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ROBUST DESIGN OF GEARS WITH MATERIAL AND LOAD UNCERTAINTIES

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ABSTRACT

Traditionally gears are designed using design standards such as AGMA, ISO, etc. These design standards include a large number of "design factors" accounting for various uncertainties related to geometry, load and material uncertainties. As the knowledge about these uncertainties increases, it becomes possible to include them systematically in the gear design procedure, thereby reducing the number of empirical design factors. In this paper a method is proposed to eliminate two design factors (viz., factor of safety in contact and reliability factor) used in standard AGMA-based design procedures through the formal introduction of uncertainty in the magnitude of load and material properties. The proposed method is illustrated via the design of an automotive gear with a desired reliability, cost, and robustness. The solutions obtained are encouraging and in-line with the existing knowledge about gear design, and thus reinforces the possibility of schematically reducing the aforementioned design factors.

Keywords: Robust Design, Uncertainty, Design Factors, Reliability.

NOMENCLATURE

b	=	face width, mm
C	=	cost, INR
d	=	center distance, mm
d_1^+, d_2^+, d_3^+	=	deviation variables to account for over-achievement
d_1^-, d_2^-, d_3^-	=	deviation variables to account for under-achievement
G	=	gear ratio
K	=	design capability index
K_H	=	load distribution factor
K_O	=	overload factor
K_R	=	reliability factor
K_S	=	size factor
K_V	=	dynamic load factor
m	=	module of gear, mm
N	=	number of teeth
R	=	reliability, %
R_c	=	contact ratio
r_{HPSTC}	=	radius of highest point in single tooth contact
ρ	=	density, Kg/m^3
S_U	=	tensile strength
$S_{U_{MIN}}$	=	minimum tensile strength
$S_{U_{MAX}}$	=	maximum tensile strength
S_c	=	factor of safety in contact
$\sigma_{allowable}^{bending}$	=	allowable bending stresses
$\sigma_{induced}^{bending}$	=	induced bending stresses
$\sigma_{allowable}^{contact}$	=	allowable contact stresses
$\sigma_{induced}^{contact}$	=	induced contact stresses
σ_{S_U}	=	standard deviation in tensile strength

σ_{S_T}	=	standard deviation in torque
T	=	torque, Nm
W_1, W_2, W_3	=	weights associated with the design goals
W^t	=	tangential load, N
Y_θ	=	temperature factor
Z	=	deviation function
Z_E	=	elastic coefficient
Z_I	=	geometry factor
Z_N	=	stress cycle factor
Z_R	=	surface condition factor for pitting
Z_W	=	factor for hardness ratio

1 DESIGN OF GEARS

1.1 Traditional Gear Design Procedure and Design Factors

In current industrial practice gear design is driven by the guidelines from AGMA [1], ISO [2], etc. These design procedures are similar with minor variations and contain a number of empirical design factors obtained from experiments and experience. These factors account for various uncertainties and idealizations related to operating conditions and load, material and manufacturing processes, design parameters, precision and quality requirements, etc. A list of parameters used for steel spur gear design is given in Table 1 along with their dependence on various uncertainties related to geometrical design, material, manufacturing processing and operating conditions. In the table, a tick mark represents the consideration of given class of uncertainty by the factor.

Table 1: List of Design Factors and Their Dependence

Factor	Material	Manuf.	Oper. Cond	Design
Stress factor for bending fatigue	✔	✔		✔
Stress factor for contact fatigue	✔	✔	✔	✔
Factor for temperature effects	✔		✔	
Factor for reliability	✔			
Factor for overload			✔	
Factor for dynamic load		✔	✔	
Factor for size effects	✔			✔
Factor for load distribution		✔	✔	✔
Factor for hardness ratio	✔	✔		
Factor for geometry				✔
Factor for surface condition for pitting	✔	✔		
Factor for rim effects				✔
Elasticity constant	✔			
Factor of safety in bending	✔			✔
Factor of safety in contact	✔			✔

Factor of safety in contact and the reliability factor are the two key factors with significant influence on the design outcome and depend on the load conditions and material properties. These factors, in essence, account for uncertainty in load and material properties. AGMA in its standards states the following: "*as design practices become more comprehensive, some influence factors have been removed from the unknown area of the "safety factor" and are introduced as predictable portions of the design method. The reliability factor, K_R, is an example*" [1]. Hewitt [3] discusses selection of gear materials and estimation of reliability. He considers a statistical distribution of factor of safety having some part of the range less than unity and states that though purists may baulk at the concept of a safety factor less than unity, the procedure will be of engineering utility. Accordingly, our focus in this paper is on systematically accounting for variability in load (coming out of deviations from the rated load) and material properties (deviations from the nominal properties) to eliminate the dependence on these two factors in the AGMA design procedure, and yet lead to a robust design.

1.2 Gear Design for Reliability and Robustness

Reliability based design of gears has received some attention in the literature. Stoker and co-authors [4] have studied the impact of errors in gear geometry on AGMA equations (analytical) and compared them with FEM results. Houser [5] reported on the effect of accuracy requirements on the calculation of stresses using statistical design of experiments and Monte Carlo simulation techniques to quantify the effects of different manufacturing and assembly errors on root and contact stresses. Reliability is one of the empirical design factors used, which considers the statistical nature of failure of materials.

In recent years, considerable attention has been given to reliability-based design of gears. Zhang et al. [6] used perturbations and reliability-based design theory and employed them for practical and effective method for the design of gear pairs. Aziz and Chassapis [7] explored application of the Stress-Strength Interference (SSI) theory within the context of a "Design for Reliability" for detailed gear design through evaluation the tooth-root strength with FEM-based verification.

Sensitivity analysis for reliability is discussed in References [8-13]. Yang and co-authors [8] studied reliability sensitivity based on reliability design theory, the Edgeworth series method, and the sensitivity analysis method. They discussed the reliability sensitivity of the cylindrical gear pairs with non-Gaussian random parameters and presented a numerical method for reliability sensitivity based design. Sun and co-authors [9] used limit state theory and studied the sensitivity of gear reliability with the change of various random factors. A procedure is developed to find sensible random factors that have larger influence on gear reliability for the convenience of strict control of gear design, manufacturing, use and future maintenance in order to meet the requirement for gear reliability. Zhang and co-authors [10] proposed a method to calculate the reliability and the reliability sensitivity of gear pairs. Zhang and Cui [11] worked on reliability design, a Kriging approximation model and an optimization method to build an optimization model, which can be used with the objective of minimizing the volume and cost of the gears while meeting the design criteria including reliability.

Robust design of gears has received less attention. MacAldener [12] has proposed methods for robust design of gears addressing slender teeth gears. Kulkarni, and co-authors [13] have used the compromise Decision Support Problem construct for robust design of gears and have explored the design and material space for gear design.

2 ROBUST DESIGN UNDER UNCERTAINTY

We recognize two approaches to robust design, namely, one anchored in optimization theory and the other anchored in the notion of *satisficing* and the compromise Decision Support Problem. Each is described in turn.

Papers by Bryne [14] and Taguchi and co-authors [15] represent first efforts at creating robust designs. They introduce a method to minimize the effects of uncontrollable parameters during design. Ross, and co-authors [16] used the Taguchi loss function to make a design more tolerable to the model variations. Other researchers [17-20] use optimization to minimize the variation of input parameters to obtain designs with lower sensitivity of performance to design parameters. They propose a robust design optimization with Taguchi loss function as an objective function subject to the model constraints. Implementing this, the constant and variable sensitivity from controllable and uncontrollable parameters are respectively minimized using nonlinear programming. Padulo [21] discusses two approaches for robust optimization in which design parameters are stochastic.

Robust design involves achieving the system performance while minimizing the sensitivity of performance objectives with respect to the design variables. Achieving robustness in the presence of uncertainty may lead to designs that differ substantially from those anchored in traditional optimization that inherently does not consider uncertainty. The objective is to achieve ‘satisficing’ solutions that provide good performance despite the presence of uncertainty, as opposed to solutions that are optimum in a narrow range of conditions but perform poorly when the conditions change slightly.

Mistree and co-authors have proposed the compromise Decision Support Problem (cDSP) construct for robust design with multiple goals [22-24]. The cDSP is a hybrid formulation based on mathematical programming and goal programming concepts. It enables the construction of different practical scenarios in a multi-objective formulation. By giving appropriate weight to different goals, the compromises among them can be explored. The cDSP minimizes the difference, d_i between the desired (the target G_i) and the achieved ($A_i(x)$), value of a goal. The difference between these values is the deviation value, which represents overachievement, d_i^+ , or underachievement d_i^- of each goal. The details of the cDSP can be found in References [22 - 24].

The cDSP construct has been successfully used to perform robust design by simultaneously maximizing the expected performance and minimizing the deviation from the expected performance for various engineering systems. Within a cDSP, these two objectives are often treated as two separate goals that are traded against one another. The cDSP construct allows a designer to specify different levels of robustness and helps in finding Pareto families of solutions by changing the weights for performance and robustness goals. In the following section, we discuss the cDSP based robust design formulation for gears.

3 ROBUST GEAR DESIGN WITH cDSP – FORMULATION

3.1 AGMA Based Design and Assumptions

During a preliminary analysis, it was observed that minimizing the contact factor of safety is more critical than the bending factor of safety, as it generally tends to be higher. Hence we concentrate in this paper in dealing with the factor of safety in the contact region. This is seen to be a general overriding factor and would also result in a higher factor of safety in bending. As in Reference [1], the induced stress in the contact is given by:

$$\sigma_{induced}^{contact} = Z_E \sqrt{W^t K_O K_V K_S \frac{K_H}{mNb} \frac{Z_R}{Z_I}} \qquad (1)$$

The methods for computing various factors, required in Eq. (1), can be found in References [1] and [25]. The tangential load W^t is given by:

$$W^t = T / r_{HPSTC} \qquad (2)$$

The design parameters are selected such that:

$$\sigma_{induced}^{contact} \leq \sigma_{allowable}^{contact} \qquad (3)$$

where the allowable contact stress is given by:

$$\sigma_{allowable}^{contact} = \frac{0.5 S_U}{S_C} \frac{Z_N Z_W}{Y_\theta K_R} \qquad (4)$$

In the above expression, the reliability factor K_R “*accounts for the effect of the normal statistical distribution of failures found in materials testing*” [1]. The factor S_C is “*an additional safety factor [that] should be considered to allow for safety and economic risk considerations along with other* *unquantifiable aspects of the specific design and application (variations in manufacturing, analysis etc.)*" [1].

Our aim in this study is to eliminate the need for the factors, i.e., reliability factor K_R, and factor of safety in contact S_C. As per [25], it is the resultant contact factor of safety which governs the design of a gear, i.e., if gear design is safe in contact induced stresses it is implied that it will be safe in bending induced stresses. In view of this, though the proposed method can easily be extended to eliminate the factor of safety in bending, in this paper we focus only on the contact factor of safety, effectively dealing with two parameters.

3.2 Uncertainty in Load and Material Properties

AGMA provides an expression for K_R as a function of desired reliability and recommends methods for the selection of factor of safety. In this work we set $K_R = 1$ and express factor of safety as below by equating the induced stress (Eq. (1)) with the allowable stress (Eq. (4)) and used this in constraints and goals. Through this, we have eliminated the use of two "design factors" from AGMA.

$$S_C = \frac{0.5 S_U Z_N Z_W}{Y_\theta Z_E} \sqrt{\frac{mNbZ_I}{W^t K_O K_V K_S K_H Z_R}} \quad (5)$$

In order to account for uncertainty in material properties and load-related economic risk, we assume that S_U and T have random Gaussian distributions with standard deviations σ_{S_U} and σ_T, respectively. Accounting for uncertainty in material properties would eliminate the need for the reliability factor K_R discussed in Section 3.1; and accounting for the uncertainty in the load would effectively help set the factors of safety in contact to unity, effectively eliminating the need for these two design factors that are otherwise based on AGMA guidelines or experience.

The standard deviation of S_C from Eq. (5) can be written as:

$$\sigma_{S_C} = \sqrt{\left(\frac{\partial S_C}{\partial S_U}\right)^2 \left(\sigma_{S_U}\right)^2 + \left(\frac{\partial S_C}{\partial T}\right)^2 \left(\sigma_{T_U}\right)^2} \quad (6)$$

This can be reduced to:

$$\sigma_{S_C} = \frac{C_1}{\sqrt{T}} \sqrt{\left(\sigma_{S_U}\right)^2 + \frac{S_U^2 \sigma_T^2}{T^2}} \quad (7)$$

Where C_1 is constant, which is derived from Eq. (2), (5) and (6).

Under ideal conditions, when there is no uncertainty in load and material properties, a value of S_C marginally more than one should suffice. A factor of safety less than or equal to unity would lead to failure.

Based on the description of the concepts of stochastic analysis of induced stress and allowable material strength given in [25], the allowable material strength can be described by a Gaussian distribution and the induced contact stress (which is proportional to $\sqrt{W^t}$), can be described by Chi distribution. Factor of safety, which is defined as the ratio of allowable material strength to induced stress, will have a different distribution. In order to simplify further analysis, we have assumed that the factor of safety S_C follows Gaussian distribution. This is in general, in line with standard practices in design. Figure 1 shows the schematic of distribution for factor of safety.

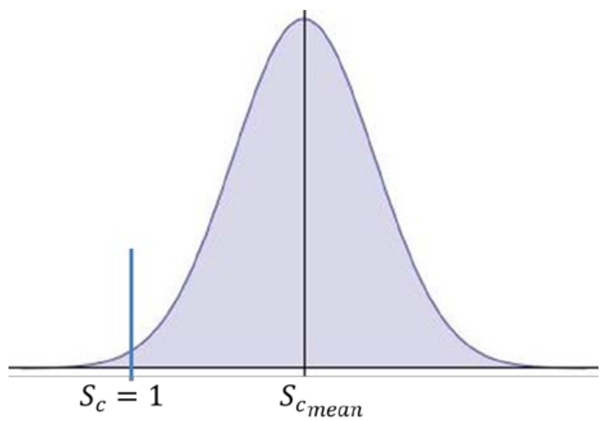

Figure 1: Schematic of Probability Density Function for Contact Factor of Safety Distribution (*P(S$_c$)* ⩾1)

Design capability index, *K* [24], is based on the concepts of process capability index from statistical process control and provide an approach for robust design. It is used to determine if a range of design solutions is capable of satisfying given ranged set of requirements. It is possible to define reliability [3] in terms of design capability index *K* as:

$$K = \frac{S_{c_{mean}} - 1}{\sigma_{S_C}} \quad (8)$$

In this equation the value of desired factor of safety is 1.0, and our objective is to bring the mean of achieved factor of safety (the variation in factor of safety is due to load and material uncertainties; other types of uncertainties are ignored at this stage and the corresponding design factors are used) closer to the desired factor of safety. Maximizing this value of *K* will give the value corresponding to maximum reliability. *K* is related to the reliability as given in Table 2.

Table 2: Reliability as a Function of Design Capability Index

K	Reliability (%)
1.65	95.00
2.00	97.73
2.50	99.38
3.00	99.87
3.50	99.97
3.72	99.99

3.3 The cDSP Formulation

In the current work, we are interested in the design of a pinion for first gear reduction for a compact sized automobile. The problem statement is as follows:

"Design a robust, highly reliable and low cost pinion of a commercial spur gear system of AGMA precision no. 8 to carry a torque of 113 Nm @ 4500 rpm with gear ratio of 3.5 having a target for reliability of 99.99% but not less than 95%. The gear teeth have a pressure angle of 20° and they are cut using rack cutter. The expected fatigue life is of 10^9 *cycles. The materials for gear are available in the range of tensile strength of 800 MPa to 1600 MPa. Standard deviations in torque T and ultimate tensile strength* S_U *are* σ_T *= 30 Nm and* σ_{S_U} *= 50 MPa respectively. In this gear design study, the factor of safety should be taken as unity and the AGMA reliability factor (which represents the statistical nature of failure of materials) should be considered based on the given probability distribution of material strength and load"*

Following the example in Reference [22], the cDSP for robust design of gear is formulated as given below.

Given

The spur gear pinion is required to be designed based on AGMA [1] standards based on *20°* standard pressure angle system. The gear is required to carry 113 Nm torque (with specified standard deviation in torque σ_T = 30 Nm). The required gear ratio, *G*, is 3.5. The geometry of a gear is defined by *m*, *N* and *b*. Additionally, the material required for a gear is defined by S_U (with specified standard deviation in material strength σ_{S_U} *= 50 MPa*). Hence there are four design variables for the design problem. There are three design goals (i.e., maximization of reliability, maximization of robustness and minimization of cost). There are six deviation variables, one each to take care of over-achievement and under-achievement for each design goal. The design space for the gear is bounded by eight constraints, which are described in details below.

Assumptions: The tensile strength of the available material, as well as the torque to be transmitted by the gear follow Gaussian distribution. The factor of safety is assumed to follow a Gaussian distribution and the reliability is determined based on this assumption. The modules for the gear are available in discreet set due to tooling limitations, however for simplicity, it is assumed to be a continuous variable.

Find

Find the design variables that meet the design requirements.

- module (*m*) in mm,
- number of teeth (*N*),
- face width (*b*) in mm. We are using these geometry variables along with
- material strength (S_U) in MPa

The description of module, number of teeth and face width can be found in Reference [24].

Find the deviation variables, d_i^+, d_i^-, such that $d_i^+ . d_i^- = 0$ and $d_i^+,\ d_i^- \geq 0$ where *i* = 1 to 3

Satisfy

Bounds on design variables:

- B1: $4 \leq m \leq 8$ (mm)
- B2: $18 \leq N \leq 40$
- B3: $40 \leq b \leq 80$ (mm)
- B4: $800 \leq S_U \leq 1600$ (MPa)
- B5 – B10: deviation variables, d_i^+, d_i^-, such that $d_i^+ . d_i^- = 0$ and $d_i^+,\ d_i^- \geq 0$ where *i* = 1 to 3

Design constraints:

- C1: Minimum face-width*: $b \geq 3\pi m$
- C2: Maximum face-width*: $b \leq 5\pi m$
- C3: Maximum limit on center distance*: s $d = m(1+G)N/2 \leq 300$ mm
- C4: Bending stress induced: $\sigma_{allowable}^{bending} - \sigma_{induced}^{bending} \geq 0$; Taken from Reference [25] with the factor of safety and reliability factor set to 1.
- C5: Contact stress induced: $\sigma_{allowable}^{contact} - \sigma_{induced}^{contact} \geq 0$; from equations (1) and (4)
- C6: Minimum contact ratio*: $R_c \geq 1.4$
- C7: Maximum contact ratio*: $R_c \leq 1.8$
- C8: Minimum reliability: $K \geq 1.65$; (corresponding to minimum permissible reliability of 95%; see Table 2).

* Based on references [1, 25].

Goals:

- G1: Maximize the design capability index *K* (which corresponds to maximizing reliability defined in terms of *K*), given in Eq. (8), with a target value of 3.72

$$\frac{K}{3.72} + d_1^- - d_1^+ = 1$$

- G2: Minimize σ_{S_c} of Eq. (7) (with a target value of 0.1 Maximize robustness)

$$\frac{0.1}{\sigma_{S_c}} - d_2^- + d_2^+ = 1$$

- G3: Minimize cost C with target cost is INR 57

$$\frac{57}{C} - d_3^- + d_3^+ = 1$$

Total cost consists of material cost and manufacturing cost. The expression for cost is given by Eq. (9), where the first term in the square brackets accounts for the manufacturing cost per unit weight, the second term accounts for the material cost per unit weight and ρ is the density, taken as 7800kg/m^3 for this study.

$$C = \left[5 + 40\left\{ 1 + \left(\frac{S_U - S_{U_{MIN}}}{S_{U_{MAX}} - S_{U_{MIN}}} \right)^{1.5} \right\} \right] \pi \rho b \left(\frac{mN}{2} \right)^2 \tag{9}$$

Minimize

In the cDSP the deviation function is to be minimized. The deviation function is constructed as shown below, where the system goals and constraints are normalized as per Reference [22].

$$Z = \{(d_1^- + d_1^+), (d_2^- + d_2^+), (d_3^- + d_3^+)\} \tag{10}$$

As we are minimizing for multiple goals (maximization of reliability (defined as maximization of corresponding design capability index (K)) is considered as minimization of (1-K)), the negative deviation is always 0 and Eq. (10) simplifies as

$$Z = \{d_1^+, d_2^+, d_3^+\} \tag{11}$$

An Archimedean approach [22] is used to construct the deviation function and the cDSP is solved using the DSIDES[1] software.

$$Z(d^-, d^+) = \sum W_i^+ d_i^+ \quad i = 1.......3 \tag{12}$$

$$\sum_{i=1}^{3} W_i = 1 \ \text{ and } \ W_i \geq 0 \ \text{ for all } i \tag{13}$$

4 RESULTS & DISCUSSION

The gear design problem described in the earlier section is formulated as a compromise DSP and is solved under different scenarios of weights for goals and the outcome is discussed in this section. Further, a comparison with traditional gear design is made to show the effectiveness of the method in dealing without the three design factors of AGMA.

4.1 Design Scenarios

In order to explore the design space and evaluate the proposed method, various scenarios of target goals are studied by assigning different weights to the goals as shown schematically in Figure 2. These goals are generally conflicting and compromise between these is to be explored for making design decisions. This study has primarily looked at seven scenarios as given Figure 2. The corresponding weights are given in Table 3.

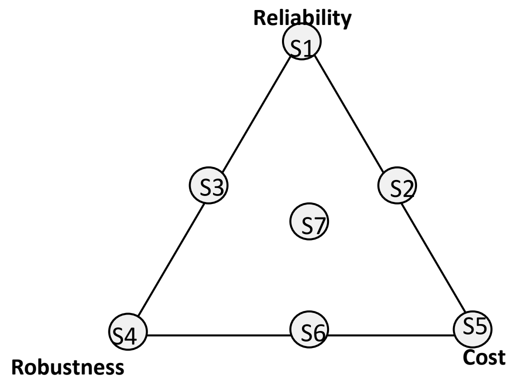

Figure 2: Scenarios Analyzed for Goals.

4.2 Discussion of Scenarios

Design parameters and goal achievement for different scenarios as discussed above are evaluated using the cDSP construct. Goal achievement for different scenarios is shown in Figure 3 below. The goal achievement values are normalized representing a value of 1 correspond to achieving lowest cost, highest reliability and highest robustness (low σ_{S_c}).

Scenarios S1, S4 and S5 have unique goals and the outcome represents the same. It can be seen here that for S4 and S5 cases, where robustness has to be maximized or cost has to be minimized without targets on reliability, the reliability has hit the lower bound. When reliability is the only target, the cost and robustness goal achievement is lower. This is in line with expectations.

Table 3: Weights of Goals for the Scenarios which are Considered

Scenario	Reliability	Robustness	Cost
S1	1	0	0
S2	0.5	0	0.5
S3	0.5	0.5	0
S4	0	1	0
S5	0	0	1
S6	0.5	0.5	0.5
S7	0.3333	0.3333	0.3333

In Figure 3, it can be seen that Scenario S4, S5 and S6 converge essentially to the same goal achievement. This is due to the solution reaching the lower limit of reliability (used as constraint) allowing for satisfaction of robustness and cost goals together.

Scenarios S2 and S3 represent compromise situations between reliability and robustness, and reliability and cost respectively. In both these cases, the reliability achieved is reasonably high,

[1] Mistree F., "DSIDES- Decision Support in the Design of Engineering Systems", User manual.

indicating its dominance. However, of these two, in Scenario S2, the robustness and cost goals are better achieved as compared to S3. However this is at a cost of lower reliability. In Scenario S6, where reliability is not a goal, its achievement is the same solution as in S4 and S6 and this is because the solution hits the reliability constraint. In Scenario S7, where all three goals are weighted equally, target reliability is achieved and the solution also performs well on the other two goals. We can see that Scenarios S2 and S7 lead to the best solutions as reliability values are close to the maximum value and both cost and variation in factor of safety are lower. Of these two, Scenario S2 represents a situation, which has minimum variation in factor of safety, comparatively higher reliability and lower cost. If higher reliability is desired, S7 would be the best scenario.

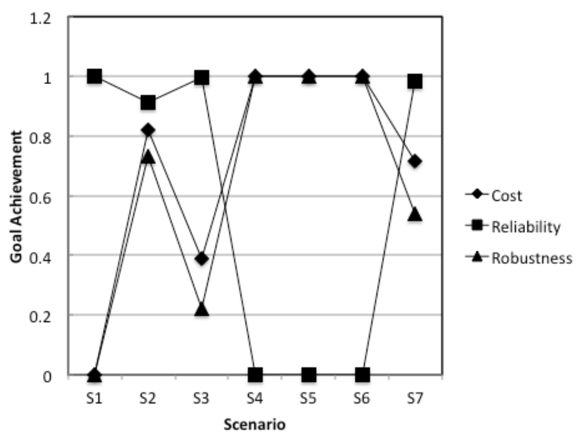

Figure 3: Goal achievement for Different Scenarios.

The minimum value for σ_{S_c} achieved during these studies is 0.183 while the target is 0.10, indicating that this target value was practically unachievable with a mean factor of safety greater than 1 (satisfying constrains C2 and C3) and simultaneously keeping the reliability greater than 95% (constraint C8). The lowest mean factor of safety observed is as low as 1.3. This is in contrast to the recommended factor of safety by AGMA of 2. Though it is not clear up to what extent AGMA considered uncertainty in parameters, it can now be easily quantified and taken into account while designing. The current design is seen to be robust as we try to minimize the standard deviation of the factor of safety.

4.3 Comparison with AGMA Based Design

Scenario S2 and S7 are considered for validation with AGMA based designs accounting for the values of reliability factor and factor of safety as recommended. The reliability factor, as recommended by AGMA is given by Eq. (15). The reliability factor was taken as per the value obtained in the earlier studies and the factor of safety as 2. A design following the AGMA prescribed methods was followed for the same problem and the design parameters were compared. The outcome in terms of design parameters was found to be reasonably close to the proposed method as seen from Tables 4 and 5.

$$K_R = 0.658 - 0.0759 \ln 1 - R \qquad \text{for } 0.5 < R < 0.99$$
$$K_R = 0.5 - 0.109 \ln 1 - R \qquad \text{for } 0.99 < R < 0.9999 \qquad (15)$$

Our approach resulted in gear designs, which are compact (i.e., fewer teeth and face width) and of lower cost. The required strength of the material is marginally higher for the design obtained using the proposed method in case of Scenario S2; however the net impact on the cost is beneficial.

It is noted that the comparison of the proposed method with AGMA depends on the values of the standard deviation for torque and ultimate tensile strength of the materials. Here, the authors have taken these from their past experience and they have no details of ranges considered by AGMA while deriving these factors. The good agreement with the outcome shows the possible utility of the proposed method, while more studies with actual data have to be carried out.

Table 4: Comparison for Scenario S2 with AGMA-based Design

Design Variable	AGMA	Current
m (mm)	4	4
b (mm)	67.3	62.9
N	19	18
UTS (MPa)	1550	1600
COST (INR)	192	178

Table 5: Comparison for Scenario S7 with AGMA-Based Design.

Design Variable	AGMA	Current
m (mm)	4	4
b (mm)	39.9	40
N	29	26
UTS (MPa)	1590	1550
COST (INR)	277	215

5 SUMMARY

In this paper a method is presented for the robust design of gears based on the compromise Decision Support Problem construct. Two important design factors (factors of safety in contact and reliability factor) used in standard AGMA based design procedures are eliminated through formal introduction of uncertainty in the magnitude of load and material properties. The outcome of the solution is in line with that which is expected based on experience and for select cases are in close agreement with what can be obtained using AGMA standards. The solution thus obtained reinforces the possibility of systematically reducing the number of empirical design factors. As we get more information about material properties and manufacturing processes as well as a knowledge of loads, the proposed method can be used improve the design procedures with reduced number of factors from the AGMA based design without lengthy experimentation to modify the factors under new conditions. This would save a significant amount of time of experimentation for the industry and also help make continual changes to design when more precise information of variability in load and material variability become available.

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