An Upper Bound on the Probability of Instability of a DVB-T/H Repeater with a Digital Echo Canceller

Flavio Zabini
Univ. of Bologna, Italy
flavio.zabini@unibo.it

Matteo Mazzotti
Univ. of Bologna, Italy
mazzotti.matteo@unibo.it

Davide Dardari
Univ. of Bologna, Italy
davide.dardari@unibo.it

Oreste Andrisano
Univ. of Bologna, Italy
o.andrisano@ieee.org

Abstract—The architecture of a digital On-Channel Repeater (OCR) for DVB-T/H signals is described in this paper. The presence of a coupling channel between the transmitting and the receiving antennas gives origin to one or more echoes, having detrimental effects on the quality of the repeated signal and critically affecting the overall system stability. A low-complexity echo canceller unit is then proposed, performing a coupling channel estimation based on the local transmission of low-power training signals. In particular, in this paper we focus on the stability issues which arise due to the non perfect echo cancellation. An upper bound on the probability of instability of the system is analytically found, providing useful guidelines for conservative OCR design, and some performance figures concerning different propagation scenarios are provided.

I. INTRODUCTION

An advantage of recent Digital Video Broadcasting-Terrestrial/Handheld (DVB-T/H) standards [1] is the possibility to realize a Single Frequency Network (SFN) to broadcast the video signal. As a consequence, the delicate task of planning the service coverage for a given geographical area is greatly simplified with respect to more traditional Multi Frequency Networks (MFNs), both because the frequency distribution on the territory stops being a critical issue and also because proper On-Channel Repeaters (OCRs) can be easily introduced as gap-fillers to extend or enhance the coverage. An important phenomenon to consider when designing and installing an OCR is the coupling between the transmitting and the receiving antennas, which inevitably causes detrimental echoes in the received signal. In practice these echoes degrade the signal and limit the amplifier gain of the repeater, since dangerous oscillations with potential system instability have to be avoided. To address this critical problem, several architectures for digital echo cancellers have been proposed, which mainly differ on the basis of the technique adopted to estimate the coupling channel [2]-[5]. In this work we focus on the low-complexity technique proposed in [6], based on the transmission of a low power training signal by the OCR, to estimate the echoes and correspondently set the cancelling unit.

The main problem caused by the presence of the coupling channel between the transmitting and the receiving antennas is the possibility that the repeater becomes unstable due to non perfect channel estimation and noise. The rate of occurrence of this condition has to be minimized through a careful design of the OCR, since, at best, an instability situation forces the OCR in an out-of-service state for several seconds. Even worse, if the mechanisms to detect possible system instabilities are not rapid enough, the signal amplifier may completely burn out. To the best of our knowledge, no mathematical analysis has been developed to quantify the probability that a repeater becomes unstable due to echoes and related cancelling techniques. Here we exploit the results of the pioneeristic work in [8], about the zeroes distribution of a random polynomial, to develop an analytical framework to express an upper bound on the probability of instability as a function of the coupling channel. The goal is to evaluate how the insertion of an echo canceller improves the performance of a repeater in terms of overall system stability. A low probability of instability, in fact, permits to increase the gain of the HPA and, as a consequence, to cover a wider area (reaching more users).

II. SYSTEM DESCRIPTION

Let us consider the general OCR architecture depicted in Fig. 1, where we have adopted the low-pass equivalent representation of the different processing blocks. The received DVB-T/H signal is sampled by an ideal A/D converter, with sampling frequency $f_s = 1/T_s$, chosen high enough to avoid aliasing. Then, it is passed through a digital filter aimed at reducing the amount of noise and possible interference from adjacent channels. Dually, at the output, the D/A conversion is preceded by a digital transmission filter whose main goal is to make the transmitted signal compliant with the electromagnetic compatibility mask. It is worth noticing that, if
the filtering process within the OCR is realized through a multirate architecture, the receiving filter may coincide with the first cascade of filtering and decimation stages, while the transmitting filter may be constituted by the subsequent cascade of interpolation and filtering stages [9]. This solution would imply also the advantage that the echo cancelling unit would work at the lowest sample rate of the system, reducing the requirements in terms of processing capabilities. After the D/A conversion, the signal to be transmitted is amplified by a high power amplifier (HPA) with gain $G$. In this work we assume the amplifier as ideal, i.e. not affected by non-linear effects, and not frequency-selective within the considered bandwidth.

Due to coupling effects between the transmitting and receiving antennas, a part of the (re)transmitted signal returns at the OCR receiver side, giving origin to one or more echoes. The purpose of the echo cancelling unit is to realize a negative feedback capable to compensate the effects of the coupling channel between the OCR antennas. The cancellation mechanism is based on the possibility to estimate the coupling channel impulse response and to locally reproduce it through a digital FIR filter. The OCR operates in two different modes: in the first one (referred to as start-up mode), the switch $S$ (see Fig. 1) is open and the repeater estimates the coupling channel without retransmitting the received signal. This initial channel estimation is realized through open-loop transmission of short pulses and a proper estimator unit operating on the received samples, as explained more in detail in Section III. Once the channel has been estimated, the echo canceller filter is initialized, the switch $S$ is closed and the OCR starts repeating the received DVB-T/H signal (steady-state mode). While operating in the steady-state mode, the OCR may keep tracking possible channel variations by continuously transmitting the estimation signal superimposed to the useful signal. It has to be noticed that the disturbance affecting the estimation is due to both thermal noise and received DVB-T/H data signal.

In practice, to realize the digital echo canceller we employ a shift register with $D$ stages followed by a FIR filter with $P$ taps, indicated with $w[k]$ for $k = 0 \ldots P - 1$. In fact the receiving and transmitting filters introduce a deterministic delay for the echoes, which can be easily compensated through a shift register of proper length. The cancelling window covered by the FIR filter has a length of $P \cdot T_s$ seconds, starting from instant $D \cdot T_s$. Thus, the transfer function $W(z)$ of the overall echo canceller is:
\[
W(z) = z^{-D} \sum_{k=0}^{P-1} w[k] z^{-k}.
\]

Let us indicate with
\[
h_c(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l)
\]
the continuous-time impulse response of the coupling channel, where $L$ is the total number of echo paths and $\tau_l$ and $h_l$ are the delay and the complex gain of the $l$-th path, respectively. For signals in the bandwidth $[-\frac{1}{2T_s}, \frac{1}{2T_s}]$, the cascade of Tx filter, D/A converter, HPA, coupling channel, A/D converter and Rx filter represents a linear system, that can be described through its discrete-time impulse response $h_{eq}[k]$ for $k = 0, 1, \ldots$ and the correspondent transfer function $H_{eq}[z] = Z\{h_{eq}[k]\}$. For the following analysis of stability it is convenient to consider the discrete-time equivalent scheme depicted in Fig. 2, which models the general OCR architecture of Fig. 1 under the assumptions of ideal HPA, A/D and D/A converters. In Fig. 2 we have indicated with $x[k] = d[k] + n[k]$ the sum of the received DVB-T/H samples and the thermal noise entering the OCR, with $r_1[k]$ the samples of the echoes due to the coupling channel and with $r_2[k]$ the samples of the local replica aimed at cancelling the echoes.

### III. Coupling Channel Estimation

In this work we adopt the low-complexity and sub-optimal approach proposed in [6], where coupling channel estimates are obtained by transmitting a series of short training pulses and by observing the corresponding channel response. In the following we will explicitly refer to the estimation during the start-up phase (switch $S$ open), but similar results hold also for the steady-state mode, if we assume a slowly varying coupling channel.

In order to reduce the estimation errors, the training pulses are repeated $N$ times with interval $T_p = K \cdot T_s$, with $K$ integer, and the samples of the corresponding channel responses are averaged. The training signal before the transmitting filter and the D/A converter is
\[
s_T[n] = \sum_{m=0}^{N-1} p_m \delta_{mK,n}
\]
with $\delta_{k,n}$ representing the Kronecker symbol and $p_m$ the complex amplitude of the $m$-th pulse. The received signal,
after the Rx filter, is:

\[ s_R[k] = \sum_{n=-\infty}^{+\infty} s_T[n] h_{eq}[k-n] + d[k] + n[k] \]

\[ = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{N-1} p_m \delta_{mK,n} h_{eq}[k-n] + d[k] + n[k] \]

\[ = \sum_{m=0}^{N-1} p_m h_{eq}[k-mK] + d[k] + n[k], \]  

(4)

where we have indicated with \( d[k] \) and \( n[k] \) the received DVB-T/H samples and the thermal noise, respectively. Denoting with \( p_n = A e^{j\phi_n} \) the complex amplitude of the \( n \)-th pulse, where \( A > 0 \) is a fixed amplitude and \( \phi_n \) a proper phase shift, the samples of the equivalent channel impulse response can be estimated by averaging the normalized received pulses. After some computations we obtain:

\[ h_{eq}[k] = \frac{1}{N} \sum_{n=0}^{N-1} \frac{p_n^*}{A^2} s_R[nK+k] = h_{eq}(kT_s) + v[k], \]  

(5)

where we have assumed that the interval \( K \) is long enough to have \( h_{eq}[n] \approx 0, \forall n \) such that \( n \geq K \), and where:

\[ v[k] \triangleq \frac{1}{NA} \sum_{n=0}^{N-1} e^{-j\phi_n} (d[nK+k] + n[nK+k]), \]  

(6)

It can be shown that, if we assume a low oversampling factor within the repeater and a non-selective channel between the main broadcast station and the OCR, the estimation errors \( v[k] \) are (approximately) statistically i.i.d. and distributed according to a Circular Symmetric Gaussian Distribution (CSGD), i.e.,

\[ v[k] \sim \mathcal{C}\mathcal{N} \left( 0, \frac{2N_0 B_{eq}}{N A^2} + \frac{P_{DVB}}{N A^2} \right), \]  

(7)

where \( N_0 \) is the one-sided power spectral density of the thermal noise, \( P_{DVB} \) is the power of the signal \( d(t) \) and \( B_{eq} \) is the equivalent noise bandwidth of the Rx filter [7]. We verified numerically that this Gaussian approximation well suits the case under study.

After the estimation process, we set the filter taps to mimic the behaviour of the equivalent coupling channel, i.e.

\[ w[k] = h_{eq}[k+D] = h_{eq}[k+D] + v[k+D]. \]  

(8)

Clearly, the condition for a perfect echo cancellation is \( W(z) = H_{eq}(z) \), which is attained when \( w[k] = h_{eq}[k+D] \), for \( k = 0 \ldots P - 1 \), and \( h_{eq}[k] = 0 \) for \( k \leq D - 1 \) and \( k \geq D + P \). In this case the overall transfer function of the OCR would be non-selective in the bandwidth of interest. Unfortunately, the presence of the noise component \( v[k] \) in (8) leads to a non-perfect channel estimation and a consequent non-perfect echo cancellation, resulting in potential instability of the OCR.

IV. UPPER BOUND ON THE PROBABILITY OF INSTABILITY

A. Transfer Function of the OCR

According to Fig. 2, the analysis of the repeater stability can be carried out considering the following transfer function in the Z-transform domain:

\[ H_{OCR}(z) = \frac{\mathcal{Z}\{v[k]\}}{\mathcal{Z}\{x[k]\}} = \frac{1}{1 - [H_{eq}(z) - W(z)]}, \]  

(9)

where \( H_{eq}(z) = \mathcal{Z}\{h_{eq}[k]\} \) and \( W(z) = \mathcal{Z}\{w[k-D]\} \). Assume that the equivalent channel impulse response is different from zero in the window \([k_m T_s, k_M T_s]\). In other words, we can find a couple of indices \((k_m, k_M)\) so that \( h_{eq}[k] \approx 0 \) for \( k < k_m \) and \( k > k_M \). For simplicity, \(^1\) we consider a cancelling windows large enough to fit completely the pulse response of the equivalent channel, i.e. \( P = k_M - k_m + 1 \) and \( D = k_m \).

Define the estimation signal-to-noise ratio as:

\[ \gamma_e \triangleq \frac{1}{\mathbb{E}\{v[k]^2\}} = \frac{NA^2}{2N_0 B_{eq} + P_{DVB}}. \]  

(10)

Thanks to the previous definitions and assumptions, we can write:

\[ H_{OCR}(z) = \frac{1}{1 + \sum_{k=k_m}^{k_M} v[k] z^{-k}} = \frac{z^{k_M}}{z^{k_M} + \sum_{l=0}^{k_M-k_m} v[k-M-l] z^{-l}} = \frac{z^{k_M}}{\sum_{l=0}^{k_M} p_l z^{-l}}, \]  

(11)

where

\[ p_l \triangleq \begin{cases} v[k_m-l] & \text{for } 0 \leq l \leq k_m - k_m, \\ 0 & \text{for } k_m - k_m < l \leq k_M - 1, \\ 1 & \text{for } l = k_M, \end{cases} \]  

and \( v[l] \sim \mathcal{C}\mathcal{N} \left( 0, \frac{1}{\gamma_e} \right) \forall l, \) and \( \mathbb{E}\{v^*[l]v[m]\} = \frac{1}{\gamma_e} \delta_{l,m}. \)

B. Random Polynomial

We note from (11) that the stability of the repeater depends on the zeroes distribution of the \( k_M \)-degree polynomial \( p(z) \) at the denominator, i.e.,

\[ p(z) \triangleq \sum_{l=0}^{k_M} p_l z^{-l} \]  

(13)

Thanks to this observation, we can apply the method presented in [8] to evaluate the zeroes density function \( f(z) \) in the complex domain. To this aim, in the following we will define and evaluate some stochastic parameters.

By defining the random vector \( \mathbf{p} \triangleq [p_0, p_1, \ldots, p_{k_M}]^T \), it is evident that the polynomial (13) can be written as:

\[ p(z) = \sum_{l=0}^{k_M} p_l z^{-l} = \mathbf{p}^T \mathbf{v}(z), \]  

(14)

\(^1\)Note that our approach is valid also without this assumption, but the following expressions become more complicated.
where \( v(z) \triangleq [1, z, \ldots, z^{km}] \). The zeroes of this polynomial are shown in [8] to be distributed in the complex domain according to the following zeroes density function:

\[
\begin{align*}
\tilde{f}_z(z) & = \frac{1}{\pi l_0(z)} \exp \left( -\frac{|v^T(z)u_p|^2}{l_0(z)} \right) \times \\
& \left( v'(z) - \frac{l_2(z)}{z l_0(z)} v(z) \right)^T \Phi_{pp} \left( v'(z^*) - \frac{l_2(z)}{z^* l_0(z)} v(z^*) \right),
\end{align*}
\]

where \( v'(z) \triangleq [0, 1, 2z, \ldots, kmz^{km-1}]^T \) and

\[
\begin{align*}
u_p & \triangleq \mathbb{E} \{ p \}, \\
\Phi_{pp} & \triangleq \mathbb{E} \{ pp^H \},
\end{align*}
\]

\[
l_0(z) \triangleq v^H(z) C_{pp} v(z), \\
l_1(z) \triangleq v^H(z) C'_{pp} v(z), \\
l_2(z) \triangleq v^H(z) C''_{pp} v(z),
\]

being \( C_{pp} \triangleq \Phi_{pp} - u_p u_p^H \), and \( C'_{pp} \) and \( C''_{pp} \) \( L \times L \) matrices whose generic elements at row \( l \) and column \( m \) are defined as \( C'_{pp}[l, m] \triangleq \text{Im} C_{pp}[l, m] \) and \( C''_{pp}[l, m] \triangleq \text{Im} C_{pp}[l, m] \), respectively. More precisely, integrating (15) over a region \( C \) of the complex plain provides the average number of polynomial zeroes falling in \( C \). After some calculations, it is possible to prove that:

\[
\tilde{f}_z(z) = \frac{1}{\pi |z|^2} \exp \left( -\gamma e \frac{\sum_{l=0}^{km} k_{m-l-k} |z|^{2l}}{\sum_{l=0}^{km} k_{m-l-k} |z|^{2l}} \right) \left[ \Lambda(|z|) + \gamma e \Psi(|z|) \right],
\]

where we have defined the following functions of real variables:

\[
\begin{align*}
\Lambda(x) & \triangleq \sum_{l=0}^{km-k_0} \left( -\frac{\sum_{l=0}^{km-k_0} k_{m-l-k} x^{2k}}{\sum_{l=0}^{km-k_0} k_{m-l-k} x^{2k}} \right)^2 x^{2l}, \\
\Psi(x) & \triangleq \sum_{l=0}^{km-k_0} \left( -\frac{\sum_{l=0}^{km-k_0} k_{m-l-k} x^{2k}}{\sum_{l=0}^{km-k_0} k_{m-l-k} x^{2k}} \right)^2 x^{2km}.
\end{align*}
\]

Writing \( z = r e^{j\theta} \), where \( r = |z| \) and \( \theta = \arg(z) \), the zeroes density and zeroes cumulative functions are respectively:

\[
\tilde{f}_r(r) \triangleq r \int_0^{2\pi} f_z(r \cos(\theta) + j r \sin(\theta)) d\theta
\]

\[
= r \int_0^{2\pi} \frac{1}{\pi r^2} \exp \left( -\gamma e \frac{\sum_{l=0}^{km} k_{m-l-k} r^{2l}}{\sum_{l=0}^{km} k_{m-l-k} r^{2l}} \right) \left[ \Lambda(r) + \gamma e \Psi(r) \right] d\theta
\]

\[
= r \exp \left( -\gamma e \frac{\sum_{l=0}^{km} k_{m-l-k} r^{2l}}{\sum_{l=0}^{km} k_{m-l-k} r^{2l}} \right) \left[ \Lambda(r) + \gamma e \Psi(r) \right]
\]

\[
F_R(R) = \int_0^R f_r(r) dr.
\]

In particular, the zeroes cumulative function \( (25) \) represents the average number of polynomial zeroes contained in the circle of radius \( R \) and center in the origin of the complex plain. Since its value for \( R = 1 \) gives the mean number of zeroes inside the unit circle, it is clear that, for a polynomial with degree \( km \), the average number \( \pi \) of the zeroes of \( p(z) \) outside the unit circle is:

\[
\pi \triangleq km - F_R(1) = km - \int_0^1 f_r(r) dr.
\]

Since the number of zeroes is always integer, it is also true that:

\[
\pi = \sum_{i=1}^{km} \frac{1}{\pi} \left[ \left[ \Lambda(r) + \gamma e \Psi(r) \right] d\theta \right] = \sum_{i=1}^{km} \frac{1}{\pi} \left[ \Lambda(r) + \gamma e \Psi(r) \right] d\theta.
\]

On the other side, the probability that the system is unstable is the probability that at least one zero is outside the unit circle:

\[
P_{\text{uns}} = \sum_{i=1}^{km} \frac{1}{\pi} \left[ \Lambda(r) + \gamma e \Psi(r) \right] d\theta.
\]

Thus we are interested in the evaluation of \( \pi = km - F_R(1) \) because it constitutes an upper bound of the probability of instability. By substituting (24) in (25) we have:

\[
\pi = km - \int_0^1 \frac{1}{\pi} \exp \left( -\gamma e \frac{\sum_{l=0}^{km} k_{m-l-k} r^{2l}}{\sum_{l=0}^{km} k_{m-l-k} r^{2l}} \right) \left[ \Lambda(r) + \gamma e \Psi(r) \right] d\theta.
\]

that can be evaluated numerically.

V. NUMERICAL RESULTS

In Fig. 3 we show the upper bound (29) on the probability of instability as a function of the estimation signal-to-noise ratio \( \gamma e \). We have considered several propagation scenarios, where the coupling channel is characterized by different delay spreads. We have taken into account a typical urban environment and the different curves depicted in Fig. 3 refer to increasing number \( L \) of echo paths. In particular, the coupling channel delay spreads have been modelled according to the TU-12 channel used in [4]. As expected, we can observe that, increasing \( \gamma e \) (i.e. the quality of the coupling channel estimate), \( P_{\text{uns}} \) tends to zero, determining a more robust behaviour for the OCR. As can be noticed from the definition (10), the signal-to-noise ratio \( \gamma e \) can be increased by augmenting the number \( N \) of channel responses being averaged or the pulse amplitude \( A \). In the former case, the duration of the estimation process becomes longer and the capacity of the OCR to react to possible echo variations decreases. In the latter case, the

\[
2p(z) \text{has always km zeroes, because, from (12), the km-th order coefficient is always 1.}
\]
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

Fig. 3. The Upper Bound of the Probability of Instability as a function of the estimation Signal-to-Noise Ratio for different number \( L \) of considered echoes.

Fig. 4. Comparison between the upper bound (29) and some estimates of the probability of instability obtained through simulation. In particular, each simulated point has been obtained through randomly generating OCR transfer functions until 100 unstable realizations have been obtained.

disturbance introduced by the training pulses over the DVB-T/H signal increases, reducing the SNR experienced by the final users. Some of these aspects have been investigated in [5] and [6]. A correct OCR design should jointly take into account all these issues, determining the proper trade-off between the probability of instability of the system, the quality of the repeated signal and the echo tracking capacity.

In Fig. 4, we report the comparison between the bound (29) and simulation results obtained by randomly generating many transfer functions (9) and verifying their stability through the analysis of their poles. As it can be noticed from the figure, for low values of \( \gamma_e \) (and \( L \)), the upper bound is not extremely tight, but, for increasing values of \( \gamma_e \), simulation results tend to the analytical bound (29). However, we observe that a good OCR design should guarantee a very low probability of instability and, as a consequence, the most interesting region from a design perspective is exactly where our bound becomes tight. In any case, the bound (29) can provide useful guidelines for a conservative design of the OCR parameters. In Fig. 4, we have also reported some simulation results (indicated with triangles and circles) considering correlation between the estimation noise samples, due to moderate oversampling (\( \approx 1.6 \)). In principle, the approach of section IV can be extended to obtain a bound accounting also for the correlated noise. However, from the results in Fig. 4, it can be noted that its effect is negligible for the values of \( P_{\text{uns}} \) of interest and the bound (29) is still useful.

VI. CONCLUSIONS

In this paper we have shown the stability performance of a low-complexity digital echo canceller based on the transfer function of a locally generated training signal. An analytical framework has been developed to evaluate an upper bound on the probability of instability of the repeater, as a function of the signal-to-noise estimation ratio, for different coupling channels.

ACKNOWLEDGMENT

This work has been performed within the Project DVB2006 supported by MetaSystem S.p.A.

REFERENCES


3Notice that the simulated points constitute just estimates of the probability of instability, thus, when the bound becomes very tight it is possible that the simulation results fall a little below or also above the upper bound. Moreover, increasing \( L \) the analysis of the poles of the transfer functions requires to find the roots of high-degree polynomials and some numerical inaccuracies inevitably arise.