

A method for optimal design of automotive body assembly using multi-material construction

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Abstract

This paper proposes a new method for designing lightweight automotive body assemblies using multi-material construction with low cost penalty. Current constructions of automotive structures are based on single types of materials, e.g., steel or aluminium. The principle of the multi-material construction concept is that proper materials are selected for their intended functions. The design problem is formulated as a multi-objective nonlinear mathematical programming problem involving both discrete and continuous variables. The discrete variables are the material types and continuous variables are the thicknesses of the panels. This problem is then solved using a multi-objective genetic algorithm. An artificial neural network is employed to approximate the constraint functions and reduce the number of finite element runs. The proposed method is illustrated through a case study of lightweight design of an automotive door assembly. © 2007 Elsevier Ltd. All rights reserved.

Keywords: Material selection; Multi-material structure; Lightweight

1. Introduction

The increasing number of automobiles has led to various societal and environmental concerns, such as fuel efficiency, emission, and global warming. The automobile industry is under considerable pressure to reduce the fuel consumption and the emissions of their vehicles. Reducing the weight of the vehicles is one key approach to achieving fuel efficiency, since every 56.69 kg weight reduction results in a gain of 0.09–0.21 km per liter fuel economy [1].

In general, the car body and its interior accounts for approximately 40 percent of the vehicle weight. Thus, weight reduction of car bodies offers a promising way of improving the fuel economy of automobiles. One basic approach to reducing the weight of car bodies is using lightweight materials. Yet, one key obstacle of the substitution of lightweight materials for commonly used steel is the high cost of these materials as compared to steel [2]. For instance, the costs

of a sophisticated all-aluminium car body like that of the Audi A8 is significantly higher than that of an ordinary steel body. Therefore, some researchers [3–5] have suggested that the car body of the future will be a composite of steel and several lightweight materials, i.e., multi-material structures.

In the past several years, some of the major auto body panels have been developed with new advanced materials, such as high strength steels [6,7], composite materials [8,9] and aluminium alloys [4,10]. These materials allow for lighter weight than traditional steel car bodies. However, most work done in lightweight materials car body design has been limited to single-material construction method. There have been very few researches that explicitly address the detailed methods for multi-material structure design and construction.

The concept of the multi-material construction is that proper materials are selected for the intended part functions. Compared to single material construction method, multi-material structures allow for allocating the optimal material to each independent structural component. Therefore, multi-material construction method enables designers to fully exploit the advantages of different materials and achieve the optimal production efficiencies.

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The objective of this paper is to present a new method for designing lightweight automotive body assemblies cost effectively using multi-material construction. The material type and the thickness of each independent panel are treated simultaneously as design variables. The integrated optimization problem of material selection and thickness determination is formulated as a multi-objective nonlinear mathematical programming problem involving both continuous and discrete variables. The optimization problem is solved by a multi-objective genetic algorithm, which can efficiently generate a well-spread Pareto front over multiple objectives. Neural network approximations are used to reduce computational complexity, which represent the relationships between constraints and the design variables.

2. Formulation of optimization model

To illustrate the problem, consider the following design conditions:

- (1) A thin-walled structure consisting of n components, which is an assembly of an automotive body, is being designed.
- (2) Select the optimal material from m candidate materials for each component.
- (3) The aim of the problem is to design the assembly as inexpensive and light as possible, while at the same time ensuring basic structural performances are met.
- (4) The topology and shape of the assembly is given, the thickness can be changed in order to satisfy structural performance.

It is difficult to divide a single panel from a thin-walled assembly since several panels support the loads together. Hence, it is better to select materials at the assembly level by considering all components simultaneously. Moreover, even for a small number of components and candidate materials, the number of potential material combinations to be considered can be very high. In this paper, therefore, the problem is formulated as an optimization problem and solved using a genetic algorithm, which allows the efficient exploration of multiple high-performance solutions based on given criteria.

A general optimization model for the design problem can be formulated as follows:

$$\text{Minimize } W(t_i, \rho_i) = \sum_{i=1}^n A_i t_i \rho_i \quad (1)$$

$$\text{Minimize } C(t_i, \rho_i, p_i) = \sum_{i=1}^n A_i t_i \rho_i p_i \quad (2)$$

$$\text{Subject to } g_j(t_i, B_i) \leq 0, \quad i = 1, \dots, n, \quad j = 1, \dots, k \quad (3)$$

$$t_i^L \leq t_i \leq t_i^U, \quad i = 1, \dots, n \quad (4)$$

$$B_i \in \{(\rho_1, E_1, p_1, \sigma_1, \dots), \dots, (\rho_m, E_m, p_m, \sigma_m, \dots)\}, \\ i = 1, \dots, n \quad (5)$$

where t_i and A_i denote the thickness and the area for the i th component, respectively, t_i^L and t_i^U are the lower and upper bounds of the thickness, n denotes the number of independent components, B_i are the materials properties, which include density (ρ_i), raw material price (p_i), elastic modulus (E_i), and yield strength (σ_i) etc., m is the number of candidate materials.

Eqs. (1) and (2) define the objective functions, which are weight and material cost of the assembly, respectively. Eq. (3) is the constraint function, which provides the bounds on member stress, static stiffness, frequency, dynamic response, etc. Eqs. (4) and (5) define the search region for the optimum.

In the general model, it is straightforward to take material properties as the design variables. However, such an approach introduces an enormous number of design variables and the relationships among them. For instance, assignment of 7800 kg/m^3 to the density generally specifies the value for elastic modulus to be 210 GPa. The high number of design variables as well as their relationships lead to a high computational complexity of the optimization problem.

To solve the difficulties mentioned above, the material types rather than material properties are introduced as design variables. Each candidate material type has been assigned an ID number from 1 to m , which can be in any arbitrary order. Define the material used for i th component as a design variable named M_i ($M_i \in \{1, 2, \dots, m\}$). If a material type is given to M_i , all the related properties of the material can be identified exactly. For a mathematical model, it can be stated as follows:

$$\begin{pmatrix} \rho_i \\ E_i \\ p_i \\ \vdots \end{pmatrix} = \begin{pmatrix} f_\rho(M_i) \\ f_E(M_i) \\ f_p(M_i) \\ \vdots \end{pmatrix} \quad (6)$$

Introducing Eq. (6) into Eq. (1) leads to

$$\text{Minimize } W = W(t_i, \rho_i) = W(t_i, f_\rho(M_i)) \\ = W(t_i, M_i), \quad i = 1, \dots, n \quad (7)$$

Similarly, Eqs. (2)–(5) can be rewritten as

$$\text{Minimize } C = C(t_i, M_i), \quad i = 1, \dots, n \quad (8)$$

$$\text{Subject to } g_j(t_i, M_i) \leq 0, \quad i = 1, \dots, n, \quad j = 1, \dots, k \quad (9)$$

$$t_i^L \leq t_i \leq t_i^U, \quad i = 1, \dots, n \quad (10)$$

$$M_i \in \{1, 2, \dots, m\}, \quad i = 1, \dots, n \quad (11)$$

In Eqs. (7)–(11), the thicknesses of the independent components and the materials types are taken as design variables.

The optimization model given by Eqs. (7)–(11) is a multi-objective nonlinear programming problem, in which some of the design variables are discrete and others are continuous. Due to the complexity of the problem and the multi-objective formulation without predefined weight, a multi-objective genetic algorithm named non-dominated

sorting genetic algorithm II (NSGA-II) [11], was chosen because of its robustness to discrete problems and efficiency in handling multi-objective functions. This algorithm uses the non-dominated sorting method for Pareto ranking procedure, which has been successfully applied in numerous studies such as topology optimization [12], mechanical design [13] and decomposition-based assembly synthesis [14,15].

3. Multi-objective genetic algorithms

In most real-world problems, several objectives must be satisfied simultaneously in order to obtain an optimal solution. The presence of multiple objectives in a problem usually gives rise to a set of optimal solutions, largely known as Pareto optimal solutions. A Pareto set is represented by a set of solutions such that when one moves from one solution to any other, at least one objective function improves, while the other worsens.

Genetic algorithms have been recognized to be well suited for multi-objective optimization because they have the ability to find multiple Pareto optimal solutions in one single simulation run [11]. This has led to the development of many successful multi-objective optimization genetic algorithms [11,16–18] over the past years. Among several methods available to solve multi-objective optimization problems, non-dominated sorting genetic algorithm II (NSGA II), developed by Deb et al. [11], is used here to obtain the Pareto set.

3.1. NSGA-II: A brief overview

The main idea of the non-dominated sorting genetic algorithm (NSGA) [17] is that a ranking selection method is used to emphasize good points and a niching method is used to maintain stable subpopulations of good points. The main difference between NSGA and a simple genetic algorithm is in how the selection operator works. The crossover and mutation operators remain as usual. NSGA II [11] is the modified version of NSGA with the properties of a fast non-dominated sorting procedure, an elitist strategy, a parameterless approach and a simple yet efficient constraint-handling method. The basic steps [11,14] of NSGA-II are outlined below:

- (1) Create a random population P of n chromosomes (an encoded representation of design variables).
- (2) Divide the population members into a number of subpopulations according to an increasing level of non-domination (rank 0 is Pareto optimal). Store the chromosomes with rank 0 into set O . Also, create an empty subpopulation Q .
- (3) Select two chromosomes c_i and c_j in P with probability proportional to $n - rank(c_i)$ and $n - rank(c_j)$.
- (4) Crossover c_i and c_j to generate two new chromosomes c'_i and c'_j with a certain high probability.
- (5) Mutate c'_i and c'_j with a certain low probability.
- (6) Evaluate the objective function values of c'_i and c'_j and store them Q . If Q contains less than n new chromosomes, go to 3.
- (7) Let $P \leftarrow P \cup Q$ and empty Q . Rank each chromosome in P and remove n chromosomes with lowest ranks from P .
- (8) Update set O and increment the generation counter. If the generation counter has reached a pre-specified number, terminate the process and return O . Otherwise go to 3.

Details of this method are available in the literature [11]. NSGA-II has been implemented using the iSIGHT software [19] from Engineous Inc. The NSGA-II code in C programming language is also available from the KanGAL website at <<http://www.iitk.ac.in/kangal/soft.htm>>.

4. Response modeling using artificial neural network

The multi-objective optimization using genetic algorithm often requires inordinately large amount of simulations and each simulation of finite element analysis (FEA) takes a long CPU time. Therefore, artificial neural networks (ANNs) are employed to approximate the constraint functions of Eq. (9) and reduce the number of FEA runs.

An ANN can be understood [20] as a mapping from input to output, i.e., $R^n \rightarrow R^m$, $f(X) = Y$, where $X(x_1, x_2, \dots, x_n)$ is the input vector, and $Y(y_1, y_2, \dots, y_m)$ is the output vector. As shown in Fig. 1, a conventional ANN consists of an input layer (the first layer), an output layer (the last layer), and one or more hidden layers. Each layer is composed of a large number of parallel processing elements known as neurons. Neurons in the input layer transmit the input signal x_i to neurons in the hidden layer. Each neuron j in the hidden layer sums its input signals x_i after multiplying them by the respective connection weights w_{ji} , and then computes its output h_j as a function of the sum, i.e.,

$$h_j = f\left(\sum w_{ji}x_i\right) \quad (12)$$

where f is usually a sigmoidal function. The outputs of neurons in the output layer are computed similarly.

The network starts processing the incoming training signals with arbitrary weights. Network training is the act of continuously adjusting their connection weights until they reach unique values that allow the network to achieve an accurate prediction. The training algorithms optimize the weights based upon the main principle of minimizing the sum of squared differences between the desired and actual values of the output neurons, namely:

$$e = \frac{1}{2} \sum_j (y_{dj} - y_j)^2 \quad (13)$$

where y_{dj} is the desired value of output neuron j and y_j is the actual output of that neuron.

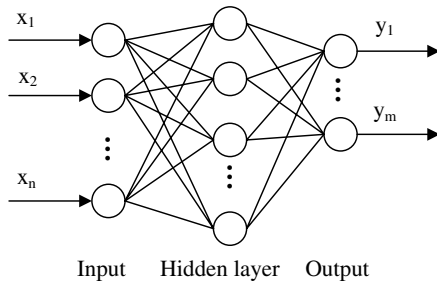


Fig. 1. Structure of an ANN with a hidden layer.

Each connection weight w_{ji} is adjusted by adding an increment Δw_{ji} to it. Δw_{ji} can be used to reduce e as rapidly as possible. The adjustment is carried out over several training iterations until an appointed small value of e is obtained or a given number of iterations is reached. How Δw_{ji} is identified depends on the training algorithm.

The training of an ANN usually adopts a back-propagation (BP) algorithm, which was first proposed by Rumelhart and McClelland [21]. The BP algorithm determines the change $\Delta w_{ji}(k)$ in the weight of the connection between neurons i and j at iteration k as:

$$\Delta w_{ji}(k) = -\alpha \frac{\partial e}{\partial w_{ji}(k)} + \mu \Delta w_{ji}(k-1) \quad (14)$$

where α is called the learning coefficient, μ is the momentum coefficient and $\Delta w_{ji}(k-1)$ is the weight change in the immediately preceding iteration.

In this study, the steps to construct the ANN approximation are as follows:

Step 1: Gather training data. The first step is to generate the data sets that can be used to train an appropriate neural network. The training set consists of pairs of input (design) vectors and associated outputs (responses). FEA is used to generate the data set. In order to obtain enough information, the Latin Hypercube Sampling method [22] is employed to distribute the sampling points uniformly over the entire design space.

Step 2: Decide the appropriate architecture of the network. The number of nodes in the input and output layer equals the number of input and output variables, respectively. The primary question for an ANN designer is how to obtain an optimal topology of the hidden layer or layers. Nowadays, the most common approach is still the trial-and-error method based on comparative performance of different network architectures.

Step 3: Train the network using BP algorithm. The training is carried out over several iterations until the desired level of accuracy is obtained or a given number of iterations are reached.

Step 4: Test the network performance. In this step, the trained network is tested with data, which were not present in the training data set.

Please note that the performance of an ANN depends strongly on the network architecture as well as the richness of the training data set. Therefore, changes in network architecture and training data set have been done and the whole training and testing process have been repeated several times before arriving at an optimal ANN.

5. Application

The proposed method is illustrated with an example of lightweight design of a car door assembly (Fig. 2). Two major panels, the outer panel and the inner panel, are considered since they make major contribution to the total weight of the door. Four candidate materials are considered for each panel and each candidate material has been assigned an ID number from 1 to 4. Subsequently, two design variables, $M_1, M_2 \in \{1,2,3,4\}$, are introduced which denote the material used for the inner panel and outer panel, respectively. The ID numbers as well as properties of these four materials are given in Table 1.

The requirement of the problem is to design the door for minimum weight and cost, while meeting the multiple constraints involving vertical sag stiffness (d_{sag}), upper lateral stiffness (d_{upper}), lower lateral stiffness (d_{lower}) and the natural frequency of the free-free normal mode (ω_1). Each stiffness is evaluated as the deflection at the load application point under the fixed load, calculated by finite element methods. The boundary conditions for the three stiffness load cases are shown in Fig. 3. For vertical sag stiffness, the hinges are fully restrained and the vertical load ($F_{\text{sag}} = 800 \text{ N}$) is applied at the latch. For both upper and lateral stiffness, the hinges are fully restrained and the latch is restrained except for roll rotation, at the same time the horizontal load ($F_{\text{upper}} = 200 \text{ N}$, $F_{\text{lower}} = 200 \text{ N}$) is applied to the upper side of the inner panel and to the lower side of the outer panel, respectively.

The optimization problem is formulated as follows:

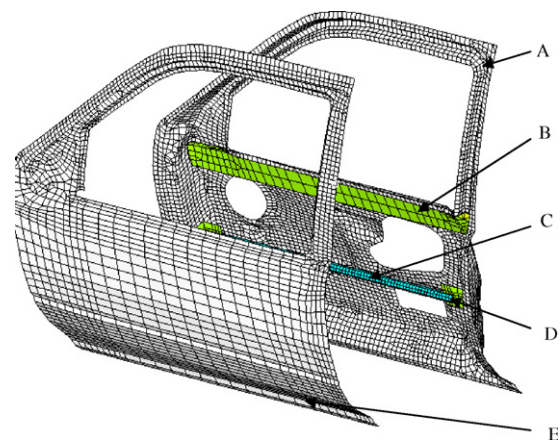


Fig. 2. Exploded view of the door assembly. A: inner panel; B: belt reinforcement; C: beam; D: beam bracket E: outer panel.

Table 1
ID number and the properties of candidate materials

ID M_i	Material	Elastic modulus (GPa)	Density (kg/m ³)	Price ^a (\$/kg)
1	Magnesium	45	1840	2.86
2	Aluminium	72	2720	2.2
3	Mild steel	210	7840	0.66
4	Carbon fiber	80	1900	17.6

^a The column presents material prices in China in 2003.

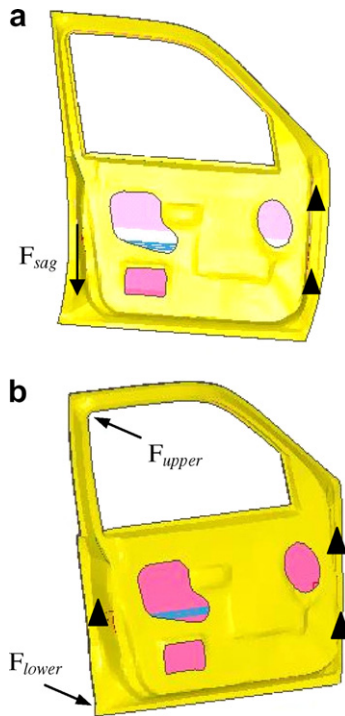


Fig. 3. Boundary conditions: (a) vertical sag stiffness analysis and (b) upper and lateral stiffness analysis.

$$\text{Minimize } W = W(t_i, M_i), \quad i = 1, 2 \quad (15)$$

$$\text{Minimize } C = C(t_i, M_i), \quad i = 1, 2 \quad (16)$$

$$\text{Subject to } d_{\text{sag}} \leq 2.7 \text{ mm}$$

$$d_{\text{upper}} \leq 3.0 \text{ mm}$$

$$d_{\text{lower}} \leq 2.7 \text{ mm} \quad (17)$$

$$\omega_1 \geq 35 \text{ Hz}$$

$$0.6 \text{ mm} \leq t_i \leq 3.0 \text{ mm}, \quad i = 1, 2 \quad (18)$$

$$M_i \in \{1, 2, 3, 4\}, \quad i = 1, 2 \quad (19)$$

where the design variables are the thicknesses of inner panel and outer panel (t_1, t_2) and the materials ID numbers (M_1, M_2).

A BP ANN was used to approximate the constraint functions of Eq. (17). In the present work, a total of 150 FE models have been run using the FEA software MSC/NASTRAN and 150 training data sets have been generated. Addition 50 data sets that excluded from the training data have been generated for future testing. It was decided to have two hidden layers and the numbers of neurons in the hidden layers were decided by trial and error. Based on the trial runs, the network architecture decided in the present case was 4-10-10-4. The input layer of the ANN consists of four neurons, corresponding to the four design variables. The output layer consists of four neurons, corresponding to $d_{\text{sag}}, d_{\text{upper}}, d_{\text{lower}}$ and ω_1 . The number of hidden layers is two and each layer includes ten neurons.

To test the precision of the ANN model, Table 2 shows the deviations between the FEA and the ANN predicted results for both training data set and testing data set. Only the vertical sag stiffness has been graphically compared as shown in Fig. 4, in which the sample data sets have been ordered in increasing value of the vertical sag stiffness (d_{sag}). The deviation of predicted results are minor and with average errors less than 5%. It has been observed from

Table 2
The prediction error of the ANN

	d_{sag}		d_{upper}		d_{lower}		ω_1	
	Training	Testing	Training	Testing	Training	Testing	Training	Testing
Avg. error (%)	1.6	2.1	1.2	1.4	3.2	4.9	0.3	1.0

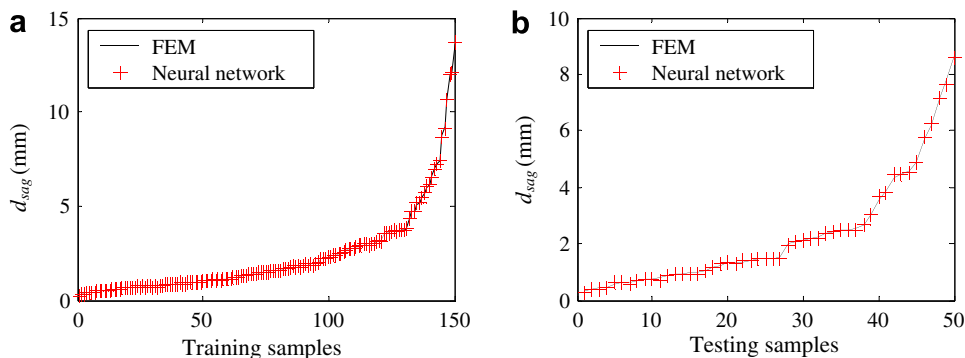


Fig. 4. Comparison of the FEA and the ANN predicted results (vertical sag stiffness): (a) training data set and (b) testing data set.

Table 3
NSGA-II parameters used in the case study

Population size	90
Number of generations	150
Crossover probability	90%
Crossover distribution index	15
Mutation distribution index	75

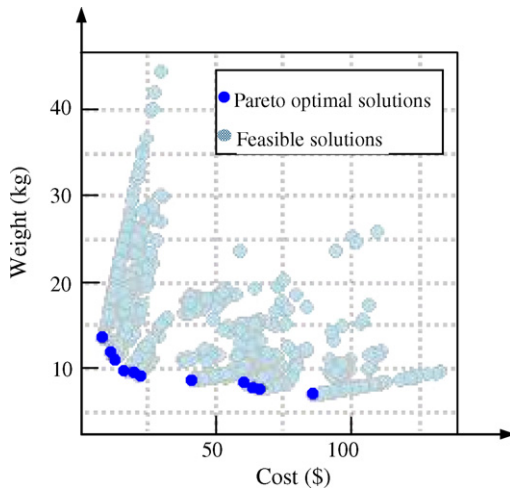


Fig. 5. Pareto optimal set for the lightweight door assembly design.

the results that the prediction of the network is accurate with very little error.

Table 3 lists the values of NSGA-II parameters used for the optimization. The values are chosen to provide the results with good repeatability. Fig. 5 shows the Pareto solutions of the problem obtained by the multi-objective GA, where the horizontal and vertical axes are the material cost and the structural weight, respectively. The highlighted points (dark blue) are the Pareto optimal solutions and the corresponding results are shown in Table 4.

Fig. 5 shows that in addition to the lowest weight and cost points, an optimal Pareto set of designs is found in one single simulation run. Since none of the non-domi-

nated solutions in the Pareto set is superior to any other, any one of them is an acceptable solution. The decision as to which solution is most suitable for the door assembly depends on its application. As could be seen from Table 4, all-aluminium and all-magnesium structures realized lightweight effects of 28.0% and 31.6% than all-steel structure, respectively, while the costs are about 101.1% and 140.7% more expensive. All-aluminium or all-magnesium structures may only be suitable for luxury vehicles that have larger profit margins since there is a considerable cost penalty associated with the lightweight materials. Moreover, if cost was not a concern, carbon fiber might be used to achieve the greatest weight reduction. In the case study, steel–aluminium hybrid door assembly achieved weight reduction of 12.9% with just 33.1% cost penalty. This design leads to a lightweight and affordable door assembly that appears to be the favorable structure for inexpensive vehicles which desire high fuel economy and are under the cost constraint. The similar situation is also true for the steel–magnesium door design. Going briefly through Table 4, one can note that several multi-material designs achieved lower cost penalties associated with unit decrease in weight, which implies that compared to single-material structures, multi-material designs provide more opportunities to achieve fuel economy and low cost penalty simultaneously. Through optimal materials selection for each independent structural member, multi-material construction method allows designers to fully exploit the advantages of each material and achieve the optimal production efficiencies.

6. Conclusions

A new synthesis method is proposed for designing lightweight automotive body assemblies using multi-material construction with low cost penalty. The novel feature of the method is that the integrated problem of material selection–thickness determination is posed as an optimization problem, in which the material types are introduced as dis-

Table 4
Pareto optimal designs for the door assembly

M_1^a	M_2^a	t_1 (mm)	t_2 (mm)	Weight (kg)	Cost ^d (\$)	Weight reduction ^b (%)	Cost penalty ^b (%)	$\frac{\Delta C^c}{\Delta W}$ (\$/kg)
3	3	0.81	0.70	13.583	8.965	0	0	–
3	2	0.88	1.09	11.835	11.934	12.9	33.1	1.70
3	1	0.90	1.30	11.498	12.335	15.4	37.6	1.62
2	2	1.80	1.07	9.778	18.027	28.0	101.1	2.38
2	1	1.77	1.57	9.683	19.545	28.7	118.0	2.71
1	1	2.60	1.35	9.285	21.577	31.6	140.7	2.93
2	4	1.77	0.98	8.765	41.791	35.5	366.2	6.81
4	2	1.68	1.05	8.022	63.284	40.9	605.9	9.77
4	1	1.75	1.28	7.704	65.958	43.3	635.7	9.69
4	4	1.61	1.00	7.027	85.336	48.3	851.9	11.65

^a 1, 2, 3 and 4 correspond to magnesium, aluminium, mild steel and carbon fiber, respectively according to Table 1.

^b Relative changes with respect to the values of all-steel (3–3–0.81–0.70) solution.

^c The increase in cost associated with unit decrease in weight.

^d The column presents cost based on raw material prices in China in 2003.

crete design variables. The proposed method enables optimal materials selection for each independent component of an assembly while simultaneously determining sizing variables. The case study demonstrated that the method successfully generate a well-spread Pareto optimal set in one single simulation run. From this Pareto optimal set, decision makers can select the most suitable design according to the vehicle program and its application. It is observed that several hybrid-material structures in the example achieved lower cost penalties associated with unit decrease in weight than all-aluminium or all-magnesium structures. From the results of the case study, it can be seen that if proper materials are selected for the appropriate parts, a lightweight, good structure performances and low cost car body assembly can be acquired.

Although the results of the case study demonstrate the potential utility of the present method, the forming and joining process aspects of fabricating the structures should be included for the method to be more practical. These aspects should be included in the design model by taking into account forming and joining process economic attributes and process constraints. These extensions will be the focus of our future work.

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