Robust Adaptive Sliding Mode Controller for Triaxial Gyroscope

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Abstract—This paper presents an adaptive variable structure controller with on-line identification of the upper bounds of uncertainties and disturbance for MEMS triaxial angular sensors device that is able to detect rotation in three orthogonal axes, using a single vibrating mass. An adaptive robust sliding mode controller is developed and the stability of the closed-loop system can be guaranteed with the proposed adaptive robust sliding mode control strategy. The on-line identification of the upper bounds of uncertainties and disturbance can be incorporated into sliding mode controller to alleviate the chartering. The proposed adaptive sliding mode controller updates estimates of all stiffness, damping errors, and input rotation parameters, upper bound of uncertainties and disturbance in real time, removing the need for any offline calibration stages. The numerical simulation for MEMS gyroscope triaxial angular sensor is investigated to verify the effectiveness of the proposed adaptive robust sliding mode control scheme.

I. INTRODUCTION

GYROSCOPE is commonly used sensor for measuring angular velocity in many areas of applications such as navigation, homing, and control stabilization. Gyroscopes are the devices that transfer energy from one axis to other axis by Coriolis force. Fabrication imperfections always result in some cross stiffness and cross damping effects and the performance of the MEMS gyroscope is also hindered by the effects of time varying parameters as well as noise sources such as thermal and mechanical noise and sensing circuitry noise, environment variations, quadrature errors, parameter variations, and external disturbance which generate a frequency of oscillation mismatch between the two vibrating axes. It is necessary to control the MEMS gyroscope.

Sliding mode control is a robust control technique which has many attractive features such as robustness to parameter variations and insensitivity to external disturbance. The sliding mode controller is composed of an equivalent control part that describes the behavior of the system when the trajectories stay over the sliding manifold and a variable structure control part that enforces the trajectories to reach the sliding manifold and prevent them leaving the sliding manifold. It has some limitation such as chattering or high frequency oscillation in practical applications. Adaptive control is an effective approach to handle parameter variations. Adaptive sliding mode control has the advantages of combining the robustness of variable structure methods with the tracking capability of adaptive control strategies.

Utkin [1] [2] introduced the variable structure system and showed that variable structure control is insensitive to parameters perturbations and external disturbances. Narendra [3], Astrom [4], Ioannou and Sun [5], Tao [6] described the adaptive controls. The adaptive control law, merging parameter identification and sliding mode was proposed and analytically studied by Andrievsky and Fradkov [12]. In the last few years, many applications have been developed using sliding mode control and adaptive control. Lee [7] developed a variable structure augmented adaptive controller for a gyro platform. Wang [8] proposed an adaptive sliding mode controller for a microgravity isolation system. Sam [10] presents a class of proportional and integral sliding mode control with application to active suspension system. Some control algorithms have been proposed to control the MEMS gyroscope. Batur [13] developed a sliding mode control for MEMS gyroscope system. Leland [14] presented an adaptive controller for tuning the natural frequency of the drive axis of a vibratory gyroscope. An adaptive controller for a MEMS gyroscope is proposed by Park [15] which drive both axes of vibration and controls the entire operation of the gyroscope. John et. al [11] extends the park’s method [15] to triaxial angular sensors and presents a novel concept for an adaptively controlled triaxial gyroscope.

The proposed adaptive sliding mode controller is different from [11] and [15] in that a sliding mode control algorithm is incorporated into the adaptive control system to control the vibration of triaxial gyroscope. However, the sliding mode control requires upper bounds of uncertainties to specify the sliding mode gain to satisfy the requirement of stability and robustness. These upper bounds are often chosen conservatively, and consequently high sliding mode gain and oscillation such as chartering may occur. The on-line identification of the upper bounds of uncertainties and disturbance can be incorporated into sliding mode controller to alleviate the chartering. The limitation regarding the information of the upper bound of the uncertainties and disturbance must be known in advance is removed in this paper. The on-line adaptive algorithm to estimate the upper bound of the uncertainties and disturbance is proposed referring to [9].

This paper extends adaptive sliding mode control from two axial angular sensors [16] to triaxial angular sensors and proposes a novel concept for an adaptively controlled triaxial angular velocity sensor device that is able to detect rotation in three orthogonal axes, using a single vibrating mass. The triaxial angular velocity sensor will be based on a surface micromachining technology capable of sensing...
angular motion about three orthogonal axes. It provides
analog outputs for angular velocity and precision
references about the X, Y, and Z-axes. The on-line
identification of the upper bounds of uncertainties and
disturbance is incorporated into sliding mode controller to
alleviate the chattering.

The paper is organized as follows. In section II, the
dynamics of triaxial MEMS vibratory gyroscope is
described. In section III, an adaptive sliding mode
controller with on-line identification of upper bounds of
uncertainties and disturbance is developed. Simulation
results are presented in section IV. Conclusion is provided
in section V.

II. DYNAMICS OF MEMS TRIAXIAL GYROSCOPE

Assume that the gyroscope is moving with a constant
linear speed; the gyroscope is rotating at a constant angular
velocity; the centrifugal forces are assumed negligible; the
gyroscope undergoes rotations about the x, y and z axis.

Refer to [16], the dynamics of triaxial gyroscopic system is
as follows

\[ m\ddot{x} + d_x \dot{x} + k_x x = u_x + 2m\Omega \dot{y} - 2m\Omega \dot{z} \]

\[ m\ddot{y} + d_y \dot{y} + k_y y = u_y - 2m\Omega \dot{x} + 2m\Omega \dot{z} \]

\[ m\ddot{z} + d_z \dot{z} + k_z z = u_z + 2m\Omega \dot{x} - 2m\Omega \dot{y} \]

where \( \Omega_x, \Omega_y \) and \( \Omega_z \) are angular velocities in the the x y
and z direction respectively.

Refer to [15], dividing the equation by the reference mass and
rewriting the dynamics in vector forms result in

\[ \ddot{q} + \frac{D}{m} \dot{q} + \frac{K}{m} q = \frac{u}{m} - 2\Omega \dot{q} \tag{2} \]

where \( q = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}, \Omega = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ -\Omega_y & 0 & -\Omega_x \\ \Omega_x & \Omega_y & 0 \end{bmatrix}, \]

\[ D = \begin{bmatrix} d_x & d_y & d_z \\ d_y & d_z & d_x \\ d_z & d_x & d_y \end{bmatrix}, K = \begin{bmatrix} k_x & k_y & k_z \\ k_y & k_z & k_x \\ k_z & k_x & k_y \end{bmatrix}. \]

Since the non-dimensional time \( t^* = \frac{t}{\omega_0}, \) dividing both
sides of equation by reference frequency \( \omega_0^2 \) and reference
length \( q_0 \) gives the final form of the non-dimensional
equation of motion for the z-axis gyroscope.

\[ \ddot{q}^* + \frac{D}{m} \dot{q}^* + \frac{K}{m} q^* = \frac{u}{m} - 2\frac{\Omega}{\omega_0} \dot{q}^* \tag{3} \]

Define new parameters as follows:

\[ q^* = \frac{q}{q_0}, D^* = \frac{D}{m \omega_0}, \Omega^* = \frac{\Omega}{\omega_0}, \]

\[ u_x^* = \frac{u_x}{m \omega_0 q_0}, u_y^* = \frac{u_y}{m \omega_0 q_0}, u_z^* = \frac{u_z}{m \omega_0 q_0}, \]

\[ w_x^* = \frac{k_x}{m \omega_0}, w_y^* = \frac{k_y}{m \omega_0}, w_z^* = \frac{k_z}{m \omega_0}, \]

\[ w_{xy} = \frac{k_{xy}}{m \omega_0}, w_{yz} = \frac{k_{yz}}{m \omega_0}, w_{xz} = \frac{k_{xz}}{m \omega_0}. \]

Ignoring the superscript (*) for notational clarity, the
non-dimensional representation of (1) and (2) is

\[ \dot{q} + D \dot{q} + K q = u - 2\Omega \dot{q} \tag{4} \]

where

\[ K = \begin{bmatrix} w_x^2 & w_{xy} & w_{xz} \\ w_{xy} & w_y^2 & w_{yz} \\ w_{xz} & w_{yz} & w_z^2 \end{bmatrix}. \tag{5} \]

III. ADAPTIVE SLIDING MODE CONTROLLER DESIGN

This section proposes an adaptive sliding mode control
strategy for MEMS triaxial gyroscopes. A detailed study of
the sliding mode control algorithm with proportional
sliding surface is presented in the presence of matched
parameter uncertainties and external disturbance with the
triangular gyroscopic model. The on-line identification of the
upper bounds of uncertainties and disturbance is proposed
with the sliding mode controller to alleviate the chartering.

The control target for MEMS gyroscopic is to maintain
the proof mass to oscillate in the x y and z direction at given
frequency and amplitude \( x_m = A_x \sin(\omega t), y_m = A_y \sin(\omega t), \)
\( z_m = A_z \sin(\omega t). \)

Figure 1: Block diagram of a direct adaptive sliding mode control for a
MEMS triaxial gyroscope

Rewriting the gyroscopic model in state space equation:

\[ X = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}, \]

\[ X = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -w_x^2 & -d_x & -w_{xy} & -(d_y - 2\Omega_x) & -w_{xz} & -(d_z + 2\Omega_x) \\ -w_{xy} & -d_y & -w_{xy} & -(d_x - 2\Omega_y) & -w_{xy} & -(d_z + 2\Omega_y) \\ -w_{xz} & -d_z & -w_{xz} & -(d_x + 2\Omega_z) & -w_{xz} & -(d_z + 2\Omega_z) \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -w_{xz} & -d_z & -w_{xz} & -(d_x + 2\Omega_z) & -w_{xz} & -(d_z + 2\Omega_z) \end{bmatrix} \]

\[ \dot{X} = AX + Bu, \]

where \( A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ u = \begin{bmatrix} u_x^* \\ u_y^* \\ u_z^* \end{bmatrix}. \]

Rewriting the reference model in state space equation:

\[ \dot{q}_m + K_m q_m = 0 \tag{7} \]

where \( K_m = \text{diag} \left\{ w_x^2, w_y^2, w_z^2 \right\}. \)
\[
X_n = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-w_2^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -w_2^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -w_2^2 & 0
\end{bmatrix} X_n
\]
(8)
which is \(\dot{X}_n = A_n X_n\), where \(X_n = [\dot{x}_n \; \dot{y}_n \; \dot{z}_n \; \dot{\theta}_n \; \dot{\phi}_n \; \dot{\psi}_n]^T\).

Consider the system in (9) with parametric uncertainties \(\Delta A\) and external disturbance \(f(t)\) as
\[
\dot{X}(t) = (A + \Delta A)X(t) + Bu + f(t)
\]  
(9)
where \(X(t) \in \mathbb{R}^n\), \(u(t) \in \mathbb{R}^m\) and \(A \in \mathbb{R}^{n \times n}\) is unknown matrix, \(\Delta A\) is the unknown parameter uncertainties of the matrix \(A\), \(f(t)\) is an uncertain extraneous disturbance.

We make the following assumptions.

Assumption 1. There exist unknown matrices of appropriate dimension \(D\), \(G\) such that (i) \(\Delta A(t) = BD(t)\), where \(BD(t)\) is the matched uncertainty and (ii) \(f(t) = BG(t)\), where \(BG(t)\) is the matched disturbance. Therefore, (9) can be rewritten as
\[
\dot{X}(t) = AX(t) + Bu(t) + Bf_m(t, X)
\]  
(10)
where \(Bf_m(t, X)\) represents the lumped, matched uncertainty and disturbance which is given by
\[
f_m(t, X) = DX(t) + G.
\]  
(11)
2. The matched lumped uncertainty and external disturbance \(f_m\) is bounded such as \(\|f_m\| \leq \bar{c}_1 + \bar{c}_2 \|X\|\)
where \(\bar{c}_1\) and \(\bar{c}_2\) are unknown positive constants.

3. There exists a constant vector \(K^* \in \mathbb{R}^n\) such that the following equation is satisfied \(A + BK^* = A_m\). The condition is called the matching condition.

The tracking error is defined as
\[
e(t) = X(t) - X_m(t)
\]  
(12)
and the derivative of tracking error is
\[
\dot{e} = A_n e + (A - A_n)X + Bu + Bf_m
\]  
(13)
The sliding surface is defined as \(s(t) = \lambda e\), where \(\lambda\) is a constant matrix as
\[
\lambda = \begin{bmatrix}
\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} & \lambda_{15} & \lambda_{16} \\
\lambda_{21} & \lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} & \lambda_{26} \\
\lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{34} & \lambda_{35} & \lambda_{36}
\end{bmatrix}
\]  
(14)
The derivative of the sliding surface is
\[
\dot{s} = \lambda A_n e + \lambda (A - A_n)X + \lambda Bu + \lambda Bf_m
\]  
(15)
Setting \(s = 0\) to solve equivalent control \(u_{eq}\) gives
\[
u_{eq} = K^T X(t) - (\lambda B)^{-1} \lambda A_n e - f_m
\]  
(16)
The adaptive controller is proposed as
\[
u(t) = -\rho (\lambda B)^{-1} s + K^T (t) X(t) - (\lambda B)^{-1} \lambda A_n e(t) - \rho \|B^T \dot{s}\| 
\]  
(17)
where \(K^T = \begin{bmatrix}
k_{11}^* & k_{12}^* & k_{13}^* & k_{14}^* & k_{15}^* & k_{16}^* \\
k_{21}^* & k_{22}^* & k_{23}^* & k_{24}^* & k_{25}^* & k_{26}^* \\
k_{31}^* & k_{32}^* & k_{33}^* & k_{34}^* & k_{35}^* & k_{36}^*
\end{bmatrix}\) \(K(t)\) is the estimate of \(K^*\), \(\rho = \alpha_1 + \alpha_2 \|X\|\) where \(\alpha_1\) and \(\alpha_2\) are estimates of unknown positive constants \(\bar{c}_1\) and \(\bar{c}_2\), \(\|\cdot\|\) is the Euclidean norm, \(u_s = u_{s1} u_{s2} = \left[\begin{bmatrix}B^T \dot{s}\right]\right]\) is the sliding mode term representing the nonlinear feedback control for suppression of the effect of the uncertainty.

Define the estimation error as \(\hat{K}(t) = K(t) - K^*\), and is positive definite matrix, \(c_1\) and \(c_2\) are constant positive.

Differentiating \(V\) with respect to time yields
\[
V = \frac{1}{2} s^T s + \frac{1}{2} tr(\tilde{K} M^{-1} \tilde{K}^T) + \frac{1}{2} [\tilde{\alpha}_1]^2 + \frac{1}{2} c_2 [\tilde{\alpha}_2]^2
\]  
(21)
where \(M = M^T > 0\), \(\tilde{\alpha}_1 = \alpha_1 - \tilde{\alpha}_1\), \(\tilde{\alpha}_2 = \alpha_2 - \tilde{\alpha}_2\), \(M\) is positive definite matrix, \(c_1\) and \(c_2\) are constant positive constants.

Then, we choose the adaptive laws for upper bound as
\[
\dot{\hat{K}}(t) = -\rho \lambda B \tilde{K}^T s + \rho B^T \dot{s}
\]  
(20)
Define a Lyapunov function
\[
V = \frac{1}{2} s^T s + tr(\tilde{K} M^{-1} \tilde{K}^T) + c_1 [\tilde{\alpha}_1 \tilde{\alpha}_1] + c_2 [\tilde{\alpha}_2 \tilde{\alpha}_2]
\]  
(22)
To make \(\dot{V} \leq 0\), we choose the adaptive laws as
\[
\dot{\hat{K}}(t) = -\rho \lambda B \tilde{K}^T s + \rho B^T \dot{s}
\]  
(23)
with \(K(0)\) being arbitrary. This adaptive law yields
\[
\dot{V} = -\rho s^T s + \rho [B^T \dot{s}]^T + \rho \lambda B \dot{f}_{m}(t) + c_1 [\tilde{\alpha}_1 \tilde{\alpha}_1] + c_2 [\tilde{\alpha}_2 \tilde{\alpha}_2]
\]  
(24)
\[
\dot{\lambda} = \frac{1}{c_1} \left[ \varphi_1 \dot{\lambda} \right] \| B^T \dot{X}^s \|, \\
\dot{\varphi}_1 = \frac{1}{c_2} \left[ \varphi_2 \dot{\varphi}_1 + \| B^T \dot{X}^s \| X \right]
\]

where 0 ≤ V implies that the closed-loop system is stable. This implies that s, \( \dot{\lambda} \) are all bounded. \( \dot{V} \) is negative definite implies that s, \( \dot{\lambda} \) all converge to zero. The property of \( \dot{V} \) is negative semi-definite ensures that V, s and \( \dot{\lambda} \) are all bounded. \( \dot{V} = 0 \) implies s = 0. LaSalle’s invariant set theorem can be used to prove that \( \lim_{t \to \infty} V(t) = 0 \). \( \dot{V} = 0 \) implies s = 0 and there is no other solution but s = 0.

According to LaSalle’s invariant set theorem and defining \( R = \{ s \in \mathbb{R}^n \mid V(x) = 0 \} \), then if R contains no other trajectories other than s = 0, the origin 0 is asymptotically stable. Consequently the sliding surface s = 0 is an invariant set which implies that any trajectory starting from an initial condition within the set remains in the set all the time, that is \( \dot{s}(t) \) will asymptotically converge to zero.

In order to prove that the parameter errors \( \hat{K} \) converge to zero, we need to make the persistence of excitation argument. From the adaptive law \( \dot{\hat{K}}^T(t) = -MB^T \hat{X}^T sX^T \), and according to the persistent excitation theory [5], if X meets persistence of excitation condition, then \( \dot{K}(t) \) will converges to its true value. It can be shown that there exist some positive scalar constants \( \alpha \) and \( T \) such that for all \( t > 0 \), \( \int_{0}^{t} XX^T \dd t \geq \alpha T \). If \( w_1 \neq w_2 \neq w_3 \), i.e. the excitation frequencies on x, y and z axes should be different, it can be shown that \( XX^T \) has full rank. Since X meets persistence of excitation condition, then \( \hat{K} \to 0 \), i.e. the unknown angular velocity as well as all other unknown parameters can be consistently estimated. In consequence, angular velocity \( \Omega_x, \Omega_y, \Omega_z \) and gyroscope parameters converge to their true values.

Remark 1. The convergence of the adaptive parameters of upper bound to real ones cannot be guaranteed, since the system trajectory is restricted to the switching surface after arriving at the sliding mode, the adaptive parameters converge to some values depending on the values of \( \alpha_1, \alpha_2, c_1 \) and \( c_2 \). In some time s will not be equal to zero all the time, this will decrease tracking accuracy of control system. The adaptive gains will slow increase boundless. The dead-zone techniques can be used to remove this implementation problem. To simplify the problem of integral wind-up in the adaptation of the upper bound of the unknown disturbance, the adaptive laws are modified as

\[
\dot{\varphi}_1 = \frac{1}{c_1} \left[ \varphi_1 \dot{\varphi}_1 \right] \left[ \varphi_2 \dot{\varphi}_2 \right] + \| B^T \dot{X}^s \| \]
\[
\lambda = \begin{bmatrix}
0 & 10 & 0 & 0 & 0 \\
0 & 0 & 10 & 0 & 0 \\
0 & 0 & 0 & 10 & 0
\end{bmatrix}.
\]

The adaptive gain of (23) is \( \{ \lambda_1 \}^{20} \) = \( \{ \lambda_2 \}^{20} \), and the adaptive gain of (27) is \( c_1 = c_2 = 20 \), \( \varphi_1 = \varphi_2 = 1 \), \( \varepsilon = 0.15 \) in (29).

Fig. 2. Convergence of the tracking error \( e(t) \)

Fig. 3. Convergence of the sliding surface \( s(t) \)

Fig. 4. Convergence of the estimated angular velocity with smooth sliding mode controller

Fig. 4. Convergence of the estimated angular velocity with non-smooth sliding mode controller

Fig. 5. Convergence of the estimated angular velocity with non-smooth sliding mode controller

Fig. 6. The adaptation of controller parameters

Fig. 7. Smooth sliding mode control force
The convergence behavior of sliding surface and tracking error are shown in Figs. 2-3. Fig. 4 and Fig. 5 compare the angular velocity estimation between the smooth sliding mode controller and non-smooth sliding mode controller. It is observed that the estimated angular velocity with smooth sliding mode controller has improved convergence performance. Fig. 6 demonstrates that the estimation of controller parameters converge to their true values with persistent excitation signal.

Fig. 7 and Fig. 8 compare the sliding mode control force between smooth sliding mode controller and non-smooth sliding mode controller. It is shown that the adaptive control system with the smooth sliding mode controller can reduce chattering significantly.

Fig. 9 shows that the upper bound estimation of uncertainties and external disturbance. It is observed that upper bound of the uncertainties and disturbance can be estimated. It can be observed from Fig. 7-9 that the on-line identification of the upper bounds of uncertainties and disturbance can be incorporated into sliding mode controller to alleviate the chattering.

V. CONCLUSION

This paper presents adaptive sliding mode control using proportional sliding surface for triaxial angular velocity sensor of MEMS gyroscope. The on-line identification of the upper bounds of uncertainties and disturbance is developed. The upper bounds on-line adaptation could lower sliding mode gain and as a result, reduce the chartering that could damage the actuator in the practical application. The proposed adaptive robust sliding mode controller drives the single mass along a controlled oscillation trajectory, removing the need for additional drive control. The proposed device includes a single suspended mass that can move in three axes, having actuation and sensing element in three orthogonal axes. The simulation for the triaxial angular velocity sensor model is implemented to verify the effectiveness of the proposed control for triaxial angular velocity sensor in the presence of unknown upper bound of external disturbance and model uncertainties.

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