p(\cdot)$ is the Probability Density Function (PDF) of the end–to–end SNR, $\gamma_l$, at the fusion center. Depending on the chosen AF relay mechanism, the latter SNR can be explicitly written as $\gamma_l = \sum_{l=1}^L \gamma_{l,t}$, where [33], [34]:

$$
\begin{align*}
\gamma_{l,t} & = \begin{cases} 
\frac{\gamma_{SR} G_l}{\gamma_{SR} + \gamma_{RD}} & \text{CSI} \\
\frac{\gamma_{SR} G_l}{\gamma_{SR} + \gamma_{RD}} + \frac{1}{\gamma_{SR} + \gamma_{RD}} & \text{FG} \end{cases} \\
& = \begin{cases} 
\frac{C_l}{\gamma_{SR} + \gamma_{RD}} & \text{CSI} \\
\frac{C_l}{\gamma_{SR} + \gamma_{RD}} + \frac{1}{\gamma_{SR} + \gamma_{RD}} & \text{FG} \end{cases} \tag{2}
\end{align*}
$$

Furthermore, in (1) we have introduced a constant term $p$. We have done this for the sake of generality. As a matter of fact, two system setups have been proposed for spectrum sensing using EDs: 1) the first one (see, e.g., [9]) where $p = 1$, and 2) the second one (see, e.g., [19]) where $p = LN/2$. These system setups are related to different settings, with 2) being used more often than 1) by many researchers. Since in the next sub--sections we will propose approximations for performance analysis, and, in general, approximations can yield a different level of accuracy for different weight functions [31], we will analyze the reliability and numerical stability of our frameworks for both system setups in what follows.

Let us also note that the typical scenario with error--free reporting channels can be readily obtained from (2) when $\{ \gamma_{l,RD} \}_{l=1}^L \rightarrow \infty$. In such a case, we will have $\gamma_{l,t} = \gamma_{l,t}^{SR}$, and $\gamma_l$ will reduce to the well–known expression used in most analysis (see, e.g., [9], [23]), i.e., $\gamma_l = \sum_{l=1}^L \gamma_{l,t}^{SR}$. 

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Finally, as $P_{th}$ in (1) is independent from channel statistics, in the next sections we will be mainly interested in developing a simple but effective framework to compute $P_d$ in (1), which requires a closed-form expression for the PDF of $\gamma_t$. In Section IV we will show that the computation of $f_{\gamma_t} (\cdot)$ boils down to have accurate and simple methods for approximating the power-sum of generically correlated Log–Normal RVs. In particular, a novel two-step method will be proposed to handle this problem in the specific system setup under analysis.

IV. MODELING CORRELATED LOG–NORMAL POWER–SUM FOR COOPERATIVE SPECTRUM SENSING

As mentioned in Section I, even though there are several methods for approximating the power–sum of Log–Normal RVs for both scenarios with and without distributed diversity [31], none of them can handle correlation among all diversity branches of the distributed spectrum sensing problem depicted in Fig. 1. In particular, we have recently proposed a method for taking into account shadowing correlation for multi–hop networks [37], but we have always assumed uncorrelated branches, while correlation among sensing and reporting branches is the main focus of the present contribution. In order to solve this problem, we propose a novel two–step approximation method, and show that, with respect to [37], the approximation of a Log–Normal power–sum with another Log–Normal RV might not be accurate for most system setups. So, we will exploit non–Log–Normal approximation methods, by providing advances along three main directions [32]: i) these methods have been analyzed only for uncorrelated terms so far, while our interest is in correlated links, ii) these methods have not been analyzed for distributed detection problems and scenarios with cooperative diversity, while this is the main focus of this contrition, and iii) the accuracy of Log–Normal power–sum approximations, either with Log–Normal or non–Log–Normal RVs, has never been tested for computing either $P_d$ or the Miss Probability $P_{th} = 1 - P_d$, which are the performance metrics we are considering. In the subsequent sub–sections, we will propose a two–step approximation method and substantiate its accuracy in typical operating system setups. In particular, in Section IV-A we will show that for all system setups analyzed in this contribution $\gamma_t$ can be written as the power–sum of correlated Log–Normal RVs in a unified way; then, in Section IV-B we will introduce the two–step approximation method.

A. $f_{\gamma_t} (\cdot)$ as Power–Sum of Log–Normal RVs

By carefully looking at $\gamma_t^{\text{CSI}}$ and $\gamma_t^{\text{FG}}$ defined in Section III–B, we can easily figure out that in both cases the inverse of end–to–end SNR in every branch is given by the summation of correlated Log–Normal RVs. As a consequence, modeling the distribution of the SNRs in Section III–B is equivalent to find the distribution of the inverse of a linear combination of generically correlated Log–Normal RVs.

Before proceeding with the analytical derivation, it is convenient to re–write the SNRs $\gamma_t^{\text{CSI}}$ and $\gamma_t^{\text{FG}}$ in a unified way. In particular, they are expressible as

$$\gamma_{t,n} = \left[ \sum_{n=1}^{2} X_{(i,n)} \right]^{-1} = \left[ \sum_{n=1}^{2} 10^{\frac{1}{2} Y_{i,n}} \right]^{-1}, \text{ where } \{ Y_{i,n} \}_{n=1}^{2} \text{ is a vector of Normal RVs with mean vector } (\mu_{Y_{i}}) \text{ and covariance matrix } (\Sigma_{Y_{i}}) \text{ given in (3) and (4) for CSI and FG relays, respectively}^{1}:$$

\[
\begin{align*}
\text{CSI} : \\
\mu_{Y_i}(1) &= -\mu_{I,SR} - 10 \log_{10} \left( \frac{E_s}{N_0} \right) \\
\mu_{Y_i}(2) &= -\mu_{I,RD} - 10 \log_{10} \left( \frac{E_s}{N_0} \right) \\
\Sigma_{Y_i}(1,1) &= \sigma_{I,SR}^2 \\
\Sigma_{Y_i}(2,2) &= \sigma_{I,RD}^2 \\
\Sigma_{Y_i}(1,2) &= \Sigma_{Y_i}(2,1) = \rho_{I,SR,RD} \sigma_{I,SR} \sigma_{I,RD} \\
\end{align*}
\]

\[
\begin{align*}
\text{FG} : \\
\mu_{Y_i}(1) &= -\mu_{I,SR} - 10 \log_{10} \left( \frac{E_s}{N_0} \right) \\
\mu_{Y_i}(2) &= -\mu_{I,RD} - 10 \log_{10} \left( \frac{E_s}{N_0} \right) \\
\Sigma_{Y_i}(1,1) &= \sigma_{I,SR}^2 \\
\Sigma_{Y_i}(2,2) &= \sigma_{I,RD}^2 + 2 \rho_{I,SR,RD} \sigma_{I,SR} \sigma_{I,RD} \\
\Sigma_{Y_i}(1,2) &= \Sigma_{Y_i}(2,1) = \sigma_{I,SR}^2 + \rho_{I,SR,RD} \sigma_{I,SR} \sigma_{I,RD} \\
\end{align*}
\]

where i) $\mu_{I,SR}$ and $\mu_{I,RD}$ are the mean values, ii) $\sigma_{I,SR}^2$ and $\sigma_{I,RD}^2$ are the variances, and iii) $\rho_{I,SR,RD}$ is the correlation coefficient of RVs $\chi_{SR} = 10 \log_{10} \left( \frac{\alpha_{I,SR}}{\alpha_{I,RD}} \right) \text{ and } \chi_{RD} = 10 \log_{10} \left( \alpha_{I,SR}^2 \right)$, i.e., the Normal RVs associated to the Log–Normal power gains $\alpha_{I,SR}$ and $\alpha_{I,RD}$, respectively.

According to the above analysis, the problem of computing $P_d$ (or $P_{th}$) boils down to have general and flexible methods for managing the power–sum of correlated Log–Normal RVs. In particular, two main problems need to be addressed: i) firstly, the PDF of $\{ \gamma_{t,n} \}_{n=1}^{L}$ needs to be estimated, which results in the need to have efficient tools for approximating the power–sum of generically correlated Log–Normal RVs, and ii) secondly, the PDF of $\gamma_t = \sum_{n=1}^{L} \gamma_{t,n}$ needs to be computed, which results in dealing with the power–sum of correlated either Log–Normal or non–Log–Normal RVs depending on the assumptions done to compute the PDF of $\{ \gamma_{t,n} \}_{n=1}^{L}$ [32].

B. A Two–Step Approximation for Computing $f_{\gamma_t} (\cdot)$

To handle correlation among every pair of links in Fig. 1, we propose a simple yet accurate two–step procedure for computing $f_{\gamma_t} (\cdot)$, and then use it for the estimation of $P_d$ in (1). Although several methods are available in the literature [31], we will consider and generalize two of them: the Improved Schwartz–Yeh (I–SY) [37] and Pearson Type IV (P–IV) [32] methods. The reason for using these methods is twofold: i) they offer simple non–recursive closed–form (using Gauss Quadrature Rules, GQRs) formulas to compute the approximating parameters, without the need to use non–linear least–squares methods, and ii) when error–free reporting channels are considered, they have been shown to be very accurate for approximating $P_d$ [23].

1) Step 1 – Improved Schwartz–Yeh (I–SY) Approximation for $\{ \gamma_{t,n} \}_{n=1}^{L}$: The I–SY approximation method has been recently introduced in [32], only for independent Log–Normal RVs, as an improved version of the SY method proposed in 1982 [39]. The main idea of Step 1 is to approximate the Log–Normal power–sum $\gamma_t$, with another Log–Normal RV by using I–SY, as follows:

\[
f_{\gamma_t}(\xi) \equiv \frac{10}{\ln(10)} \exp \left[ -\frac{(10 \log_{10} (\xi - \mu_{I,SY}))^2}{2 \sigma_{I,SY}^2} \right]
\]

1We denote with $v(i)$ the i–th element of vector $v$, and with $M(i,j)$ the element in the i–th row and j–th column of matrix $M$.\]
where \( \mu_{1.1-SY} \) and \( \sigma_{1.1-SY} \) are the parameters of the approximating PDF, which are obtained via a moment matching in the logarithmic domain between \( \gamma_{l,t} \) and the approximating Log–Normal RV, i.e.:

\[
\begin{align*}
\mu_{1.1-SY} &= m_{(n)}^{(1)}(\Gamma_{\gamma_{l,t}^m}, \gamma_{l,t}), \\
\sigma_{1.1-SY} &= \sqrt{\frac{m_{(n)}^{(2)}(\Gamma_{\gamma_{l,t}^m}, \gamma_{l,t}) - m_{(n)}^{(1)}(\Gamma_{\gamma_{l,t}^m}, \gamma_{l,t})^2}{\Phi_p}}.
\end{align*}
\]  

(6)

where \( m_{(n)}^{(1)}(\Gamma_{\gamma_{l,t}^m}, \gamma_{l,t}) = (-1)^n E\{10 \log_{10}\left(1/\gamma_{l,t}\right)^n\} \)

and \( E\{\cdot\} \) denotes statistical expectation.

From (5), (6), it turns out that the I–SY method requires the computation of the log–moments \( m_{(n)}^{(1)} \) of the power–sum \( 1/\gamma_{l,t} \). These log–moments have been computed in closed–form by using GQR methods. By avoiding the lengthy but simple computation of the log–moments \( m_{(n)}^{(1)} \) \( (\Psi_{p}^{\prime}) \) of the power–sum \( 1/\gamma_{l,t} \), the analysis of the CSI and MRC setups in [37] can confirm. As a matter of fact, in [37] correlation arises only because of the second moment dependencies, and it needs to be generalized to the correlated scenario to be used in our context. As a matter of fact, in [37] correlation arises only because of the second moment dependencies, and it needs to be generalized to the correlated scenario to be used in our context.

Using Step 1, we can now approximate in closed–form the PDF of \( \gamma_{l,t} \) via a Log–Normal RV. We observe that even though in [31] it is pointed out that a Log–Normal approximation for the Log–Normal power–sum may fail to be accurate in some circumstances, Step 1 needs to deal only with the power–sum of two correlated Log–Normal RVs. It is well–known in the literature (see, e.g., [32] and references therein for Log–Normal power–sum approximation) that non–Log–Normal approximation methods should be preferred when the number of summands increases. So, in the present context a Log–Normal approximation seems to be a good compromise in terms of approximation accuracy and simplicity. We will prove this claim in Section V, where Monte Carlo numerical simulations will be presented to substantiate the accuracy of our two–step method.

2) Step 2 – Pearson Type IV Approximation for \( \gamma_{l,t} \): As a result of the I–SY approximation for \( \gamma_{l,t} \) in Step 1, the computation of \( f_{\gamma_{l,t}}(\cdot) \) boils down to the estimation of the PDF of the power–sum of generically correlated Log–Normal RVs. However, in this case the number of RVs, i.e., \( L \), involved in the summation is, in general, greater than two. As a consequence, to get very accurate results, we propose to use a non–Log–Normal approximation method to estimate \( f_{\gamma_{l,t}}(\cdot) \). Moving from the excellent matching accuracy at the PDF level shown by the P–IV method introduced in [32] for independent summands, we will generalize it to deal with correlated terms to be used in Step 2.

The P–IV method allows to approximate the PDF of \( \gamma_{l,t} \), \( f_{\gamma_{l,t}}(\cdot) \), as follows:

\[
f_{\gamma_{l,t}}(\xi) \approx \frac{10}{\ln(10)} h \left[ 1 + \frac{10 \log_{10}(\xi) + u}{d^2} \right]^{-m} \times \exp \left[ -\nu \tan^{-1} \left( \frac{10 \log_{10}(\xi) + u}{d} \right) \right]
\]  

(9)

where \( h \) is a normalization factor, and \( u, m, d, \nu \) are the parameters that define the P–IV distribution [32]. These latter parameters can be computed from the non–central moments of \( \gamma_{l,t} \) as shown in (10) on top of next page, where \( \mu_{(n)}^{(1)}(\cdot) \) and \( \Omega_{\gamma_{l,t}^m}(\cdot) \) are defined in (8) on top of this page, where \( \Sigma_{\gamma_{l,t}^m}^{eq} = \tilde{U}\tilde{V}^{1/2} \), and \( \tilde{U} \) and \( \tilde{V} \) are the matrices containing the eigenvectors and eigenvalues of \( \Sigma_{\gamma_{l,t}^m} \), which is the covariance matrix of SNRs \( \{\gamma_{l,t}\}_{l=1}^{L} \) in the logarithmic domain, i.e.,
\[ \Pi_{\gamma_{\text{all}}} (p) = \prod_{i=1}^{L} \frac{H_{p_i}}{\sqrt{\pi}} \]

\[ \Omega_{\gamma_{\text{all}}} (p) = \sum_{i=1}^{L} \exp \left[ \frac{\ln (10)}{10} \left( \sqrt{2} \sum_{j=1}^{L} \Sigma_{\gamma_{\text{all}}}^q (i,j) x_{p_j} + \mu_{\gamma_{\text{all}}} (i) \right) \right] \]  

(10)

\[ R_{\gamma_{\text{all}}} (l, n) = \sum_{p_1=1}^{N_p} \sum_{p_2=1}^{N_p} \sum_{p_3=1}^{N_p} \sum_{p_4=1}^{N_p} \left\{ \frac{1}{\pi} \right\}^2 H_{p_1} H_{p_2} H_{p_3} H_{p_4} \cdot \left[ 10 \log_{10} (\Psi_1 + \Psi_2) \right] \cdot \left[ 10 \log_{10} (\Psi_3 + \Psi_4) \right] \]  

(12)

\[ R_{\gamma_{\text{all}}} (l, n) = \sum_{p_1=1}^{N_p} \sum_{p_2=1}^{N_p} \sum_{p_1=1}^{N_p} \sum_{p_4=1}^{N_p} \left\{ \frac{1}{\pi} \right\}^2 H_{p_1} H_{p_2} H_{p_3} H_{p_4} \cdot \left[ 10 \log_{10} (\Psi_1 + C_l \Psi_2) \right] \cdot \left[ 10 \log_{10} (\Psi_3 + C_l \Psi_4) \right] \]  

(13)

\[ \Sigma_{Z_{l,n}} = \begin{bmatrix} \sigma_{SR}^2 & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} \\ \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \sigma_{SR}^2 & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} \\ \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \sigma_{SR}^2 & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} \\ \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \sigma_{SR}^2 \\ \end{bmatrix} \]

(14)

\[ \Sigma_{Z_{l,n}} = \begin{bmatrix} \sigma_{SR}^2 & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} \\ \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \sigma_{SR}^2 & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} \\ \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \sigma_{SR}^2 & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} \\ \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \rho_{(SR,SR)} \sigma_{SR} \sigma_{SR} & \sigma_{SR}^2 \\ \end{bmatrix} \]

In formulas, we have:

\[ \{ \mu_{\gamma_{\text{all}}} (l) \}_{l=1}^{L} = E \left\{ 10 \log_{10} (\gamma_l) \right\} = m_{l,\text{all}}^{(1)} \]

\[ \{ \Sigma_{\gamma_{\text{all}}} (l, n) \}_{l,n=1}^{L} = E \left\{ 10 \log_{10} (\gamma_{l,n}) \cdot 10 \log_{10} (\gamma_{l,n}) \right\} - \frac{m_{l,\text{all}}^{(1)} m_{n,\text{all}}^{(1)}}{L} \]

(11)

From (11), we can readily figure out that while the moments, \[ \{ \mu_{\gamma_{\text{all}}} (l) \}_{l=1}^{L} \] in (10) can be obtained from (7) with \( Q = L \), the expression of the covariance, \[ \{ \Sigma_{\gamma_{\text{all}}} (l, n) \}_{l,n=1}^{L} \] is not available in the open technical literature for both the independent and correlated scenario, but needs to be obtained from the correlation matrix \( \mathbf{R}_{\gamma_{\text{all}}} \). So, we need to derive it in closed–form in order to develop our P–IV approximation to solve distributed data fusion problems. The final expression of \( \mathbf{R}_{\gamma_{\text{all}}} \) can be found in (12) and (13) on top of this page for CSI and FG relaying, respectively. In particular, in (12) and (13) we have defined \( \Sigma_{\gamma_{\text{all}}} (l,n) \) with \( k = 1, 2, 3, 4 \), \( \mu_{Z_{l,n}} (1) = \mu_{SR}, \mu_{Z_{l,n}} (2) = \mu_{RD}, \mu_{Z_{l,n}} (3) = \mu_{SR}, \mu_{Z_{l,n}} (4) = \mu_{RD}, \Sigma_{Z_{l,n}} \) which is defined in (14) on top of this page\(^1\). Due to space constraints, the proof of (12), (13) is not shown in the present paper, but can be found in [41].

V. NUMERICAL AND SIMULATION RESULTS

The aim of this section is to analyze the accuracy of the proposed approximations to compute \( P_m \), as a function of

\(^2\)Note that \( \Sigma_{\gamma_{\text{all}}} (l,l) \) is the variance of \( 10 \log_{10} (\gamma_{l,l}) \), and can be obtained from (7) using its definition.

\(^1\)\( \rho_{(a,b)} \) is the correlation coefficient between link \( a \) and \( b \)

the number of reporting dual–hop links \( (L) \), and shadowing correlation among them. In particular, \( P_m \) is obtained, via straightforward numerical integration techniques, from (1) by approximating the PDF of the SNR \( \gamma_l \) by using the two–step method described in Section IV-B. Analysis is compared with Monte Carlo simulations to assess the accuracy.

With regard to the system setup, most settings are shown directly in the captions of figures, as the robustness of the approximations has been substantiated for several system setups and correlation values. However, in general, the common assumptions for every system setup are, unless otherwise stated, as follows. i) The detection/decision threshold \( \lambda \) is computed according to a Constant False Alarm (CFA) criterion [19] by using the formula for \( P_{fa} \) in (1) with \( P_{fa} = \left\{ 10^{-3}, 10^{-4} \right\} \). ii) In those scenarios where the shadowing is assumed to be identically distributed (but not necessarily uncorrelated), we denote with \( \mu, \sigma, \) and \( \rho \) the mean, standard deviation, and correlation coefficient of the Log–Normal power gains in every wireless link, respectively. Moreover, in this scenario \( C \) is used to denote the constant for FG relaying. iii) \( N = 10 \).

In Figs. 2, 3, we have analyzed robustness and numerical accuracy of the proposed approximations for both setups available in the literature for ED (i.e., \( p = 1 \) and \( p = LN/2 \)), as well as for different gains \( G_f \) (CSI, blind, and semi–blind setups) that the SUs might use to forward the signal to the fusion center on the reporting channel. Overall, we can observe a very good accuracy of the proposed approximation method for several system setups and \( P_{fa} \) requirements. In general, we observe that increasing the number, \( L \), of SUs is useful only when shadowing on both sensing and reporting channels is uncorrelated. Otherwise, the performance degradation is significant and can limit the achievable \( P_m \) for a given
where all nodes in the network do not have the same view of it, thus possibly experiencing unbalanced shadowing conditions and correlation coefficients. In order to simulate arbitrary unbalanced system setups, when the shadowing parameters are not fixed and the same in all links, we have extracted them from uniform RVs, i.e., $c \sim U(a, b)$ in the figures means that $c$ is uniformly distributed in the interval $[a, b]$. In particular, in Fig. 4 we can observe the effect, on the system performance, of having unbalanced means and standard deviations, when all links have the same correlation coefficient. We can see a very good agreement of our framework with Monte Carlo simulations, even for the scenario with unbalanced standard deviations, which is, in general, the most complicated to be modeled [37]. Finally, in Fig. 5 we have investigated the most important system setup: we have analyzed the effect of different correlation coefficients on the wireless links, as well as considered unbalanced standard deviations, which is, in general, very different performance.

Finally, in Fig. 4, we have analyzed a more realistic setup for data reporting to the classical system configuration used in most literature. We observe that the setups $p = 1$ and $p = LN/2$ yield, in general, very different performance, but our framework can track both of them. Finally, let us note that in Fig. 2 we have also verified the accuracy for different values of $C$ when blind relays are considered. We observe a good accuracy in a wide range of values of this parameter. Moreover, when $C = 0$ the system setup boils down to the classical system configuration used in most literature with ideal reporting channel [23]. As expected, in this case the system yields the best performance, as neither noise accumulation at the SUs nor channel fading on the reporting channel are considered. In other words, this figure clearly shows that, even in the absence of shadowing correlation, considering a more realistic setup for data reporting to the fusion center can yield a non-negligible performance drop.

Finally, in Fig. 4, 5 we have analyzed a more realistic setup where all nodes in the network do not have the same view of it, thus possibly experiencing unbalanced shadowing conditions and correlation coefficients. In order to simulate arbitrary unbalanced system setups, when the shadowing parameters are not fixed and the same in all links, we have extracted them from uniform RVs, i.e., $c \sim U(a, b)$ in the figures means that $c$ is uniformly distributed in the interval $[a, b]$. In particular, in Fig. 4 we can observe the effect, on the system performance, of having unbalanced means and standard deviations, when all links have the same correlation coefficient. We can see a very good agreement of our framework with Monte Carlo simulations, even for the scenario with unbalanced standard deviations, which is, in general, the most complicated to be modeled [37]. Finally, in Fig. 5 we have investigated the most important system setup: we have analyzed the effect of different correlation coefficients on the wireless links, as well as considered unbalanced standard deviations, since the number of possibilities to analyze is large.

We have considered three sets of correlation coefficients: i) the correlation coefficient of wireless links in the sensing channel, which is denoted by $\rho_{SR}$, ii) the correlation coefficient of wireless links in the reporting channel, which is denoted by $\rho_{RD}$, and iii) the correlation coefficient between a wireless receiver and a reference channel, which is denoted by $\rho_{RE}$. As expected, in this case the system yields the best performance, as neither noise accumulation at the SUs nor channel fading on the reporting channel are considered.

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link in the sensing channel and a wireless link in the reporting channel, which is denoted by $\rho_{RD}$. The most important result shown in Fig. 5 is as follows: we can see that even though the observations sensed by the SUs are independent, i.e., $\rho_{SR} = 0$, (which offers the best diversity gain for the classical system setup), the reporting channel might degrade the system performance due to shadowing effects that might make the data reported to the fusion center correlated, i.e., $\rho_{RD} \neq 0$. In other words, when considering a realistic system setup, shadowing correlation on the reporting channel can yield similar performance degradations as shadowing correlation on the sensing channel. So, uncorrelated observations might become correlated due to reporting operations regardless of the degree of correlation of SUs' observations. This is an important result as it shows that, for the accurate analysis and design of decentralized data fusion problems, the reporting channel should be carefully taken into account, as well as good reporting strategies should be designed. Moreover, we can also observe from Fig. 5 a very good match of our framework in a very challenging setup for Log–Normal power–sum approximation, i.e., unbalanced standard deviations and unbalanced correlation coefficients. Finally, we can readily figure out how the system performance get worse when more links get correlated.

VI. CONCLUSIONS

In this paper, we have provided an advanced framework for the analysis and design of cooperative spectrum problems over realistic sensing and reporting channels, which might experience correlated Log–Normal shadow–fading. Besides having shown the good accuracy, with a moderate computational complexity, of the analytical framework proposed for performance analysis, we have also highlighted the deleterious effects of correlated Log–Normal shadowing on the reporting channel, which might yield similar performance degradations as shadowing correlation on the sensing channel.

REFERENCES


