A non-cooperative quantum game is a function $Q$ such that the pure strategy sets $S_i$ of player $i$ and image called the set of outcomes $O$ such that the players entertain non-identical preferences over these outcomes.

Example (Prisoners’ Dilemma):
$$G : \{D, H\} \times \{D, H\} \rightarrow \{(3,3), (0,5), (5,0), (1,1)\}$$

Players’ Preferences
- Player I: (5,0) > (3,3) > (1,1) > (0,5)
- Player II: (0,5) > (3,3) > (1,1) > (5,0)

The symbol $\triangleright$ captures “more preferred”

Assuming rational players seeking outcomes consistent with their preferences, the optimal outcome here is (3,3).

However, in attempting to satisfy their preferences, the likely outcome for rational players is the Nash equilibrium (1,1) where each player gives a best reply to the choice of his opponent’s pure strategy.

A choice of pure quantum strategies $(S,T)$ in a two player strictly competitive game $Q$ is a Nash equilibrium if $Q(S,T)$ is a quantum superposition that simultaneously minimizes the distance between itself and the most preferred basis element of each player.

A quantum control mechanism at Nash equilibrium

$$\gamma = Q(S,T), \text{ a Nash equilibrium quantum superposition}$$

Gaming the quantum allows one to speak of optimal control of quantum systems in the game-theoretic sense of Nash equilibrium.

One potential application of this idea is to quantum circuit design for quantum algorithms. Grover’s search algorithm for example exhibits strictly competitive preferences over the outcomes, with the searched item being the most preferred outcome of “Player I”. This setting may also provide insights into equilibrium behavior of quantum effects found in biological systems such as photosynthesis.

Another possible application is in the area of quantum neural networks. An ongoing project with Ahmed al Hady at Max-Planck-Institut für Dynamik und Selbstorganisation in Germany attempts to identify optimal learning strategies for quantum mechanical models of a simple neural network called the perceptron.

A further application of gaming the quantum could be in the study of relativistic quantum mechanics. The geometry of Nash equilibrium, at least in strictly competitive games, is based on the metric of the Hilbert space of quantum superpositions. Could it be the case that some appropriate functor in the form of a topological quantum field theory be constructed between the category of (projective) Hilbert spaces used in quantum mechanics and the category of differential manifolds used in general relativity? Such a functor would allow to speak meaningfully of relativistic quantum control at Nash equilibrium.

References