Reliability and Efficiency Analysis of Distributed Source Coding in Wireless Sensor Networks

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Abstract

We offer a complete theoretical framework to evaluate reliability and energy consumption of distributed source coding (DSC) in wireless sensor networks (WSNs) applications. Specifically, the amount of measurements that can be successfully decoded in tree-based WSNs employing DSC in the presence of different coding topologies and packet aggregation schemes (PA), and DSC energy efficiency are accurately characterized. The system model includes a realistic network architecture with multi-hop communication, automatic repeat request protocol (ARQ), packet losses due to channel impairments and collisions, and correlation properties of the sensed phenomena. Four DSC topologies and three alternatives of PA are considered. The analysis is carried out by evaluating the expressions of reliability of DSC in terms of probability of measurements that cannot be decoded (loss factor), and the efficiency in terms of average energy consumption of the network. Numerical results show that the best choice of DSC topology and packet aggregation depends highly on the network parameters and source characteristics. Therefore, the analytical results developed in this paper can be used to optimize DSC operations with respect to the numerous network parameters that interact with DSC.

Keywords: Wireless Sensor Networks (WSNs), Distributed Source Coding (DSC), Packet Aggregation (PA).

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I. INTRODUCTION

Wireless Sensor Networks (WSNs) consist of sensing devices often deployed with limited energy resources. Distributed Source Coding (DSC) claims to decrease the energy consumption for lifetime extension of WSNs: to this end it exploits the richness of information usually provided by spatial correlation of measurements taken by different sensors, to compress data without loss of information.

Although DSC has recently found in WSNs a relevant application domain, the theoretical foundations date back to the pioneering work by Slepian and Wolf [1]. A practical code construction to achieve the theoretical bounds devised in [1] has been proposed by Pradhan et al. in [2] and [3]. Those works have been extended by Chou et al. [4], who have proposed an algorithm to achieve DSC over WSNs using a single codebook with variable compression rate. Marco and Neuhoff [5] and Ganesan et al. [6] have pointed out that performance of DSC is largely influenced by the DSC topology, i.e., the nodes’ position with respect to the spatial correlation profile of the sensed phenomena. The problem of optimizing the DSC topology has been investigated by Cristescu et al. [7], [8] for tree-like networks. The interaction between DSC and routing has been studied in [9]–[11]. Petrovic et al. [9] have also noted that routing with packet aggregation allow for decreasing protocol overhead, with reduction of energy consumption. Baek et al. [12] have dealt with the problem of finding the optimal packet aggregation alternative for WSNs with DSC.

These contributions assume that DSC acts above the typical protocol stack of a sensor network platform. However, many other mechanisms for WSNs (e.g., routing and MAC protocols) are targeted to the same goal of achieving better energy usage. As it was observed in [13], DSC interactions with lower layers of the protocol stack (network, data link, and physical layer) deserve particular attention to fully exploit the claimed benefits. For instance, header overheads that appear in any protocol data unit (PDU) format should be taken into account. A node may aggregate packets coming from other nodes with its own data in order to reduce the impact of packet overhead in multi-hop routing. It can be argued that DSC has a relevant impact on
network, data link, and physical layers, and vice versa. Although the above mentioned relevant contributions address fundamental topics of DSC, they neglect some important aspects that relate to network design and operations in realistic scenarios, namely: packet losses, packet aggregation and fragmentation, overhead reduction, and cross-layer interaction. We believe that a crucial concern is represented by a proper (joint) combination of DSC topologies and packet aggregation (PA) in realistic multi-hop communication scenarios.

In this paper we investigate the interplay of main communication system parameters that affect DSC performance. We develop a complete framework for a joint analysis of the reliability (loss factor) and overall energy consumption of DSC. Our contribution is related to [5] because we include the four alternatives of coding topology therein proposed. It is also related to [9], because we consider three PA techniques. However, we simultaneously consider DSC in a system scenario including an accurate model of the physical layer, a data link layer, packet aggregation, multi-hop routing, and the correlation pattern of the sensed phenomena.

The remainder of the paper is organized as follows: in Section II, the system model is described. Section III is dedicated to the analysis of packet loss probability, which is then used first in Section IV to characterize the network-wide loss factor, and subsequently in Section V to analyze the energy consumption. Our theoretical framework is then used in Section VI to derive and discuss numerical results in selected scenarios. Finally, in Section VII, we summarize the main conclusions and evidence future perspectives.

II. SYSTEM MODEL

Consider an area wherein $N$ nodes are deployed: each of them takes some measurements and attempts to transmit reports towards a sink. The network topology is assumed to be organized in a tree, where each node belongs to a generic level $i$ (with $i = 1, \ldots, L$), $i$ being the distance of a node from the sink in terms of number of hops (see Fig. 1). We denote with $i = 0$ the sink node, which is placed at the level 0. Tree based architecture is a relevant network topology for the IEEE 802.15.4 standard [14, pag. 15–16]. Furthermore, the interest for such a network topology
is confirmed by a large number of relevant contributions that can be found in the literature (see, for instance, [5] – [8], [15] – [17]), which all deal with tree-like networks. Denote with $l_i$ the number of nodes belonging to level $i$. We set $l_0 = 1$, which is the number of nodes of level 0 (only the sink). The equality $\sum_{i=1}^{L} l_i = N$ holds true. Each node is connected to a parent node and takes $m$ measurements of $m$ different physical phenomena per sensing interval. We assume that the measurements of different phenomena are continuous i.i.d. random variables, and that the measurements of a given phenomenon by different sensor nodes are spatially correlated. The measurements are quantized by an analog-to-digital converter, which is assumed to have the same resolution in each sensor. Denote with $Y_{i,j}$, for $i = 1, \ldots, L$, and for $j = 1, \ldots, l_i$, the quantized measurement taken from the generic node $j$ of level $i$ at a given time instant for one of the $m$ phenomena. Coherently with the purpose of this paper, we retain $Y_{i,j}$ as a discrete random variable taken from random process, which has a countable discrete alphabet [4], [6], [7]. Furthermore, a set of variables $Y_{i,j}$s at a given time instant are generically spatially correlated, the correlation pattern being constant in time under the assumption of stationarity [5].

Each node performs the following tasks: it encodes the measurements according to the DSC algorithm proposed in [4], receives PDUs from the child nodes, aggregates them, and then transmits them towards its parent node. Let $R(Y_{ij})$ be the coding rate of the DSC technique [4] of the considered source. When an ARQ protocol is employed, retransmissions are attempted until either successful reception is attained or a maximum number of tries is reached.

For defining and computing performance indexes, in the following we refer to a packet as a PDU, which results from application of major headers at the various layers of the protocol stack. Hence, a packet is composed by a frame length indicator plus a preamble and a synchronization symbol of $P$ bits; a data link header of $O$ bits, a network header of $Q$ bits; a payload incorporating coded measurements taken by nodes, and $ID$ bits for node identification; $CRC$ bits for the forward error detection [18]. The payload may have variable size as a consequence of the DSC topology and aggregation mechanism, as it will be clear in the following subsections. The packet structure is adherent to specifications by the IEEE 802.15.4 communication standard [14] for the
data link layer and physical layer of the protocol stack, while we refer to Tmote Sky sensors [19] for the network layer.

In the next subsections, we first describe the DSC topologies and the aggregation mechanisms we intend to analyze; next we define the performance indexes we intend to adopt.

A. DSC Topology and Packet Aggregation

Three prominent DSC coding schemes are here studied: No DSC (NODSC), Sequential DSC (SEQ), Clustered DSC (CL), and DSC Master Slave (MS). These schemes were firstly proposed in [5]. In the following, we briefly summarize their characteristics.

In the NODSC topology, each sensor encodes measurements independently from other nodes. No distributed source coding is adopted.

In the SEQ scheme (see Fig. 2), node \((1,1)\) performs encoding of \(Y_{1,1}\) with \(R_{1,1} = R(Y_{1,1})\) bits; node \((1,2)\) encodes \(Y_{1,2}\) with \(R_{1,2} = R(Y_{1,2}|Y_{1,1})\) bits, provided that the decoder knows \(Y_{1,1}\); node \((1,j)\) encodes \(Y_{1,j}\) with \(R_{1,j} = R(Y_{1,j}|Y_{1,1}...Y_{1,j-1})\) bits, provided that the decoder knows \(Y_{1,1}...Y_{1,j-1}\); in general, node \((i,j)\) encodes \(Y_{i,j}\) using the following number of bits \(R_{i,j} = R(Y_{i,j}|Y_{1,1}...Y_{1,l_1}Y_{2,1}...Y_{2,l_2}...Y_{i,1}...Y_{i,j-1})\), provided that the decoder knows \(Y_{1,1}...Y_{1,l_1}Y_{2,1}...Y_{2,l_2}...Y_{i,1}...Y_{i,j-1}\).

In the CL topology (see Fig. 3), nodes are grouped in \(K = l_1\) clusters, where each cluster consists of a sub-tree having a node of level 1 as a root. Each cluster includes a number of nodes, which is denoted by \(N_k\). For each node of a cluster, a SEQ coding topology is adopted independently from other clusters and starting from the root node \((1,k)\), for \(k = 1,...,l_1\), of cluster \(k\).

In the MS topology (see Fig. 4), nodes are grouped in clusters as in the case of CL, but each root \((1,k)\), for \(k = 1,...,l_1\), of a sub-tree is elected as master node. However, in contrast to CL, for each node of a cluster \(k\), the DSC is performed only with respect to the master \((1,k)\) of the cluster the node belongs to.

We consider three alternatives for aggregation: the classical multi-hop (CMH), where nodes relay packets without any aggregation; the aggregated multi-hop (AMH), where nodes collect
received packets, aggregate them at the MAC layer into a single frame and relay it; the Fragmented Aggregated Multihop (FAMH), which differs from AMH as it allows us to do packet fragmentation: when an aggregated packet reaches a size that exceeds a maximum threshold, it is fragmented in multiple packets.

### B. Performance Indexes

Two performance indexes are considered in this paper, namely: the loss factor and the energy consumption of the network, which we denote with $\Lambda$ and $E_N$, respectively. The loss factor is defined as the fraction of measurements generated by the network that cannot be reconstructed at the sink. The energy consumption of the network is instead expressed as the average number of bits (per reporting interval) transmitted across the network. In computing $E_N$, we will neglect the impact of acknowledgement packets, which is a rather common assumption in literature, and is motivated by the fact that they have marginal impact on overall performance.\(^1\)

The characterization of both $\Lambda$ and $E_N$ is founded on the packet error probability. In the next sections we derive this probability for the various combinations of DSC topologies and PA mechanisms. Such an analysis provides us with the analytical expressions of $\Lambda$ and $E_N$, as reported in Tab. I, which summarizes the core contribution of the paper.

### III. Packet Loss Probability

Consider a wireless channel which exhibits non-selective fading behavior both in frequency and in time, and a O-QPSK (offset quadrature phase shift keying) modulation. These assumptions are consistent with transceivers operating in the ISM frequency band according to the IEEE 802.15.4 communication standard [14]. Therefore, the bit error probability is [18]

$$P_e(d) = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma(d)}{1 + \gamma(d)}} \right), \quad (III.1)$$

\(^1\)Note that the performance indexes do not include the packet delay caused by the coding topology. Indeed, under the assumptions that the correlation properties of the sensed phenomena are stationary, that the number of measurements per sampling rate is the typical one provided by sensors off-the-shelf [19] (i.e. less than ten), and that the number of levels is not large, then the delays are negligible.
where $\gamma(d)$ is the average value of the Signal-to-Noise Ratio (SNR) computed at distance $d$ from the signal source. We adopt the following model for $\gamma(d)$ in dB [20]: $\gamma(d)_{dB} = P_t \, dB - P(d)_{dB} - P_n \, dB$, where $P_t$ is the transmit power, $P(d)$ is the path loss at distance $d$ from the transmitter, and $P_n$ is the noise floor at the receiver. The path loss is expressed as $P(d) = P(d_0) + 10 \cdot \alpha \cdot \log_{10}(d/d_0)$, where $P(d_0)$ is the path loss computed at the reference distance $d_0$ (see e.g. [21]); while $\alpha$ is the path-loss decay constant with typical values belonging to the interval $[2, 4]$. Under the assumption that the CRC code is always able to detect perfectly corrupted packets (see [22] for an experimental support), the average packet loss probability at level $i$ is defined as

$$\hat{\Psi}(d, s_i) = 1 - \left\{ (1 - \phi_i) \cdot [1 - P_e(d)] \right\}^{2(P+O+Q+CRC+s_i)},$$  \hspace{1cm} (III.2)$$

where $\phi_i$ is the collision probability, and the factor 2 accounts for Manchester encoding. We remark that the collision probability is computed on a bit basis in order to account for the effects of the packet size. The term $s_i$ is defined as the average payload size among the nodes of level $i$. The average packet loss probability across one hop of level $i$ is defined as the average of (III.2) with respect to the hop length:

$$\Psi(s_i) = \frac{1}{d_{\text{max}} - d_{\text{min}}} \int_{d_{\text{min}}}^{d_{\text{max}}} \hat{\Psi}(\rho, s_i) d\rho,$$  \hspace{1cm} (III.3)$$

where $d_{\text{min}}$ and $d_{\text{max}}$ are, respectively, the minimum and maximum distances between any pair of child-parent nodes. We call (III.3) single-hop packet loss probability. When an ARQ protocol is implemented at level $j$, with $M_j$ maximum number of retransmissions, we can easily express the packet loss probability of a packet generated at level $i$ across one hop as follows:

$$\tilde{\Psi}(s_i, M_j) = \Psi(s_i)^{M_j}.$$  \hspace{1cm} (III.4)$$

The packet loss probability over a multi-hop path from node $i$ up to the sink can be computed with

$$\bar{\Psi}(s_i) = 1 - \prod_{j=1}^{i} \left[ 1 - \tilde{\Psi}(s_i, M_j) \right].$$  \hspace{1cm} (III.5)$$
Remark 3.1: Observe that (III.4) depends on the coding topology and aggregation scheme through the payload size $s_i$, which will be characterized in the next subsections for the cases of coding NODSC, SEQ, CL, and MS, and aggregation scheme CMH, AMH, and FAMH.

For the sake of the performance analysis, we define the average coding rate of the level $i$ as

$$R_i = \frac{1}{l_i} \sum_{l=1}^{l_i} R_{i,l}.$$ 

We define the average connectivity of a node belonging to level $i$ as the average number of children of that node, namely $C_i = l_{i+1}/l_i$, $i = 0, \ldots, L - 1$, where we impose $C_L = 0$.

A. CMH

The contribution to the average payload size for the CMH scheme at level $i$, as given from measurements generated by a node of level $j$, is

$$s_i = ID + m \cdot H_j, \quad i = 1, \ldots, L \text{ and } j \geq i,$$ 

(III.6)

Therefore, the single-hop packet loss probability is given by (III.4), where $s_i$ is given by (III.6).

B. AMH

Proposition 3.2: Consider the AMH scheme. Then, the average payload size at the $i$th level is

$$s_i = \begin{cases} 
ID \cdot \left(1 + \sum_{j=1}^{L-i} \prod_{k=1}^{j} n_{i+k}\right) + m \cdot \left(R_i + \sum_{j=1}^{L-i} R_{i+j} \cdot \prod_{k=1}^{j} n_{i+k}\right) & i = 1, \ldots, L - 1, \\
ID + m \cdot R_L & i = L.
\end{cases}$$ 

(III.7)

where $n_i = [1 - \bar{\Psi}(s_i, M_i)] \cdot C_{i-1}$, for $i = 2, \ldots, L$, is the average number of packets of level $i$ successfully received by the parent node of level $i - 1$.

Proof: See Appendix.

According to the previous result, the single-hop packet loss probability is given by (III.4) and (III.7).
C. FAMH

In the FAMH case, the payload size is limited to a maximum length \( s_{\text{max}} \). When the limit is exceeded, the payload is split into \( F_i \) fragments, with \( F_i = \lceil s_i/s_{\text{max}} \rceil \), where \( \lceil \cdot \rceil \) denotes the upper integer (ceiling). Each fragment is then mapped onto a packet and let \( \tilde{s}_i = (Q + s_i)/F_i \) denote the length of the related PDU at the data link layer. Since next hop node is known at the moment of the fragmentation, it is not necessary to transmit \( F_i \) times the same network header overhead, which can be split into \( F_i \) fragments. The fragmented packets, having payload \( \tilde{s}_i \), need to have all other headers (preamble, MAC and CRC) to be transmitted, however, they do not need to have the network header (for next hop is known at each fragment). The loss probability of a fragmented packet, averaged over a single-hop distance, is given as follows:

\[
\Psi_F(\tilde{s}_i) = \frac{1}{d_{\text{max}} - d_{\text{min}}} \int_{d_{\text{min}}}^{d_{\text{max}}} 1 - \{(1 - \phi_i) \cdot [1 - P_e(\rho)]\}^{2(P + O + CRC + \tilde{s}_i)} \, d\rho . \tag{III.8}
\]

The packet loss probability over a single-hop is

\[
\Psi(s_i) = 1 - [1 - \Psi_F(\tilde{s}_i)]^{F_i} . \tag{III.9}
\]

Finally, expression (III.9) can be used in (III.4) to obtain the single-hop packet loss probability.

IV. LOSS FACTOR

Here we build on the packet loss probability analysis of previous section to derive the loss factor. First, let us introduce some definitions. Denote with \( D_i \) the average number of descendants of the generic node of level \( i \) whose measurements are successfully received at that node. Denote with \( D_{i,j} \) the average number of descendants at level \( j \) whose measurements are successfully received at the generic node of level \( i \). We also impose \( D_{i,i} = 1 \), and that \( D_{i,j} = 0 \) if \( i > j \).

Then,

**Proposition 4.1:** At level \( i \) of the network, the average number of descendants is

\[
D_i = \sum_{j=i+1}^{L} D_{i,j} , \quad i = 0, \ldots, (L - 1) , \tag{IV.1}
\]
where
\[ D_{i,j} = \prod_{k=i+1}^{j} \left[ 1 - \tilde{\Psi}(s_j, M_k) \right] \cdot C_{k-1}. \] (IV.2)

Proof: See Appendix.

In the next subsections, we characterize the loss factor for each case of coding topology and aggregation mechanism.

A. NODSC

In the CMH scenario, the probability that a packet generated at level \( i \) does not reach the sink is given by (III.5). Hence, after averaging over all levels and number of nodes, the loss factor is
\[ \Lambda_{\text{NODSC}}^{\text{CMH}} = \frac{1}{N} \sum_{i=1}^{L} \left\{ 1 - \prod_{j=1}^{i} \left[ 1 - \tilde{\Psi}(s_i, M_j) \right] \right\} \cdot l_i. \] (IV.3)

When aggregation procedures are adopted, derivation of the loss factor has to take into account that loosing a packet generated from a node of level \( i \) determines the avalanche effect of losing measurements successfully received by that node and coming from lower levels of the network. A packet loss at level \( i \) causes an average loss of \( 1 + D_i \) measurements. Therefore, the loss factor for the AMH (b) and FAMH (c) cases is
\[ \Lambda_{\text{NODSC}}^{(b,c)} = \frac{1}{N} \sum_{i=1}^{L} \left\{ 1 - \prod_{j=1}^{i} \left[ 1 - \tilde{\Psi}(s_i, M_j) \right] \right\} \cdot (1 + D_i) \cdot l_i, \] (IV.4)

where the computation of \( \tilde{\Psi}(s_i, M_j) \), which appears also in the \( D_i \), is performed by using (III.3), (III.4), and (III.7) for AMH (b), and using (III.4), (III.8), and (III.9) for FAMH (c), respectively.

B. SEQ

Let us firstly consider the CMH case:

Proposition 4.2: The loss factor of the SEQ topology in the CMH aggregation is
\[ \Lambda_{\text{SEQ}}^{\text{CMH}} = \frac{1}{N} \sum_{i=1}^{L} A_{\text{SEQ},i}^{\text{CMH}} \cdot l_i \cdot B_{\text{SEQ},i,j} \cdot C_{\text{SEQ},i,j}. \] (IV.5)
where

\[
A^\text{CMH}_{\text{SEQ},i} = \left\{ 1 - \prod_{m=1}^{i} \left[ 1 - \tilde{\Psi}(s_i, M_m) \right] \right\},
\]

\[
B^\text{SEQ,i,j} = N + 1 - j - \sum_{m=1}^{i-1} l_m,
\]

\[
C^\text{CMH}_{\text{SEQ},i,j} = \prod_{k=1}^{i} \prod_{m=1}^{k} \left[ 1 - \tilde{\Psi}(s_k, M_m) \right] l_k \cdot \prod_{m=1}^{i} \left[ 1 - \tilde{\Psi}(s_i, M_m) \right]^{j-1}.
\]

and where \( \tilde{\Psi}(s_i, M_k) \) is computed with (III.4).

**Proof:** See Appendix.

In the cases of AMH and FAMH, expressions for the loss factors are similar to those given by Proposition 4.2. In fact, the loss of a packet coming from node \((i, j)\) induces the loss of all measurements aggregated from lower layers of the network until \((i, j)\): its average value is given by \(D_i\) in (IV.1). However, this number of losses is already included in the number of measurements that, being coded with respect to \(Y_{i,j}\), cannot be reconstructed. From Fig. 1, one sees that this metric is \(N + 1 - j - \sum_{k=1}^{i-1} l_k\). Therefore, we can readily express the loss factor for the AMH (b) and FAMH (c) cases as

\[
\Lambda^{(b,c)}_{\text{SEQ}} = \frac{1}{N} \sum_{i=1}^{L} A^{(b,c)}_{\text{SEQ},i} \cdot \sum_{j=1}^{l_i} B^\text{SEQ,i,j} \cdot C^{(b,c)}_{\text{SEQ},i,j},
\]

where the superscript \((b)\) denotes that the probability \(\tilde{\Psi}(s_i, M_j)\) has to be computed using (III.3), (III.4), and (III.7), whereas the superscript \((c)\) denotes that the computation of \(\tilde{\Psi}(s_i, M_j)\) is accomplished using (III.4), (III.8), and (III.9).

**C. CL**

When the network is partitioned into clusters, for each cluster the loss factor can be computed as in the SEQ case. Hence, the loss factor for CMH (a), AMH (b), and FAMH (c) is provided by the following expression:

\[
\Lambda^{(a,b,c)}_{\text{CL}} = \frac{1}{K} \sum_{k=1}^{K} \Lambda^{(a,b,c)}_{\text{SEQ},k},
\]

(IV.7)
where the subscript $k$ denotes the loss factor of cluster number $k$, and it is computed using (IV.5) for the case CMH (a), (IV.6) for the cases AMH (b) and FAMH (c). When computing $\Lambda^{(a,b,c)}_{\text{SEQ},k}$, $N$ and $l_i$ must be replaced (whenever they appear in (IV.5) and (IV.6)) with $N_k$ and $l_i^{(k)}$ respectively, where $l_i^{(k)}$ is defined as the number of nodes of level $i$ of cluster $k$. Notice that $l_0^{(k)} = 1$ is the number of nodes of level 0 (only the master node) of cluster $k$.

**D. MS**

Let us consider first the CMH scheme. The following result holds:

**Proposition 4.3:** The loss factor of the MS topology in the CMH aggregation is

$$\Lambda^{\text{CMH}}_{\text{MS}} = \frac{1}{K} \sum_{k=1}^{K} \Lambda^{\text{CMH}}_{\text{MS},k}$$  \hfill (IV.8)

where $\Lambda^{\text{CMH}}_{\text{MS},k}$ is the loss factor for cluster $k$, and it is given by

$$\Lambda^{\text{CMH}}_{\text{MS},k} = \tilde{\Psi}(s_1, M_1) + \frac{1}{N_k} \sum_{i=2}^{L} l_i^{(k)} \cdot \left[ 1 - \tilde{\Psi}(s_1, M_1) \right] \cdot \left[ 1 - \prod_{m=1}^{i} \left[ 1 - \tilde{\Psi}(s_i, M_m) \right] \right]$$

and where $\tilde{\Psi}(s_i, M_m)$ is computed using (III.4).

**Proof:** See Appendix.

In the case of packet aggregation, the procedure to derive the loss factor is the same as in the CMH scheme. However, the loss of a measurement taken by a node $(i, j)$ in cluster $k$, induces to lose further $D_i^{(k)}$ measurements on the average, where $D_i^{(k)}$ is defined as in (IV.1), but using $C_i^{(k)}$ instead of $C_i$, with $C_i^{(k)}$ defined as $C_i^{(k)} = l_{i+1}^{(k)}/l_i^{(k)}$, $i = 0, \ldots, L - 1$, and that $C_L^{(k)} = 0$. Therefore, the loss factor for the AMH (b) and FAMH (c) cases is given by

$$\Lambda^{(b,c)}_{\text{MS}} = \frac{1}{K} \sum_{k=1}^{K} \Lambda^{(b,c)}_{\text{MS},k}$$  \hfill (IV.9)

where

$$\Lambda^{(b,c)}_{\text{MS},k} = \tilde{\Psi}(s_1, M_1) + \frac{1}{N_k} \sum_{i=2}^{L} l_i^{(k)} \cdot (1 + D_i^{(k)}) \cdot \left[ 1 - \tilde{\Psi}(s_1, M_1) \right] \cdot \left[ 1 - \prod_{m=1}^{i} \left[ 1 - \tilde{\Psi}(s_i, M_m) \right] \right]$$

(IV.10)

In (IV.10), $\tilde{\Psi}(s_i, M_m)$ and $D_i^{(k)}$ are computed by resorting to (III.3), (III.4), and (III.7) for the AMH (b) case, and to (III.4), (III.8), and (III.9) for the FAMH (c) case. Obviously, (III.7) is computed by replacing $N, l_i$, and $C_i$ with $N_k, l_i^{(k)}$, and $C_i^{(k)}$ whenever they appear.
V. AVERAGE ENERGY CONSUMPTION

Expressing the average number of bits transmitted by the network requires characterization of the average number of transmitted packets at the generic level \(i\). Recall that such a number depends on the coding topology by the payload size. In the following, we derive the energy consumption for the CMH, AMH and FAMH schemes.

The number of retransmissions of a packet sent from a node of level \(i\) over the hop between level \(i\) and \(i-1\) is denoted as \(\Theta_i(s_i)\). It is expressed as [18]

\[
\Theta_i(s_i) = \frac{1 + M_i \cdot \Psi(s_i)^{M_i+1} - (M_i + 1) \cdot \Psi(s_i)^{M_i}}{1 - \Psi(s_i)}.
\]

Therefore, the average number of transmitted bits from the generic node of level \(i\), when an ARQ protocol is employed, is

\[
B_i = \sum_{j=i}^{L} (P + O + Q + s_j + CRC) \cdot D_{i,j} \cdot \Theta_j(s_j),
\]

where \(D_{i,j}\) has been defined in (IV.2).

Consider the CMH case. The average number of transmitted bits is

\[
E_{N}^{\text{CMH}} = \sum_{i=1}^{L} B_i \cdot l_i,
\]

(V.1)

where \(\Theta_i(s_i)\) is computed using (III.3).

Let us consider now the AMH case. Each node of level \(i\) transmits only one packet containing both the measurements taken from local sensing and the measurements coming from lower levels of the network and encapsulated therein. Therefore, \(E_N\) is readily expressed as follows:

\[
E_{N}^{\text{AMH}} = \sum_{i=1}^{L} (P + O + Q + s_i + CRC) \cdot \Theta_i(s_i) \cdot l_i,
\]

(V.2)

where \(\Theta_i(s_i)\) is computed using (III.3) and (III.7).

Finally, consider the FAMH case. The average number of transmitted bits is

\[
E_{N}^{\text{FAMH}} = \sum_{i=1}^{L} (P + O + \tilde{s}_i + CRC) \cdot F_i \cdot \Theta_i(\tilde{s}_i) \cdot l_i,
\]

(V.3)

where \(\Theta_i(\tilde{s}_i)\) is computed using the payload size \(\tilde{s}_i\) for the FAMH case, with (III.8) and (III.9).
VI. NUMERICAL EXAMPLES

In this section we report numerical results obtained by a Matlab simulation of the analytical framework developed in previous sections.

A. Simulation Parameters

The parameters setting adopted for simulations is representative of the Tmote Sky sensors [19] and the communication standard IEEE 802.15.4 [14], and are introduced in the sequel. We consider a network deployed in a square area having facet of 11m with the sink located in the middle and $N = 64$ nodes distributed in $L = 4$ levels, where each node is randomly located within a sub-square of 1.2m. The number of nodes for each level and average connectivity are $[l_0, \ldots, l_L] = [1, 4, 12, 20, 28]$ and $[C_0, \ldots, C_4] = [4, 3, 20/12, 28/20, 0]$, respectively. Each node senses $m = 8$ measurements per sensing event. The packet frame format is as follows: $P = 48$, $O = 184$, $Q = 56$, $ID = 32$, $CRC = 16$, and $s_{max} = 760$, where the unit is intended in bit. Transmission power has been set to $-10$ dBm. The noise floor is $P_n = -120$ dBm [19].

The collision probability has been set to $\phi_i = 10^{-5}$ (this corresponds to a packet collision probability of about 0.005, which is quite large if related to the sampling rate, the packet duration, and the number of nodes). Smaller values for the collision probability basically induce similar effects. Without loss of generality, and coherently with [14], we set the maximum retransmission iterations of the ARQ protocol for levels $1, \ldots, L = 4$, to $3, 3, 1, 1$, respectively. This choice is motivated by the fact that nodes closer to the sink are subject to larger packet losses as a consequence of larger traffic load.

Measurements are characterized with a $N$ dimensional multi-variate normal distribution having average $\mu$ and covariance matrix $K = [K_{k,l}]$, where $K_{kk} = \sigma^2$ and $K_{k,l} = \sigma^2 e^{-rd_{kl}}$ for $k \neq l$, where $r$ is the spatial correlation decay parameter, and $d_{kl}$ is the distance between nodes $k$ and $l$. The correlation decay parameter is defined such that it tends to 0 for highly correlated sources, whereas tends to 1 for low correlated sources, as discussed in [7]. Consider the measurement $Y_{i,j}$, and the vector $Y$ collecting the measurements used as side information for the encoding of $Y_{i,j}$. 
Let \( \mu_Y \) be the portion of \( \mu \) corresponding to \( Y \). Denote by \( K_{Yij} \) the vector constructed by taking the entries of \( K \) corresponding to \( Y_{ij} \) and \( Y \), and let \( K_Y \) the sub-matrix of \( K \) corresponding to the vector \( Y \). Then, the following upper bound for the DSC coding rate can be derived [4], [23]

\[
R(Y_{ij}|Y) \leq \frac{1}{2} \log_2 E\{N_j^2\} - \frac{1}{2} \log_2 P_e + 1 - \log_2 \Delta, \tag{VI.1}
\]

where

\[
E\{N_j^2\} = E\{Y_{ij}^2\} - E\{Y_{ij} Y^T\}[E\{YY^T\}]^{-1} E\{Y_{ij} Y\},
\]

\[
E\{Y_{ij}^2\} = \sigma^2 + \mu_{ij}^2,
\]

\[
E\{Y_{ij} Y\} = K_{Yij} + \mu_{ij} \mu_Y^T,
\]

and

\[
E\{YY^T\} = K_Y + \mu_Y \mu_Y^T.
\]

In Equation (VI.1), \( \Delta \) is the quantization step, and \( P_e \) is the desired probability of decoding error. We set \( \Delta = \sigma/10 \) and \( P_e = 0.0001 \).

**B. Simulation Results**

We consider two representative scenarios: the first one refers to large spatial correlations among measurements, while the second one refers to low correlation patterns.

In Fig. 5, the loss factor is plotted for various cases of the coding topology (NODSC, SEQ, CL, and MS) and aggregation scheme (CMH, AMH, and FAMH) as obtained in Section IV (see Tab. I), for a highly correlated measurements scenario, where \( r = 10^{-5} \). We observe that ARQ decreases significantly the amount of measurements that cannot be decoded at the sink node. This is particularly evident in the MS case, where the decrease of loss factor due to ARQ is relevant. When looking at the SEQ case, the loss factor takes on the largest value among DSC topologies. Indeed, recalling the behavior of the SEQ topology summarized in Section II.A, losing a measurement at node \( (i, j) \) determines the avalanche effect of losing all measurements \( t, q \), with \( t = i + 1, \ldots, L \) and \( q = j + 1, \ldots, l_i \). The same effect appears also in the CL
topology, but clustering reduces the amount of performance loss. Indeed, no relevant avalanche effect is present in this topology. When looking at aggregation mechanisms, we observe that they cause a slight increase of the loss factor, since packets have larger sizes if compared to the CMH case. A relevant conclusion that can be drawn from Fig. 5 is that MS seems to be the most appealing coding topology in terms of reliability, and that aggregation mechanisms exhibit similar performance within the same coding topology.

In Fig. 6, values of the energy consumption $E_N$ for the various DSC topologies and aggregation schemes are plotted as obtained for highly correlated sources. They have been computed through the analysis presented in Section V (see Tab. I). The right column is referred to the ARQ protocol, while the left column is referred to the case where ARQ is not adopted. In the CMH case, although ARQ introduces a larger number of packets per node to be relayed, we can clearly observe that it determines a small energy rise. By contrast, the ARQ affects more sensibly the energy efficiency for the AMH and FAMH in percentage terms. By considering that the ARQ protocol is beneficial in terms of loss factor, we conclude that the number of retransmissions must be carefully selected. The NODSC topology exhibits the worst performance, since it uses no compression at all. Observe that SEQ, CL and MS topologies do not show large differences. This is mainly due to the large spatial correlation, so that measurements from a few nodes are enough to achieve good compression rates. However, the difference among the topologies is more remarked for low correlated sources, as we see later. By considering the aggregation mechanisms, we see clearly that they achieve significant performance improvement. Specifically, CMH exhibits the worst performance among aggregation mechanisms, whatever DSC scheme is adopted. This is due to the fact that packet overhead introduces extra bits in the packet transmission.

For a better understanding of energy consumption, define the following coding efficiency metric:

$$\eta_{CT} = 1 - \frac{E_{N,CT}}{E_{N,NODSC}},$$

where the subscript CT denotes one of the coding topologies SEQ, CL, or MS. Large values of
\( \eta_{CT} \) mean that the corresponding coding topology exhibits reduced energy consumptions when compared to the NODSC case.

In Fig. 7, the coding efficiency is plotted versus the different coding topologies (x axis) and aggregation algorithms for the case of highly correlated sources. The coding efficiency rises remarkably when we move from CMH to FAMH, with AMH providing intermediate performance. We can observe that the performance provided by the three topologies, SEQ, CL and MS, are quite different for the cases of AMH and FAMH, while they are less evident in CMH. MS coding topology shows the worst performance. Indeed we recall that the compression is performed with respect to only one measurement for each cluster.

To compare both efficiency and reliability, in Fig. 8 the coding efficiency is reported as a function of the loss factor, for the case of highly correlated sources. It turns out that the SEQ alternative often offers the largest coding efficiency, but also induces larger loss factors. On the contrary, the MS case exhibits poor energy savings, but good loss factor. The CL topology presents fair joint performance. Finally, it is confirmed that aggregation mechanisms significantly impact on the energy consumption.

Let us now consider the case of low correlated measurements, in which we set \( r = 10^{-3} \). In Fig. 9, we plot the coding efficiency. As expected, we see that the maximum achievable coding efficiency is smaller than the maximum values obtained for highly correlated measurements in Fig. 7. Notice that the ARQ protocol induces a significant degradation of the energy efficiency in the AMH and FAMH cases, meaning once again that the number of retransmissions must be carefully selected. In Fig. 10, we reported the coding efficiency as a function of the loss factor. The remarks done for Fig. 8 still hold true. However, a drop of efficiency of the AMH and FAMH mechanisms is evident, particularly in the MS case when compared to the CL CMH.

VII. CONCLUSIONS AND FUTURE PERSPECTIVES

A comprehensive theoretical framework to evaluate the loss factor and energy efficiency of distributed source coding in the presence of four alternatives of coding topology (no DSC, sequen-
tial, clustered and master slave) of three alternatives of packet aggregation mechanisms (classic multi-hop, aggregated multi-hop and aggregated and fragmented multi-hop) was proposed.

Our analysis showed that reliability and energy efficiency of DSC is highly influenced by the numerous protocol and source parameters (packet size, fragment size, coding topology, ARQ, correlation pattern, etc.). Numerical results obtained by exploitation of our analytical framework showed that SEQ coding topology performs better in terms of energy consumption, yet showing poor reliability performance. On the contrary, the MS case offers better performance in terms of measurements that are successfully decoded, but it is poorer in terms of energy efficiency. Furthermore, while the ARQ protocol decreases significantly the amount of measurements which cannot be decoded at the sink, retransmissions affect remarkably energy efficiency of the AMH and FAMH cases. Packet overheads and packet loss probability played a significant role in network energy consumption.

Given a network topology and correlation pattern of the sensed phenomena, and considering the application requirements (reliability and energy consumption), our analysis can be used for an efficient network deployment. Since the analytical results capture the trade-off between the network parameters, they can be employed effectively by network designers to provide a global optimization of the DSC operations. Therefore, our analysis is a major tool to guarantee a long lifetime of the network under reliable communications.

While in the present paper we considered a tree-like network, we plan to extend the analysis to a more general network topology to optimize jointly routing, aggregation scheme, and source coding with respect to the correlation pattern of the distributed source. We believe that non-linear mixed integer-real optimization theory will be useful for the solution of this challenging problem.


APPENDIX

A. Proof of Proposition 3.2

To prove (III.7) consider the average number of packets that are successfully transmitted to each parent-node of level $L - 1$: $n_L = [1 - \Psi(s_L, M_L)] \cdot C_{L-1}$. At level $L - 1$, each node aggregates the received packets and relays a unique packet having an average payload $s_{L-1} = ID + m \cdot R_{L-1} + s_L \cdot n_L = ID + ID \cdot n_L + m \cdot (R_{L-1} + H_L \cdot n_L)$. At level $L - 2$, the average number of received packets is $n_{L-1} = [1 - \Psi(s_{L-1}, M_{L-1})] \cdot C_{L-2}$. Therefore, the average payload size and the average number of packets coming to this level are

$$s_{L-2} = ID + ID \cdot n_{L-1} \cdot n_L + ID \cdot n_{L-1} + m \cdot (R_{L-2} + R_{L-1} \cdot n_{L-1} + R_L \cdot n_L \cdot n_{L-1}),$$

$$n_{L-2} = [1 - \Psi(s_{L-2}, M_{L-2})] \cdot C_{L-3}.$$

Generalization of these expressions to obtain (III.7) turns out from iteration.

B. Proof of Proposition 4.1

The proof is by induction. At level $L - 1$, $D_{L-1}$ is easily obtained by $D_{L-1} = D_{L-1,L} = [1 - \Psi(s_L, M_L)] \cdot C_{L-1}$. At level $L - 2$, $D_{L-2}$ depends on the number of packets that are generated from level $L$ and level $L - 1$ and that are correctly received at level $L - 2$:

$$D_{L-2} = D_{L-2,L-1} + D_{L-2,L}$$

$$= [1 - \Psi(s_{L-1}, M_{L-1})] \cdot C_{L-2} + [1 - \Psi(s_L, M_{L-1})] \cdot [1 - \Psi(s_L, M_L)] \cdot C_{L-2} \cdot C_{L-1}.$$

For the generic level $i$, this expression can be easily extended to obtain (IV.1).

C. Proof of Proposition 4.2

We prove (IV.5) by induction. Denote with $\Phi_{1,1}$ the event of losing $Y_{1,1}$, so that measurements encoded with respect to $Y_{1,1}$ cannot be decoded. Then, in the case of the event $\Phi_{1,1}$, it causes the avalanche effect of losing $N$ measurements, since $Y_{1,1}$ is lost along with all measurements taken by the other $N - 1$ nodes that were encoded with respect to $Y_{1,1}$. Therefore, the amount of measurements lost when only the event $\Phi_{1,1}$ happens, is given by $\Lambda_{\Phi_{1,1}} = \Psi(s_1, M_1) \cdot N$. 
Denote with \( \Phi_{1,2} \) the event of successfully receiving \( Y_{1,1} \) while losing \( Y_{1,2} \). Then, when \( \Phi_{1,2} \) occurs, \( N - 1 \) measurements are lost, since \( Y_{1,2} \) is lost along with the \( N - 2 \) measurements that were coded with respect to \( Y_{1,2} \). Therefore, the amount of measurements lost when only \( \Phi_{1,2} \) takes place is given by
\[
\Lambda_{\Phi_{1,2}} = \Psi(s_1, M_1) \cdot (N - 1) \cdot [1 - \Psi(s_1, M_1)].
\]

Denote with \( \Phi_{1,l_1} \) the event of losing \( Y_{1,l_1} \), while \( Y_{1,1}, Y_{1,2}, \ldots, Y_{1,l_1-1} \) were successfully received. If \( \Phi_{1,l_1} \) happens, then \( N - l_1 + 1 \) measurements are lost, since measurements \( Y_{2,1}, \ldots, Y_{2,l_2}, Y_{3,1}, \ldots, Y_{L,l_L} \) were coded with respect to \( Y_{1,l_1} \). Therefore, the amount of measurements lost in correspondence of the only event \( \Phi_{1,l_1} \) is given by
\[
\Lambda_{\Phi_{1,l_1}} = \Psi(s_1, M_1) \cdot (N + 1 - l_1) \cdot 
\left[1 - \Psi(s_1, M_1)\right]^{l_1-1}.
\]

Consider now the general case of the event \( \Phi_{i,j} \), when \( Y_{i,j} \) is lost, while \( Y_{i,1}, Y_{i,2}, \ldots, Y_{i,l_1}, Y_{i,2,1}, \ldots, Y_{i,1}, \ldots, Y_{i,j-1} \) are successfully received. When \( \Phi_{i,j} \) occurs, then \( Y_{i,j+1}, \ldots, Y_{i,l_1}, Y_{i+1,1}, \ldots, Y_{i+1,1}, \ldots, Y_{L,l_L} \) are lost, since they were coded with respect to \( Y_{i,j} \). Hence, the total number of lost measurements is \( N + 1 - j - \sum_{k=1}^{l_1-1} l_k \), and the amount of measurements lost when the only event \( \Phi_{i,j} \) takes place is
\[
\Lambda_{\Phi_{i,j}} = A_{SEQ,i}^{CMH} \cdot B_{SEQ,i,j} \cdot C_{SEQ,i,j}^{CMH}.
\]

By averaging with respect to all measurements, Eq. (IV.5) is obtained.

\[ D. \text{ Proof of Proposition 4.3} \]

Denote with \( \Omega_{1,k} \), for \( k = 1, \ldots, l_1 \), the event of losing \( Y_{1,k} \). When \( \Omega_{1,k} \) takes place, all \( N_k \) measurements of clusters \( k \) are lost, since they were coded with respect to \( Y_{1,k} \). Therefore, the amount of lost measurements induced by the only event \( \Omega_{1,k} \) is given by
\[
\Lambda_{\Omega_{1,k}} = N_k \Psi(s_1, M_1).
\]

Consider now the event \( \Omega_{i,j} \), for \( i = 2, \ldots, L \) and \( j = j_k, \ldots, \), where \( j_k \) is the index of the first node of level \( i \) of cluster \( k \), and \( j_k + l^{(k)} - 1 \) is the index of the last node of the same level of the same cluster. The event \( \Omega_{i,j} \) occurs when \( Y_{1,k} \) is successfully received and \( Y_{i,j} \) is lost. Therefore, the loss factor induced by the only event \( \Omega_{i,j} \) is given as
\[
\Lambda_{\Omega_{i,j}} = \left[1 - \Psi(s_1, M_1)\right] \cdot \left\{1 - \prod_{m=1}^{j} \left[1 - \Psi(s_i, M_m)\right]\right\}.
\]

By averaging with respect to \( \Lambda_{\Omega_{i,j}} \), we get expression (IV.9).
REFERENCES


<table>
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<tr>
<th></th>
<th>Loss Factor - $\Lambda$</th>
<th>Energy Consumption - $E_N$</th>
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<tr>
<td>CMH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AMH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FAMH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NODSC</td>
<td>(IV.3)</td>
<td>(V.1)</td>
</tr>
<tr>
<td></td>
<td>(III.3)</td>
<td>(III.3, (III.7)</td>
</tr>
<tr>
<td>SEQ</td>
<td>(IV.5)</td>
<td>(V.1)</td>
</tr>
<tr>
<td></td>
<td>(III.3)</td>
<td>(III.3, (III.7)</td>
</tr>
<tr>
<td>CL</td>
<td>(IV.7)</td>
<td>(V.1)</td>
</tr>
<tr>
<td></td>
<td>(IV.5, (III.3)</td>
<td>(III.3, (III.7)</td>
</tr>
<tr>
<td>MS</td>
<td>(IV.8)</td>
<td>(V.1)</td>
</tr>
<tr>
<td></td>
<td>(III.3)</td>
<td>(III.3, (III.7)</td>
</tr>
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**TABLE I**

LOSS FACTOR AND ENERGY CONSUMPTION AS FUNCTION OF THE CODING TOPOLOGY AND AGGREGATION MECHANISM.


<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>N</td>
<td>Number of nodes</td>
</tr>
<tr>
<td>N_k</td>
<td>Number of nodes in the cluster k</td>
</tr>
<tr>
<td>L</td>
<td>Number of levels of the network</td>
</tr>
<tr>
<td>l_i</td>
<td>Number of nodes of level i of the network</td>
</tr>
<tr>
<td>C_i</td>
<td>Average connectivity of a node of level i</td>
</tr>
<tr>
<td>D_i</td>
<td>Average number of descendants of a node of level i</td>
</tr>
<tr>
<td>D_{i,j}</td>
<td>Average number of descendants at level j whose measurements are successfully received at a node of level i</td>
</tr>
<tr>
<td>K</td>
<td>Number of clusters of nodes</td>
</tr>
<tr>
<td>d_{min}</td>
<td>Minimum distance among nodes</td>
</tr>
<tr>
<td>d_{max}</td>
<td>Maximum distance among nodes</td>
</tr>
<tr>
<td>Y_{i,j}</td>
<td>Data measured by node j of level i</td>
</tr>
<tr>
<td>m</td>
<td>Number of sensed phenomena per node per sampling time</td>
</tr>
<tr>
<td>R(·)</td>
<td>Coding rate achieved by DSC</td>
</tr>
<tr>
<td>P</td>
<td>Frame length indicator + Preamble+Sync size (bits)</td>
</tr>
<tr>
<td>O</td>
<td>DataLink section size (bits)</td>
</tr>
<tr>
<td>Q</td>
<td>Network section size (bits)</td>
</tr>
<tr>
<td>s_i</td>
<td>Average payload size (bits)</td>
</tr>
<tr>
<td>s_{i}</td>
<td>Average fragmented payload size (bits)</td>
</tr>
<tr>
<td>s_{max}</td>
<td>Maximum payload size (bits)</td>
</tr>
<tr>
<td>n_i</td>
<td>Average number of packets of level i that are successfully received by a parent node of level i − 1</td>
</tr>
<tr>
<td>CRC</td>
<td>Correction code section size (bits)</td>
</tr>
<tr>
<td>ID</td>
<td>Identification code size (bits)</td>
</tr>
<tr>
<td>F_i</td>
<td>Number of packet fragments at level i</td>
</tr>
<tr>
<td>M_i</td>
<td>Maximum number of retransmissions at level i</td>
</tr>
<tr>
<td>γ(·)</td>
<td>SNR function</td>
</tr>
<tr>
<td>φ_i</td>
<td>Collision probability at level i</td>
</tr>
<tr>
<td>P_e(·)</td>
<td>Bit error probability function</td>
</tr>
<tr>
<td>̂Ψ(d, s_i)</td>
<td>Single-hop packet loss probability function</td>
</tr>
<tr>
<td>Ψ(s_i)</td>
<td>Average single-hop packet loss probability function</td>
</tr>
<tr>
<td>̂Ψ(s_i, M_j)</td>
<td>Single-hop packet loss probability function with ARQ</td>
</tr>
<tr>
<td>Ψ(s_i)</td>
<td>Multi-hop packet loss probability function with ARQ</td>
</tr>
<tr>
<td>Θ_i(s_i)</td>
<td>Average number of retransmissions for a packet of level i</td>
</tr>
<tr>
<td>B_i</td>
<td>Average number of transmitted bits for a node of level i</td>
</tr>
<tr>
<td>Λ</td>
<td>Loss Factor of the network</td>
</tr>
<tr>
<td>E_N</td>
<td>Average number of transmitted bits of the network</td>
</tr>
<tr>
<td>η_{CT}</td>
<td>Coding efficiency</td>
</tr>
</tbody>
</table>

**TABLE II**

**Symbols.**
Fig. 1. Network topology. Nodes are organized as a tree with $L$ levels. The root node acts as sink.

Fig. 2. Sequential coding topology (SEQ).
Fig. 3. Clustered coding topology (CL). In each cluster, a SEQ coding is employed. The superscript in the indexes corresponds to the cluster the node belongs to. Notice that the level 1 of each cluster contains only a node.

Fig. 4. Master Slave coding topology (MS). It is similar to the CL, but the coding is done only with respect to a cluster’ master. The superscript in the indexes corresponds to the cluster the node belongs to. Notice that the level 1 of each cluster contains only a node.
Fig. 5. Loss factor for CMH, AMH, and FAMH with and without ARQ, for the case of highly correlated measurements. The right bars are referred to ARQ case, whereas the left ones are referred to the case without ARQ.

Fig. 6. Average number of byte transmitted by the network for the case of highly correlated measurements. The right bars are referred to ARQ case, whereas the left ones are referred to the case without ARQ.
Fig. 7. Coding Efficiency for CMH, AMH, and FAMH with and without ARQ for the case of highly correlated measurements. On the abscissa the cases SEQ, CL, and MS are reported.

Fig. 8. Coding Efficiency vs Loss factor for CMH, AMH, and FAMH with and without ARQ, for the case of highly correlated measurements. On each curve, the markers on the left are referred to MS, in the middle to CL, and on the right to SEQ.
Fig. 9. Coding Efficiency for CMH, AMH, and FAMH with and without ARQ, for the case of weakly correlated measurements. On the abscissa the cases SEQ, CL, and MS are reported.

Fig. 10. Coding Efficiency vs Loss factor for CMH, AMH, and FAMH with and without ARQ, for the case of weakly correlated measurements. On each curve, the markers on the left are referred to MS, in the middle to CL, and on the right to SEQ.