Using phase space reconstruction for patient independent heartbeat classification in comparison with some benchmark methods

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ARTICLE INFO

Article history:
Received 25 September 2010
Accepted 11 April 2011

Keywords:
Automatic heartbeat classification
Patient independent ECG classification
Reconstructed phase space
Gaussian mixture model
TDNN

ABSTRACT

Many methods for automatic heartbeat classification have been applied and reported in literature, but relatively few of them concerned with patient independent classification because of the less significant results compared to patient dependent ones. In this work, using phase space reconstruction in order to classify five heartbeat types can fill this gap to some extent. In the first and second method, Reconstructed phase space (RPS) is modeled by the Gaussian mixture model (GMM) and bins, respectively, and then classified by classic Bayesian classifier. In the third method, RPS is directly used to train predictor time-delayed neural networks (TDNN) and classified based on minimum prediction error. All three methods highly outperform the results reported before, for patient independent heartbeat classification. The best result is achieved using GMM–Bayes method with 92.5% classification accuracy.

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1. Introduction

Heart arrhythmias result from any disturbance in the rate, regularity, and site of origin or conduction of the cardiac electric impulse. Broadly speaking, arrhythmias can be divided into two groups. The first group includes ventricular fibrillation and tachycardia, which are life-threatening and require immediate therapy with a defibrillator. Detection of these arrhythmias is well researched and successful detectors have been designed with high sensitivity and specificity [1–4]. This study investigated the second group which includes arrhythmias that are not imminently life-threatening but may require therapy to prevent further problems. The electrocardiogram (ECG) is a low-cost, noninvasive, and effective test for arrhythmia analysis and has become the standard diagnostic tool. Some arrhythmias appear infrequently and up to a week of ECG activity may need to be recorded using a Holter ECG monitor to successfully capture them. Many arrhythmias manifest as sequences of heartbeats with unusual timing or ECG morphology.

An important step toward identifying an arrhythmia is the classification of heartbeats. Classification of heartbeats can be very time-consuming and hence any automated processing of the ECG that assists this process would be of assistance and is the focus of this study.

The literature investigated on automatic heartbeat classification can be grouped to different categories depending on the applied features, classification methods, database, and strategy of training and testing. Among a large variety of features used to represent ECG we can name: morphological features [5,6], heartbeat interval features [7,8], frequency-based features [9], hermit polynomials [10], and independent component analysis [11]. The classification methods include, back propagation neural networks [5–7], self-organizing networks with learning vector quantization [8], self-organized maps [10], support vector machine [12], etc. However, the fact that the information existing in the power spectrum of the signal may be more discriminative than the information in the frequency spectrum is less considered in the past researches. In this work, extracted features from power spectrum have been investigated and compared with previous methods.

Because of using different classifiers and different features, an accurate comparison of various methods used for heartbeat classification was impossible before. Recently a study compared the classification abilities of four classification methods, the Kth nearest neighbor classification rule (Knn), neural networks (NN), discriminant analysis (DA), and fuzzy logic (FL) [13]. This work tested the classification performance and learning capacity of these four classification methods for five heartbeat classes, which feature with particular QRS behavior (N1, PVC2, LBBB3, RBBB4, and PBB5). In order to investigate learning capacity, two main training strategies are considered in this study: In the first

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doi:10.1016/j.compbio.2011.04.003


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| 1 | Normal. |
| 2 | Premature ventricular contractions. |
| 3 | Left bundle branch blocks. |
| 4 | Right bundle branch blocks. |
| 5 | Paced beats. |
approach (local learning) a small database that is customized to the test patient is applied. In the sense of local set, specific strategies are adopted for local learning in some arrhythmia monitors, for example in intensive care rooms, where only the normal or predominant beats are identified by the cardiologist, and any irregular heartbeats, like premature ventricular contractions, are classified automatically on the basis of pattern differences with the annotated beats. However, in many commercial applications preferable classifiers avoid the need for time-consuming human expert annotations, associated with manual editing of patient-specific ECG records [13]. To this aim, global learning approach is implemented. Global learning set is built from a large ECG database and a patient independent classifier is achieved. The significant improvement reported in Ref. [13] is in the case of a patient-adapted local learning set. But in the global set approach, which is stated as patient independent case, weak results are reported.

An important issue to be considered in heartbeat classification is to use patient independent features and methods. From a stochastic process perspective, traditionally linear approaches to signal analysis and classification, which are typically based on frequency domain characteristics, only present first- and second-order statistics of the signal, and it may be insufficient to separate different classes. Additionally the morphological features are patient dependent. Signal classification techniques based on power spectral information cannot distinguish between different signals that have the same power spectrum, but such signals may be distinguishable in a reconstructed phase space (RPS). Additionally, the underlying theory, outlined in more detail later, guarantees that the dynamics of any system are fully described by an RPS generated from any single state variable, provided that the dimension of the RPS is greater than twice the box counting dimension of the original system [14]. Because, the RPS captures the important differentiation phase information of the signal, and not the local characteristics of that, it will be suited to derive models with less patient dependency.

In this work we use RPS to train statistical distributions and then classify them. The distributions learned over RPSs are a parametric one based on the Gaussian mixture model and a nonparametric one based on binning and occurrence counts. The classifiers are a Bayes maximum likelihood classifier and an artificial neural network. RPS has been used widely on estimating dynamical invariants, which are not sensitive to initial conditions or smooth transformations of the space. Various methods of estimating dimension have been proposed, such as correlation dimension [15], minimum phase space volume [16], and box counting [17]. The work presented in Ref. [18] develops statistical features of RPSs whereas topological analysis approach builds global vector reconstructions. In this study we apply the RPS method presented in Ref. [18] to improve the classification abilities reported previously, on global training set and prove that in this way an automatic patient independent classifier is achieved. This classifier has high performance despite being patient independent.

This paper is organized as follows. In Section 2 methods and theoretical requirements are presented. Section 3 describes the learning strategy and dataset. Then, in Section 4 the experimental results are reported, and finally, Section 5 concludes the paper.

2. Materials and methods

As mentioned, in this study we use RPS to estimate a joint probability density function of the space and train two statistical models. Then classifiers will be adopted to recognize the heartbeat type. The multistep process of the algorithms can be illustrated as follows: In the first step the optimum value of the phase space parameters, time lag, and dimension, is estimated. Then the RPS is created using these values. For two of the methods, it is necessary to model the RPS. This modeling is implemented by GMM or bins. In both methods, then the parameters of the obtained model are given to a Bayes classifier. However, in the third method the samples of the RPS are directly fed to a TDNN and classified. Fig. 1 shows the multistep algorithm of each method. This section describes the nonlinear signal processing concepts used in this work.

2.1. Reconstructed phase space

In a time series, such as the ECG signal, it is sometimes necessary to search for patterns not only in the time series itself, but also in a higher dimensional transformation of the time series. The reconstructed phase space is an example of such a transformation. An RPS is an n-dimensional space in which a signal is plotted against time-delayed versions of itself. Each point in the phase space is calculated according to

\[ x_n = [x_0, x_\tau, \ldots, x_{n-(d-1)\tau}] = (1 + (d-1)\tau) \cdot \ldots \cdot N \]

where \( x_n \) is the nth point in the time series, \( \tau \) is the time lag, \( N \) is the number of points in the time series, and \( d \) is the dimension

![Fig. 1. Block diagram of 3 methods: (a) GMM–Bayes (b) Bin–Bayes (c)TDNN.](image-url)
of the phase space. The entire phase space is generated by

\[
X = \begin{bmatrix}
X_1 \cdots X_{1+\tau} \\
X_2 \cdots X_{2+\tau} \\
\vdots \\
X_N \cdots X_{N-\tau}
\end{bmatrix}
\]

(2)

In order to create a reconstructed phase space, the dimension and the lag need to be determined. The proper determination of these parameters can have a significant effect on the classification performance. Selecting values that are too high adds complexity and increases the amount of data needed for an experiment. If the values selected are too low, the complex nature of the problem may not be captured in the phase space.

Two common methods used to estimate the proper lag are: the first minimum of mutual information and an empirical method. The mutual information method determines the information that is shared between consecutive integer lag choices. The first minimum of the mutual information is used as an estimate of the optimal lag. A second method estimates the lag empirically. The lag is selected by examining the results of multiple classification experiments. In each experiment a different lag is studied and the classification accuracy is determined. The lag that provides the highest accuracy is the one that is selected.

The process for choosing the dimension is similar to that of determining the lag. The dimension can be selected using the false nearest neighbors or by an empirical method. The false nearest neighbors are points in a phase space of n dimension which are near each other, but are not near each other in a phase space of dimension n+1. The number of points falsely near each other indicates whether a higher dimension should be used. A threshold of 0.001 on the percentage of false nearest neighbors is used to select the dimension. The empirical method operates the same as the lag determination. Multiple experiments are conducted and the experiment with the highest accuracy indicates the dimension to be chosen.

2.2. Gaussian mixture models

A Gaussian mixture model (GMM) is a set of N multidimensional Gaussian distributions. The set of GMMs approximately models the distribution of the data. The GMM is defined as

\[
p(X) = \sum_{m = 1}^{M} W_m p_m(X) = \sum_{m = 1}^{M} W_m N(x; \mu_m, \Sigma_m)
\]

(3)

where \(W_m\) is the mixture weights (\(\sum_{m = 1}^{M} W_m = 1\)), \(M\) is the number of mixtures, and \(N(x; \mu_m, \Sigma_m)\) is a normal distribution with mean \(\mu_m\) and covariance matrix \(\Sigma_m\). Using multiple Gaussian distributions, any distribution of data can be modeled. The number of distributions to use in a GMM model is a parameter dependent on the complexity of the distribution of the data to be modeled. This number can be determined empirically by running multiple classifications that utilize different numbers of GMMs. The classification with the highest accuracy is the one that is selected. The weights, mean, and covariance matrix of the GMM are estimated using the Expectation Maximization (EM) algorithm [22].

The method begins with initial values for each parameter, then iterates through the available data to find the Maximum Likelihood (ML) estimate. The formulas used for the estimation are:

\[
\mu_m = \frac{\sum_{n = 1}^{N} X_n p_m(X_n)}{\sum_{n = 1}^{N} p_m(X_n)}, \quad \Sigma_m = \frac{\sum_{n = 1}^{N} (X_n - \mu_m)(X_n - \mu_m)^T p_m(X_n)}{\sum_{n = 1}^{N} p_m(X_n)}
\]

(4)

where \(M\) is the number of mixtures, \(p_m\) is the probability distribution function, and \(N\) is the number of points in the signal [22].

2.3. Bin-based model

The use of histograms as an estimate of the discrete probability mass function (pmf) of the attractor is straightforward. The space is divided into bins and occurrence counts in each bin over the training examples, divided by the total number of points, provide a direct estimate of the posterior probability within each bin

\[
p(b) = \frac{\text{number of example points in bin } b}{\text{Total number of bins}}
\]

(5)

Given \(x_n\) in bin \(b\), the posterior probability for \(x_n\) is \(\hat{p}(b)\).

In order to improve the reliability of pmf estimate, it is desirable to establish a binning system that more accurately reflects the underlying distribution. The distribution of points throughout the phase space is not uniform; therefore, no uniformly spaced bins are used. A two-step process is used to form the bins. First, along each dimension a set of intercepts is computed such that in that dimension the histogram formed by the intercepts is uniform. The outlying bins of this one-dimensional (1D) histogram extend to infinity. Second, the higher dimensional bins are formed as hypercubes, whose boundaries are formed by the intercepts determined in the first step. In forming an RPS for a signal of reasonable length, the intercepts are approximately the same in each dimension. Disadvantages of the bin-based system include an exponentially increasing number of bins as RPS dimension increases and fixed bin boundaries.

2.4. Bayes classifier

The Bayes classifier [23] uses the principles of Bayes' law to compute the class with the highest conditional likelihood. Bayes' laws states:

\[
p(\text{class}|x_n) = \frac{p(x_n|\text{class})p(\text{class})}{p(x_n)}
\]

(6)

where \(p(x_n|\text{class})\) is the conditional probability of a specific class given a point in the RPS. Since the \(p(x)\) term is simply a normalizing value, it is left out of the equation when comparing class probabilities. So, the Bayes classifier becomes:

\[
\text{class} = \arg \max_{\text{class}, \in \text{AllClasses}} p(x_n|\text{class})
\]

(7)

where \(N\) is the number of points in the RPS and \(d\) is the dimension of the RPS. The conditional probabilities \(p(x_n|\text{class})\) can be learned from the training data of the experiment. In this way, the class of a given RPS is compared with the learned models and then classified.

2.5. Predictors artificial neural network (ANN)

The second classifier used in this study is a time delay neural network (TDNN), which is used as a nonlinear one step predictor.
The TDNN classifications are made using minimum prediction error. A 10-fold cross-validation approach, which is discussed in detail in the next section, is used to implement the methods. The folds are formed in a statistically balanced manner across classes. The same folds are used to train both learners and test both classifiers.

The number of inputs to the TDNN is the dimension of the RPS. Thus, both methods are working with the same number of inputs. The TDNN has two hidden layers. Given $d$ inputs to the TDNN, there are $d$ neurons in the first hidden layer $\sqrt{d}$ neurons in the second hidden layer, and one output neuron. The hidden layers use tan-sig transfer functions, and a linear transfer function is used for the output.

The folds are embedded with dimension $d + 1$. For every point in the embedded space the first $d$ elements are used as the inputs of TDNN and their output is set to be the $(d + 1)$th element. A set of ANNs is trained, one for each class, using the above strips of the first dimension rows as the training and the last row as the target. Testing is accomplished by computing the features over a new signal sample, and selecting the class label corresponding to the ANN with the lowest prediction error.

3. Data, learning strategy, and methods validation

3.1. Dataset

In order to compare our results with that of Ref. [13], the study involved all 48 ECG recordings from the MIT-BIH arrhythmia database. In this dataset each recording has duration of 30 min and includes two leads. The sampling frequency is 360 Hz and the resolution is 200 samples per mV. The heartbeats were recognized by the fiducial points in the database and the original database annotations were accepted. The wide variations in the heartbeat waveforms among the individual patients make the selection of the learning set a critical choice. For this fact, two main training strategies are considered: global learning set, independent from the tested patient and built from a large ECG database, and the local learning set, in case it is customized to the tested patient [13]. Consequently, in order to adopt a patient independent heartbeat classifier, we formed a global learning set. The study was focused on the classification of the five largest heartbeat classes in the MIT-BIH arrhythmia database: (1) normal beats (N)—about 74820 cases; (2) premature ventricular contractions (PVC)—about 6970; (3) left bundle branch blocks (LBBB)—about 8050; (4) right bundle branch blocks (RBBB)—about 7220; (5) paced beats (PB)—about 7000. In our study, we analyzed the N, PVC, LBBB, RBBB, and PB in the full-length MIT-BIH files. 10-fold cross validation was applied in order to achieve an independent learning from the tested patient (global learning). As indicated in Ref. [13] using this selection of data, referred to as global training set versus the local training set, a 10-fold cross validation leads to a patient independent method.

In 10-fold cross validation method, described in the next subsection, in each experiment 90% of the global data is used to train and the rest of the data applied to test. In practice, we observed that this large training dataset is too redundant in the phase space, unlike the time space. So, to form the reconstructed phase space we do not need all the training data and a very short signal could be sufficient to model the phase space. Therefore, from each record in training dataset we selected only 2 QRS complexes and a total of 376 QRS complexes were collected and used in training process. There are 80 normal beats, 76 PVC beats, 76 PB cases, 68 LBBB beats, and 78 RBBB cases in this training dataset. In order to compare the results with that of Ref. [13], the testing dataset is exactly the same (10,406 beats).

However, in Ref. [13] the authors stated that their methods could not show acceptable performance on global dataset. In this work we applied the global training set, thus our results would be evaluated in comparison with the results reported in Ref. [13] for patient independent classification.

3.2. Ten-fold cross validation

In experiments where the Test Set is not available, it is necessary to use a method of algorithm validation that is data/patient independent, but does not need a separate set of data for testing. The method used in this research is called ten-fold cross validation. The first step in this method is to separate the Training Set into ten groups (folds). The classifier is trained on nine of the ten folds. Then the learned models are used to classify the tenth fold. This is repeated 9 more times, each time leaving out a different fold. In the end, 10 patient independent classifications have been conducted. The classification results are combined to calculate the overall accuracies.

Using ten-fold cross validation maintains patient independent classification without the availability of the ‘Test Set’. This allows for an effective validation of possible algorithms while not analyzing the ‘Test Set’. Thus, ten-fold cross validation is an accurate way of simulating ‘Test Data’ results when a testing dataset is not available.

Implementing this procedure increases the likelihood that if the developed algorithm has high classification accuracy on the ‘Test Set’, it will be able to generalize other datasets. It also allows results from classifications run on the Training Set to generally be compared to the Test Set results.

3.3. Methods validation

In order to compare our results with that of Ref. [13] we have to validate the classification abilities of RPS method with the same criteria used in that study. Four statistical indices were calculated for each heartbeat class $i$ ($i = \text{N}, \text{PVC}, \text{LBBB}, \text{PB}, \text{and} \ \text{RBBB}$): sensitivity $Se$, specificity $Sp$, positive predictive value $PPV_i$, and negative predictive value $NPV_i$. They are computed according to the following relations:

$$Sp_i = \frac{TN_i}{TN_i + FP_i}$$

$$PPV_i = \frac{TP_i}{TP_i + FP_i}$$

$$NPV_i = \frac{TN_i}{TN_i + FN_i}$$

$$Se_i = \frac{TP_i}{TP_i + FN_i}$$

(8)

$TP_i$ (true positives) is the number of heartbeats of the $i$th class, which are correctly classified (e.g., N classified as N); $TN_i$ (true negatives) is the number of heartbeats not belonging to the $i$th class and not classified in the $i$th class (e.g., PVC, LBBB, RBBB, and PB not classified as N); $FP_i$ (false positives) is the number of heartbeats classified erroneously in the $i$th class (e.g., PVC, LBBB, RBBB, and PB classified as N); $FN_i$ (false negatives) is the number of heartbeats of class $i$, classified in a different class (e.g., N not classified as N).

4. Experiments and results

In this work a rapid and accurate classification of 5 heartbeat types is done using reconstructed phase space. To preprocess the signals, at first they are segmented into 2 s intervals. The electrocardiogram signals should be normalized in the time domain as follows: $x'_t = (x_t - \mu_x)/\sigma_x$. Then the following procedures are realized: a notch filter for elimination of the power-line interference, implemented by moving averaging of samples in one period of the
interference, a low-pass filter for suppression of the tremor noise, realized by moving averaging of samples in 30 ms time interval, and a high-pass recursive filter for drift suppression with cut-off frequency of 2.2 Hz.

### 4.1. Embedded space construction

The first step to implement the above methods is to construct an RPS from a signal. The dimension of the RPS and the time lag at which to sample the signal must be selected to embed the space of the signal. As proposed in Ref. [17], the mode of distribution of the first minimum of the automutual information function across all signals is used as an initial estimate of the time lag. Similarly, the mean plus two times standard deviations of the distribution of false nearest neighbor dimensions across all signals is used as an initial estimate of the dimension.

Although the accurate assessment of dimension and time lag is very important to get the highest classification abilities, in practice the methods used here are effective across a range of dimensions and time lags [20]. The initial estimates of time lag and dimension are 4 and 10, respectively, using the method proposed in Ref. [17]. Figs. 2 and 3, respectively, show classification accuracy versus dimension with lag held constant and accuracy versus time lag with dimension held constant using an 8 mixture GMM and a Bayes classifier using a 10-fold cross validation for all experiments. Classification accuracy is the total number of correctly classified signals divided by the total number of signals. According to empirical results, we selected 15 as dimension and 25 as time lag and the embedded space is constructed using the equations described before. A 2D RPS for class Normal is shown in Fig. 4 and those of the other classes in Fig. 5. These figures show that the patterns existed in higher dimensional transformations can be more discriminate than those in time series itself. This discrimination is clearly observable in Figs. 4 and 5 in RPS for ECG signals.

### 4.2. GMM–Bayes classifier

In order to implement this method, a GMM probability distribution is learned for each signal class. This is done by creating an RPS using the time lag and dimension determined in the previous step and inserting all the signals for a particular class into this space as described before.

The required number of mixtures is related to the underlying distribution of the RPS density. The classification accuracy tends toward an asymptote as the number of mixtures increases provided there is sufficient training data [17]. Fig. 6 shows the accuracy versus number of mixtures. In this figure we see that the accuracy between 16 and 32 mixtures remains relatively constant. However, using more mixtures dramatically increases the computational cost. So, we used 16 mixtures to model the phase space of the signal. An illustration of a GMM over a 2D RPS is shown in Fig. 7, where the principle axes of the ellipses indicate one standard deviation of each mixture in the model.

Using the best empirical time lag, dimension and 16 mixtures, GMM–Bayes method results in 92.5% classification accuracy. The other classification indices of this method compared to the results reported in Ref. [13] are illustrated in Fig. 8. In Ref. [13], four classification methods are used: the K nearest neighbor classification rule (Knn), neural networks (NN), discriminant analysis (DA), and fuzzy logic (FL). This figure illustrates that the performance of GMM–Bayes method is much better than the other methods used for classification of heartbeats with global set data.

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4.3. Bin–Bayes classifier

Although the concept of bin-based RPS modeling is very simple, because of the enormous computational cost, its implementation is not very easy. The GMM–Bayes is able to successfully model a 21-dimensional space. To apply the bin-based method to such a space with only one division per dimension would require over one million bins [18]. Using a three-dimensional space with time lag 25 and 1000 bins (10 strips over each dimension) the classification accuracy is obtained 70.93%. Increasing the dimension to 5 with 100,000 bins the accuracy is gained 84.83%. The classification ability of this method over each class is listed in Tables 1 and 2. It is seen that, although the performance of bin–Bayes method is less than GMM–Bayes, still it is much better than the other methods used for classification of heartbeats with global set data.

4.4. Neural network classifier

As illustrated before, in this method the points of \((d+1)\) dimensional embedded space are directly used to train 5 TDNNs (one for every class). Theses Networks are trained to predict the values of \((d+1)\)th dimension by getting the first \(d\) elements in their input [25]. For the test data, a new signal belongs to the class with minimum mean squared prediction error.

In practice, the networks cannot be trained on spaces with high time lags. However, the experiments show that increasing the time lag improves the classification accuracy. Table 3 shows the results. The last column is the Training predictive error of the TDNN for normal class trained with folds 2 to 10 and tested on fold 1. Classification accuracy is computed on all TDNNs. Training epochs is 30 for all networks. So, we used 5 as time lag and reconstructed a 16 dimensional space to train and test the TDNNs. The results gained with this method are presented in Table 4.

5. Discussion and conclusion

This study is carried out in order to investigate on the discriminating capabilities of the information existing in the power spectrum of the heartbeat signal and to compare these features with previous methods. For the sake of comparison, the results reported in Ref. [13] are considered and the same datasets are used, because
in that study some of the most successful classifiers have been compared using the same features and two approaches called as patient dependent and patient independent has been used. Those methods had led to weak results in the patient independent case and the method used in this work could improve these results.

In this study three classification methods based on reconstructed phase space theory are applied to improve the classification abilities of patient independent heartbeat classifiers. For all three methods at first the best time lag and dimension should be estimated. The first estimated values for time lag and dimension are reasonable but not optimum. Some experiments are carried out to find the best practical values. Using the best empirical time lag and dimension the phase space of the signal is constructed. In the first two methods, this RPS is modeled using Gaussian mixtures or bins. Then Bayesian classifier is used to classify these models. In the last method the RPS is directly used to train TDNNs as one step forward predictors. The classification criterion is chosen to be minimum prediction error.

According to the high performance of RPS representations, all methods result in high classification accuracies comparing to our benchmark reports. GMM–Bayes has the best classification ability and clearly outperforms the best results reported here. The classification indices of other methods are lower than those of GMM–Bayes but still are higher than or comparable to the benchmark’s results.

These high abilities are due to the large advantages of using RPSs as a theoretically well founded method to represent nonlinear signals. Because the RPSs contain the salient information about signal variability, statistical methods are able to capture the information in such spaces. When we are limited by amount of data and lack of suitable tools to deal with high dimensions, the phase space of a signal is a very capable representation to indicate the full dynamic structure of any finite-dimensional generating system. On the other hand, in the applications such as heartbeat classification, the short duration of a QRS complex lessens the ability of classifiers using common representations of data. But it is shown that using RPS, only a 2 s signal is enough to obtain an accurate classification.

These advantages of RPS make a high performance patient independent heartbeat classifier attainable. The success of RPS
method to classify heartbeat types indicates that phase is very important for differentiating between these classes. Especially for PB class the RPS methods highly outperform other methods, so this type of heartbeat has a very distinguishable phase space among other types.

In future works the local information of the signal consisting of frequency representations and morphological descriptors can be combined with features captured in RPSs. This combination can be very beneficial, since some information exists in phase and some in shape and frequency domain. Additional RBF network may be a very suitable classifier for heartbeat RPSs classification application. This network can combine the classification abilities of GMM and neural networks. Also, fuzzy logic classifiers can be good options to classify RPSs because of their high performance in dealing with qualities which is indicated by an RPS.

Conflict of interest statement

We all authors do declare that we do not have any financial and personal relationships with other people or organizations that could inappropriately influence our work. The study is carried out in the

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**Table 1**

Results of 3 D bin–Bayes method.

<table>
<thead>
<tr>
<th></th>
<th>Se (%)</th>
<th>Sp (%)</th>
<th>PPV (%)</th>
<th>NPV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>68.47</td>
<td>81.9</td>
<td>64.41</td>
<td>84.44</td>
</tr>
<tr>
<td>PVC</td>
<td>75.29</td>
<td>93.77</td>
<td>55.65</td>
<td>90.58</td>
</tr>
<tr>
<td>LBBB</td>
<td>70.00</td>
<td>96.55</td>
<td>84.00</td>
<td>92.56</td>
</tr>
<tr>
<td>PB</td>
<td>90.00</td>
<td>100</td>
<td>100</td>
<td>97.25</td>
</tr>
<tr>
<td>RBBB</td>
<td>50.85</td>
<td>79.84</td>
<td>78.95</td>
<td>89.06</td>
</tr>
</tbody>
</table>

**Table 2**

Results of 5D bin–Bayes method.

<table>
<thead>
<tr>
<th></th>
<th>Se (%)</th>
<th>Sp (%)</th>
<th>PPV (%)</th>
<th>NPV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>95.50</td>
<td>84.80</td>
<td>73.61</td>
<td>97.70</td>
</tr>
<tr>
<td>PVC</td>
<td>69.41</td>
<td>94.53</td>
<td>79.73</td>
<td>90.88</td>
</tr>
<tr>
<td>LBBB</td>
<td>80.00</td>
<td>99.26</td>
<td>96.00</td>
<td>95.74</td>
</tr>
<tr>
<td>PB</td>
<td>98.33</td>
<td>100</td>
<td>100</td>
<td>99.62</td>
</tr>
<tr>
<td>RBBB</td>
<td>77.97</td>
<td>99.27</td>
<td>95.83</td>
<td>95.44</td>
</tr>
</tbody>
</table>

**Table 3**

Results of NN method using different lags and dimension.

<table>
<thead>
<tr>
<th>NN method</th>
<th>Time lag</th>
<th>Dimension</th>
<th>Classification accuracy (%)</th>
<th>Training predictive error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>15</td>
<td>52.82</td>
<td>0.01</td>
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<td>0.10</td>
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<td></td>
<td>5</td>
<td>15</td>
<td>76.27</td>
<td>0.29</td>
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<tr>
<td></td>
<td>7</td>
<td>15</td>
<td>68.42</td>
<td>0.49</td>
</tr>
</tbody>
</table>

**Table 4**

Results of RPS–NN method.

<table>
<thead>
<tr>
<th>RPS–NN</th>
<th>Se (%)</th>
<th>Sp (%)</th>
<th>PPV (%)</th>
<th>NPV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>66.67</td>
<td>94.64</td>
<td>86.05</td>
<td>93.14</td>
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<tr>
<td>PVC</td>
<td>74.12</td>
<td>80.80</td>
<td>54.31</td>
<td>91.02</td>
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<td>LBBB</td>
<td>78.33</td>
<td>95.98</td>
<td>82.46</td>
<td>94.84</td>
</tr>
<tr>
<td>PB</td>
<td>98.33</td>
<td>98.70</td>
<td>95.16</td>
<td>99.56</td>
</tr>
<tr>
<td>RBBB</td>
<td>72.88</td>
<td>95.67</td>
<td>79.63</td>
<td>93.82</td>
</tr>
</tbody>
</table>

Fig. 8. Comparison of characteristics of 5 methods, neural networks (NN), fuzzy logic (FL), discriminant analysis (DA), Kth nearest neighbor (Knn), and GMM–Bayes classifier versus classes. (a) Specificity, (b) sensitivity, (c) positive predictive value, and (d) negative predictive value.
Amirkabir University by students, Isar Nejadgholi and Fatemeh Abdolali and under supervision of the professor, Dr. Moradi.

References
