Streamwise and spanwise vortex interaction in an axisymmetric jet. A computational and experimental study

F. F. Grinstein
Laboratory for Computational Physics and Fluid Dynamics, Code 6410, NRL, Washington, DC 20375-5344

E. J. Gutmark, a) T. P. Parr, and D. M. Hanson-Parr
Propulsion Research Branch, Code 3892, NAWC, China Lake, California 93555-6001

U. Obeysekare
Center for Computational Science, Code 5594, NRL, Washington, DC 20375-5344

(Received 2 October 1995; accepted 21 February 1996)

The near-field of an azimuthally excited round jet was investigated in a combined computational/experimental study. The reaction zones in the jet were visualized using OH Planar-Laser-Induced-Fluorescence (PLIF) diagnostics. Both axisymmetric and azimuthal modes of the jet were excited to stabilize its spatial structure. Three-dimensional flame visualization of the laboratory jet reconstructed from multiple two-dimensional images acquired at constant phase angle, reveal a complex structure of the reaction zone. Time-dependent numerical simulations provided insight into the underlying fluid-dynamical processes leading to this flame structure. Simulations of reactive and non-reactive free jets used a Monotonically Integrated Large-Eddy-Simulation (MILES) approach, multi-species diffusive transport, global finite-rate chemistry and appropriate inflow/outflow boundary conditions. The flow visualizations of the experimental and computational jets strongly resemble each other, revealing tight coupling between axisymmetric vortex rings and braid (rib) vortices. The jet vorticity evolution is dominated by the dynamics of vortex-ring self-deformation induced by the azimuthal excitation imposed at the jet exit, the dynamics of rib vortices forming in the braid regions between undulating vortex rings, and strong interactions between rings and ribs. The observed topological features of the flow are directly related to the nearly-inviscid jet vorticity dynamics. These processes affect the mixing pattern of the jet, resulting in localized regions of high fuel concentration leading to combustion inactive regions in the flame, and other regions with enhanced mixing and a proper air-to-fuel ratio in the flame where the combustion process is intense. The vorticity dynamics and ensuing mixing processes determine the regions of combustion within the flame and thus the overall heat release pattern. © 1996 American Institute of Physics.

I. BACKGROUND

Large-scale coherent structures (CS)—characterized by organized vorticity distributions—are intrinsic features of mixing layers at high enough Reynolds numbers (Re) and govern the entrainment and mixing in jet flows. Control of the jet development in the practical applications is strongly dependent on understanding the dynamics and topology of CS; in particular, how the jet properties can be affected through control of the formation, interaction, merging, and breakdown of CS. Two-dimensional vortex rings and their sequential mergings dominate the shear-layer growth and entrainment in round jets at moderately-high Re. For higher Re, and/or when the axisymmetry of the jet is broken or not present, three-dimensionality becomes an important feature, and mechanisms such as self-induction and vortex stretching, as well as vortex reconnection, become major fluid-dynamical processes involved in the transition from the laminar to the turbulent jet regime.

In order to passively enhance the three-dimensionality of the flow, and thus entrainment and mixing, there have been

\(^{a}\)Present address: Department of Mechanical Engineering, Louisiana State University, Baton Rouge, Louisiana 70803-6413.

10 Phys. Fluids 8 (6), June 1996
1070-6631/96/8(6)/1515/10/$10.00 © 1996 American Institute of Physics
prime importance, even at low Re. In fact, the 3-D interaction between large-scale spanwise vortices and streamwise vortices, in a simple axisymmetric nozzle combustion system, is apparently a major mechanism in the breakdown of 2-D CS into fine scale 3-D turbulence required for molecular mixing and good combustion efficiency. The experimental results, which can show only the end outcome of the complex physical processes, exhibit a complex three-dimensional structure of the flame with many unexplained features such as the distribution of the reaction zones, extinction locations, and the evolution of the geometry of the reaction zones. These observations can be interpreted by the numerical simulations which give insight into the detailed fluid dynamics, the vorticity field, and the other mixing processes leading to the measured flame structure.

In the present work, the interaction between streamwise and axisymmetric vortices in an azimuthally-excited circular jet was studied experimentally and computationally. Three-dimensional images of a reacting jet based on planar laser-induced-fluorescence diagnostics were constructed from multiple two-dimensional images acquired at a constant phase angle. Reactive and non-reactive free jets developing in both space and time, were investigated computationally using a monotonically integrated large-eddy-simulation approach, and appropriate inflow/outflow boundary conditions. The laboratory spanwise vortex structures were stabilized acoustically in a turbulent circular jet at a moderate Re, while streamwise vorticity was introduced by mounting small vortex generators around the nozzle circumference in the boundary layer upstream of the exit. The numerical simulations involve weak axial forcing, and emulate the effects of the excitation introduced by the vortex generators in the laboratory jets by including an azimuthally varying, time-independent radial velocity among the jet initial conditions.

In what follows, we first describe the experimental setup and results on visualization of the azimuthal structure of the flame and reconstructed three-dimensional images. Next, we describe the numerical model, and include a detailed discussion of the jet dynamics based on the numerical database. It is shown both in the experimental results and the simulations that the interaction between the axisymmetric vortices and the imposed streamwise excitation leads to the deformation of the vortex rings affecting their local curvature. The subsequent self-induction processes result in a complex three-dimensional structure which modifies the mixing and combustion processes.

II. LABORATORY EXPERIMENTS

A system for PLIF imaging of OH radicals in flames using a XeCl excimer laser at 308 nm was used (Fig. 1). The laser beam was expanded into a planar sheet, passed through the flow in a chosen direction: parallel to the jet axis for studying the streamwise evolution of the structures or perpendicular to the axis for azimuthal structures measurements. The resonance fluorescence from OH was imaged with a gated intensified diode array camera. The sheet was approximately 0.5 mm wide.

The OH radical produced by combustion reactions is used to indicate regions where combustion or hot combustion products are present. The OH molecule is an important flame radical participating in combustion reactions in the flame front, but it is also present as an equilibrium product in the hot burnt gas region. Thus OH PLIF images might show not only the active combustion zones, but regions of burnt gases as well. However, several studies have shown superequilibrium concentrations of OH in the flame front regions. Cattolica showed OH concentrations in the flame front of lean and stoichiometric methane-air flat flames as much as 6 to 8 above equilibrium, and up to 40 times above equilibrium for rich flames. Drake et al. show 4 to 5 times superequilibrium concentrations of OH in turbulent non-premixed flames. Thus OH concentration maps would be expected to favor the flame front in turbulent combusting flows, and OH PLIF images should give a good indication of regions of active combustion. This is especially true if, in addition to superequilibrium effects, the OH concentration in the burnt gas regions are reduced by cooling caused by entrainment of surrounding cool air.

The air jet issued at a velocity of 5 m/s from a circular nozzle with diameter $D=22$ mm, ratio $D/\theta_0=21$, and Reynolds number $Re_\theta$ in the range of 350 to 4000-based on the initial momentum thickness $\theta_0$, jet exit velocity $U_j$, and kinematic viscosity $\nu$ of air at room temperature.

Propane fuel was injected circumferentially around the air jet nozzle into the initial shear layer, via 24 1.6 mm diameter holes, on a 23.5 mm diameter circle, which are slightly angled into the shear layer. This burner configuration formed an annular diffusion flame, typical to a ramjet combustor, for example. The azimuthal continuity of the OH distribution, was not affected significantly by the fuel injection through multiple discrete holes: the flame sheet and the flow became circumferentially uniform at a distance of 0.3D downstream from the nozzle. PLIF OH images taken in a plane perpendicular to the flame’s axis at this distance showed a uniform annular distribution, and so did azimuthal hot-wire measurements of the flow. The fuel exit velocity was 0.2 m/s. The initial rms turbulence level measured at the exit of the nozzle on the centerline was 3.6%. The exit velocity profile was typical of a turbulent pipe flow.

The Damköhler number $Da$, defined as the ratio of characteristic convective mixing and chemical times is estimated
to be between 2 and 420. This is based on an estimated chemical time of 85 μs calculated as the ratio of the cold kinematic viscosity to the square of the laminar flame speed. The mixing time is calculated as the ratio of a characteristic length scale to the turbulent fluctuation velocity, which in the present case is about 0.7 m/s. The larger length scale $l_0$, leading to the larger $Da$, was taken as the wavelength of the preferred mode vortices. The shorter length scale used, leading to the smaller $Da$, was the Kolmogorov scale $l_k$. The Kolmogorov scale was estimated here to be $l_k=0.12$, using $l_k=(nu/\varepsilon)^{1/4}$, where $\varepsilon$ is the dissipation rate. The dissipation rate was approximated with $\varepsilon=3/4(u'\nu)^{1/4}$, where $u'$ is the peak rms level of the turbulent fluctuations in the center of the jet shear layer.

The flow was excited by an acoustic driver mounted on the circular pipe leading to the nozzle. The driver operated at controlled frequencies and amplitudes using a dual phase locked loop and audio power amplifier. The acoustic forcing level resulted in coherent velocity fluctuations of 15% of the mean velocity. To construct the turbulent structure at a specific phase, the laser system was phase-locked to the acoustic excitation of the flame jet.

The azimuthal structure was stabilized by vortex generators in the jet boundary layer upstream of the exit. Previous experiments showed that the preferred azimuthal structure has a five-fold symmetry. Therefore, a set of five pairs of split delta-wings were installed at the exit. The configuration of wings discussed in this paper, introduce radial velocity perturbations directed towards the jet axis. The triangles forming the semi-delta wing had a base of 4 mm and an apex angle of 30°. Two configurations of vortex generators were used: delta-wing and “mushroom” vortex generators. The differences between them are explained below when Fig. 2 is discussed.

The data was taken in two different modes. Short time exposures were taken yielding instantaneous OH PLIF signal level images (18 ns). Alternately, multiple frames (about 100), taken at a constant phase angle, were averaged together for each image to reduce the chaotic nature of the flow and bring out the coherent portion. Further discussion of the data, including distribution of the rms velocity fluctuations, were given in Ref. 19.

Multiple planar cross-sections of the reaction zones along the flame, as indicated by OH concentration, were visualized at different axial distances using PLIF. By acquiring multiple images with the laser sheet at increasing $x/D$ locations and a constant phase angle in the vortex roll up process, the 3-D structure of the flow was reconstructed based on up to twenty planar slices measured at successive streamwise locations. By using this technique, it was possible to follow the generation of streamwise structures in the braid region, their growth, and subsequent interaction with the large-scale spanwise structures.

The streamwise vorticity introduced by the vortex generators has a strong effect on the flame structure. The initial shaping of the jet shear layer and that of the incipient vortices is affected to a great extent by the streamwise vortices which are produced by the semi-delta winglets. The orientation of the vorticity vector determines the direction of the induced velocity field and the subsequent initial distortion of the vortex ring. This process was demonstrated experimentally by forcing the jet with two different sets of semi-delta wings: one that produces a pair of counter-rotating vortices as in Fig. 2(A), resembling separating vortices over a delta wing at a high angle of attack, and a mushroom-shaped pair of vortices as in Fig. 2(B). The two pairs feature opposite orientation of the vorticity vector: the delta-wing vortices induce radial velocity towards the jet axis between the two, while the mushroom vortices induce outward velocity. The opposite effect of these pairs of streamwise vortices on the incipient spanwise vortices is depicted in Figs. 2(A), (B), based on visualization of the cross-sectional shape of the annular diffusion flame at the vicinity of the nozzle, at $x/D =0.15$. The vortex generators were placed in a five-fold symmetry forming a pentagonal shape. The delta-vortices induce outward velocity field on the vortex sections between them and inward velocity at their location. This induction pattern results in the formation of corners midway between the vortex generators and flattening at their position. The pentagonal shape which is formed leads to the subsequent vortex axis-rotation farther downstream, as discussed later. The mushroom vortices induce strong outward velocity at the location of the vortex generators and inward velocity between them. The vortex developed a different pentagonal shape which is rotated relative to the previous one by 36° and has concave regions between the vortex generators. The additional mixing of the fuel with the ambient air, provided by the streamwise vortices, enhances the combustion rate within the vortex rings and increases the thickness of the reaction zone.

Figure 3 shows multiple two-dimensional images of radial cross-sections of the jet obtained at increasing axial distances from the flameholder using the delta vortex generators (frames A–C) and the “mushroom” vortex generators (frames D–F) at the jet exit. The initial braid region acquires a pentagonal shape due to the flow induced by the streamwise vortices. Subsequently, the downstream spanwise vortices are pinched off with a five-fold symmetry. The interac-

FIG. 2. Planar cross-sectional cuts of the flame at $x/D=0.15$, with delta-wing vortex generators (A) and “mushroom” vortex generators (B). The corresponding sketches of the vortex patterns and induced velocities are depicted.
tion between streamwise and spanwise vortices results in the deformation of the latter leading to their breakdown into small scale eddies. The cross-sectional cut through the center of the pinched vortices shows that the reaction zone is in the outer parts of the vortex.

III. NUMERICAL JET SIMULATIONS

Previous numerical simulations of axisymmetric jets investigated the unsteady vortex-ring dynamics and associated near-field statistical jet properties in the transitional subsonic regime, and the three-dimensional vortex-ring instabilities in spatially-periodic configurations. The present three-dimensional numerical simulations address the development of a jet in both space and time, and are thus capable of capturing the inherently spatially-developing global features of the laboratory jets. The numerical model involves multi-species, temperature-dependent diffusive transport, and finite-rate (single-step, irreversible) chemistry. For convenience, well established global chemistry modelling appropriate for hydrogen combustion was used here.

At the convection stage, the conservation equations for density, momentum, energy, and species concentrations (or passive mixing scalar) are solved numerically using explicit, finite-difference Flux-Corrected Transport (FCT) algorithms on structured grids, which are fourth-order accurate in space and second-order accurate in time. Other physical processes are coupled to convection using timestep splitting. A monotonically-integrated large-eddy simulation (MILES) approach based on the solution of the unfiltered inviscid equations is used, in which the FCT nonlinear high-frequency filtering provides a minimal subgrid model. Properties of the FCT effective numerical viscosity relevant to the simulation of the large-scale features of free shear flows have been reported, and it has been shown that this approach is capable of capturing the dominant inertial subrange of the energy spectrum. The $(nx \times ny \times nz)$ grids are held fixed in time, and are evenly spaced throughout the streamwise ($x$) direction and in the jet regions, and geometrically stretched outside as the cross-stream boundaries are approached. The simulations involve $CFL = 0.25 – 0.5$ and typically between $112 \times 87 \times 87$ and $225 \times 174 \times 174$ cells, with most simulations performed using an intermediate-resolution grid with $150 \times 120 \times 120$ cells. Convergence tests illustrating the effectiveness of the grid resolution used in jet simulations such as reported here, can be found in Ref. 27. Validation of the numerical reactive/diffusive models has been reported in Ref. 24.

A. Chemistry modelling

Solving a detailed set of chemical kinetic rate equations in conjunction with the unsteady, three-dimensional (3D) fluid flow equations is prohibitively expensive with current computer resources. We adopt here the approach of using global chemistry models. The simplest reasonable global chemistry model one can use is a single-step irreversible chemical reaction model. For the hydrogen chemistry considered, we use $2H_2 + O_2 \rightarrow 2H_2O$, with the reaction rate given by an Arrhenious form,

$$k_r = A_r \exp(-E_{act}/kT),$$

and the fuel burning rate given by

$$\omega = -2k_r[H_2]^2[O_2],$$

where the $[i] (i = H_2, O_2, H_2O)$, denote species concentrations.

For a global chemistry model to be truly useful, it should be able to simulate at least some aspects of the detailed chemistry model. In our simplified model we fit the heat release profile from a detailed chemistry model for an idealized one-dimensional problem. Profiles of the temperature, heat release, fuel, and oxidizer distributions are obtained at a characteristic time (typical of those in the multidimensional simulation). A least squares fit for a normalized heat release profile involving the parameters mentioned above gives the pre-exponential factor $A_r$ and activation energy $E_{act}$. The effective enthalpy of formation also needs to be corrected because the global model does not contain the radicals which are present in the detailed model.

B. Initial and boundary conditions

The numerical simulations considered initially laminar gas jets emerging from circular nozzles into quiescent background. Both jet and background are initially at the same uniform temperature ($T_0$) and pressure ($P_0 = 1 \text{ atm}$). The numerical studies focused on the large-scale shear-flow dynamics downstream of the jet exit.

The jets studied here with the MILES approach had $D/\theta_0 = 28$ and estimated Reynolds number $Re_{\theta_0} > 4300$ (based on $\theta_0$, $U_j$, and on upper bounds for the effective numerical viscosity of the algorithm).
shows the initial profiles of the mixing scalar relative to the neighborhood of scalar was initialized as unity within a thin annular region in the laboratory experiments. At the jet exit, the mixing of the injected fuel with the jet and its surroundings was simulated in conserved form to simulate the convective scalar

\[ \rho \frac{\partial \tilde{u}}{\partial t} + \nabla \cdot (\rho \tilde{u} \tilde{u}) = \nabla \cdot (\tau_{ij} + \rho f_i) + S_{\text{chem}} \]

where \( \tilde{u} \) is the velocity, \( \rho \) is the mass density, \( f_i \) is the body force, and \( S_{\text{chem}} \) is the chemical source term. In the laboratory jets, a mixing scalar, as well as fuel and oxidizer concentrations were “injected” in the computational domain with the local jet velocity [Eqs. (3) and (4)]. The non-reactive case involved air everywhere, at \( T_0 = 298 \text{ K} \), and jet Mach number (based on free-stream jet velocity, which was the same for all cases, and sound speed

\[ a = \sqrt{k \rho_0 / R} \]

where \( \rho_0 \) is the air density, \( R \) is the gas constant, and \( k \) is the specific heat ratio. The non-reactive case involved an initial temperature \( T_0 = 1400 \text{ K} \) and consequently lower \( \rho/\rho_j \) for the injected fuel to case, because of the lighter nature of the injected fuel.

The effects of the excitation introduced by the vortex generators in the laboratory jets are simulated by imposing an azimuthally varying, time-independent radial velocity at the jet exit. This radial excitation vanishes outside of a thin layer in the neighborhood of \( r = R \) with half-thickness \( 4 \theta \), and for computational convenience, was chosen to have a simple sinusoidal dependence on the azimuthal angle \( \phi \).

\[ U_{\text{rad}}(r, \phi) = U_{\text{rad}}^0(r) \left( 1 + \sin(2 \pi M \phi) \right). \]

With the choices \( M = 5 \), and \( U_{\text{rad}}^0(R) = \pm 0.05 U_j \), this prescribes a five-fold-symmetric radial velocity perturbation directed either towards \( U_{\text{rad}}^0 < 0 \) or away \( U_{\text{rad}}^0 > 0 \) the jet axis, emulating the azimuthal forcing through delta-wing and mushroom vortex generators, respectively, in the jet experiments. In Fig. 4c we show the distributions of the instantaneous radial-velocity magnitude at the jet exit indicating directions of minimum (ray 1) and maximum (ray 2) azimuthal excitation perturbation, and for reference a contour corresponding to the half-width streamwise velocity has been superimposed.

A heat release parameter

\[ C_e = \frac{T_{\text{peak}} - T_0}{T_0 - T_{\text{ref}}} \]

for given axial coordinate \( \zeta \), this pre-scribes a five-fold-symmetric radial velocity perturbation. In the present work, since Arrhenius kinetics, and temperature- and species-dependent diffusive transport parameters are used, non-dimensional numbers such as \( Da \) and Lewis (Le) numbers vary both spatially and temporally as in laboratory experiments. However, an assessment of the values of these non-dimensional numbers in the simulations can be obtained based on the initial (free stream) conditions and other typical values for the chemical and transport properties implicit in the model. Based on the initial \( T_0 = 1400 \text{ K} \), \( Le \) varies from \( Le_j = 0.3 \) for the injected fuel to \( Le_j = 1.1 \) for the jet and surroundings. It is more difficult to estimate \( Da \) in general, since reaction rate is zero initially, and at later times reaction rates— and hence chemical times—are a strong function of the instantaneous local species concentrations and temperature. Typical peak instantaneous values \( Da = 1800 \) result, for example, using a characteristic convective time \( \tau_c = D/ \dot{\zeta} \), in terms of the initial jet shear-layer mean velocity \( \dot{\zeta} = U_j/2 \), and based on local chemical times evaluated with

\[ \dot{\zeta} = U_j/2 \]

FIG. 4. Initial conditions for the simulated jets. Radial profiles of (a) streamwise velocity and mixing scalar (Runs 1–3), and (b) streamwise velocity, mass density, fuel, and oxidizer concentrations (Runs 4–5). (c) contours of the radial velocity magnitude imposed at the jet exit as azimuthal excitation; directions of minimum (ray 1) and maximum (ray 2) radial excitation magnitude are indicated. The contour corresponding to half-width streamwise velocity is indicated for reference.

The axisymmetric jet-exit velocity profile used in the simulations was defined by

\[ U_{\text{eff}}(r) = \left( \frac{U_j}{2} \right) \left[ 1 + r \frac{g h(r)}{R} \right], \]

where \( R = D/2 \).

In the case of the non-reactive simulated jet, a mixing scalar (Schmidt number unity) convected with the fluid velocity in conserved form was used to simulate the convective mixing of the injected fuel with the jet and its surroundings in the laboratory experiments. At the jet exit, the mixing scalar was initialized as unity within a thin annular region in the neighborhood of \( r = 0.8 R \), and zero otherwise. Figure 4a shows the initial profiles of the mixing scalar relative to those of the jet streamwise velocity used in the present studies. Initial profiles for the fuel and oxidizer concentrations in the reactive jet simulations are shown in Fig. 4b, where \( N_0 \) is the initial reactant molecular concentration. Mixing scalar, as well as fuel and oxidizer concentrations were “injected” in the computational domain with the local jet velocity [Eqs. (3) and (4)]. The non-reactive case involved air everywhere, at \( T_0 = 298 \text{ K} \), and jet Mach number (based on free-stream jet velocity, which was the same for all cases, and sound speed

\[ a = \sqrt{k \rho_0 / R} \]

where \( \rho_0 \) is the air density, \( R \) is the gas constant, and \( k \) is the specific heat ratio. The non-reactive case involved an initial temperature \( T_0 = 1400 \text{ K} \) and consequently lower \( \rho/\rho_j \) for the injected fuel to case, because of the lighter nature of the injected fuel.

The effects of the excitation introduced by the vortex generators in the laboratory jets are simulated by imposing an azimuthally varying, time-independent radial velocity at the jet exit. This radial excitation vanishes outside of a thin layer in the neighborhood of \( r = R \) with half-thickness \( 4 \theta \), and for computational convenience, was chosen to have a simple sinusoidal dependence on the azimuthal angle \( \phi \).

\[ U_{\text{rad}}(r, \phi) = U_{\text{rad}}^0(r) \left( 1 + \sin(2 \pi M \phi) \right). \]

With the choices \( M = 5 \), and \( U_{\text{rad}}^0(R) = \pm 0.05 U_j \), this prescribes a five-fold-symmetric radial velocity perturbation directed either towards \( U_{\text{rad}}^0 < 0 \) or away \( U_{\text{rad}}^0 > 0 \) the jet axis, emulating the azimuthal forcing through delta-wing and mushroom vortex generators, respectively, in the jet experiments. In Fig. 4c we show the distributions of the instantaneous radial-velocity magnitude at the jet exit indicating directions of minimum (ray 1) and maximum (ray 2) azimuthal excitation perturbation, and for reference a contour corresponding to the half-width streamwise velocity has been superimposed.

A heat release parameter

\[ C_e = \frac{T_{\text{peak}} - T_0}{T_0 - T_{\text{ref}}} \]

for given axial coordinate \( \zeta \), this pre-scribes a five-fold-symmetric radial velocity perturbation. In the present work, since Arrhenius kinetics, and temperature- and species-dependent diffusive transport parameters are used, non-dimensional numbers such as \( Da \) and Lewis (Le) numbers vary both spatially and temporally as in laboratory experiments. However, an assessment of the values of these non-dimensional numbers in the simulations can be obtained based on the initial (free stream) conditions and other typical values for the chemical and transport properties implicit in the model. Based on the initial \( T_0 = 1400 \text{ K} \), \( Le \) varies from \( Le_j = 0.3 \) for the injected fuel to \( Le_j = 1.1 \) for the jet and surroundings. It is more difficult to estimate \( Da \) in general, since reaction rate is zero initially, and at later times reaction rates—and hence chemical times—are a strong function of the instantaneous local species concentrations and temperature. Typical peak instantaneous values \( Da = 1800 \) result, for example, using a characteristic convective time \( \tau_c = D/ \dot{\zeta} \), in terms of the initial jet shear-layer mean velocity \( \dot{\zeta} = U_j/2 \), and based on local chemical times evaluated with

\[ \dot{\zeta} = U_j/2 \]
actual instantaneous chemical reaction rates and reactant concentrations at intermediate temperatures between initial \( (T_0) \) and peak temperature value \( (T_{\text{peak}}) \).

Specifics of the computer simulations considered in this work are listed in Table I, where \( s = \rho_d/\rho_{j,s} \) denotes the initial ratio of mass density in the center of the annular injection region to that in the jet and surroundings.

Inflow/outflow conditions are imposed at the open boundaries in the streamwise direction, and stagnant flow conditions are imposed at the cross-stream boundaries. Inflow boundary conditions modelling the initial (jet exit) conditions for the jet investigated here specify the guard-cell values for the mass density, species concentrations (or passive mixing scalar, as appropriate), and velocities:

\[
\rho_G = \rho_{in}(r), \tag{5a}
\]

\[
[i]_G = [i]_{in}(r), \tag{5b}
\]

\[
u_G = U_{in}(r)\left[1 + F(t)\right], \tag{5c}
\]

\[
U_G = U_{rad}(r, \phi), \tag{5d}
\]

\[
u_0 = 0. \tag{5e}
\]

For \( s=0.6 \), profiles of \( \rho_{in}, [H_2]_{in} \), and \( [O_2]_{in} \) are exemplified in Fig. 4b, and \( [H_2O]_{in}=0; \) for \( s=1, \rho_{in} \) is constant and the mixing scalar initial profile is shown in Fig. 4a. Using (5c), a controlled time-dependent axial excitation at the jet exit can be introduced through \( F(t) \) to facilitate the analysis of the results; the simulated jets were forced with a weak sinusoidal perturbation with rms-level 2\% and frequency \( St = fD/U_j = 0.4 \). The latter frequency was chosen within the range of observed jet preferred frequencies in the laboratory experiments. The inflow guard-cell pressures are obtained using a characteristic-analysis approach\(^{30}\) as solutions of a one-sided finite-difference expression based on the one-dimensional, unsteady, inviscid pressure equation:\(^{31}\)

\[
\frac{\partial P}{\partial t} + (u - a) \frac{\partial P}{\partial x} = \rho a \frac{\partial u}{\partial x}, \tag{6}
\]

where \( P \) is the static pressure and \( a \) is the local sound speed. A one-dimensional non-reflecting boundary condition\(^{30}\) on the pressure is specified at the outflow boundary

\[
\frac{\partial P}{\partial x} - \rho a \frac{\partial u}{\partial x} = 0, \tag{7}
\]

where the additional numerical conditions required for closure of the discretized equations are introduced by requiring that the mass and momentum densities, and species concentrations be advected with the local streamwise velocity. Using boundary conditions for which the dependence on pressure involves only derivatives, as in Eqs. (6) and (7), potentially allows the secular global drifting of the pressure of the system. Relaxation of the outflow pressure to ambient pressure can be used to avoid this problem (e.g., Refs. 26 and 30), but was not implemented in the present work. Pressure drifting and sensitivity of the jet solutions discussed here to actual choice of downstream boundary conditions were negligible for the duration of the simulations.

<table>
<thead>
<tr>
<th>Run</th>
<th>( M )</th>
<th>( \rho_d/\rho_{j,s} )</th>
<th>Chem</th>
<th>( U_{rad}^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>1</td>
<td>No</td>
<td>&lt;0</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>1</td>
<td>No</td>
<td>&gt;0</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>1</td>
<td>No</td>
<td>&lt;0</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>0.6</td>
<td>No</td>
<td>&lt;0</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>0.6</td>
<td>Yes</td>
<td>&lt;0</td>
</tr>
<tr>
<td>6</td>
<td>0.3</td>
<td>0.6</td>
<td>Yes</td>
<td>&gt;0</td>
</tr>
</tbody>
</table>

\[
\text{FIG. 5. Instantaneous flow visualization of the (non-reactive) simulated jet Run 1 for the case } U_{rad}^0<0. \text{ Flow direction is from bottom to top, and time-interval between snapshots is } 0.2 \frac{t}{s}.
\]

**C. Non-reactive \( s=1 \) jet**

Instantaneous flow visualizations of non-reactive jets with \( s=1 \) were investigated to gain insights on the basic near-jet convective mixing patterns resulting from azimuthal excitation such as imposed in the laboratory experiments. Results from these simulations are exemplified in Fig. 5 for the case \( U_{rad}^0<0 \) (Run 1), in terms of isosurfaces of the vorticity magnitude \( \Omega \) (Fig. 5a) and of the passive mixing scalar (Fig. 5b). The induced mixing patterns resulting from the two different types of azimuthal radial velocity excitations \( (U_{rad}^0<0 \text{ or } U_{rad}^0>0) \) appear rotated a 36° azimuthal-angle relative to each other but are otherwise similar. This is illustrated in Fig. 6 based on instantaneous mixing distributions for Run 1 and Run 2 at selected cross-stream planes in the vortex roll-up region, which can be directly related to the experimental visualizations in Fig. 3. The lobed structures in Fig. 5b surround regions of high concentration of the injected scalar (bottom frames in Fig. 6). There is a clear similarity between the streamwise evolution of the mixedness distributions in Fig. 6 and those suggested by the laboratory experiments in Fig. 3: initial thin circular mixing (combusting) layer deforming into a pentagon-shaped layer, then thickening, with high-concentration (low-mixing) isolated
regions—corresponding to low-combustion (also isolated) experimental regions—aligned with the pentagon corners as in the laboratory observations.

The isosurfaces of $\Omega$ in Fig. 5a serve to characterize the major topological features of the flow: (1) vortex rings rolling up in the initial jet shear layer and undergoing undulating distortion as they are convected, and (2) pairs of counter-rotating streamwise ribs strongly coupled to the rings farther downstream.

The imposed azimuthal excitation leaves its imprint on the vortex ring as it rolls up, inducing on it a five-fold undulation which grows as it is convected downstream. The fluid-dynamical mechanisms involved in this process can be first viewed as the result of having portions of the ring move towards (or away from) the axis, due to the radial-velocity imposed at the jet inflow, while others where the initial radial-velocity perturbation is minimum tend to be unaffected, leading to pentagon-shaped rings; the rings then become non-planar as they are further distorted: (1) the corner portions of the ring move faster downstream and towards the jet axis due to local self-induction; (2) the centers of the flat-side portions of the ring left behind, eventually develop high-curvature, and by the same self-induction mechanism, move first away from the jet axis and then ahead; (3) the vortex ring recovers again an approximately pentagonal shape but with axis rotated 36° relative to those in the previous planar configuration.

The longitudinal ribs appear as a result of stretching of the streamwise vorticity—introduced by the azimuthal excitation—in the ‘‘braid’’ regions between vortex rings. The rib generation mechanism involves distortion of the braid vortex loops induced by neighboring undulating vortex rings, followed by concentration and stretching of these loops into ‘‘braid vortices,’’ as the upstream and downstream rings become substantially deformed. In Fig. 7, contours of the azimuthal excitation on the inflow cross-stream plane are shown for reference, and we illustrate the loop-distortion dynamics in terms of selected vortex lines overlayed to isosurfaces of $\Omega$ for the time of the first snapshot in Fig. 5 (Run 1).

The loop deformation leading to the formation of pairs of counter rotating streamwise ribs involves alignment of the upstream loop portions with rays 2, having the downstream loop portions pulled inside the jet and aligned with rays 1, and the upstream and downstream tips of the loops eventually wrapping around corresponding portions of the rings. This mechanism for the formation of braid vortices in jets has been also observed in square and elliptical jets. Further vortex deformation is generated on the rings by their strong interaction with the streamwise ribs, by self-induction effects due to local curvature changes, and due to azimuthal ring-instabilities.

D. Reactive jet

Figure 8 compares the initial vortex ring deformation in the jet near-field for Runs 3–5 in terms of instantaneous
isosurfaces of $\Omega$ at the same phase (time). The figure depicts a vorticity dynamics basically similar to that discussed above, involving a pentagonally-shaped jet shear layer rolling up into a new ring (I), and a previously formed ring (II) in its deforming phase due to self-induced motions associated with the high curvature corner regions. Differences between these jets and the ones discussed in the previous subsection are due to lower Mach number, lower mass-density in the annular injection region at the jet exit, and exothermic effects.

Comparison of the non-reactive $s=1$ cases (Run 1 and Run 3) as a function of Mach number, indicates peak $\Omega$ values for $M=0.3$ about 20% larger than those for $M=0.6$. This reflects on larger Kelvin-Helmholtz (KH) growth rates for lower $M$, which are expected because of the stabilizing nature of compressibility effects. On the other hand, the KH growth rates can also be affected by non-uniform mass density distributions at the jet exit. Recalling the effects of density differences on mixing layer growth, and noting the initial relative density and streamwise velocity variations indicated in Fig. 4b, we can expect, relative to the uniform density case: (1) a potential growth reduction due to the density drop (from jet to center of annular region) in the inner edge of the shear layer, and, (2) a potential growth augmentation due to density increase (from center of annular region to jet surroundings) in its outer edge—where the velocity drop is also the largest. Figure 9 compares instantaneous distributions of $\Omega_z$ in the $(x,y)$-plane passing through the jet axis, corresponding to the volume visualizations in Fig. 8. Direct comparison of the snapshots on the left and center of Fig. 9, shows slight density effects when chemical exothermicity is absent: thin inner vorticity layers of sign opposite to that of the thicker main (outer) layer in Fig. 9b, suggesting density effect (1); somewhat stronger overall characteristic vortex strengths (e.g., near-jet $\Omega_{peak}$ for Run 4 about 9% larger than for Run 3) suggestive of density effect (2).

Profiles of the streamwise velocity and mass density near the jet exit (at $x/D=0.1$) for the $s=0.6$ non-reactive (Run 4) and reactive (Run 5) jets are shown in Fig. 10. Their comparison indicates that the initial profiles are significantly affected by the exothermicity of combustion associated with the slight reactant premixedness at the jet exit (Fig. 4b) and further (diffusive) mixing immediately downstream. Figure 10 shows that expansion effects due to exothermicity produce velocity overshoots enhancing the steepness of its profiles. Footprints of these features of the effectively modified velocity profiles near the jet exit are apparent on the vorticity development downstream. Comparing the frames on the center and right of Fig. 9, in the reactive case (Fig. 9c) we note much more pronounced inner vorticity layers and thicker outer vorticity layers than in the non-reactive case, effects which can be consistently related to the overshoots and steeper effective initial velocity profiles. Exothermicity also affects the near jet development through expansion and baroclinic torque effects on the large scale vortical structures themselves (e.g., as discussed in Ref. 33).

FIG. 9. Azimuthal vorticity distributions for Runs 3–5, corresponding to the frames in Fig. 8, in a plane passing through the jet axis, containing ray 1 and ray 2 (cf. Fig. 4c).
E. Computational vs. experimental reactive jets

Figure 11 compares isosurfaces of the instantaneous magnitude of the fuel burning rate $\psi$ (level $\sim 0.1$ of the peak value) depicting the combustion regions in the reactive jet simulations (for $U^0_{\text{rad}} < 0$), with isosurfaces of the phase-averaged concentration (level $\sim 0.7$ peak-value) of the intermediate chemical product (OH) used to characterize the experimental combustion regions in the vortex roll-up region. Multiple cross-stream planar images acquired with the laser sheet at successive $x/D$ locations were used to reconstruct the 3-D image of the phase-locked structures in the laboratory jet. Isosurface levels were chosen for 3-D visualization enhancement, and representative cross-stream distributions from the streamwise ranges compared are also shown. The computational surface in Fig. 11 was based on a data sub-volume $28 \times 90 \times 90$ (with uniform resolution, $0.028D$/cell); the experimental surface was reconstructed based on $5 \times 240 \times 624$ data points (with corresponding resolutions $0.115D$/pixel, $0.0108D$/pixel, and $0.0045D$/pixel).

Figure 11 indicates good qualitative agreement on the topological features of the experimental and computational combustion regions, confirming the preliminary suggestion presented above that the topology of the laboratory combustion regions is directly related to convective mixing topology, in turn directly related to (inviscid) jet vorticity dynamics. Minor differences (not unexpected) between surfaces are also observed. Among possible sources of differences, we note: (1) non-identical azimuthal excitations and fuel injection locations—closer to the outer edges of the jet shear layer in the experiments and near the inner edge in the simulations, (2) inherently smoother laboratory data due to the phase-averaging procedure and possible OH convection effects, (3) lower characteristic $Da$ due to the slower propane chemistry in the laboratory experiments. In addition, the jet experiments involved larger mass density in the annular injection region $s \sim 1.2$, compared to $s = 0.6$ in the simulations; however, this density non-uniformity presumably had a lesser effect on actual laboratory jet growth rates because of the fuel injection locations and of much stronger axial forcing.

IV. FINAL CONCLUSIONS

We have presented results of a combined computational and experimental study of azimuthally-excited round jets. The focus has been on investigating the dynamics of large scale vortical structures developing from axisymmetric vortex sheets with imposed time-independent, five-fold symmetric, azimuthally non-uniform excitation.

The results of the spatially evolving simulations show that the basic spatial and temporal features of the transitional jet are mainly controlled by the dynamics of formation, and development of vortex rings and rib vortices. Vortex interactions and azimuthal instabilities lead to more contorted vortex rings. The interaction between the vortex rings and the streamwise vorticity introduced at the jet exit results in a deformation of the flame into a pentagonal shape which triggers processes of self-induction and vortex interactions; the self-deformation of the vortex rings due to the rapid change in azimuthal curvature at the corners of the pentagonal vortex sheet leads to a subsequent 36-degree axis-rotation of the jet cross-section. Rib (braid) vortices are formed in the high-strain regions between successive vortex rings as a result of the deformation, concentration and stretching of braid vortex loops induced by the undulating rings.

The fluid dynamical processes leading to the complex three-dimensional flame structure observed in the laboratory jets were elucidated using the database of the numerical simulations. The dynamics of large-scale vortical structures affects the mixing pattern of the jet, resulting in combustion inactive localized regions of high fuel concentration, and other regions with enhanced mixing and proper air-to-fuel ratio in the flame where the combustion is intense. The mix-
ing pattern affects the combustion process and eventually determines the energy release. By understanding these combined processes it is possible to select optimal fuel injection locations and patterns to improve the jet combustion.

ACKNOWLEDGMENTS

This research was supported by the Mechanics Division of the Office of Naval Research, Gabriel Roy, Scientific Officer, and by the Naval Research Laboratory. The calculations were performed at the computing facilities of NAS/NASA and HPC-MP/CEWES.