A correction note on “Three-Step Iterative Methods with Sixth Order Convergence for Solving Nonlinear Equations”

Malik Zaka Ullah
Department of Mathematics
King Abdulaziz University, Jeddah 21589
Kingdom of Saudi Arabia
mzhussain@kau.edu.sa

Lala Muhammad Assas
Department of Mathematics
Umm al Qura University , Makkah
Kingdom of Saudi Arabia
lalamuhammadassas

Fayyaz Ahmad
Dept. de Física i Enginyeria Nuclear
Universitat Politècnica de Catalunya
Compte d’Urgell 187, 08036 Barcelona, Spain
fayyaz.ahmad@upc.edu

A.S. Al-Fhaid
Department of Mathematics
King Abdulaziz University, Jeddah 21589
Kingdom of Saudi Arabia
aalfhaid@hotmail.com
aalfhaid@kau.edu.sa

Copyright © 2013 Malik Zaka Ullah, Laila M Assas, Fayyaz Ahmad and A.S. Al-Fhaid. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract
In M. Rafiullah and M. Haleem (2010) [M. Rafiullah, M. Haleem, Three-Step Iterative Methods with Sixth Order Convergence for Solving Nonlinear Equations, Int. Journal of Math. Analysis, Vol. 4, 2010, no. 50, 2459-2463], an iterative method with sixth-order convergence has been proposed. In this correction note, we report that the proof of order of convergence is wrong and hence claimed six order of convergence is incorrect. we show that the correct order of convergence is four.

**Mathematics Subject Classification:** 65N99

**Keywords:** Nonlinear equations, Iterative methods, Convergence order, Computational order of convergence.

## 1 Introduction

In 2010, M. Rafiullah and M. Haleem [1] proposed an iterative scheme. Unfortunately the convergence proof of iterative scheme is incorrect. Author constructed Algorithm 1 on page 2461, which is again stated here.

\[
\begin{align*}
    y_n &= x_n - \frac{f(x_n)}{f'(x_n)}, \\
    z_n &= x_n - \frac{f(x_n)}{f'(x_n)} + \frac{f(x_n)(f'(y_n)-f'(x_n))}{2(f'(x_n))^2}, \\
    x_{n+1} &= z_n + \frac{(f'(x_n))^2-4f'(x_n)f'(y_n)+(f'(y_n))^2}{2f(y_n)}.
\end{align*}
\]  

(1)

Author claimed that (1) has sixth order of convergence. we show in the following section that the correct order of convergence is four by using Maple based program and finally provide computational order of convergence in the favor of our claim.

## 2 Convergence analysis

A Maple based proof in Fig 1 shows that the order of convergence is four and we obtain the error equation

\[ e_{n+1} = c_2(c_3 + 4c_2^2)e_n^4 + O(e_n^5). \]  

(2)

## 3 Numerical examples and computational order of convergence

In this section, we provide numerical examples to show the computational order of convergence of iterative scheme (1). Let \( x_{n-1}, x_n \) and \( x_{n+1} \) be successive
Iterative methods

iterations in the vicinity of root $\alpha$ of $f(x) = 0$, the computational order of convergence (COC) [2], can be approximated by

$$\text{COC} \approx \frac{\ln|\alpha(x_n - \alpha)^{-1}|}{\ln|\alpha(x_{n-1} - \alpha)^{-1}|}$$

(3)

A set of five functions is listed in Table 1. The absolute error in numerically calculated root is depicted in Table 1. Total number of function evaluations per iteration are four for iterative scheme (1). Table 2 also shows total number of iterations and computational order of convergence.

Table 1: List of test functions

<table>
<thead>
<tr>
<th>Functions</th>
<th>Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x) = \sin(x)^2 - x^2 + 1$</td>
<td>$\alpha = -1.4044$</td>
</tr>
<tr>
<td>$f_2(x) = x^2 - \exp(x) - 3x + 2$</td>
<td>$\alpha = 0.257$</td>
</tr>
<tr>
<td>$f_3(x) = \cos(x) - x$</td>
<td>$\alpha = 0.7390$</td>
</tr>
<tr>
<td>$f_4(x) = (x - 1)^3 - 1$</td>
<td>$\alpha = 2$</td>
</tr>
<tr>
<td>$f_5(x) = x^3 - 10$</td>
<td>$\alpha = 2.1544$</td>
</tr>
</tbody>
</table>

4 Conclusion

we have shown that claimed order of convergence is incorrect. It is four instead of six. we have provided a Maple based program to verify fourth-order convergence of iterative scheme 1 and finally the computational order of convergence also favor our reported order convergence.

ACKNOWLEDGEMENTS. This work has been partially supported by the Spanish MEC grants AYA2010-15685.
Figure 1: Maple program to verify the order of convergence of iterative scheme (1).
References


Received: Month xx, 20xx