On the Capacity of UWB over Multipath Channels

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Abstract—In this work we compute the information theoretic capacity C of binary orthogonal pulse position modulated (PPM) signals for ultra wideband (UWB) communications over multipath channels. We consider binary PPM signals with random energy and correlation values. Numerical examples are given to illustrate the capacity results.

Index Terms—Ultra wideband communications, pulse position modulation, channel capacity, multipath channels.

I. INTRODUCTION

COMMUNICATIONS using UWB with time hopping (TH) and PPM has been studied extensively [1]-[3]. It also has been proposed for consideration in IEEE standard bodies [4]. This work calculates C for orthogonal UWB PPM signals over multipath channels considering the random variations in both the received signal energy and the signal correlation values.

In contrast, previous work calculated C for the additive white Gaussian noise (AWGN) channel [5] [6], and the work in [7] studied random variations in the received signal energy. We assume detection using a receiver perfectly synchronized and matched to the received signals, with time-invariant channel conditions valid during a bit interval. Fig. 1 shows the situation considered in this work.

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II. SYSTEM MODEL

We assume detection using a receiver perfectly synchronized and matched to the received signals, with time-invariant channel conditions valid during a bit interval. Fig. 1 shows the situation considered in this work.

Under free space propagation conditions (i.e., the AWGN channel with n(t) having two-sided power spectrum density N0/2), the received signal Ψi(t), i = 1, 2, is the derivative of the transmitted signal ΨTx(t), modified by amplitude Ao and delay τo factors that depend on the transmitter-receiver separation distance D,1 where

\[ E^\Psi \triangleq \int_{-\infty}^{\infty} [\Psi_i(t)]^2 dt \]

is the received signal energy and

\[ beta \triangleq \frac{1}{E^\Psi} \int_{-\infty}^{\infty} [\Psi_1(t)]\Psi_2(t) dt \]

is the normalized correlation value between Ψ1(t) and Ψ2(t), with β = 0 for orthogonal signals. In this case C depends solely on the signal-to-noise ratio (SNR) [8].

Under multipath conditions (e.g., a slowly varying indoor radio channel, where the transmitter is placed at a certain fixed location, and the receiver is placed at a variable location denoted uo), the received signal Ψi(uo, t) is a multipath spread version consisting of multiple replicas Ψi(k)(uo, t), each one with different amplitude Ak(uo, t), delay τk(uo), and frequency content [9], where

\[ E^\Psi_k(uo) \triangleq \int_{-\infty}^{\infty} [\Psi_i(uo, \xi)]^2 d\xi \]

is the random signal energy, and

\[ beta(uo) \triangleq \frac{1}{E^\Psi(uo)} \int_{-\infty}^{\infty} [\Psi_i(uo, \xi)] [\Psi_2(uo, \xi)] d\xi \]

is the random normalized correlation value between Ψ1(uo, t) and Ψ2(uo, t). Even when β = 0, β(uo) is a random variable that is not necessarily zero [10].

A. PPM Signals over the Gaussian Channel

The binary orthogonal PPM signals considered here are [2]

\[ \Psi_i(t) = \sum_{k=0}^{N_s-1} w(t - kT_f - (i - 1)\delta), \quad i = 1, 2. \] (1)

The T_f is the frame repetition period. The duration of Ψi(t) is T_s = N_sT_f. The signal w(t) is the basic UWB pulse used to convey information. It has duration T_w and energy

\[ E_w = \int_{-\infty}^{\infty} [w(t)]^2 dt \]

The normalized signal correlation function of w(t) is

\[ gamma(\tau) \triangleq \frac{1}{T_w} \int_{-\infty}^{\infty} w(t)w(t - \tau)dt > -1 \\forall \tau. \]

By assuming T_f > T_w + δ we get that Ψi(t) in (1) have

\[ E^\Psi = E_w N_s \]

and also that β = γ(δ). For δ ≥ T_w, the signals become orthogonal with β = 0.

B. PPM Signals over the Multipath Channel

The transmitted pulse is the same pulse \( w_{\text{TH}}(t) \) used in the Gaussian channel case, and the received “pulse” is \( \sqrt{E_w w}(u_o, t) \).

\[ C \triangleq E_u \{ C(u) \} \]

of C(uo) over the multipath effects.

The transmitted pulse is the same pulse \( w_{\text{TH}}(t) \) used in the Gaussian channel case, and the received “pulse” is \( \sqrt{E_w w}(u_o, t) \).

1In our analysis we will assume A_o = 1 and τ_o = 0.
is a multipath spread version of \( w(t) \) received at position \( u_o \), it has average duration \( T_a > T_w \) and random energy \( E_w(u_o) \triangleq E_o \alpha^2(u_o) \), where \( E_o \) is the average energy and \( \alpha^2(u_o) \triangleq \int_0^\infty [\hat{w}(u_o,t)^2] dt \). The 
\[
\sqrt{E_w}(u_o,t) \hspace{1cm} \text{has random correlation function } \gamma(u_o,\tau) \triangleq \frac{\int_{-\infty}^{\infty} \sqrt{E_w(s)} \sqrt{E_w(u_o,t-\tau)} ds}{E_w(u_o)} > -1 \hspace{1cm} \forall \tau. \]
Clearly, the multipath effects change for different \( u_o \), and therefore \( E_w(u_o) \) and \( \gamma(u_o,\tau) \) both change with \( u_o \) [10].

Using the previous pulse definition, the PPM signals containing the multipath effects can be denoted 
\[
\hat{\Psi}_i(u_o,t) = \sum_{k=0}^{N_s-1} \sqrt{E_o} \hat{w}(u_o,t-ktf-(i-1)\delta), \quad i = 1, 2, \quad (2) \]
For simplicity we assume that \( \hat{\Psi}_i(u_o,t) \) has fixed duration \( T_f \sim N_s T_f \). By assuming \( T_f > T_a + \delta \) we get 
\( \hat{\Psi}_i(u_o,t) \) in (2) have \( E_w(u_o) \approx E_o \alpha^2(u_o) \), where \( E_o = N_s E_w \) is the average signal energy, and also that the \( \hat{\Psi}_1(u_o,t) \) and \( \hat{\Psi}_2(u_o,t) \) have \( \beta(u_o) \approx \gamma(u_o,\delta) \). Even though \( \beta = 0 \) for \( \delta \geq T_w \), \( \beta(u_o) \) is a random variable with values in \( (-1, 1) \).

C. The Choice of \( \delta \) and \( T_f \)
In a “typical” UWB TH PPM system design [1]-[3], \( \delta \) lasts a few ns (similar to \( T_o \)), \( T_f \) lasts a few hundred of ns, and \( N_s \) is a random variable of dozens or even a few hundred. The symbol rate is \( R_s = \frac{1}{T_f N_s} \). In a multi-user environment with TH present, the pulse hops over a range \( T_f - \delta - T_w \), and the processing gain is \( G \approx N_s T_f T_w^{-1} \).

In this work we use \( \delta = T_w \). To avoid interpulse interference we use \( T_f > T_a + \delta \) (for consistency, this condition is applied to both Gaussian and multipath channels). This assumption simplifies the calculation of \( \beta(u_o) \) for \( \hat{\Psi}_i(u_o,t) \) to get a value approximately similar to \( \gamma(u_o, T_w) \) for \( \hat{w}(u_o,t) \). Even if interpulse interference were present, intersymbol interference could be neglected for large \( N_s \).

D. Vector Model for Capacity Calculations
Capacity calculations are based on energy and correlation values of the received signals. We consider, for the time being, that the multipath conditions are being kept fixed, i.e., the capacity calculations are conditioned on a particular value \( u_o \). To calculate the capacity we use the channel model in Fig. 2. We consider a binary PPM modulator with input \( V \) and output \( \Psi_i(u_o), \ i = 1, 2 \). The scalar \( V \) is the output of a 1-bit source, with zeros and ones being equally likely. The 2-dimensional vector \( \Psi_1(u_o) \triangleq \sqrt{E_o} \psi(u_o) (\psi_1(u_o), \psi_2(u_o)) \) represents the UWB signal \( \hat{\Psi}_1(u_o,t) \), and the vector \( \Psi_2(u_o) \triangleq \sqrt{E_o} \psi(u_o) (\psi_1(u_o), \psi_2(u_o)) \) represents \( \hat{\Psi}_2(u_o,t) \), where 
\[
\psi_1(u_o) \triangleq \int \frac{1+\gamma(u_o,\delta)}{2} ds \quad \text{and} \quad \psi_2(u_o) \triangleq \int \frac{1-\gamma(u_o,\delta)}{2} ds. \]
These vectors are the projections of the received signals with respect to an orthonormal basis whose elements are 
\[
\Phi_1(u_o,t) \triangleq \Phi_1(u_o,t) + \Phi_2(u_o,t) \quad \text{and} \quad \Phi_2(u_o,t) \triangleq \frac{\Phi_1(u_o,t) - \Phi_2(u_o,t)}{2 \sqrt{E_o} \psi(u_o)} \psi(u_o). \]
The reader can verify that \( \Psi_1(u_o), \Psi_2(u_o) \) indeed have energy \( E_o \psi(u_o) \) and correlation value \( \beta(u_o) = \gamma(u_o,\delta) \).

The vector \( \Psi_i(u_o) \) is sent through the vector Gaussian channel in Fig. 2. The output of the channel is \( \Psi_i(u_o) = \bar{W}(1) + \bar{W}, \) where \( \bar{W}(1) \triangleq \langle y_1(u_o), y_2(u_o) \rangle \) and \( \bar{W} \triangleq \langle \eta_1, \eta_2 \rangle \) is a real Gaussian noise vector with zero mean and variance \( \sigma^2 = \frac{N_o}{2} \) in each dimension.

III. Calculation of Channel Capacity
In this section we calculate the information-theoretic channel capacity for the binary PPM vectors \( \Psi_i(u_o) \). The capacity derivation generalizes the calculations done for orthogonal signals in [8]. The channel capacity with input signals restricted to a discrete set of binary equally-likely non-orthogonal PPM signals, and continuous-valued outputs, can be found to be 
\[
C(u_o) = \frac{1}{2} \left( \log_2 \left( 1 + \frac{p(\bar{W}(1) \mid \Psi_1(u_o))}{p(\bar{W}(1) \mid \Psi_2(u_o))} \right) \right) \]
\[
C(u_o) = \frac{1}{2} \left( \log_2 \left( 1 + \frac{p(\bar{W}(2) \mid \Psi_1(u_o))}{p(\bar{W}(2) \mid \Psi_2(u_o))} \right) \right) \]
\[
C(u_o) = 1 \]
\[
C(u_o) = \left( 1 + \frac{\sqrt{E_o}(u_o) \psi_2(u_o) + y_2(u_o)}{N_o/2} \right) \]
\[
C(u_o) = \left( 1 + \frac{\sqrt{E_o}(u_o) \psi_2(u_o) + y_2(u_o)}{N_o/2} \right) \cdot (3) \]
where \( p(\bar{W}(u_o) \mid \Psi_1(u_o)) \) is the probability density function (p.d.f.) of \( \bar{W}(u_o) \) conditioned on \( \Psi_1(u_o) \), and \( E[\cdot] = E(\bar{W}(u_o) \mid \Psi_1(u_o)) \{ \} \) is the expected value with respect to \( \bar{W}(u_o) \) conditioned on \( \Psi_1(u_o) \).

To calculate the capacity \( C(u_o) \) in (3) we need to calculate the expectations \( E[\bar{W}(u_o) \mid \Psi_1(u_o)] \{ \} \). These expectations can be estimated via Monte Carlo simulation [8]. The method is to generate pseudorandom 2-dimensional vectors \( \bar{W}(u_o) \) according to the p.d.f. \( p(\bar{W}(u_o) \mid \Psi_1(u_o)) \). For each generated
Finally the sample average of the logarithm is calculated.

\[ \bar{C} \approx \frac{1}{u_s} \sum_{u_o=1}^{u_s} C(u_o). \]  

IV. NUMERICAL EXAMPLE

The UWB signals considered in this example are based on pulsed sine waves. The received pulse is modeled as

\[ w(t) = \sin(2\pi \frac{10}{T_w} t), \quad 0 \leq t \leq T_w, \]  

with autocorrelation

\[ \gamma(\tau) = \frac{1}{E_w} \frac{T_w}{T} \cos(2\pi \frac{10}{T_w} \tau), \quad -T_w \leq \tau \leq T_w. \]  

The duration of \( w(t) \) is \( T_w = 2.0 \) ns, with \( E_w = \left( \frac{T_w}{T} \right) \). The spectrum of \( w(t) \) is centered at \( \left( \frac{10}{T_w} \right) = 5 \) GHz, with a 10 dB bandwidth of about 700 MHz, satisfying the new definition of UWB signal stating that the 10 dB bandwidth of the signal should be at least 500 MHz [12].

To characterize the multipath channel we use an autoregressive channel model [13] [14] to form and ensemble of modeled channel pulse responses \( \hat{w}(u_o, t) \) as described in [15]. We consider line-of-sight (LOS) scenarios with \( D = 3, 6, 9 \) m, and non-line-of-sight (NLOS) scenarios with \( D = 1, 2, 3 \) m. The simulated \( \hat{w}(u_o, t) \) has \( T_a \approx 160 \) ns. By selecting \( T_f = 170 \) ns we make sure that \( T_f > T_a + \delta \).

A total of \( u_s = 294 \) channel pulse responses \( \hat{w}(u_o, t) \) are used (49 per each distance value). An equal number of pairs \( E_{\hat{w}}(u_o), \beta(u_o) \) are calculated. These \( u_o \) sets of values are then used to compute (4). Figs. 3(a) show examples of energy values. Figs. 3(b) and 3(c) show examples of correlation values. Fig. 3(d) shows \( \bar{C} \) vs. \( E_{\hat{w}}/N_o \triangleq E_{\hat{w}}/(N_0/2\pi) \) for both AWGN and multipath channels. Compared with the AWGN channel, in the LOS multipath channel the 99 percent capacity is reached with an SNR disadvantage of about 4 dB. For the NLOS the SNR disadvantage is about 11 dB.

V. CONCLUSIONS

This work computes the information theoretic capacity of binary orthogonal PPM signals for UWB communications over multipath channels. We consider binary PPM signals with random energy and correlation. A numerical example is given to illustrate the capacity results and quantify the SNR losses in the presence of multipath.

VI. ACKNOWLEDGMENTS

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