A Purchasing Framework for B2B Pricing Decisions and Risk-sharing in Supply Chains*

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ABSTRACT

This paper presents a common modelling structure for (i) the implementation of operational policies by individual purchasing managers of risk-sharing agreements among supply-chain partners, and (ii) the integration of brick and click purchasing policies in a B2B. The problem of price uncertainty created within these two environments is modelled as a stochastic repetitive-sales problem, applicable to any probability distribution. The model identifies sufficient conditions for regenerative ordering cycles, which allows for the use of the renewal reward theorem. The end result is a two-price purchasing policy, which may substantially ease implementation problems across a global corporation's purchasing managers world-wide and across B2B markets.

Subject Areas: Global Operations Management, Inventory Management, Stochastic Processes, and Supply Chain Management.

INTRODUCTION

This paper focuses on two specific problems observed by the authors to occur while putting into practice solutions to issues arising from the interface of e-commerce and supply chains. One of these problems arises when attempting to operationalize risk-sharing agreements between two components of the supply chain. The other is

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salient in the process of coordinating pricing policies for an Internet-based (or "click") and for a more traditional (or "brick") B2B market. Those two problems, to be detailed next, give rise to a common structure that leads to the development of the stochastic repetitive-sales problem, subject of this paper.

A Foreign-Exchange Risk-sharing Problem

The first problem takes place within a floating-exchange regime, where long-term supply contracts create operating exposure to the parties, as the effective purchase price of a given component at the time of execution may fluctuate due to the change in the relative value of the currencies. To mitigate such an effect, supply chain partners often enter into risk-sharing agreements, whereby "the buyer and the seller agree to share or split currency movement impacts on payments between them" (Eiteman et al., 2001, p. 219). In this way, the parties share the favourable/ adverse effects of changes in exchange rates as long as the fluctuations remain within agreed-upon limits. Otherwise, a new risk-sharing agreement must be negotiated, as it often happens when dealing in countries with hyperinflation conditions (see Austin, 1990). The basic rationale for the use of risk-sharing contracts is well known in international finance, since floating exchange rates have been a fixture in world markets since the 1970s and even before the 1950s (see Eiteman et al., 2001, p. 220). Nonetheless, their impact on operations has been largely neglected up to the last few years, even if reports of their saliency in Global Supply-Chain Management were being published as early as the late '80s (see Carter & Vickery, 1988; Dornier et al., 1998). Even today, the literature on global supply-chain issues in general (see Cohen & Huchzermeir, 1999; Cohen & Mallik, 1997) and on supply-chain contracts in particular (see Tsay et al., 1999) emphasise the development of multi-sourcing models, with hedging mechanisms designed to shift the risk to the other parties, rather than intended to share the risk among the members of the chain. Examples include contracts with quantity flexibility (Li & Kouvelis, 1999; Tsay & Lovejoy, 1999), special sourcing strategies (see Kouvelis, 1999), price protection (Lee et al., 2000) and options (Huchzermeir & Loch, 2001).

Nevertheless, for the numerous purchasing managers of a global manufacturing concern, the presence of risk-sharing agreements still implies purchasing price uncertainty, even if there exists a contract at an agreed-upon purchase price in the buyer's home currency. However, letting individual purchasing managers deal continuously with a whole range of uncertain prices without appropriate methodological support tends to seriously undermine the salutary (at least for the firm) effect of risk-sharing contracts. Furthermore, the nature of exchange rate induced uncertainties is normally similar across purchasing managers world-wide. Hence, as part of the process to standardise the operationalization of these risk-sharing contracts, a standard two price structure is often developed for favourable and adverse exchange-rate situations, whereby a higher (lower) standard unit purchase price of \( C_U \) (\( C_L \)) may be set if the current exchange rate exceeds (is below) a predetermined exchange rate. As the actual exchange rate is random, so is the occurrence of \( C_U \) and \( C_L \). Hence, when designing the cost-minimising ordering policies for the acquisition of the components dictated by the production plan, the purchasing manager may take into account the exchange-rate uncertainty through the consideration of a simple two-price structure.
A Brick and Click Pricing Problem

The second problem giving rise to this paper deals with the issue of B2B pricing and with the development of brick and click pricing policies (see Gulati & Garino, 2000). Even if the emergence of the Internet is changing the landscape of the B2B marketplace, online technology has not proven itself to be the panacea to cure all of the world’s ills, nor are the traditional brick-and-mortar stores as hopelessly outdated as it was once thought. Today the picture is more balanced (see Taylor, 2001). Hence the trend is to mix these two types of distribution modes. The development of joint pricing policies is part of this integration process. Based on the type of goods (what is bought) and the type of sourcing (how it is bought), Kaplan and Sawhney (2000) identify four different types of B2B hubs. Of interest here are the on-line exchanges (e.g., redtagbiz.com, isolve.com), widely expected to solve or at least alleviate some of the distribution problems that were created through the traditional methods (see Booker, 1999) and hence result in low-cost and efficient way of displaying merchandise, attract customers and handle purchase orders (see Copacino, 2000). More importantly for the purpose at hand, these surplus-inventory websites provide information flow of temporary excess supply readily available online and hence the opportunity for the retailer to buy at a price lower than the regular purchase price either directly from the wholesalers or more and more through these e-exchanges (see Booker, 1999; Brynjolfsson & Smith, 2000). As before, this process results in a two-price, \((C_U, C_L)\), structure, where the retailer normally buys at the regular “brick” price of \(C_U\), as stipulated in the traditionally long-term contract existing in supply chains, but takes periodic advantage of the lower, \(C_L\), click prices if the situation so warrants.

The Stochastic Repetitive-Sales Model

The two problems just described exhibit a common two-price structure, which is akin to that used in solving the repetitive-sales problem (see Arcelus et al., 2000; Friend, 1960; Hunter & Kaminsky, 1968a, 1968b; Moinzadeh, 1997; Silver et al., 1993). In this situation, a retailer is confronted with periodic discount offers of random starting points and duration, but of certain occurrence. The question of price difference existing within these two environments is modelled as a stochastic repetitive-sales problem with equal-order sizes, applicable to any probability distribution. In the model of this paper, \(C_L(C_U)\) plays the role of the discounted (regular) purchase price, \(Q_L\) and \(Q_U\) represent the respective order quantities, and \(r\) is the reordering point in the discounted sub-cycles.

Some of the key assumptions of the special-sale studies are retained because they are also commonly used in the literature related to the two alluded problems. As a result, no shortages are allowed, lead times are zero, and the demand for a given component is constant and independent of its purchase price. These are reasonable assumptions because the purchasing manager under consideration is expected to develop the ordering policies within the constraints of the predetermined production plan.

Two other assumptions need to be modified. The first is that of exponentially distributed time between two consecutive discount occurrences. Its usefulness is due to its memoriless property, which in turn leads to optimal equal orders. In that
way, the problem may be formulated as a Markov process. This yields a sufficient but not necessary condition for the special order cycle to be considered as a regenerative cycle. Hence, the use of the renewal reward theorem (Ross, 1983) in order to obtain the expected cost per unit time is justified. However, when the exponential assumption is dropped in favour of a general distribution, the Markovian property is no longer valid. The resulting formulation consists of a stochastic dynamic program with a random number of cycles, different order sizes, and hence totally impractical from an applications perspective. Arcelus et al. (1999) describe this issue in more detail for a related problem. Instead, this paper presents an equal-order-size approximated model, which (i) redefines the concept of a cycle for the non-exponential case; (ii) restores the properties of repetitive cycles and of repetitive rewards for each cycle; hence, (iii) permits the use of the renewal reward theorem; and thus (iv) substantially simplifies the computational procedure and the implementation of the results. Another assumption to be dropped is that of zero duration or instantaneous end of the discount offer (see Moinzadeh, 1997, p. 336). Instead, the discount offer is assumed to last for a finite but random time period because exchange rate fluctuations or the Internet offer may keep the purchase price high or low continuously for a finite period of time. With these caveats in mind, it is the purpose of this paper to develop cost-minimising equal-order strategies that incorporate the uncertain occurrence of predetermined standard purchase prices.

The Ordering Policy
To develop the purchasing manager's ordering policy, it is necessary to first define a cycle and then provide the conditions that render a cycle regenerative for cases when the assumption of exponentially distributed time between two consecutive discount occurrences does not hold.

On the nature of the regenerative cycle
Under the exponential assumption and following current practice (see Arcelus et al., 2000; Moinzadeh, 1997; Hunter & Kaminsky, 1968b), a cycle is defined "as the time between two consecutive deal replenishments" (Moinzadeh, 1997, p. 336). As shown in Property 1 of Arcelus et al. (2000), this definition includes also the placing of consecutive discount orders within a time horizon without a nondiscount price period in between. Under these conditions, each of the consecutive discount orders covers a full regenerative cycle, irrespective of whether or not a nondiscount price is operative in between. However, in the presence of nonexponentially distributed time between any two consecutive cycles, the memoryless property breaks down and that "discounted repeated behaviour" is not necessarily valid. Hence, the definition of a regenerative cycle is modified as follows:

Definition: A cycle is defined as the time between two consecutive discount commencements.

Observe that this definition contemplates the feasibility of ordering consecutive discount orders within a cycle. However, such policy also carries the proviso that a nondiscount offering of positive length must follow before the cycle is complete, even if it is of such short duration that no order is placed. More formally, this
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A definition recognises the existence of $L$ and $U$ possible sub-cycles. An $L$ sub-cycle is the time duration between the ending of a nondiscount period and the beginning of the next nondiscount period. Also, a $U$ sub-cycle is the time duration between the beginning and the end of a discount period. Once the nature of a cycle is defined, the following property establishes the characteristics that yield the cycle repetitive.

**Property 1:** Characteristics of a regenerative cycle when the time between two consecutive discount occurrences follows a non-exponential distribution.

1a. The cycle must be a $U + L$ cycle, consisting of a discounted, $L$, and a non-discounted, $U$, sub-cycle.

1b. An order up to $Q_L + r$ is placed as soon as the discount sub-cycle begins, irrespective of inventory position.

1c. At least one discount order must be placed in each cycle, but a policy of zero non-discount orders is also feasible.

Property 1a is a direct result of the definition of a cycle, Property 1b ensures that the cycle is repetitive, and Property 1c is derived from the other two. One of the implications of this property is that there is only one way of starting a new cycle, namely, by placing an order of the size given in Property 1b as soon as the discount is ON. Another is that there are three ways of completing a cycle, as shown in Figure 1 (see also Properties 1b, 1c, and 1d of Arcelus et al., 2000). In all three, the ending inventory, if any (i.e., $i_U \geq 0$), is assumed to be part of the inventory of the next cycle's $L$ sub-cycle. The last two graphs in Figure 1 consider cases where (i) the last order is discounted, (ii) the end of the $L$ sub-cycle occurs after (middle graph) or before (bottom graph) reaching $r$, but (iii) the discount becomes available once again before reaching the zero inventory level. Hence, no $U$ order is placed and the next cycle commences with the immediate placing of an order of size $Q_L + r - i_U$ (Property 1b). In the top graph, the last order is nondiscounted and thus placed in the $U$ sub-cycle, but the discount becomes ON before running out of inventory, at which time an immediate order of size $Q_L + r - i_U$ is placed (Property 1b).

**On the nature of the ordering policy**

In order to satisfy a constant demand rate of $R$ units per year, the purchasing manager's decision process calls for the possibility of placing orders during successive ordering cycles. A cycle is composed of two consecutive and distinct time sub-cycles of random length $j = L$, $U$ (Property 1a), where the distinguishing characteristic is whether the price fluctuation is favourable ($j = L$) or unfavourable ($j = U$) to the manager. Observe that these fluctuations may be due to exchange-rate movements or to changes in either brick or click prices. Each order of size $Q_j$ carries a purchasing cost of $C_j$ per component, an ordering cost of $K_j$ per order placed, and a holding cost factor, $F$, per year per dollar of purchase, independent of $j$. Further, sub-cycle $j$'s duration is characterised by its mean ($\Theta_j$), with its finite standard deviation, its upper bound ($B_j$), its density $[g_j(.)]$, and cumulative distribution $[G_j(.)]$. 

functions. The more common distributional examples used in practice are the exponential, Weibull, and gamma for infinite ranges and the beta for finite ranges. In addition, a random number of $N_j$ orders, with a mean of $\mu_j$, are placed in sub-cycle $j$, where their upper bound, denoted by $M_j$, may be finite or infinite depending upon the size of $B_j$, the upper-bound length of sub-cycle $j$. The various characteristics are defined in Table 1, along with sub-cycle $j$'s ending inventory, $i_j$, to be discussed in section 4.
Table 1: Definitions of some key characteristics of the model.

<table>
<thead>
<tr>
<th>Favourable Exchange-Rate Sub-Cycle, ( L )</th>
<th>Unfavourable Exchange-Rate Sub-Cycle, ( U )</th>
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| \( M_L = \left[ 1 + \frac{RB_L}{Q_L} \right] \), if \( B_L \) is finite  
  \( = \infty \), otherwise  
  where \([z]\) is the greatest integer less than or equal to \( z \) | \( M_U = \left[ \frac{RB_U}{Q_U} \right] \), if \( B_U \) is finite  
  \( = \infty \), otherwise  
  where \([z]\) is the smallest integer greater than or equal to \( z \) |
| \( N_L = m \), if \( (m-1)Q_L/R < L \leq mQ_L/R \) | \( N_U = 0 \) if \( U \leq i_U/R \)  
  \( = n \) if \( [i_U + (n-1)Q_U/R] < U \leq [i_U + nQ_U/R] \) |
| \( P(N_L = m) = G_L(mQ_L/R) - G_L((m-1)Q_L/R) \) | \( P(N_U = 0) = \sum_{m=1}^{M_L} P(L + U \leq t_{m,0} | N_L = m) P(N_L = m) \) |
| \( P(N_U = n) = \sum_{m=1}^{M_L} P(t_{m,n-1} < L + U \leq t_{m,n} | N_L = m) P(N_L = m) \) |  

\( i_L = N_LQ_L + r - LR \)  
\( \mu_L = \sum_{n=0}^{M_L-1} \sum_{m=0}^{M_L-1} [1 - G_L(mQ_L/R)] \), for \( M_L - 1 \leq RB_L < M_L \)  
\( = \sum_{n=0}^{M_L-1} \left[ 1 - \sum_{m=1}^{M_L} P(L + U \leq t_{m,n} | N_L = m) P(N_L = m) \right] \) |

\( i_U = Q_U - R[U + L - i_{N_L,N_U-1}] = N_LQ_L + r + N_UQ_U - (L + U)R \)  
\( E(i_U) = \mu_LQ_L + r + \mu_UQ_U - (\theta_L + \theta_U)R \)
To develop the purchasing manager's ordering policy over each $U + L$ ordering cycle, a new variable in the form of a reordering point, $r$, is introduced. Then, at the beginning of the order cycle, that is, whenever the exchange rate first floats towards the low side, an $L$ sub-cycle starts with the purchasing manager placing an order of up to $(Q_L + r)$ units at $C_L$ per unit (Property 1b). The exact size of this first order (Property 1c) is contingent upon the size of the inventory at hand, $i_U$, when the low price comes into effect. Afterwards, every time the inventory level reaches the reorder point $r$, without an intervening $U$ sub-cycle, an order of size $Q_U$ should be placed, at a unit cost of $C_U$, for as long as the low purchase price is in effect. On the other hand, if a high price comes into effect, a $U$ sub-cycle commences. Then, the strategy is to let the remaining inventory be fully depleted before placing orders of size $Q_U$ at a cost of $C_U$ per unit for as long as the higher purchase price remains in effect. However, as soon as the discount price is ON again, even if the inventory on hand has not yet been fully depleted, revert back to the low price policy and start a new order cycle by ordering immediately $Q_L + r - i_U$ units at the lower price of $C_L$ per component (Property 1b).

Two observations about the preceding discussion are in order. First, the use of the reordering point, $r$, is normally associated in the literature with uncertain lead times. Here, the assumption is one of instantaneous delivery and hence of no lead-time. This is done, following the repetitive sales literature (see Moinzadeh, 1997), to be able to focus on the use of $r$, as a tool designed to handle the lesser known source of uncertainty, giving rise to that two-price structure of this paper. Second, the non-optimality of the equal size policy is recognised for non-exponential ordering cycle duration times. Its main advantage is that it eliminates the need for the dynamic programming formulation with random number of variables alluded to earlier.

**The Cost Minimisation Decision Model**

The purchasing manager's decision model requires first the derivation of the expression for $E(C_{U+L})$, the expected total cost function over each ordering cycle. This is done as the sum of the cycle's expected cost of ordering and purchasing costs, $E(P_{U+L})$ plus $E(H_{U+L})$, the expected cost of holding inventory during the two sub-cycles of random lengths $U$ and $L$, respectively.

**Expected holding costs per cycle**

Observe that the carryover of $i_U$ units into the next cycle may include units bought at the previous $U$ sub-cycle (top graph in Figure 1) or at the previous $L$ sub-cycle (bottom two graphs). Hence, the reward structure is not repetitive, as the carrying cost for each cycle is dependent upon the reward structure of other cycles. This can be handled by following the common practice of assuming a common value, $h$, for the unit holding cost throughout the cycle, that is, regardless of the purchasing cost of each item. Then, multiplying $h$ by the expected total inventory carried in a cycle (see Moinzadeh, 1997, eq. 8) results in the average total holding cost per cycle. This ensures that the rewards from each cycle are independent of those of any other cycle and thus restores the repetitive property of the reward process. However, such an assumption leads to the computation of these costs as a function of the inventory levels alone. Property 2 below presents an alternative that preserves the
financial nature of the holding costs by linking them to the firm's investment in inventories and also restores the repetitive property of the rewards.

To that effect, it is first necessary to derive the holding costs for a given cycle, \( j \). Start by assuming that \( L \) and \( U \) are random variables with a finite mean and a finite variance, which is of course true for almost all practical cases. Let \( N_U(j) \geq 1 \) and \( N_L(j) \geq 0 \) be the number of discount and nondiscount orders, respectively, placed during the \( j^{th} \) \((U + L)\) cycle and let \( i_U(j) \) be the ending inventory level just before placing the first discount order of the \((j + 1)^{th} \) \((U + L)\) cycle. If the \( j^{th} \) \((U + L)\) cycle were to start and end with zero inventory, then \( i_U(j) = i_U(j-1) = 0 \) and no adjustments would be needed to the standard computation of the average inventories for the \( U \) and the \( L \) sub-cycles. As a result, \( N_U(j) \) nondiscount orders of length \( Q_U/R \) would be placed in sub-cycle \( U \), for an average inventory on hand of \( Q_U^2/2 \) and a holding cost charge of \( $c_U F \) per unit. Likewise, \( N_L(j) \) discount orders of length \( Q_L/R \) would be placed in sub-cycle \( L \), for an average inventory on hand of \( (r + Q_L/2) \) and a holding cost charge of \( $c_L F \) per unit. Finally, when the discount price is no longer in effect, the \( r \) units of the reordering point are expected to be depleted uniformly over a \( r/R \) time period, for an average inventory of \( r/2 \) and a holding cost charge of also \( $c_L F \) per unit. Then the standard inventory carrying costs, \( W(j) \) may be expressed as follows:

\[
W(j) = C_U F[N_U(j)(r + Q_U/2)Q_U/R + (r^2 - i_U^2)/(2R)] + C_U FN_U(j)Q_U^2/(2R).
\] (1)

Some readjustment to \( W(j) \) may be needed if additional holding costs arise from the inventory position at the beginning and at the end of the \( j^{th} \) \((U + L)\) cycle, that is, from the size of \( i_U(j - 1) \) and \( i_U(j) \), respectively. Property 2 establishes a procedure to cost these ending inventories, which may have been bought at discounted or at non-discounted prices.

**Property 2:** A regenerative reward process dependent upon investment in inventory. For the purpose of holding-cost estimation, if any cycle \( j \) has some beginning inventory, \( i_U(j - 1) \), it is treated as if coming from the previous cycle's \( U \) sub-cycle and is valued at \( C_U \) per unit.

Property 2 is clearly an approximation. As shown in the lemma presented below, its justification stems from the fact that the expected value of the deviation or error per cycle by making this assumption reduces to zero in the long run. Observe that an adjustment is needed if \( i_U(j - 1) > 0 \), in which case \( N_U(j - 1) \geq 1 \) and/or if \( i_U(j) > 0 \), in which case \( N_U(j) \geq 1 \). Denote the corresponding holding cost adjustment values as \( V(j - 1) \) and \( V(j) \). It can readily be seen that the values of these adjustments are given by

\[
V(s) = \chi[N_U(s) \geq 1](C_U - C_L)F i_U^2(s)/(2R), s = j - 1, j,
\]
where
\[ \chi[A] = \begin{cases} 1, & \text{if } A \text{ occurs} \\ 0, & \text{otherwise.} \end{cases} \] (2)

Then, using (2) and (3), the holding costs for the \( j \)th \((U + L)\) cycle, \( H_{U+L}(j) \), are given by
\[ H_{U+L}(j) = W(j) + V(j - 1) - V(j). \] (3)

Note that the actual costs (or rewards in the language of renewal processes) in \( H_{U+L}(j) \) of (3) have a mixture of random variables from the previous and the present cycles. Thus, the rewards need to be modified in order to apply the renewal reward results and to get a regenerative cycle. Let \( t = 0 \) be the time point at which the discount price comes into force and the inventory level is \( Q_L + r \). Then, over the time horizon, observe the successive occurrence of nonnegative iid random variables \( L \) and \( U \), where the mean of the random variable \((L + U)\) is non-zero, that is, positive. This generates (i) a renewal process if the initial \( L + U \) cycle starts with initial inventory of zero, that is, if \( i_U = 0 \), then \( V(0) = 0 \); or (ii) a delayed renewal process if the first \((L + U)\) cycle differs from the rest. For simplicity, since the results of the following lemma apply (Ross, 1983) in either case, the existence of a reward renewal process is assumed.

**Lemma:**
Let
\[ S_n = \sum_{j=1}^{n} [L(j) + U(j)] \]
a counting process
\[ N(t) = \sup \{ n: S_n \leq t \} \]
and
\[ Y(t) = \sum_{j=1}^{N(t)} H_{U+L}(j) = \sum_{j=1}^{N(t)} [W(j) - V(j) + V(j - 1)]. \] (4)

Then
\[ \{ E[Y(t)] \}/t \to \{ E[W(1)] \}/E(U + L), \text{ as } t \to \infty \text{ with probability one.} \]
Theorem: Consider a renewal process \( \{N(t), t \geq 0\} \) having interarrival times (i.e., of time lengths) \( L_j + U_j \) with reward \( W(j) \) for \( j \geq 1 \). It can be seen that the \( W(j) \)'s are independent and identically distributed for \( j = 1, 2, 3 \). Further, rewards and random variables have finite expectations. Hence, \( N(t) \) with rewards \( W(j) \) for \( j = 1, 2, 3 \) forms a renewal reward process. Define

\[
Z(t) = \sum_{j=1}^{N(t)} W(j).
\]

Using the renewal reward theorem (Ross, 1983), the following holds,

\[
\{(E[Z(t)])/t\} \to \{E[W(1)]/E(U + L)\}, \text{ as } t \to \infty. \tag{6}
\]

Hence,

\[
\left[ \sum_{j=1}^{N(t)} H_{U + L}(j) \right] / t = \left[ \sum_{j=1}^{N(t)} W(j) \right] / t + \{V(0) - V[N(t)]\} / t \to
\]

\[
\frac{E[W(1)]}{E(U + L)}, \text{ as } t \to \infty. \tag{7}
\]

The above result proves that the average cost/reward per unit time could be obtained by using a renewal reward process with rewards \( W(j) \). However, the difference between actual, \( H_{U + L}(j) \), and the adjusted, \( W(j) \), rewards is nullified in the long run. Clearly an inventory system with inventory holding costs as \( H_{U + L}(j) \) for \( j = 1, 2, 3 \), for every \( (L + U) \) cycle forms a regenerative reward process, as the terms \( V(j) \) and \( V(j - 1) \) are probabilistically equivalent and the process probabilistically restarts itself at the beginning of each cycle. In addition, the cycles of the inventory system under consideration are also regenerative, as each cycle starts from the placement of the first discount order of size up to \( Q_L + r \) (Property 1b), following a \( U \) sub-cycle (Property 1a).

Then, \( E(H_{U + L}) \), the expected holding cost function per ordering cycle, is given by

\[
E(H_{U + L}) = C_L F\{\mu_L(r + Q_L/2)Q_L/R + [r^2 - E(t_U^2)]/(2R)\} + C_U F^2 \mu_U Q_U^2/(2R). \tag{8}
\]

**Expected ordering and purchasing costs per cycle**

The ordering policy described earlier calls for a switch to the discount order policy as soon as the discount price is once again available (Property 1b). This implies
that the purchasing manager is expected to place in the \( j \)th cycle (i) a first order, at \( C_L \) per unit, of \( Q_L + r \) units net of the \( i_U(j - 1) \) units available from the previous cycle (Property 1b), whatever inventory level is still on hand at the time the low purchase price is back in force; (ii) \( N_L(j) - 1 \geq 0 \) orders of size \( Q_L \), at \( C_L \) per unit; plus (iii) \( N_U(j) \geq 0 \) orders of size \( Q_U \) at \( C_U \) per unit. Furthermore, since all cycles exhibit the same initial inventory position of \( Q_L + r \) units, they all have the same probability of reaching the three ending inventory positions depicted graphically in Figure 1. However, the reward structure associated with ordering and purchasing costs for each cycle is dependent upon the reward structure of other cycles through \( i_U(j - 1) \). Nevertheless, (9) below shows that the resulting reward process is repetitive, even if all cycles do not have the same ending inventory. To prove this claim, the expected purchasing and ordering costs, \( E(P_{U+L}) \), over the ordering cycle may be written as:

\[
E(P_{U+L}) = E\{K_L N_L(j) + K_U N_U(j) + C_L [N_L(j)Q_L + r - i_U(j - 1)]
\]

\[
+ C_U N_U(j)Q_U \}
\]

\[
= E\{K_L N_L(j) + K_U N_U(j) + C_L [N_L(j)Q_L + r - i_U(j)]
\]

\[
+ C_U N_U(j)Q_U + C_L [i_U(j) - i_U(j - 1)] \}
\]

\[
= K_L \mu_L + K_U \mu_U + C_L [\mu_L Q_L + r - E(i_U)] + C_U \mu_U Q_U
\]

\[
= K_L \mu_L + K_U \mu_U + (C_U - C_L) \mu_U Q_U + C_L R[\theta_L + \theta_U].
\]

The first line of (9) represents the purchasing/ordering policy described earlier. The second results from adding and subtracting \( C_L i_U(j) \) and rearranging terms. Since they are i.i.d., the expected value of the difference between \( i_U(j) \) and \( i_U(j - 1) \) is zero and condition (4) of the lemma applies. This justifies the third line of (9) and thus the existence of a regenerative reward process. Further, the fourth ensues when using the definition of \( i_U \) and \( E(i_U) \) from Table 1. It also provides an alternate economic interpretation to that of the third line. Rather than counting the units bought at the two different prices, the fourth line cost all units demanded at the discounted price of \( CL \) and adds the surcharge to those acquired in the non-discounted \( U \) sub-cycle.

**Expected total costs per cycle**

Finally, using the lemma and (8) and (9), the expected cost of the ordering cycle per unit time, \( E[C_{U+L}/(U + L)] \), may be written as follows

\[
E[C_{U+L}/(U + L)] = E(C_{U+L})/E(U + L)
\]

\[
= [E(P_{U+L}) + E(H_{U+L})]/(\theta_U + \theta_L).
\]

with \( E(P_{U+L}) \) and \( E(H_{U+L}) \) defined in (1) and (2), respectively. The purchasing manager's objective can now be clearly stated, namely, find the order sizes during
the L and U sub-cycles, $Q_L^*$ and $Q_U^*$, and the reorder point for the L sub-cycle, $r^*$, that minimise the expected cost per unit time of each ordering cycle, $E[C_U + L/(L + U)]$, as defined by (10). It should be observed that the existence of a global optimum couldn't be proven theoretically due to the analytical complexity of $E[C_U + L/(U + L)]$, as defined in (10). Nevertheless, the existence of excellent global optimisation methods, such as simulated annealing or the neighborhood search heuristic algorithm (see Eiselt, et al., 2000) that are available to find numerically the global optimum, almost ensures that if one exists it will be found. For these reasons, the constraint optimisation routines of the Matlab Optimisation Toolbox (Matlab, 1999) have been used in the computation of these solutions.

A computational constraint has also been added, as an upper bound on $Q_L + r$, in the form of the optimal order size, $MAXQ_L$, if the retailer knows with certainty that the next order is the last of the L sub-cycle. In the model of the present paper, the retailer only has a probabilistic knowledge of this event. The constraint may be expressed as follows (see Arcelus & Srinivasan, 1992, 1995; Lev & Weiss, 1990):

$$Q_L + r \leq \text{MAX } Q_L = Rd/C_LF + C_U[2RK/(C_UF)]^{1/2}/C_L.$$  \hspace{1cm} (11)

For most of the cases, such knowledge is at best probabilistic in nature. This is captured by the assumption that both $j = U, L$ have a Weibull probability distribution with a shape parameter, $a_j$, and a scale parameter, $b_j$; that is,

$$g(w_j) = b_j a_j w_j^{a_j-1} \exp(-b_j w_j^{a_j})$$

for $j = U, L; w_j > 0; a_j > 0$ and $b_j > 0$

with a mean of $\theta_j = [\Gamma(1/a_j + 1)]/b_j^{1/a_j}$

and $E(W_j)^2 = [\Gamma(2/a_1 + 1)]/b_j^{2/a_j}$. \hspace{1cm} (12)

**NUMERICAL EXAMPLE**

This section is intended to illustrate via a numerical example how the optimal policy is determined and how changes in the key parameters impact these policies. Table 2 provides a sample of results from an extensive set of sensitivity analysis conducted with several examples and with wide variability in the ranges of the parameters. The parameter values for the base case appears at the bottom of the table. For comparability purposes across sub-cycles, the $\theta_j$s, $j = L, U$, are set as given in the base case, along with the $a_j$s. Then, the $b_j$s are computed given the expression for $\theta_j$ in (11). The first row contains the optimal values for the decision variables, for the expected total cost per unit of cycle time, and for assorted intermediate results. The remaining rows contain also optimal policies, but for different
Table 2: A numerical example.

<table>
<thead>
<tr>
<th>Parameter $^1$ $^2$</th>
<th>$\mu_L$</th>
<th>$\mu_U$</th>
<th>$Q_L$</th>
<th>$r$</th>
<th>$Q_L + r$</th>
<th>$Q_U$</th>
<th>PUL</th>
<th>HUL</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case $^3$</td>
<td>4.09</td>
<td>44.77</td>
<td>1,670</td>
<td>4,232</td>
<td>5,902</td>
<td>20.84</td>
<td>100,309</td>
<td>5,304</td>
<td>88,011</td>
</tr>
<tr>
<td>$(K_L, K_U)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(25, 50)</td>
<td>93.89</td>
<td>100.11</td>
<td>107.54</td>
<td>99.01</td>
<td>101.42</td>
<td>97.65</td>
<td>98.83</td>
<td>101.02</td>
<td>98.94</td>
</tr>
<tr>
<td>(75, 50)</td>
<td>100.98</td>
<td>99.91</td>
<td>99.10</td>
<td>99.46</td>
<td>99.36</td>
<td>103.41</td>
<td>101.18</td>
<td>98.85</td>
<td>101.06</td>
</tr>
<tr>
<td>$(K_L, K_U)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(50, 25)</td>
<td>122.98</td>
<td>99.87</td>
<td>79.34</td>
<td>104.11</td>
<td>97.10</td>
<td>100.00</td>
<td>99.92</td>
<td>99.49</td>
<td>99.89</td>
</tr>
<tr>
<td>(50, 75)</td>
<td>89.00</td>
<td>100.11</td>
<td>114.49</td>
<td>97.12</td>
<td>102.03</td>
<td>100.00</td>
<td>100.07</td>
<td>100.45</td>
<td>100.10</td>
</tr>
<tr>
<td>$(K_L, K_U)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(25, 25)</td>
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<td>99.93</td>
<td>80.66</td>
<td>100.29</td>
<td>97.80</td>
<td>96.74</td>
<td>98.74</td>
<td>100.62</td>
<td>98.84</td>
</tr>
<tr>
<td>(75, 75)</td>
<td>90.22</td>
<td>100.02</td>
<td>112.75</td>
<td>96.76</td>
<td>101.29</td>
<td>103.21</td>
<td>101.25</td>
<td>99.32</td>
<td>101.15</td>
</tr>
<tr>
<td>$F$</td>
<td>.15</td>
<td>79.22</td>
<td>0.00</td>
<td>131.14</td>
<td>115.90</td>
<td>120.21</td>
<td>0.00</td>
<td>98.15</td>
<td>71.66</td>
</tr>
<tr>
<td></td>
<td>.45</td>
<td>109.29</td>
<td>97.88</td>
<td>90.48</td>
<td>82.14</td>
<td>84.50</td>
<td>193.67</td>
<td>101.64</td>
<td>111.73</td>
</tr>
<tr>
<td>$R$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5,000</td>
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<td>100.04</td>
<td>61.14</td>
<td>47.09</td>
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<td>51.15</td>
</tr>
<tr>
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<td>99.96</td>
<td>132.46</td>
<td>154.21</td>
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<td>146.74</td>
<td>148.74</td>
<td>150.64</td>
<td>148.84</td>
</tr>
<tr>
<td>$(c, d, d/c)$</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
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<td>0.00</td>
<td>131.20</td>
<td>115.88</td>
<td>120.21</td>
<td>0.00</td>
<td>38.33</td>
<td>53.75</td>
<td>39.11</td>
</tr>
<tr>
<td>(15, 2, .13)</td>
<td>110.76</td>
<td>97.48</td>
<td>89.04</td>
<td>78.47</td>
<td>81.46</td>
<td>212.81</td>
<td>161.78</td>
<td>114.59</td>
<td>159.41</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10, 1, .1)</td>
<td>98.53</td>
<td>95.31</td>
<td>101.74</td>
<td>50.40</td>
<td>64.93</td>
<td>339.30</td>
<td>113.00</td>
<td>57.26</td>
<td>110.20</td>
</tr>
<tr>
<td>(10, 3, .3)</td>
<td>96.58</td>
<td>0.00</td>
<td>104.13</td>
<td>121.32</td>
<td>116.38</td>
<td>0.00</td>
<td>86.22</td>
<td>124.53</td>
<td>88.14</td>
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<tr>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5, 1, .2)</td>
<td>84.11</td>
<td>100.04</td>
<td>122.28</td>
<td>94.19</td>
<td>102.13</td>
<td>106.43</td>
<td>51.25</td>
<td>49.28</td>
<td>51.15</td>
</tr>
<tr>
<td>(15, 3, .2)</td>
<td>111.74</td>
<td>99.96</td>
<td>88.26</td>
<td>102.79</td>
<td>98.68</td>
<td>97.84</td>
<td>148.74</td>
<td>150.64</td>
<td>148.84</td>
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<tr>
<td>$(a_U, \theta_U, b_U)$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1, .6, .67)</td>
<td>98.53</td>
<td>76.75</td>
<td>101.80</td>
<td>94.52</td>
<td>96.58</td>
<td>160.56</td>
<td>99.91</td>
<td>31.56</td>
<td>96.48</td>
</tr>
<tr>
<td>(3, .6, .30)</td>
<td>98.78</td>
<td>100.06</td>
<td>101.44</td>
<td>104.30</td>
<td>103.49</td>
<td>72.31</td>
<td>99.82</td>
<td>120.02</td>
<td>100.84</td>
</tr>
</tbody>
</table>
Table 2: (continued) A numerical example.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1</th>
<th>2</th>
<th>( Q_{L} + r )</th>
<th>( Q_{U} )</th>
<th>( PUL )</th>
<th>HUL</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
<td>(1, 6, 1.67)</td>
<td>75.79</td>
<td>100.34</td>
<td>139.82</td>
<td>92.32</td>
<td>104.25</td>
<td>100.29</td>
</tr>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
<td>(3, 6, 3.30)</td>
<td>100.01</td>
<td>99.97</td>
<td>100.54</td>
<td>99.91</td>
<td>100.62</td>
<td>99.97</td>
</tr>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
<td>(1, 6, 1.67)</td>
<td>75.31</td>
<td>76.99</td>
<td>141.26</td>
<td>84.57</td>
<td>100.61</td>
<td>162.04</td>
</tr>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
<td>(3, 6, 3.30)</td>
<td>82.65</td>
<td>109.49</td>
<td>212.35</td>
<td>101.20</td>
<td>104.37</td>
<td>103.47</td>
</tr>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
<td>(2, 3.8, 7.3)</td>
<td>67.73</td>
<td>101.59</td>
<td>169.32</td>
<td>79.22</td>
<td>121.48</td>
<td>109.52</td>
</tr>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
<td>(1, 5.0, 0.97)</td>
<td>124.45</td>
<td>97.88</td>
<td>222.33</td>
<td>117.31</td>
<td>73.87</td>
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<tr>
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<td>106.60</td>
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<td>208.45</td>
<td>93.05</td>
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</tr>
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<td>86.19</td>
<td>56.17</td>
<td>142.36</td>
<td>138.78</td>
<td>74.58</td>
<td>138.78</td>
</tr>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
<td>(2, 3.8, 7.3)</td>
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<td>0.00</td>
<td>71.15</td>
<td>50.28</td>
<td>63.03</td>
<td>99.14</td>
</tr>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
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<td>122.25</td>
<td>97.86</td>
<td>220.11</td>
<td>125.06</td>
<td>127.13</td>
<td>125.06</td>
</tr>
<tr>
<td>((a_{L}, \theta_{L}, b_{L}))</td>
<td>(2, 3.8, 7.3)</td>
<td>97.86</td>
<td>119.76</td>
<td>217.62</td>
<td>125.06</td>
<td>127.13</td>
<td>125.06</td>
</tr>
</tbody>
</table>

Except for those of the base-case, all values for these variables are equal to 100 times the ratio of the corresponding optimal values to those of the base case.
values of a given parameter, keeping all others at the values of the base case. Also for comparability purposes, the values of these remaining rows are the ratios of the optimal values to those of the base case.

Consider the base case. Table 2 suggests that the purchasing manager’s optimal ordering policy consists of placing \( p, = 4.09 \) orders of size \( Q_L = 1,670 \) each (of \( Q_L = 5,902 \) in the first order) at the discounted unit price of \( c - d = 10 - 2 = $8 \) and \( \mu_U = 44.74 \) orders of size \( Q_U = 20.84 \) units each at the regular unit price of \( c = $10 \). With such policy, the manager is expected to incur a cost of \( E(P_U + L) = $100,309 \) in ordering and purchasing and of \( E(H_U + L) = 5,304 \) in holding inventory throughout the \( \theta_L + \theta_U = 14.4 \) months of the expected planning horizon. The end result is an optimal yearly expected cost per ordering cycle of \( E[C_{U + L}(U + L)] = [E(H_{U + L}) + E(P_{U + L})]/(\theta_L + \theta_U) = $88,011 \).

The rest of the table contains selected results related to the sensitivity of fluctuations in the values of the base-case parameters on optimal policies. Only parametric changes of \( +50\% \) and \( -50\% \) of base-case values are reported because they represent a sufficiently wide variation in the data for the purposes of this section. Even a cursory look at the table suggests the following observations. The first three sets of results are designed to ascertain the effect of absolute and relative changes in the ordering costs. In the first (second) case, \( K_L(K_U) \) changes keeping \( K_U(K_L) \) constant at \( $50 \). In the third, both change simultaneously in the same proportion of either \( -50\% \) or \( +50\% \). As stated before, the effect of these changes on the expected cost per cycle are minute but in the expected direction, with the fluctuations never exceeding two percentages point either way. Their effect on policy varies depending upon the relative magnitude of the costs. When both \( K_L \) and \( K_U \) move in the same direction (from \( K_L = K_U = 25 \) to \( K_L = K_U = 75 \)), \( K_L \) induces the normal EOQ-type effect. The higher its value, the fewer the number (\( \mu_L \) from 121.03 to 90.22) of discounted orders placed but the larger their size (\( Q_L \) from 80.66 to 112.75). Further, to counteract, at least partially, the gain (loss) in the size of \( Q_L, r \) falls (rises), even though the all-important \( Q_L + r \) rises (falls) slightly and hence the probability of being able to place a new discount order rises (falls) also slightly. The effect of \( K_U \) is similar to \( K_L = s \), but the magnitude of the changes is smaller. At the end, there is a compensating change in the expected holding, purchasing, and ordering costs. Hence, increases (decreases) in the order quantities result in much lower increases (decreases) in holding (ordering and total) costs.

A similar effect occurs when analysing the effect of fluctuations in \( F, Q_L, r, \) and \( Q_L + r \) are more sensitive to changes in \( F \) than \( Q_U \). Their ultimate impact of the holding costs is subject to the compensating effect alluded to earlier. According to the evidence from the table, holding costs increases (decreases) due to rises (reductions) in \( Q_U \) are clearly offset by the corresponding decreases in the holding costs associated with lower (higher) magnitudes in \( Q_L, r, \) and \( Q_L + r \). The net effect is a much smaller movement in all three costs, but in the direction concordant with the tenets of economic theory. Also, as expected, higher (lower) demand rates, \( R, \) are accompanied by substantial increases (decreases) in both order sizes, \( Q_U \) and \( Q_L, \) and in the reordering point. Since the fluctuations are all in the same direction, there is no compensating mechanism. Rather, their effect on costs is compounded, which explains the large variability in the three types of expected costs.
A set of parameters that cause still larger changes than those of \( R \) in the expected cycle costs is \((c, d, dlc)\). The first (second) set consider the changes in \( c(d) \) for a constant \( d(c) \), with \( dlc \) varying accordingly. The last pair keeps \( dlc \) constant and allows for movements in \( d \) and \( c \). It is clear from the total column's results that changes in the regular purchasing cost, \( c \), account for the main share of the large fluctuations in the cycle expected cost (39.11, 159.41). This is followed by the effect of the discount, \( d \), factor (51.15, 148.84). Nevertheless, the middle pair (with \( c \) constant) still exhibits relatives cost changes of a much higher magnitude (110.2, 88.14) than those for all other parameters except \( R \). All changes appear to be in accordance with standard economic theory, and their magnitude reflects the fact that these movements directly impact both the purchasing and the holding costs. Less pronounced but still in line with EOQ-type economic policy is the effect on the discount ordering policies. Higher unit discounts, \( d \), for a constant purchase price, leads to increases in \( Q_L + r \), but at a decreasing rate (64.93, 100.00, 116.38). Similarly, raises in the purchase price, with a constant discount, leads to reductions in \( Q_L + r \) at a decreasing rate. Further, for sufficiently small values for purchase price (i.e., \( c = 5 \)) and/or the discount (i.e., \( d = 1 \)), the manager can do away with placing non-discount orders \((Q_U = \mu_U = 0)\), through adroit increases in \( Q_L \) and in the reordering point.

The last half of the table is devoted to study the sensitivity of the optimal policies to changes in the uncertainty parameters, \( a_j \), \( b_j \), and \( \theta_j, j = L, U \). In the first (second) three sets, the \( \theta_j \)'s (\( a_j \)'s) are kept constant at .6(2), the \( a_j \)'s (\( \theta_j \)'s) allow to fluctuate by 50% and \(-50\%\) and the \( b_j \)'s computed from the expression for \( \theta_j \) in (12). Once again, the cycle costs remain relatively immune to differences in the average length of each sub-cycle or in its variability. Most exhibit changes of less than 1 point and none are above 4 points. In contrast, the ordering policies fluctuate widely, especially in the last three sets, where the average lengths of the sub-cycles are allowed to vary, either independently of each other or simultaneously.

Several important generalisations may be made on the basis of our computational experience. First, except for \( R \) and \((c, d, dlc)\), \( E[C_{U+L}(U + L)] \) appears to be rather impervious to fluctuations in all parameters, even if the resulting policies, in the form of the optimal values of the remaining variables, quite often experience substantial changes. Such consistency in the results is quite encouraging, especially in the case of the Weibull distribution parameters, given the notorious and well-known difficulties embedded in their estimation. Second, the results in all cases appear to satisfy the usual tenets of economics, at least as far as the policy on discount orders is concerned. Hence, increases (decreases) in costs or in uncertainty tend to increase (decrease) the expected cycle costs. Such results also provide some measure of indirect validation to the model. Third, discount and nondiscount policies tend to move in the opposite direction. As expected result, the assumption of constant demand indicates that increases in the number of discounted units acquired must necessarily lead to decreases in the volume of the nondiscounted purchases. Fourth, from results not shown here, we observe that on average the inventory on hand at the end of each cycle is nil (i.e., \( i_U = 0 \)). This is not an unexpected occurrence. There exists scant incentive to carry forward any units, given that it is feasible to buy cheaper as many as needed at the beginning of the next cycle. The only cases where we have encountered a positive \( i_U \) are for very
small expected planning horizons (i.e., $\theta_L + \theta_U = .2, .3, .4$), where a larger order size may needed to compensate for the ordering costs.

Finally, the role of the reordering point, $r$, deserves further discussion. Ceteris paribus, a larger $r$ has the advantage of increasing the amount of discount buying, because the first order ensures that the inventory level is up to $Q_L + r$. It also leads to checking sooner for a change in price, hence increasing the probability of getting an additional discount order. The negative side lies on the increases in holding costs. The evidence is clear as to when a larger $r$ is needed, namely, for longer and/or more variable nondiscount sub-cycles and/or for cases where nondiscount costs are more expensive relative to their discount counterparts and/or for higher holding costs.

**IMPLICATIONS AND CONCLUSIONS**

This section highlights important implications of this study that deserve further consideration, as they represent salient contributions of the model to the current literature.

**Switching suppliers as a hedge against risk**

With respect to the problem of risk sharing, it should be observed that the multiple-sourcing methodology alluded to earlier as forming part of the mainstream supply-chain literature differs substantially from the current model. Switching suppliers may very well eventually counteract swings in foreign exchange, as the global sourcing literature indicates. However, if the long-term contracts so favoured in supply-chain management today are to be enforced, sharing these fluctuations among the parties may be an intermediate step necessary to avoid the negative long-term consequences of switching, even at current contract's end. Modelling such long-term consequences often is impossible. However, risk sharing, as defined earlier, is in practice an acceptable "satisficing" substitute, observed quite saliently in international finance.

Further, another reason for the existence of risk-sharing models is that many products, namely, in high-technology fields, exhibit proprietary designs. Under these conditions, it may be extremely difficult, if not downright impossible, to have multiple sources for such items. In such cases, sharing the risk, rather than shifting it to alternate supply-chain partners, may be the only feasible alternative. Another argument in favour of risk-sharing agreements lies in their contribution towards obviating the need for the development of hedging policies against risk. Avoidance of carrying such a task is most desirable given the relative ease with which operating managers may misunderstand the process of operationalizing what are essentially financial tools.

Finally, it should be observed that exchange-rate fluctuations of the type discussed in this paper are random and without drift. As a result, exposure to risk or the use of hedging policies or of risk-sharing agreements all lead towards the same expected cost in the long run. However, risk sharing tends to reduce variability in cash and income flows, always a desirable occurrence for planning or forecasting

**Switching Suppliers in the B2B Case**

A similar set of arguments can be made with respect to the second problem. Long-term contracts at known prices, plus the uncertainty in supply availability at the
expected prices in e-changes, may very well prevent the purchaser from switching suppliers. Besides, relative prices do not have to always go in the same direction, that is, with Internet prices lower than their brick-and-mortar counterparts. If the current fluctuations in California's energy prices are any indication, prices may go either way. Thus, in much the same way as in the exchange case, switching B2B partners may not lead to the desired and expected decreases in the variability of cash or income flows.

Modelling Contributions with Managerial Implications

The most important modelling contribution relates to cases where the probability distribution of the time between two consecutive cycles is not exponential. Its managerial implications stem from the ability to identify and solve with relative ease additional practical problems like those described in section 2 that can now benefit from the methodology described in the paper. For these purposes, the stochastic repetitive-sales problem with equal-order sizes has been modified in three significant ways. The first provides a new definition of a cycle, by looking at periods of times with opportunities for placing both discount and nondiscount orders, even if no nondiscount orders have to be placed. The second renders the cycles regenerative by (Property 1) reverting back to the discount policy as soon as feasible. The third (Property 2) provides a way to make the reward structure also regenerative. This is done by assuming, for holding cost estimation purposes, that the beginning inventory for any cycle was purchased at \( C_U \) per component in the unfavourable portion of the previous cycle (Property 2). It is worth noting once again that Property 2 is not the only way to obtain a regenerative reward process. However, its advantage stems from the fact that it does preserve the financial nature of the holding costs by linking them to the firm's investment in inventories. In any case, these three features allow for the use of the renewal reward theorem (see Ross 1983). Then, the expected cost of the ordering policy per unit of time over the length of the ordering cycle can be written as the ratio of the expected cost per cycle over its expected duration, without compromising the financial nature of the inventory holding costs.

Concluding Comments

As the process of globalisation of business practices inexorably increases its pace, so does the need to coordinate and operationalize the implementation of these practices world-wide. This paper models two such situations within a common structure, namely, that of standardising a purchasing manager's reaction to the price uncertainty inherent in risk-sharing contracts or in the two-market environment of the click-and-brick store. An operational purchasing policy based on equal-size orders is always preferable to purchasing managers, due to its implementation ease. Finally, a study of this type is subject to various extensions, which serve as linkages to other issues associated with risk-sharing contracts. The incorporation of price-uncertainty considerations into existing theoretical models, such as that of Li and Kouvelis (1999), as well as the study of implementation problems of these more complex models deserve special consideration. The study of these and other issues justifies additional research. [Received: October 10, 2001. Accepted: July 24, 2002.]
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