A GROUP DECISION MAKING METHOD BASED ON TOPSIS UNDER FUZZY SOFT ENVIRONMENT

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Abstract – In this paper, we first briefly present conventional TOPSIS method developed by Hwang and Yoon[10, 19] as a multi-criteria decision making technique. We then give a group decision making method based on TOPSIS method under fuzzy soft environment, and finally give an application of proposed method to show operation and effectiveness of method.

Keywords – Soft sets, Fuzzy set, Fuzzy soft sets, TOPSIS, Multi-criteria decision making.

1 Introduction

Decision making is one of important processes that human being encounters many areas of the real world such as business, service, management, military, etc. But in real life, necessary informations for decision making may not be certain always. First step of the decision making process is to model such information involving uncertainty. Hence, in 1965, fuzzy set theory was suggested to model fuzzy data as mathematically by Zadeh [20]. However, in this theory, determining of membership function is rather difficult sometimes. Therefore, in 1999, Molodtsov [14] proposed a completely new approach for modeling uncertainty, free from this difficulty. Then Maji et al. [12] gave some operations of soft sets and their properties. To make some modifications to the operations of soft sets some researchers such

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Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) being one of classical multi-attributive decision making (MADM) methods such as PROMETHEE [2], VIKOR [15], ELECTRE [16], developed by Hwang and Yoon [10]. Chen et al. [5] extended the TOPSIS method for solving multi-criteria decision making (MCDM) problems in fuzzy environment. Boran et al. [3] developed TOPSIS method for MCDM problems based on intuitionistic fuzzy sets. Chi and Liu [6] extended TOPSIS to interval neutrosophic sets (INSs), and with respect to the multiple attribute decision making problems in which the attribute weights were unknown and the attribute values take the form of INSs. Eraslan [4] gave a decision making method by using TOPSIS on soft set theory.

In this paper, we extend TOPSIS method to deal with group decision making problems in fuzzy soft environment. Then, we give an illustrative example to show the effectiveness of the suggested method.

The study is organized as follows: Section 2 introduces the basic definitions of soft set and fuzzy soft set with their basic operations. The main procedure for the conventional TOPSIS is described in a series of steps in Section 3. In Section 4, a group decision making method is developed by using TOPSIS on fuzzy soft set theory. Afterwards, an application of method is given to illustrate effectiveness of the method.

2 Preliminary

In this section, we summarize the preliminary definitions which are fuzzy set [20], soft set [14, 9], fuzzy soft set and their results that are required in this paper.

2.1 Fuzzy Sets

Definition 2.1. [20] Let $U$ be a initial universe. A fuzzy set $\mu$ over $U$ is defined by a membership

$$\mu : U \to [0, 1]$$

For $u \in U$; the membership value $\mu(u)$ essentially specifies the degree to which $u \in U$ belongs to the fuzzy set $\mu$. Thus, a fuzzy set $\mu$ over $U$ can be represented as follows,

$$\mu = \{(\mu(u)/u) : u \in U, \ \mu(u) \in [0, 1]\}$$

Note that the set of all the fuzzy sets over $U$ will be denoted by $F(U)$.

Example 2.2. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set. Let be a fuzzy set $\mu$ over $U$ can be represented as follows,

$$\mu = \{0.2/u_1, 0.5/u_2, 0.7/u_3, 0.9/u_4, 1.0/u_5\}$$
2.2 Soft Sets

**Definition 2.3.** [14] Consider a nonempty set $A$ such that $A \subseteq E$. A pair $(f, A)$ is called a soft set over $U$, where $f$ is a mapping given by

$$f : A \rightarrow \mathcal{P}(U)$$

In this paper, we will benefit following definition defined by Çağman [9] for basic set operations on soft sets.

**Definition 2.4.** [9] A soft set $f$ over $U$ is a set valued function from $E$ to $\mathcal{P}(U)$. It can be written a set of ordered pairs

$$f = \{(e, f(e)) : e \in E\}.$$ 

Note that if $f(e) = \emptyset$, then the element $(e, f(e))$ won’t be appeared in soft set $f$. Set of all soft sets over $U$ will be denoted by $\mathcal{S}(U)$.

**Example 2.5.** Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ be the universe containing eight houses and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the set of parameters. Here, $e_i$ ($i = 1, 2, 3, 4, 5, 6$) stand for the parameters “modern”, “with parking”, “expensive”, “cheap”, “large” and “near to city” respectively. Then, following soft sets are described by Mr. A and Mr. B who want to buy a house, respectively

$$f = \{(e_1, \{u_1, u_3, u_4\}), (e_2, \{u_1, u_4, u_7, u_8\}), (e_3, \{u_1, u_2, u_3, u_8\})\}$$

$$g = \{(e_2, \{u_1, u_3, u_6\}), (e_3, U), (e_5, \{u_2, u_4, u_6\})\}.$$ 

**Definition 2.6.** [9] Let $f, g \in \mathcal{S}(U)$. Then,

1. If $f(e) = \emptyset$ for all $e \in E$, $f$ is said to be a empty soft set, denoted by $\Phi$.
2. If $f(e) = U$ for all $e \in E$, $F$ is said to be universal soft set, denoted by $\hat{U}$.
3. $f$ is soft subset of $g$, denoted by $f \subseteq g$, if $f(e) \subseteq g(e)$ for all $e \in E$.
4. $f = g$, if $f \subseteq g$ and $g \subseteq f$.
5. Soft union of $f$ and $g$, denoted by $f \cup g$, is a soft set over $U$ and defined by $f \cup g : E \rightarrow \mathcal{P}(U)$ such that $(f \cup g)(e) = f(e) \cup g(e)$ for all $e \in E$.
6. Soft intersection of $f$ and $g$, denoted by $f \cap g$, is a soft set over $U$ and defined by $f \cap g : E \rightarrow \mathcal{P}(U)$ such that $(f \cap g)(e) = f(e) \cap g(e)$ for all $e \in E$.
7. Soft complement of $f$ is denoted by $f^c$ and defined by $f^c : E \rightarrow \mathcal{P}(U)$ such that $f^c(e) = U \setminus f(e)$ for all $e \in E$.

2.3 Fuzzy soft sets

**Definition 2.7.** [11] Let $U$ be an initial universe set, $X$ be a set of all parameters, $\mu$ be a fuzzy set over $U$ for every $x \in X$ and $F(U)$ denote the set of all fuzzy sets in $U$. Then, a fuzzy soft set $\gamma$ over $U$ is defined by a function $\gamma$ representing a mapping

$$\gamma : X \rightarrow F(U) \text{ such that } \gamma(x) = \emptyset \text{ if } x \notin X$$
Here, for every \( x \in X \), \( \gamma(x) \) is a fuzzy set over \( U \) and it is called fuzzy value set of parameter \( x \)-element of the \( fs \)-set. Thus, an \( fs \)-set \( \gamma \) over \( U \) can be represented by the set of ordered pairs

\[
\gamma = \{(x, \gamma(x)) : x \in X, \gamma(x) \in F(U)\}
\]

Note that from now on the sets of all \( fs \)-sets over \( U \) will be denoted by \( FS(U) \).

**Example 2.8.** Assume that \( U = \{u_1, u_2, u_3, u_4, u_5\} \) is a universal set and \( X = \{x_1, x_2, x_3\} \) is a set of all parameters. If \( \gamma(x_1) = \{0.5/u_2, 0.9/u_4\}, \gamma(x_2) = U, \gamma(x_3) = \emptyset \), then the \( fs \)-set \( \gamma \) is written by

\[
\gamma = \{(x_1, \{0.5/u_2, 0.9/u_4\}), (x_2, U)\}
\]

### 3 TOPSIS Method

TOPSIS method is a practical and useful technique for ranking and selection of a number of externally determined alternatives through distance measures. The operations within the TOPSIS process include: decision matrix normalization, distance measures, and aggregation operators [18]. For more detail of TOPSIS, we refer to the earlier studies [10, 19]. The TOPSIS process is carried out as follows.

Throughout this paper, \( I_n = \{1, 2, ..., n\} \) for all \( n \in \mathbb{N} \).

**Step 1.** Constructing of decision matrix \( D \).

\[
D = \begin{bmatrix}
A_1 & c_1 & c_2 & \cdots & c_n \\
A_2 & d_{11} & d_{12} & \cdots & d_{1n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_i & d_{i1} & d_{i2} & \cdots & d_{in} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & d_{m1} & d_{m2} & \cdots & d_{mn}
\end{bmatrix} = [d_{ij}]_{m \times n} \tag{1}
\]

Here \( A_i (i \in I_m) \) and \( c_j (j \in I_n) \) denote alternatives and criteria, respectively.

**Step 2.** Creating of standard (normalized) decision matrix \( R \).

\[
r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{k=1}^{m} d_{kj}^2}}, \quad \forall d_{ij} \neq 0 \text{ and } \forall i \in I_m, \forall j \in I_n \tag{2}
\]

\[
R = \begin{bmatrix}
r_{11} & r_{12} & \cdots & r_{1n} \\
r_{21} & r_{22} & \cdots & r_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
r_{m1} & r_{m2} & \cdots & r_{mn}
\end{bmatrix} = [r_{ij}]_{m \times n}
\]
Step 3. Creating the weighted normalized decision matrix $V$.

$$ V = [v_{ij}]_{m \times n} = [w_j r_{ij}]_{m \times n}, i \in I_n, $$

where $w_j = W_j/\sum_{j=1}^{n} W_j, j = 1, 2, ..., n$ so that $\sum_{j=1}^{n} w_j = 1$, and $W_j$ is the original weight given to the criteria $c_j, j \in I_n$.

$$ V = \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{pmatrix} = [v_{ij}]_{m \times n} $$

Step 4. Determining of positive ideal solution $A^+$ (PIS) and negative ideal solution $A^-$ (NIS).

$$ A^+ = \{v^+_1, \ldots, v^+_j, \ldots, v^+_n\} = \{\max_i v_{ij} \mid j \in J_1\}, \{\min_i v_{ij} \mid j \in J_2\}, i \in I_m $$

$$ A^- = \{v^-_1, \ldots, v^-_j, \ldots, v^-_n\} = \{\min_i v_{ij} \mid j \in J_1\}, \{\max_i v_{ij} \mid j \in J_2\}, i \in I_m $$

where $J_1$ and $J_2$ are associated with the benefit and cost attribute sets, respectively.

Step 5. Calculating of separation measurements of positive ideal ($S_i^+$) and the negative ideal ($S_i^-$) solutions.

$$ S_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^+_j)^2}, \quad \forall i \in I_m $$

and

$$ S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^-_j)^2}, \quad \forall i \in I_m $$

Step 6. Calculating of relative closeness of alternatives to the ideal solution

$$ C_i^+ = \frac{S_i^+}{(S_i^+ + S_i^-)}, \quad 0 \leq C_i^+ \leq 1, \quad \forall i \in I_m $$

Step 7. Ranking the preference order.

4 TOPSIS Method for group decision making with fuzzy soft information

In this section, we propose a new method by extending TOPSIS method to fuzzy soft environment. The main procedure of this method is presented with the following steps:
**Table 1: Linguistic terms for evaluation of parameters.**

<table>
<thead>
<tr>
<th>Linguistic Terms</th>
<th>FVs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Good / Very Important (VG/VI)</td>
<td>0.95</td>
</tr>
<tr>
<td>Good / Important (G/I)</td>
<td>0.85</td>
</tr>
<tr>
<td>Fair / Medium (F/M)</td>
<td>0.50</td>
</tr>
<tr>
<td>Bad / Unimportant (B/UI)</td>
<td>0.35</td>
</tr>
<tr>
<td>Very Bad / Very Unimportant (VB/VUI)</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Step 1. Defining of problem.**

Let us assume that $DM = \{D_p, p \in I_n\}$ is set of decision makers, $U = \{u_i, i \in I_m\}$ denotes set of alternatives and $X = \{x_j, j \in I_n\}$ is a set of all parameters (criterion). Then, a fuzzy soft set $f$ over $U$ is a function defined by

$$
\gamma : X \rightarrow F(U)
$$

**Step 2. Constructing of weighed fuzzy parameter matrix $D$ with choosing linguistic rating from Table 1.**

$$
D = \begin{pmatrix}
D_1 & x_1 & x_2 & \cdots & x_n \\
D_2 & d_{11} & d_{12} & \cdots & d_{1n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
D_i & d_{i1} & d_{i2} & \cdots & d_{in} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
D_m & d_{m1} & d_{m2} & \cdots & d_{mn} \\
\end{pmatrix} = [d_{ij}]_{m \times n}
$$

(8)

here $d_{ij}$ is linguistic rating assigned by decision maker $D_i$ the parameter $x_j$.

**Step 3. Constructing of weighted normalized fuzzy parameter matrix $R$ and forming weighed vector $W = (W_1, W_2, \ldots, W_n)$.**

The weighted normalized elements of weighted normalized fuzzy parameter matrix $R$ are calculated by using Eq (2) and weighed vector $W = (W_1, W_2, \ldots, W_n)$ is formed with aid of the formula

$$
W_j = \frac{w_j}{\sum_{k=1}^{m} w_k}, \quad w_j = \frac{1}{m} \sum_{i=1}^{m} r_{ij}
$$

(9)

$$
R = \begin{pmatrix}
r_{11} & r_{12} & \cdots & r_{1n} \\
r_{21} & r_{22} & \cdots & r_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
r_{m1} & r_{m2} & \cdots & r_{mn} \\
\end{pmatrix} = [r_{ij}]_{m \times n}
$$
Step 4. Constructing fuzzy decision matrices $D_k$ for each decision makers and building of fuzzy average decision matrix $V$

Fuzzy decision matrices $D_k$ are constructed similar way to classical TOPSIS Method (Step 2) and fuzzy average decision matrix $V$ is constructed by using Eq (10).

$$D_k = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ u_1 & d_{i1} & d_{i2} & \cdots & d_{in} \\ \vdots & \vdots & \ddots & \vdots \\ u_i & d_{m1} & d_{m2} & \cdots & d_{mn} \end{pmatrix} = [d^k_{ij}]_{m \times n}$$

where $d^k_{ij} = \gamma X_k (x_j(u_i))$.

$$V = \frac{1}{n} (D_1 \oplus D_2 \oplus \ldots \oplus D_n) = [v_{ij}]_{m \times n}$$

where $\oplus$ indicates sum of matrices

Step 5. Constructing of weighed fuzzy decision matrix $\mathcal{V}$.

$$\mathcal{V} = \begin{pmatrix} \hat{v}_{11} & \hat{v}_{12} & \cdots & \hat{v}_{1n} \\ \hat{v}_{21} & \hat{v}_{22} & \cdots & \hat{v}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{v}_{m1} & \hat{v}_{m2} & \cdots & \hat{v}_{mn} \end{pmatrix}$$

here

$$\hat{v}_{ij} = W_j \cdot v_{ij}$$

Step 6. Finding of fuzzy valued positive ideal solution (FV-PIS) and fuzzy valued negative-ideal solution (FV-NIS).

In the classic TOPSIS method, criteria are evaluated from aspect of benefit and cost. Assume that $J_1$ be a set of benefit criteria and $J_2$ be a set of cost criteria. Based on fuzzy set theory and principle of TOPSIS method, FV-PIS and FV-NIS can be found as follow respectively;

$$FV - PIS = \{ \hat{v}_{i1}^+, \hat{v}_{i2}^+, \ldots, \hat{v}_{ij}^+, \ldots, \hat{v}_{in}^+ \} = \{(\max_{i} \hat{v}_{ij} | j \in J_1), (\min_{i} \hat{v}_{ij} | j \in J_2), i \in I_m \}$$

$$FV - NIS = \{ \hat{v}_{i1}^-, \hat{v}_{i2}^-, \ldots, \hat{v}_{ij}^-, \ldots, \hat{v}_{in}^- \} = \{(\min_{i} \hat{v}_{ij} | j \in J_1), (\max_{i} \hat{v}_{ij} | j \in J_2), i \in I_m \}$$

Step 7. Calculating of the separation measurement for each parameter.

Separation measurements ($S^+_i$) and ($S^-_i$) are found by using Eq(5) and Eq(6).
Step 8. Calculating of the relative closeness of alternative to the ideal solution.

Relative closeness of alternatives to the ideal solution are calculating by using Eq(7)

\[ C_i^+ = \frac{S_i^+}{(S_i^+ + S_i^-)} , \quad 0 \leq C_i^+ \leq 1 , \quad \forall i \in I_m \]

Step 9. Ranking the preference order.

5 Application

In this section, we have presented an application for a group decision making method by using TOPSIS on fuzzy soft set theory. Now, by using the algorithm of this new group decision making method we can solve the following example (problem) step by step as follows:

Step 1. Defining the problem.
Assume that a real estate agent has a set of different types of houses \( U = \{ u_1, u_2, u_3 \} \) which may be characterized by a set of all parameters \( X = \{ x_1, x_2, x_3 \} \). For \( j = 1, 2, 3 \) the parameters \( x_j \) stand for “cheap”, “modern”, “large”, respectively. Then we can give the following examples.

Suppose that three decision-makers come to the real estate agent to buy a house. Firstly, each decision-maker has to consider their own set of parameters. Then, they can construct their fuzzy soft sets. Next, by using TOPSIS on fuzzy soft set theory decision making method we select a house on the basis for the sets of decision-makers parameters. Assume that decision-makers \( D_1, D_2 \) and \( D_3 \) construct fuzzy soft sets, respectively as follows;

\[
\gamma_X^{(1)} = \{(x_1, \{0.5/u_1, 0.2/u_2, 0.5/u_3\}), (x_2, \{0.2/u_1, 0.6/u_2, 0.1/u_3\}), (x_3, \{0.3/u_1, 0.7/u_2, 0.2/u_3\})\}
\]

\[
\gamma_X^{(2)} = \{(x_1, \{0.1/u_1, 0.6/u_2, 0.8/u_3\}), (x_2, \{0.4/u_1, 0.9/u_2, 0.2/u_3\}), (x_3, \{0.2/u_1, 0.3/u_2, 0.7/u_3\})\}
\]

\[
\gamma_X^{(3)} = \{(x_1, \{0.3/u_1, 0.2/u_2, 0.7/u_3\}), (x_2, \{0.1/u_1, 0.5/u_2, 0.6/u_3\}), (x_3, \{0.6/u_1, 0.1/u_2, 0.1/u_3\})\}
\]

Step 2. Weighed fuzzy parameter matrix \( D \) is as follow

\[
D = \begin{pmatrix}
D_1 & D_2 & D_3 \\
0.95 & 0.35 & 0.10 \\
0.50 & 0.10 & 0.10 \\
0.10 & 0.50 & 0.85 \\
\end{pmatrix} = [d_{ij}]_{3 \times 3}
\]

Step 3. Weighted normalized fuzzy parameter matrix can be obtained as follow

\[
R = \begin{pmatrix}
0.88 & 0.57 & 0.12 \\
0.46 & 0.16 & 0.12 \\
0.09 & 0.81 & 0.99 \\
\end{pmatrix}
\]

And weighed vector \( W \) can be obtained using by Eq (9), as follow

\[
W = (0.34, 0.37, 0.29)
\]
Step 4. Fuzzy decision matrices can be constructed by decision makers as follows;

\[
D_1 = \begin{pmatrix} 0.50 & 0.20 & 0.30 \\ 0.20 & 0.60 & 0.70 \\ 0.50 & 0.10 & 0.20 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 0.10 & 0.40 & 0.20 \\ 0.60 & 0.90 & 0.30 \\ 0.80 & 0.20 & 7.00 \end{pmatrix}, \quad D_3 = \begin{pmatrix} 0.30 & 0.10 & 0.60 \\ 0.20 & 0.50 & 0.10 \\ 0.70 & 0.60 & 0.10 \end{pmatrix}
\]

and from Eq (10) fuzzy average decision matrix is

\[
V = \begin{pmatrix} 0.30 & 0.23 & 0.37 \\ 0.33 & 0.67 & 0.37 \\ 0.67 & 0.30 & 2.43 \end{pmatrix}
\]

Step 5. Weighed fuzzy decision matrix \( V \) is constructed with aid of Eq (11) as follow,

\[
V = \begin{pmatrix} 0.10 & 0.09 & 0.11 \\ 0.11 & 0.24 & 0.11 \\ 0.23 & 0.11 & 0.71 \end{pmatrix}
\]

Step 6. Positive ideal solution (FV-PIS) and fuzzy valued negative-ideal solution (FV-NIS) can be obtained using the Eq (12) and (13) as follow

\[
v_+^1 = 0.23, \quad v_+^2 = 0.24, \quad v_+^3 = 0.71
\]

\[
v_1^- = 0.10, \quad v_2^- = 0.09, \quad v_3^- = 0.11
\]

Step 7. From Eq(5) and Eq(6), \( S_i^+ \) and \( S_i^- \), for \( i \in \{1, 2, 3\} \), we have

\[
S_1^+ = 0.63, \quad S_1^- = 0.61
\]

\[
S_2^+ = 0.61, \quad S_2^- = 0.16
\]

\[
S_3^+ = 0.13, \quad S_3^- = 0.61
\]

Step 8. Relative closeness of alternatives to the ideal solution as follows

\[
C_1^+ = 0.51
\]

\[
C_2^+ = 0.79
\]

\[
C_3^+ = 0.18
\]

Step 9. Ranking the preference order is \( u_3 < u_1 < u_2 \).

6 Conclusion

In this paper, we have presented a group decision making method by using TOPSIS under fuzzy soft environment. Finally, we provided an example that demonstrated that this method can be successfully worked. It can be applied to decision making problems of many fields that contain uncertainty. However, the approach should be more comprehensive in the future to solve the related problems and a large number of examples could be recommended for test in future studies.
References


