Students' constructions of dynamic geometry

Federica Olivero
Graduate School of Education, University of Bristol, UK
fede.olivero@bristol.ac.uk

Abstract. *This paper draws on a study investigating the development of the proving process in a dynamic geometry environment in the context of open geometry problems at secondary school level. Starting with a paradigmatic example, the paper will explore how the modalities of interaction with Cabri that students show, influence the construction of different 'dynamic geometry instruments' and direct the proving process in different directions. The modalities of interaction with the software will be interpreted within the instrumental framework (Verillion & Rabardel, 1995) and illustrated by students' protocols.*

A Paradigmatic example: the Cabri of Bartolomeo and the Cabri of Tiziana

The paper starts with a paradigmatic example, taken from a study (Olivero, 2002), that shows how interactions with a dynamic geometry software in the context of the same problem differ for two students working in pairs, and impact on the shaping of the proving process. Bartolomeo and Tiziana are 15-year-old Italian students who have used Cabri a couple of times before they were given the problem 'Perpendicular bisectors of a quadrilateral'[i] to tackle in Cabri in pairs[ii]. They both have an average mathematical background and considerable Cabri experience.

Tiziana has the mouse. After constructing the figure, the students start exploring the situation and quite soon they get to this extract that leads to the formulation of the conjecture 'If ABCD is a parallelogram then HKLM is a parallelogram too', through an episode of dragging.

69	Bartolomeo:	what have you done, a rectangle? (Figure 1)
70	Tiziana:	yes, well…
71	Bartolomeo:	so… it is a point… try to make it bigger…
75	Tiziana drags D up and stops to observe and think (Figure 2)	
76	Tiziana:	excuse me! This (she points at LM) follows what this (AB) does, this (LK) follows this (AD) … (she laughs)
77	Bartolomeo:	let's examine some more cases
78	Tiziana drags A up and gets Figure 3	
79	Bartolomeo:	ah, when it's a rectangle it's always a point… (he writes down the second conjecture) […]
80	Tiziana:	No, because now it's a point too. Tiziana drags B so that ABCD is no longer a rectangle but inside there is still a point (Figure 4).

Tiziana

Bartolomeo

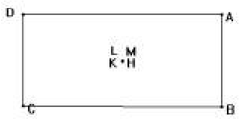

Figure 1

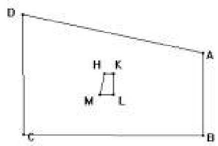

Figure 2

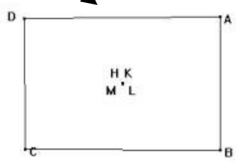

Figure 3

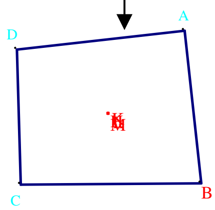

Figure 4

The students start from the same figure (a rectangle) but use Cabri in two different ways, that open a window on their aims, and potentially direct the proving process in different directions: by the end of the process Tiziana discovers the most general conjecture for this problem, while Bartolomeo goes on with a systematic exploration of particular cases, as the one explored in this extract.

The Cabri of Bartolomeo

In 72, 77 and 79 Bartolomeo shows how he wants to use Cabri: to produce and check conjectures in a very systematic way ("let's examine some more cases" – 77). During this episode of dragging, he pays attention only to the initial and final figure (Figure 1 and Figure 3), as two snapshots, as his aim is clear: checking if HKLM is always a point when ABCD is a rectangle[iii]. And, as soon as Tiziana stops in Figure 3 he formulates a conjecture (79).
This episode represents well the overall interaction with Cabri shown by Bartolomeo throughout the proving process. He shows a quite 'controlled' use of the software, and seems not to be absorbed by it. He does with Cabri something that he could have done on paper too. He has a precise strategy, which is to examine particular cases and to use dragging as a tool for validating conjectures, as is made clear at the beginning of the process ("let's see what happens in every case, shall we?"): the software is used to carry out this plan. *Photo-dragging*[iv] (Olivero, 2002) characterises his behaviour[v] because dragging itself is only used to transform a figure into another one, and the attention is focused only on the initial and final state of dragging.

This 'controlled' use of Cabri, in which the software is incorporated in a pre-determined solution process, is limited to the exploration of particular cases/ conjectures and may hinder the discovery of new properties/conjectures.

The Cabri of Tiziana

Tiziana's use of Cabri in this episode is different from Bartolomeo's. Tiziana is observing the figure over the dragging which takes her from Figure 1 to Figure 3, through Figure 2, 'reading' what the figure suggests her. She stops in 75 and reads a relationship between elements of the configuration (the sides of ABCD and the sides of HKLM), which opens up a new thread in the proving process and will be transformed into a general conjecture later on in the process[vi]. At the end of this episode, Tiziana does not stop on the rectangle configuration but moves to another 'unknown' configuration in which there is still a point inside (80), opening up another new thread for exploration[vii].

This episode represents Tiziana's prevalent type of interaction with Cabri. She does not show a pre-specified plan of action and she shows a more open use of Cabri: she uses Cabri in order to experiment, explore the situation, get ideas and discover new properties. With a metaphor we can say she is 'dragged by dragging', in that she reads what is happening in Cabri while she is dragging. Her modality is *film-dragging*[viii] (Olivero, 2002): she focuses on the intermediate state of the figure while she is dragging and stops whenever she sees something interesting. Cabri does for her something that she would not be able to do on paper and is an integral part of her actions.

This more open use of the software transforms Cabri (and dragging in particular) into a tool for discovering new relationships and facts, leading to general conjectures. The overall process is determined by what emerges from observing what happens in Cabri.

The research problem

The paradigmatic example has shown different ways in which the students exploit and incorporate the software in their solution processes and how this affects in different ways the development of the proving process. What is the research problem suggested by this example?

Educational innovations tend to take on an objectified character in popular thinking. Innovators advocate and administrators endorse the educational use of this new technology or that, as if the instrument were invariant and its use determinate (Ruthven, 2005). This shows how the risk of the "fingertip effect" (Perkins, 1985), that is simply making a support system available and expecting that people will more or less automatically take advantage of the opportunities that it affords, is always there. However, research has shown that technologies do not work by themselves and people do not automatically take on board the technology or software: "The computer is an expressive medium that different people can make their own in their own way" (Turkle & Papert, 1990). This leads to the exploration of students' constructions of dynamic geometry, to be interpreted within the instrumental framework, as developed by Verillion & Rabardel (1995) and elaborated by Mariotti (2002), that may help us understand why this is the case. The research questions we may ask are:

- What are the *instruments-Cabri* constructed by the students starting from the *artefact-Cabri*?
- How do students make Cabri their own *instrument-Cabri?* What elements play a role in the process?

The instrumental framework

The instrumental approach elaborated by Verillion & Rabardel (1995) provides a new perspective on the effect of technical devices[ix] on learning processes.

According to the instrumental approach, any technical device has a double interpretation: on the one hand it has been constructed according to a specific knowledge which assures the accomplishment of specific goals, and on the other hand, there is a user who makes his/her own use of the device. In other terms, in this perspective it is important to highlight the distinction between *artefact*, which is "the particular object with its intrinsic characteristics, designed and realised for the purpose of accomplishing a particular task" and *instrument*, that is "the artefact and the modalities of its use, as elaborated by a particular user" (Mariotti, 2002, p.702) within a given activity. "For a given individual, the artefact at the outset, does not have an instrumental value. It becomes an instrument through a process, or genesis, by the construction of *personal schemes*" (Artigue, 2002, p. 248), or *schemes of use*. As different and co-ordinated schemes of use are successively elaborated, the relationship between user and artefact evolves, in a long-term process called *instrumental genesis*, which is linked to: the characteristics of the artefact (its potentials and constraints) and those of the subject (its knowledge and former work habits) (Verillion & Rabardel, 1995). Therefore the instrument does not exist in itself, an object becomes an instrument when the subject has been able to appropriate the artefact for himself/herself and has integrated it with his/her activity. At different moments different instruments can exist even if the artefact used is the same and it may happen that an artefact is never transformed into an instrument.

The Cabri of Carla: A conflict is generated

This section shows how the *instrument-Cabri* a student constructs is not appropriate for the situation at stake. Carla and Alessandra are 15-year-old students solving the Varignon's problem[x]. They have a weak mathematical background and only used Cabri twice before this problem. After having formulated the conjecture 'if ABCD is a square then HKLM is a square', they prove it correctly on paper. Afterwards, they go back to Cabri to 'check' their proof, but the Cabri figure does not show what they have just proven. So their conclusion is that "it's all wrong".

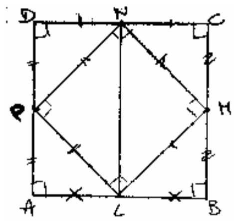

Figure 5 (on paper)

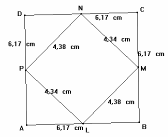

Figure 6

215 Carla: all this stuff…these…they are congruent (the halves of the sides of ABCD - **Erreur ! Source du renvoi introuvable.**). [Then Alessandra writes down the thesis: LM equals MN, equals NP, equals PL. Meanwhile Carla uses a ruler to measure the sides of LMNP]. so PL equals MN. The same for …PDN triangle and LBM triangle ⇒ PN equals LM…Should I do a cross comparison? PDN triangle and PAL triangle ⇒ PN equals PL. What's missing? These two are done, these two are done…. […] They all have equal angles. So it is a square! Ok! […]

225 Carla: the problem …is that this is not a square (ABCD) […] look… no…(**Erreur ! Source du renvoi introuvable.**) Because if you say that this equals this (PD and DN) and you say they have an equal angle (D) and then this equals this (PN and LM) and this and this (PL and MN)…then this becomes a square (LNMP), but we've just seen that it is not a square. So it's all wrong!

228 Teacher: why? What puzzles you?

227 Carla: because…if this is the midpoint (she points at P) then it divides this side in two equal parts (she points at AD and AP and PD) so it should be: if it is a square, the quadrilateral inside is a square too. Why the figure doesn't show that?

228 Teacher: What do you trust more, the figure or your proof?

In this episode, a conflict between a theoretical result (proof) and the empirical answer given by Cabri (the figure does not look like a square) arises. This happens because the students try to validate their proof in the spatio-graphical field (Laborde, 2004; Olivero, 2002). This would require looking at the figure from another point of view, not only empirical, as it may happen at the beginning of the exploration process, but also theoretical. When validating the proof the pupils ‘read’ the figure at an empirical level, they 'read' the properties of LMNP from the measurements: it has not equal sides therefore it cannot be a square. The students do not consider that their hypothesis is *ABCD square* while the Cabri figure is not a square because the angles are not right angles. Instead of 'reading' the Cabri figures, they should have looked at them from a theoretical point of view, according to which ABCD and LMNP are both 'approximations' of squares. The proof would have then been validated.
If we look at this episode in terms of the research problem highlighted above and in the context of the instrumental framework, we can see that the Cabri instrument they construct has the following characteristics: the Cabri feedback is interpreted in a visual-perceptual-numerical way and the students do not show a theoretical control over Cabri[xi]. The students take on board the software to the point that the answer they see on the screen is believed to be true even if it contradicts what they found without the software, by proving and using geometric properties of the figure. This particular instrument in the context of this problem provokes a conflict.
From this example we can see how the interaction with Cabri in the context of open geometry problems needs to be mediated by the use of the theory that allows a control over the Cabri spatio-graphical field. Also the intervention of the teacher becomes crucial to solve possible conflicts between what the software does/shows and the mathematical theory and to mediate the construction of an appropriate instrument (228).

Students’ constructions of Cabri: the role of the cabri/ mathematics experience

The two examples discussed in this paper illustrate how the process of construction of a particular Cabri instrument affects the proving process. From the analysis of the case studies that formed the research Olivero (2002), a pattern emerged in relation to what sort of Cabri instrument was constructed in the context of the problems used in the study[xii]. An instrument is constructed in order to solve the given task, which involves the construction of conjectures and proofs. Can we characterise the type of instrument that the students construct and identify what this depends on? The research showed that what play a role in the process of construction of

the instrument is the mathematical theory and the use of Cabri, which evolve together throughout the proving process.
It was observed that students with considerable Cabri experience and average mathematical background seem to manage better the interaction with Cabri, showing a wide range of dragging modalities and a successful proving process. They generally do not use paper. They are able to link the spatio-graphical and theoretical field in a productive way, which leads to the production of many conjectures and proofs. In this case, the artefact Cabri is transformed by the students into an instrument that is appropriate to deal with the situation they are presented with. There is a theoretical control over Cabri, which is used as a discovery tool within processes of conjecturing, and is then re-interpreted and used as a validation tool or support for thinking within processes related to the actual construction of proofs. This is the result of both mathematical and Cabri long term experience, which allow the students to transform Cabri into an internally oriented tool (in the sense of Vygotsky (1978)). Some students falling in this category often talk about dragging and the way they are using it, there is a control over what they are doing in Cabri and they understand well what Cabri can do and show.
A second case is when students have with very little Cabri experience but have a strong mathematical background: in this case they do not fully exploit the possibilities offered by the software. In general they show more controlled exploration in Cabri. It seems that the artefact Cabri is never transformed into an instrument for these students. It remains an artefact which is used occasionally but is not really taken on board by the students. The students prefer to use other tools (as for example paper and pencil) they are more used to and show a successful production of conjectures and proofs. Given their mathematical strength, it seems that these students are less eager to experiment with new tools they are not familiar with. This behaviour can be observed with 'experts' at different levels; Cabri offers possibilities of exploring and opening up spaces that the 'expert' does not necessarily need.
Finally, students with very little Cabri experience and weak mathematical background, like Carla and Alessandra, usually experiment a lot with Cabri but do not always use it successfully. Conflicts may arise between results produced in the spatio-graphical field and possible theoretical explanations, and the focusing process may take a wrong direction, as shown in the case of Carla above. In this case, the process of instrumental genesis develops through different steps and the intervention of the teacher is needed in order to direct students towards the construction of the appropriate instrument which allows the evolution of the focusing process in the construction of conjectures and proofs. The artefact Cabri is first turned into an instrument based on a scheme of use that relies on a visual-perceptual-numerical

interpretation of the software's feedback. This is not the instrument which serves to accomplish the goal of the problem situations. A new instrument needs to be constructed by the students, based on a theoretical way of 'reading' the Cabri figures. The role of the teacher is crucial in developing this new scheme of use and provoking students to see the same figure from a different point of view which leads them to conjecturing and finally proving.

Conclusions and Implications

To conclude, this paper shows how, given the same tool (dynamic geometry software) and the same activity (proving open problems), students develop different proving processes, both in terms of the way they interact with the software and in terms of the conjectures and proofs they produce. The instrumental approach explains this through the fact that students are constructing different instruments by transforming the same artefact (Cabri). The mathematics and Cabri experience affect and influence the instrumental genesis. Understanding the different instruments and how they are constructed is important because the construction of the instruments affects the development of the proving process in terms of production of conjectures and proofs. Further research will focus on a detailed analysis of the development of the schemes of use related to the particular elements of dynamic geometry software, as for example dragging and measurements.

The fact that students construct different instruments shows that the integration of dynamic geometry in the classroom practice is not a straightforward process but requires a careful analysis. A key challenge for the integration of technology into classrooms and curricula is to understand and to devise ways to foster the process of instrumental genesis towards the construction of the appropriate instrument for a given task. The role of the teacher emerges as important, showing that dynamic geometry per se does not guarantee a successful proving process that manages well the key relationship between the spatio-graphical field and the theoretical field. The teacher constructs different instruments too (Lins, 2003; Ruthven, 2005), which influence the instrumental genesis the students develop and their appropriation of the software. The teacher should act in ‘transforming’ the tool used by the students into a “semiotic mediator” (Mariotti, 2002) in the proving process so that a process of internalisation of the tool itself takes place and the artefact is then transformed into an appropriate instrument for the situation at stake.

References

Arsac, G., Germain, G., & Mante, M. (1988). *Problème Ouvert et Situation - Problème*. Academie de Lyon: IREM.

Artigue, M. (2002). Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work. *International Journal of Computers for Mathematical Learning, 7*(3), 245-274.

Laborde, C. (2004). The hidden role of diagrams in pupils' construction of meaning in geometry. In J. Kilpatrick, C. Hoyles & O. Skovsmose (Eds.), *Meaning in mathematics education* (pp. 1-21). Dordrecht: Kluwer Academic Publishers.

Mariotti, M. A. (2002). Influence of technologies advances on students' maths learning. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh & D. Tirosh (Eds.), *Handbook of International Research in Mathematics Education* (pp. 695-721): Lawrence Erlbaum Associates.

Lins, B. (2003). Actual meanings, possible uses: secondary mathematics teachers and Cabri-Geometre, *Proceedings of European Research In Mathematics Education*. Italy.

Olivero, F. (2002). *The proving process within a dynamic geometry environment*. Bristol: University of Bristol, ISBN 0-86292-535-5.

Perkins, D. N. (1985). The fingertip effect: How information-processing technology changes thinking. *Educational Researcher, 14*(7), 11-17.

Ruthven, K., Hennessy, S. & Deaney, R. (2005). Teacher constructions of dynamic geometry in English secondary mathematics education, paper presented at the *CAL05 conference*, University of Bristol.

Turkle, S., & Papert, S. (1990). Epistemological pluralism: styles and voices within the computer culture. *Signs, 16*(1), 345-377.

Verillion, P., & Rabardel, P. (1995). Cognition and artifacts: a contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education, 10*, 77-101.

Vygotsky, L. S. (1978). *Mind in society. The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

---

[i] You are given a quadrilateral ABCD. Construct the perpendicular bisectors of its sides: a of AB, b of BC, c of CD, d of DA. H is the intersection point of a and b, K of a and d, L of c and d, M of c and b. Investigate how HKLM changes in relation to ABCD. Prove your conjectures.

[ii] For more information about the methodology of the project this example is taken from see Olivero (2002).

[iii] For Bartolomeo "always" means in two cases only.

[iv] *Photo-dragging* incorporates "modalities which suggest a discrete sequence of images over time: the subject looks at the initial and final state of the figure, without paying attention to the intermediate instances. The aim is to get a particular figure" (Olivero, 2002, p.141).

[v] Sometimes it is 'indirect' dragging when it is Tiziana who is in fact using the mouse.

[vi] The general conjecture is: ABCD and HKLM are similar.

[vii] Which will lead to another general conjecture: if ABCD is cyclic then HKLM is a point.

[viii] *Film-dragging* incorporates "modalities which suggest a film: the subject looks at the variation of the figure while moving and the relationships among the elements of the figure. The aim of dragging is the variation of the figure itself" (Olivero, 2002, p.141)

[ix] This approach considers the use of tools in generals, not necessarily new technologies.

[x] Varignon's problem: Draw any quadrilateral ABCD. Draw the midpoints L, M, N, P of the four sides. Which properties does the quadrilateral LMNP have? Which particular configurations does LMNP assume? Which hypotheses on the quadrilateral ABCD are needed in order for LMNP to assume those particular configurations?

[xi] For a detailed analysis of this episode see Olivero (2002).

[xii] Open problems (Arsac, Germain, & Mante, 1988) requiring conjecturing and proving in geometry