THE SIMPLE EXPONENTIAL SMOOTHING MODEL

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Abstract: In the paper there is prepared a theoretical base for calculation and application of the simple exponential smoothing method. The simple exponential smoothing model is one of the most popular forecasting methods that we use to forecast the next period for a time series that have no pronounced trend or seasonality.

Keywords: simple exponential smoothing model, forecast, smoothing constant, root mean square error, MATLAB

1. Introduction

Exponential smoothing is probably the widely used class of procedures for smoothing discrete time series in order to forecast the immediate future. This popularity can be attributed to its simplicity, its computational efficiency, the ease of adjusting its responsiveness to changes in the process being forecast, and its reasonable accuracy [3].

The idea of exponential smoothing is to smooth the original series the way the moving average does and to use the smoothed series in forecasting future values of the variable of interest. In exponential smoothing, however, we want to allow the more recent values of the series to have greater influence on the forecast of future values than the more distant observations.

Exponential smoothing is a simple and pragmatic approach to forecasting, whereby the forecast is constructed from an exponentially weighted average of past observations. The largest weight is given to the present observation, less weight to the immediately preceding observation, even less weight to the observation before that, and so on (exponential decay of influence of past data [1].

2. Forecasting with the simple exponential smoothing (SES) model

This forecasting method is most widely used of all forecasting techniques. It requires little computation. This method is used when data pattern is approximately horizontal (i.e., there is no neither cyclic variation nor pronounced trend in the historical data).

Let an observed time series be $y_1, y_2, \ldots, y_n$. Formally, the simple exponential smoothing equation takes the form of

$$\hat{y}_{i+1} = \alpha y_i + (1 - \alpha) \hat{y}_i$$

where $y_i$ is the actual, known series value for time period $i$, $\hat{y}_i$ is the forecast value of the variable $Y$ for time period $i$, $\hat{y}_{i+1}$ is the forecast value for time period $i+1$ and $\alpha$ is the

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smoothing constant [2]. The forecast \( \hat{y}_{i+1} \) is based on weighting the most recent observation \( y_i \) with a weight \( \alpha \) and weighting the most recent forecast \( \hat{y}_i \) with a weight of \( 1 - \alpha \).

To get started the algorithm, we need an initial forecast, an actual value and a smoothing constant.

Since \( \hat{y}_1 \) is not known, we can:

- Set the first estimate equal to the first observation. Thus we can use \( \hat{y}_1 = y_1 \).
- Use the average of the first five or six observations for the initial smoothed value.

Smoothing constant \( \alpha \) is a selected number between zero and one, \( 0 < \alpha < 1 \). When \( \alpha = 1 \), the original and smoothed version of the series are identical. At the other extreme, when \( \alpha = 0 \), the series is smoothed flat.

Rewriting the model (1) to see one of the neat things about the SES model

\[
\hat{y}_{i+1} = \hat{y}_i + \alpha e_i,
\]

where residual \( e_i = y_i - \hat{y}_i \) is forecast error for time period \( i \). So, the exponential smoothing forecast is the old forecast plus an adjustment for the error that occurred in the last forecast [1, 4].

By continuing to substitute previous forecasting values back to the starting point of the data in model (1) we receive:

\[
\hat{y}_{i+1} = \alpha y_i + (1-\alpha) [\alpha y_{i-1} + (1-\alpha) \hat{y}_{i-1}] = \alpha y_i + \alpha (1-\alpha) y_{i-1} + (1-\alpha)^2 \hat{y}_{i-1},
\]

\[
\hat{y}_{i+1} = \alpha y_i + \alpha (1-\alpha) y_{i-1} + \alpha (1-\alpha)^2 y_{i-2} + (1-\alpha)^3 \hat{y}_{i-2},
\]

\[
\hat{y}_{i+1} = \alpha y_i + \alpha (1-\alpha) y_{i-1} + \alpha (1-\alpha)^2 y_{i-2} + \alpha (1-\alpha)^3 y_{i-3} + (1-\alpha)^4 \hat{y}_{i-3},
\]

\[
\ldots.
\]

The forecast equation in general form is

\[
\hat{y}_{i+1} = \alpha y_i + \alpha (1-\alpha) y_{i-1} + \alpha (1-\alpha)^2 y_{i-2} + \cdots + \alpha (1-\alpha)^{i-2} y_{2} + \alpha (1-\alpha)^{i-1} y_{1} =
\]

\[
= \alpha \sum_{k=0}^{i-1} (1-\alpha)^k y_{i-k}
\]

(4)

where \( \hat{y}_{i+1} \) is the forecast value of the variable \( Y \) at time period \( i+1 \) from knowledge of the actual series values \( y_i, y_{i-1}, y_{i-2} \) and so on back in time to the first known value of the time series, \( y_1 \) [2, 3]. Therefore, \( \hat{y}_{i+1} \) is the weighted moving average of all past observations.

The series of weights used in producing the forecast \( \hat{y}_{i+1} \) is \( \alpha, \alpha (1-\alpha), \alpha (1-\alpha)^2, \ldots \). These weights decline toward zero in an exponential fashion; thus, as we go back in the series, each value has a smaller weight in terms of its effect on the forecast. The exponential decline of the weights toward zero is evident [1].

After the model specified, its performance characteristics should be verified or validated by comparison of its forecast with historical data for the process it was designed to forecast. We can use the error measures such as MAPE (Mean absolute percentage error), MSE (Mean square error) or RMSE (Root mean square error):
Selection of an error measure has an important effect on the conclusions about which of a set of forecasting methods is most accurate.

The speed at which the older responses are dampened (smoothed) is a function of the value of $\alpha$. When smoothing constant $\alpha$ is close to 1, dampening is quick and when $\alpha$ is close to 0, dampening is slow [3]. If we want predictions to be stable and random variation smoothed, use a small $\alpha$. If we want a rapid response a larger $\alpha$ value is required.

Usually the $MSE$ or $RMSE$ can be used as the criterion for selecting an appropriate smoothing constant. For instance, by assigning $\alpha$ values from 0.1 to 0.99, we select the value that produces the smallest $MSE$ or $RMSE$ [1].

3. SES example with MATLAB

The simple exponential smoothing model can be illustrated by using data about number of personnel in the industrial production in Slovakia over the years 2001 – 2010 that are in the Table 1 below [5]:

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of personnel</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>486459</td>
</tr>
<tr>
<td>2002</td>
<td>488240</td>
</tr>
<tr>
<td>2003</td>
<td>493259</td>
</tr>
<tr>
<td>2004</td>
<td>497302</td>
</tr>
<tr>
<td>2005</td>
<td>518142</td>
</tr>
<tr>
<td>2006</td>
<td>510698</td>
</tr>
<tr>
<td>2007</td>
<td>526075</td>
</tr>
<tr>
<td>2008</td>
<td>532853</td>
</tr>
<tr>
<td>2009</td>
<td>447685</td>
</tr>
<tr>
<td>2010</td>
<td>430658</td>
</tr>
</tbody>
</table>

We want to forecast the number of personnel in industrial production in Slovakia for the year 2011.

By plotting of observed values we can see that we can use SES method. Since no forecast is available for the first period, we will set the first estimate equal to the first observation. For illustration here we try $\alpha = 0.4$, $\alpha = 0.8$ and $\alpha = 0.9$.

The MATLAB script appears as [4]:

```matlab
z=[486459,488240,493259,497302,518142,510698,526075,532853,447685,430658];
n=length(z);a=[0.4 0.8 0.9];
yi1=[];
yi1(1)=NaN;
yi1(2)=z(1);
for i=3:n
    yi1(i)=a(1)*z(i-1)+(1-a(1))*yi1(i-1);
end
yi2=[];
yi2(1)=NaN;
yi2(2)=z(1);
for i=3:n
    yi2(i)=a(2)*z(i-1)+(1-a(2))*yi2(i-1);
end
yi3=[];
yi3(1)=NaN;
yi3(2)=z(1);
```
for i=3:11
    yi3(i)=a(3)*z(i-1)+(1-a(3))*yi3(i-1);
end
RMSE1=...
sqrt((1/(n-1)*sum((z(2:end)-yi1(2:10)).^2)));
RMSE2=...
sqrt((1/(n-1)*sum((z(2:end)-yi2(2:10)).^2)));
RMSE3=...
sqrt((1/(n-1)*sum((z(2:end)-yi3(2:10)).^2)));
RM=[NaN,NaN,RMSE1,RMSE2,RMSE3];
i=1:11;
fprintf('	i	Yi	Y^i(0.4) Y^i(0.8) Y^i(0.9)
')
t=[i;z,NaN;yi1;yi2;yi3];
t=[t;RM];
disp(t)
hold on
plot(i(1:10),z,'o:');
plot(i(2:11),yi1(2:11),'*g-');
plot(i(2:11),yi2(2:11),'+r-.');
plot(i(2:11),yi3(2:11),'xk--');
legend('actual values','forecast:0.4','forecast:0.8','forecast:0.9');
hold off
Using SES with MATLAB we have received following data (see Table 2):

<table>
<thead>
<tr>
<th>i</th>
<th>Yi</th>
<th>Y^i (0.4)</th>
<th>Y^i (0.8)</th>
<th>Y^i (0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>4.8646</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
</tr>
<tr>
<td>2.0000</td>
<td>4.8824</td>
<td>4.8646</td>
<td>4.8646</td>
<td>4.8646</td>
</tr>
<tr>
<td>3.0000</td>
<td>4.9326</td>
<td>4.8717</td>
<td>4.8788</td>
<td>4.8806</td>
</tr>
<tr>
<td>4.0000</td>
<td>4.9730</td>
<td>4.8961</td>
<td>4.9218</td>
<td>4.9274</td>
</tr>
<tr>
<td>5.0000</td>
<td>5.1814</td>
<td>4.9268</td>
<td>4.9628</td>
<td>4.9685</td>
</tr>
<tr>
<td>6.0000</td>
<td>5.1070</td>
<td>5.0287</td>
<td>5.1377</td>
<td>5.1601</td>
</tr>
<tr>
<td>7.0000</td>
<td>5.2607</td>
<td>5.0600</td>
<td>5.1131</td>
<td>5.1123</td>
</tr>
<tr>
<td>8.0000</td>
<td>5.3285</td>
<td>5.1403</td>
<td>5.2312</td>
<td>5.2459</td>
</tr>
<tr>
<td>9.0000</td>
<td>4.4768</td>
<td>5.2156</td>
<td>5.3091</td>
<td>5.3203</td>
</tr>
<tr>
<td>10.0000</td>
<td>4.3066</td>
<td>4.9201</td>
<td>4.6433</td>
<td>4.5612</td>
</tr>
<tr>
<td>11.0000</td>
<td>NaN</td>
<td><strong>4.6747</strong></td>
<td><strong>4.3739</strong></td>
<td><strong>4.3320</strong></td>
</tr>
<tr>
<td></td>
<td>NaN</td>
<td>0.3462</td>
<td>0.3148</td>
<td>0.3088</td>
</tr>
</tbody>
</table>

By calculating the smoothing values, we obtain the values presented in the columns 3 – 5 in the Table 2, for illustration by assigning $\alpha = 0.4$, $\alpha = 0.8$ and $\alpha = 0.9$.

In the next to the last line in the Table 2 are forecast results for the time period 11, therefore for year 2011. The resulting $RMSE$ is in the last line in the Table 2.

So the forecast for year 2011 by using $\alpha = 0.9$ is about 433200 personnel in the industrial production in Slovakia.
This concept is illustrated in Figure 1, which shows a time series observed for periods 1 to 10 and the forecast for period 11.

![Plot for actual values and forecast values using α = 0.4, α = 0.8 and α = 0.9](image)

**Fig. 1 Plot for actual values and forecast values using α = 0.4, α = 0.8 and α = 0.9**

4. **Summary**

With exponential smoothing the idea is that the most recent observations will usually provide the best guide as to the future, so we want a weighting scheme that has decreasing weights as the observations get older. The choice of the smoothing constant is important in determining the operating characteristics of exponential smoothing. The smaller the value of \( \alpha \), the slower the response. Larger values of \( \alpha \) cause the smoothed value to react quickly – not only to real changes but also random fluctuations [3]. Simple exponential smoothing model is only good for non-seasonal patterns with approximately zero-trend and for short-term forecasting because if we extend past the next period, the forecasted value for that period has to be used as a surrogate for the actual demand for any forecast past the next period. Consequently, there is no ability to add corrective information (the actual demand) and any error grows exponentially.

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**References**


