Micro Black Hole Candidates and the Planck Scale: Schwarzschild micro black holes can only match a few properties of the Planck scale, while a Reissner-Nordström and Kerr micro black hole matches the properties of the Planck scale

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Abstract

One of the main challenges in modern physics has been to understand the Planck scale and to detect it even indirectly. Micro black holes are well known from the literature to be closely connected to the Planck mass. To gain further insight into the Planck scale, a precise study and comparison of already-suggested candidates for micro black holes could be useful. As we will demonstrate, the analyses done in the past did not go into too much depth and have not been very precise, something that has even led to incorrect conclusions and misconceptions around the Planck mass and micro black holes.

A series of mass candidates, all close to or equal to the Planck mass, have been suggested in the literature for micro black holes. All of these candidates have masses that are in proximity of the Planck mass. When predicted and analyzed using the Schwarzschild metric, we show that no mass candidate can match more than a few properties of the Planck scale. However, the extremal solution of the the Reissner-Nordström metric as well as the extremal solution of the Kerr metric allows a Planck mass black hole to encompass all aspects of the Planck scale. Additionally, in the relativistic modified Newton theory suggested by Bagge and Phipps, the Planck mass "black hole" fits all aspects of the Planck scale.

Key Words: Planck scale, micro black hole candidates, black holes, Schwarzschild metric, Reissner-Nordström metric, Kerr metric, Lorentz relativistic mass.

1 Micro black holes seem to be linked to the Planck scale

Max Planck [1, 2] in 1899 and 1906 suggested there were a unique length $l_p = \sqrt{\frac{G\hbar}{c^3}}$, time $t_p = \sqrt{\frac{G\hbar}{c^5}}$, mass $m_p = \sqrt{\frac{\hbar c}{G}}$, and temperature $T_p = \sqrt{\frac{\hbar c^5}{Gk_b^2}}$, today known as the Planck units. They were found by assuming there were three important universal constants: the Newton gravitational constant G, the Planck constant $\hbar$, and the speed of light c and by then combining this with dimensional analysis.

Einstein [3] had already suggested in 1916, in one of his most famous papers on general relativity theory, that a quantum gravity would likely be the next big step in gravitational theory. Then in 1918, Eddington [4] suggested that the Planck length would likely play an important role in such a theory, but without suggesting how. Today, most physicists working on quantum gravity theories think the Planck scale will play an important role in a unified quantum gravity theory; see, for example, [5–7].

Max Planck said little about what the Planck mass represented in the physical world or why it was so special. Loyd Motz [8, 9] in 1962, when working at the Rutherford laboratories, was likely the first to suggest there could be a Planck mass particle. He was well aware that this particle had much larger mass ($m_p \approx 2.17 \times 10^{-8}$ kg) than any observed particle. Therefore, he suggested it had been created in the big bang and that the Planck mass particles had radiated into all the particles we have today.

Soon it was also suggested that micro black holes could be indirectly or directly linked to the Planck scale. This was suggested in 1966 by Markov [10] and in 1971 by Hawking [11]. So, to analyse micro black holes in more depth, at least for some aspects that have not been done before, could potentially add important insight into understanding the Planck scale and even quantum gravitation.

Why have micro black holes not been detected? One possibility is they only existed just after the big bang, but many other speculative ideas exist. Jacobs and Seitzer [12] suggested in 1977 that micro black holes are forming today in the cores of condensed, stellar-mass objects, but this has not been observed directly or indirectly. Chapline [13], MacGibbon [14] and Obermair [15] have suggested black holes as candidates for dark matter; see also [16, 17]. A good start would be more direct detection of dark matter, but also dark matter has not yet been confirmed, and alternative theories not needing dark matter also exist, like Milgrom's [18] modified Newton dynamics as well as other minimum acceleration models; see [19, 20]. Dvali [21] suggested that properties of the micro black holes could likely be detected in the Large Hydron Collider (LHC). Wondrak et. al [22] discussed various ways to detect signatures of micro black holes through particle colliders such as LHC; see also [23, 24]. Bleicher [25] suggested even the possibility of creating micro black holes in the Large Hadron Collider. Still no micro black hole has been detected or created in the LHC so far. One possibility is that one needs stronger accelerators to detect and/or create micro black holes. Sokolov and Pshirkov [26] have suggested one needs a 100 TeV collider to create micro black holes, LHC is only approximately 10TeV; in other words, a particle accelerator 10 times more powerful than the LHC may be needed.

Mack, Song, and Vincent [27] have recently suggested how to detect signatures of micro black holes in the next generation of large-scale neutrino observatories. However, they relied on theories of higher dimensions that are in no way fully accepted, but are naturally still interesting.

Already in 1938, Einstein, Infeld, and Hoffmann [28] showed that elementary particles, such as electrons, could be treated as singularities, a bit similar to micro black holes. Penrose [29] specifically mentioned the idea of electrons with black hole properties. Akhavan [30] has discussed in more detail why micro black holes potentially could be found in electrons, something that also was suggested by Haug [31], but with a somewhat different perspectives.

Hawking [11] was likely the first to suggest that charged black holes may play a role similar to the atomic nuclei. Flambaum and Berengut [32] suggested protons and electrons and even a single quark can be captured by black holes, and may lead to colored black holes that play the role of an extremely heavy quark. Dokuchaev and Eroshenko [33] suggested black hole- type atoms. Nagatani [34] suggested a charged singularity at the center and quantum fluctuations of fields around the singularity quite similar to atoms. Somewhat similarly, Haramein [35] suggested micro black holes could be related to protons.

Ding and Hao [36] have suggested there could be micro black holes passing by us that comes from the big bang and that one should try to detect them. Harms and Micu [37] have suggested that microscopic black holes could be a source of ultra-high energy γ-rays, and that, if this is the case, then detection of ultra-high gamma rays would be an indirect detection of micro black holes.

Potential signatures of micro black holes have been searched for in many places. Ball lighting is a phenomena not fully understood, so Burov and Sheleg [38] even put forward the speculative hypothesis that ball lightning can represent the "micro black hole" signature, but based on analysis of data from ball lighting against predictions from micro black holes, they reject this hypothesis. So, that seems one less place less to look for micro black holes.

Despite more than 70 years passing since the idea of micro black holes was suggested, and more than 100 years since the Planck scale was suggested, little progress has been made in finding them, at least until perhaps very recently. One thing is clear: further theoretical and experimental studies related to micro black holes are clearly warranted. Bah et. al [39] suggested looking for special gravitational footprints related to micro black holes.

Potentially important in the analysis of micro black holes could be a series of additional properties linked to the Planck scale that were derived in the years after Max Planck and after the first introduction of micro black holes. Harrison in 1970 [40] seems to be the first to have described the Planck mass density. Hawking [41] in 1978 was likely the first to mention Planck volume. Scarpetta [42] introduced Planck acceleration in 1984; see also Fala and Landsberg [43], who assumes the Planck acceleration is the maximum acceleration possible. That the Planck acceleration is the maximum makes sense as, if the Planck acceleration acts on a rest mass particle even for a duration as short as the Planck time, then the velocity goes from zero to that of the speed of light in the Planck time. So, if the Planck time is the shortest possible time, then the Planck acceleration must be the highest possible acceleration. Planck power was likely first introduced by Gerlach [44] in 1996. Christian [45][1] introduced what we can call the Planck speed, $\frac{l_p}{t_p} = c$ in 2004. Table 1 shows some Planck scale properties.

When it comes to the Planck density, this is often expressed as simply $\frac{E_p}{l_p^3} = \frac{m_p c^2}{l_p^3}$. This will, however, give the density if a Planck mass (energy) was inside a cube (box) with sides equal to the Planck length. When working with black holes in the spherical metrics such as the Schwarzschild metric it is, however, natural to work with spherical

[1] or we would suspect some even much earlier, but have not found a direct reference showing so, but then it is difficult to know all that has been published.

Planck property	Formula:
Planck length	$l_p = \sqrt{\frac{G\hbar}{c^3}}$
Planck time	$t_p = \sqrt{\frac{G\hbar}{c^5}}$
Planck mass	$m_p = \sqrt{\frac{\hbar c}{G}}$
Planck energy	$E_p = \sqrt{\frac{\hbar c^5}{G}}$
Planck temperature	$T_p = \sqrt{\frac{\hbar c^5}{Gk_B^2}}$
Planck surface area sphere	$A = 4\pi r^2 = 4\pi l_p^2$
Planck surface area cube	$A = 6l_p^2$
Planck volume sphere	$V_p = \frac{4}{3}\pi l_p^3$
Planck volume cube	$V_{p,c} = r^3 = l_p^3$
Planck acceleration	$a_p = \frac{Gm_p}{l_p^2}$
Planck density	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$
Planck power	$\frac{m_p c^2}{t_p} = \frac{c^4}{G}$
Planck pressure	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$
Planck speed	$\frac{l_p}{t_p} = c$
Planck charge	$q_p = \frac{e}{\alpha} = \sqrt{\frac{\hbar}{c}10^7}$
Planck angular momentum	$m_p c l_p$

Table 1: The table shows a series of properties linked to the Planck scale.

objects, so we will assume the micro black hole is inside a sphere. To have the Planck energy inside a sphere gives Planck density $\frac{E_p}{\frac{4}{3}\pi l_p^3} = \frac{m_p c^2}{\frac{4}{3}\pi l_p^3}$.

In this paper, we will investigate to what degree different micro black hole candidates that have been suggested in the literature match the different Planck properties. Is there one mass candidate that is outstanding or is there something about the theory that is not yet understood?

2 Black holes: a short historical background

The idea of a mass with a gravity field so strong that not even light can escape goes all the way back to Michell [46] in 1784. He had calculated the radius of a hypothetical star based on Newtonian gravity and the radius was identical to the Schwarzschild radius. Michell called these objects dark stars, as the gravity just inside this radius was so strong that not even light could escape; that is, the escape velocity would be above c just inside this radius (horizon). The escape velocity in Newton [47] mechanics can be found by simply solving the following equation with respect to v:

$$\frac{1}{2}mv^2 - G\frac{Mm}{R} = 0 \tag{1}$$

This gives

$$v = \sqrt{\frac{2GM}{R}} \tag{2}$$

which is the well-known escape velocity, and normally therefore given the symbol v_e. This is exactly the same escape velocity one can derive from the Schwarzschild metric in general relativity theory; see Augousti and Radosz [48]. Next we can set $v = c$ and solve with respect to R and we get $R = \frac{2GM}{c^2}$, which is identical to the Schwarzschild radius. The escape velocity and the Schwarzschild radius derived from Einstein's [3] field equation are identical to that of the Newton theory. A black hole is basically just an object where the mass is inside this radius. This means, in general, not even light can escape. The term "black hole" was likely first suggested in 1967 by a student at a lecture by John Wheeler.

3 The Compton wavelength and the Schwarzschild relation

A series of researchers claim the Schwarzschild radius has to be equal to, or larger than, the Compton [49] wavelength of the mass in question. For example, Lake and Carr [50, 51] stated:

> "*The Compton wavelength gives the minimum radius within which the mass of a particle may be localized due to quantum effects, while the Schwarzschild radius gives the maximum radius within which the mass of a black hole may be localized due to classical gravity.*"

The same Carr [52] in 2017 seemed to claim a black hole must be larger than the reduced Compton wavelength of the particle. However, there seems to be no full agreement if the black hole of the Schwarzschild radius must be bigger or equal to the Compton wavelength or the reduced Compton wavelength. The difference is quite large as the Compton wavelength is 2π times the reduced Compton wavelength. We will, in this paper, try to come to a more precise answer for this.

The Planck mass has a Schwarzschild radius of:

$$R_s = \frac{2Gm_p}{c^2} = 2l_p \tag{3}$$

while the reduced Compton wavelength of the Planck mass must be

$$\bar{\lambda} = \frac{\hbar}{m_p c} = l_p \tag{4}$$

The Compton wavelength is then $\lambda = 2\pi l_p$. That is, a Planck mass as a candidate for a micro black hole has a larger Schwarzschild radius than its reduced Compton wavelength and thereby fits the requirement that the Schwarzschild radius is bigger or equal to this. It is smaller than the Compton wavelength, so if it is a criterion that the Schwarzschild radius must be larger than the Compton wavelength, then the Planck mass does not fit this requirement. This means the Planck mass cannot be a black hole under the criteria that the Schwarzschild radius must be larger than the Compton wavelength, but it can be a black hole based on the criteria that the Schwarzschild radius must be larger, or equal to, the reduced Compton wavelength. One can also naturally wonder why the Schwarzschild radius of a Planck mass micro black hole should be exactly twice that of the reduced Compton wavelength of the Planck mass. Is it a coincidence, or does it mean something special? We will try to also answer such questions.

Lake and Carr [50] even suggested a modified reduced Compton wavelength given by $\lambda_C = \frac{\hbar}{2mc} = 2\bar{\lambda}$. This would therefore mean the modified reduced Compton wavelength would match the Schwarzschild radius of the micro black hole with a mass equal to the Planck mass. To modify the reduced Compton wavelength is not, we think, actually the way to go. As we will show, there is another and, in our view, more sound way to get a match between the reduced Compton wavelength and the radius of a micro black hole.

Lake [51] claimed "*'particles' with rest masses $m > m_p$ have sub-Planckian Compton wavelengths, $\lambda < l_p$, but super-Planckian Schwarzschild radii, $R_S > l_p$, and may be interpreted as black holes*". Only the latter is actually correct; a mass with mass higher then the Planck mass will always have a Schwarzschild radii, larger than the Planck length, but even this is unprecise as actually any mass above half the Planck mass has a Schwarzschild radii larger than the Planck length. Further, several masses with mass below the Planck mass have a Compton wavelength larger than the Planck length but have a reduced Compton wavelength smaller than the Planck length.

A series of papers gives the incorrect impression that the Schwarzschild radius is the same as the Planck length for a Planck mass size black hole; we think this is one of the reasons few, if any, have asked the question of why they are close to each other, but not the same. For example, Baez [53] incorrectly stated that the Schwarzschild radius is:

$$R_s = \frac{GM}{c^2} \tag{5}$$

and since the reduced Compton wavelength is correctly given by:

$$\bar{\lambda} = \frac{\hbar}{mc} \tag{6}$$

and since the Planck mass is given by $m_p = \sqrt{\frac{\hbar c}{G}}$, now inputting this mass into Baez incorrect Schwarzschild radius we get:

$$R_s = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \sqrt{\frac{G\hbar}{c^3}} \tag{7}$$

and for the reduced Compton wavelength we get

$$\bar{\lambda} = \frac{\hbar}{mc} = \frac{\hbar}{c\sqrt{\frac{\hbar c}{G}}} = \sqrt{\frac{G\hbar}{c^3}} = l_p \tag{8}$$

And since the Planck length is given by $\sqrt{\frac{G\hbar}{c^3}}$ this indicates the reduced Compton wavelength and the Schwarzschild radius of a Planck mass are the same. Baez [53] has, however, used the incorrect formula for the Schwarzschild radius without pointing this out. It should be $\frac{2GM}{c^2}$ rather than $\frac{GM}{c^2}$. By correcting for this we get:

$$R_s = \frac{2G\sqrt{\frac{\hbar c}{G}}}{c^2} = 2\sqrt{\frac{G\hbar}{c^3}} = 2l_p \tag{9}$$

which is twice that of the reduced Compton wavelength of the Planck mass. Also Adler [5] gave the incorrect impression that the reduced Compton wavelength of a Planck mass is equal to the Schwarzschild radius as he wrote about the Planck mass; "*when the size approaches the Schwarzschild radius* $l \approx GM/c^2$". Well, it is not directly wrong as he uses an approximation sign, but the Schwarzschild radius is 2 times this. The impression one gets from several authors is that the reduced Compton wavelength from the Planck mass is identical to the Schwarzschild radius, but the Schwarzschild is twice that. Much of the lack of precision in the micro black hole literature has led to the incorrect impression among many that the Planck mass perfectly fits the criteria of a micro black hole under general relativity theory, which is not the case, as we will demonstrate in this paper.

There is only one mass for which the reduced Compton wavelength is equal to the Schwarzschild radius. It can be found by solving the following formula with respect to x:

$$\frac{\hbar}{xm_pc} = \frac{2Gxm_p}{c^2} \tag{10}$$

This gives $x = \frac{1}{\sqrt{2}}$; that is, only a micro black hole with mass $m_b = \frac{1}{\sqrt{2}}m_p$ has a reduced Compton wavelength equal to the Schwarzschild radius. Such a micro black hole was likely first suggested in 2016 by Haug[2] [55], but indirectly suggested already by Harrison [40] in 1970.

Alternatively, one could look at the Compton wavelength rather than the reduced Compton wavelength. Then the only mass that has a Schwarzschild radius equal to the Compton wavelength of the mass is given by solving the formula:

$$\frac{h}{xm_pc} = \frac{\hbar 2\pi}{xm_pc} = \frac{2Gxm_p}{c^2} \tag{11}$$

This leads to a micro black hole mass of $m_b = \sqrt{\pi}m_p$. This candidate for a micro black hole was first suggested by Haug in 2016 [55] and discussed in more detail by Faraoni [56] in 2017.

So, to summarize this section, the Planck mass as a candidate for a micro black hole gives a Schwarzschild radius twice the reduced Compton wavelength of the Planck mass and a radius that is shorter than the Compton wavelength. The micro black hole candidate with mass $\frac{1}{\sqrt{2}}m_p$ has an identical Schwarzschild radius and a reduced Compton wavelength of $R_s = \bar{\lambda} = \sqrt{2}l_p$, while a micro black hole with mass $\sqrt{\pi}m_p$ has a Compton wavelength equal to the Schwarzschild radius $R_s = \lambda = 2\sqrt{\pi}l_p$. However, this last micro black hole mass candidate will have a reduced Compton wavelength below the Planck length. Another mass candidate for a micro black hole is half the Planck mass, which was first suggested by Motz and Epstein [57] in 1979. Here, the Schwarzschild radius is exactly equal to the Planck length, but it does not match the reduced Compton wavelength that is now $2l_p$ or the Compton wavelength that is $4\pi l_p$. So, an important question is if some of these mass candidates are more likely to be a micro black hole than the other candidates and, if so, why? These are important questions and we think we may have a plausible answer for them.

[2] See also [54]

4 Schwarzschild Planck mass micro black hole; does it fit the Planck scale?

First, we return to the Planck mass as a candidate for the micro black hole. Here, the micro black hole clearly matches the Planck mass per definition; that is, one aspect of the Planck scale. However, we have already seen that the Schwarzschild radius is twice that of the Planck length, while the reduced Compton wavelength of this black hole is the Planck length. The time for light to cross the radius of the micro black hole is $2t_p$; further, the gravitational acceleration at the black hole surface (Schwarzschild radius) is given by:

$$g = \frac{Gm_p}{R_s^2} = \frac{1}{4}\frac{Gm_p}{l_p^2} \tag{12}$$

In other words, its gravitational acceleration is $\frac{1}{4}$ of the Planck acceleration, so it also does not seem to match this property of the Planck scale. The energy density of this micro black hole is:

$$\rho_E = \frac{m_p c^2}{\frac{4}{3}\pi R_s^3} = \frac{1}{8}\frac{m_p c^2}{\frac{4}{3}\pi l_p^3} \tag{13}$$

That is, the energy density is only $\frac{1}{8}$ of the Planck density.

The escape velocity at the Schwarzschild radius is naturally c, actually per definition as R_s is where the escape velocity is c. An interesting question to ask is what the escape velocity at the reduced Compton wavelength is, which is the Planck length. It is:

$$v_e = \sqrt{\frac{2Gm_p}{l_p}} = c\sqrt{2} \tag{14}$$

That is, the escape velocity at the Planck length is above c, which is impossible according to the special and general relativity theory that this is derived from. One could, and perhaps should, argue that since l_p and the reduced Compton wavelength are inside the black hole, it makes no sense to calculate the escape velocity inside the surface of the black hole. But before we exclude predictions for what happens at the Planck length radius, let us also look at the predicted gravitational time dilation at the Planck length distance. In general relativity, the gravitational time dilation is given by:

$$T_h = T_L\sqrt{1 - \frac{v_{e,h}^2}{c^2}} \tag{15}$$

That is when T_h is when T_h is the time very far away from the gravitational field, when both T_h and T_L are in the gravitational field we have

$$\begin{aligned} \frac{T_h}{\sqrt{1-\frac{v_{e,h}^2}{c^2}}} &= \frac{T_L}{\sqrt{1-\frac{v_{e,L}^2}{c^2}}} \\ T_h &= T_L\frac{\sqrt{1-\frac{v_{e,h}^2}{c^2}}}{\sqrt{1-\frac{v_{e,L}^2}{c^2}}} \end{aligned} \tag{16}$$

For general relativity, the escape velocity is given by $v_e = \sqrt{2GM/R^2}$, so here we have $v_{e,h} = \sqrt{2GM/R_h^2}$ and $v_{e,L} = \sqrt{2GM/R_L^2}$. This means we have the well-known formula:

$$T_h = T_L\frac{\sqrt{1-\frac{2GM}{R_h c^2}}}{\sqrt{1-\frac{2GM}{R_L c^2}}} \tag{17}$$

We can call $\sqrt{1-\frac{2GM}{Rc^2}}$ the gravitational time dilation factor. In the special case of a Planck mass size object, the reduced Compton wavelength of this mass is the Planck length. That is $\bar{\lambda} = \frac{\hbar}{m_p c} = l_p$. It is often assumed a mass can be constrained inside its reduced Compton wavelength. Assume we now stand at the surface of this mass, that is at $R = l_p$, then we get a time dilation factor of:

$$\sqrt{1-\frac{2GM}{Rc^2}} = \sqrt{1-\frac{2Gm_p}{l_pc^2}} = \sqrt{1-2} = \sqrt{-1} = i \tag{18}$$

In other words, the time dilation there is imaginary. Imaginary numbers in quantum mechanics are not that abnormal, but imaginary output in gravitational analysis like we have here is abnormal. How should this be interpreted? Actually, in general relativity theory, the simplest solution is to say that this is an irrelevant result as the Schwarzschild radius of the Planck mass is, as we have shown before, $R_s = \frac{2Gm_p}{c^2} = 2l_p$. It is often typically considered meaningless to try to explain what is happening inside a black hole. This makes some sense if one only looks at general relativity and not, at the same time, at alternative or extended theories, something we will soon do. Let us also look at what the time dilation factor is at the Schwarzschild radius; it is then given by:

$$\sqrt{1-\frac{2GM}{R_sc^2}} = \sqrt{1-\frac{2Gm_p}{2l_pc^2}} = \sqrt{1-1} = \sqrt{0} = 0 \tag{19}$$

In other words, time ceases to exist at the Schwarzschild radius; something that makes logical sense.

Our main point is simply that the Planck mass as a candidate for a micro black hole does not match any many properties of the Planck scale. It matches the Planck mass and then naturally the Planck energy as the Planck energy is simply the mass multiplied with c^2. On the other hand, the surface acceleration, the time to cross the radius by a light beam, and the density are, for the Planck mass micro black hole, all different than the properties of the Planck scale. Still, they are relatively close to the properties of the Planck scale, so is this a coincidence or does it mean something special?

5 The suggested masses for micro black holes and their match to the Planck scale

As already indicated, in addition to the Planck mass, a series of candidates has been suggested as masses for a micro black hole. All these candidates have masses close to the Planck mass. Markov suggested the micro black hole was of the size of a Planck mass, Hawking indicated it was approximately equal to the Planck mass. Motz and Epstein in 1979 tried to get the micro black hole to match the Planck length rather than the Planck mass and then got a mass equal to half the Planck mass. Haug and Faraoni suggested a mass candidate equal to $\sqrt{\pi}$ times the Planck mass; this gives an identical Schwarzschild radius and a Compton wavelength. One can also suggest a mass equal to $\frac{1}{\sqrt{\pi}}$ as this gives a match between the Schwarzschild radius and the reduced Compton wavelength. Still, none of these candidates matches more than, at maximum, a few aspects of the Planck scale. For example, I could say I think it is an important property that the micro black hole surface acceleration should match the Planck acceleration. The only mass that does this has a mass off $\frac{1}{4}$ of the Planck mass. Such a micro black hole would, however, have a Schwarzschild radius smaller than the Planck length, which is impossible if the Planck length imposes a limit on minimum size.

Table 2 shows the masses of the micro black hole candidates we have found suggested in the literature. The candidates have typically been selected to match a property of the Planck scale, or to have the Schwarzschild radius match the Compton or reduced Compton wavelength. As we can see, none of the suggested candidates, including a few new candidates suggested in this paper, such as the mass candidate to match the Planck density, fits more than one to three aspects of the Planck scale when analyzed from the Schwarzschild metric. Properties of the micro black hole that match the Planck scale or other suggested important aspects are marked in bold.

So how should this be interpreted? Are there many types of micro black holes, or do micro black holes even change their mass over time due to some radiation and thereby represent one or several of these micro black hole candidates? It is not obvious from the literature how any of these candidates are outstanding as the mass of a micro black hole relative to the others. All we can say is that a mass approximately equal to the Planck mass seems somehow to be related to the Planck scale. So why is this the case, and could there be something here that we do not yet understand?

	Fits at least one property of the Planck scale					No fits to the Planck scale				
Candidate number:	1	2	3	4	5	6	7	8	9	10
Reference first mentioned:	[15]	[57]	[55] [56]	[54]	This paper	[10]	[54]	[54]	[54]	[40] and [55]
Micro black hole mass candidate:	$\boldsymbol{m_p}$	$\frac{1}{2}m_p$	$\sqrt{\pi}m_p$	$\frac{1}{4}m_p$	$\frac{1}{\sqrt{8}}m_p$	$2m_p$	πm_p	$\frac{1}{\sqrt{\pi}}m_p$	$\sqrt{2}m_p$	$\frac{1}{\sqrt{2}}m_p$
Schwarzschild radius $R_s = \frac{2Gm}{c^2}$	$2l_p$	$\boldsymbol{l_p}$	$2\sqrt{\pi}l_p$	$\frac{1}{2}l_p$	$\frac{1}{\sqrt{2}}l_p$	$4l_p$	$2\pi l_p$	$\frac{2}{\sqrt{\pi}}l_p$	$\sqrt{8}l_p$	$l_p\sqrt{2}$
Reduced Compton wavelength $\frac{\hbar}{mc}$	$\boldsymbol{l_p}$	$2l_p$	$\frac{l_p}{\sqrt{\pi}}$	$4l_p$	$l_p\sqrt{8}$	$\frac{l_p}{2}$	$\frac{l_p}{\pi}$	$l_p\sqrt{\pi}$	$\frac{l_p}{\sqrt{2}}$	$\boldsymbol{l_p\sqrt{2}}$
Compton wavelength $\frac{h}{mc}$	$2\pi l_p$	$4\pi l_p$	$\boldsymbol{2\sqrt{\pi}l_p}$	$8\pi l_p$	$\sqrt{32}\pi l_p$	πl_p	$2l_p$	$2\sqrt{\pi^3}l_p$	$\sqrt{2}\pi l_p$	$\sqrt{8}\pi l_p$
Schwarzschild time $\frac{R_s}{c}$	$2t_p$	$\boldsymbol{t_p}$	$2\sqrt{\pi}t_p$	$\frac{1}{2}t_p$	$\frac{1}{\sqrt{2}}t_p$	$4t_p$	$2\pi t_p$	$\frac{2}{\sqrt{\pi}}t_p$	$\sqrt{8}t_p$	$\sqrt{2}t_p$
Escape velocity at $\bar{\lambda}$	$c\sqrt{2}$	$\frac{c}{\sqrt{2}}$	$c\sqrt{2\pi}$	$\frac{c}{\sqrt{8}}$	$c/2$	$c\sqrt{8}$	$c\pi\sqrt{2}$	$\frac{c\sqrt{2}}{\sqrt{\pi}}$	$2c$	$\boldsymbol{c}$
Escape velocity at λ	$\frac{c}{\sqrt{\pi}}$	$\frac{c}{2\sqrt{\pi}}$	$\boldsymbol{c}$	$\frac{c}{\sqrt{16\pi}}$	$\frac{c}{\sqrt{8\pi}}$	$\frac{2c}{\sqrt{\pi}}$	$c\sqrt{\pi}$	$\frac{c}{\pi}$	$\frac{c\sqrt{2}}{\sqrt{\pi}}$	$\frac{c}{\sqrt{2\pi}}$
Schwarzschild density $\frac{m}{\frac{4}{3}\pi R_s^3}$	$\frac{1}{8}\rho_p$	$\frac{1}{2}\rho_p$	$\frac{1}{8\pi}\rho_p$	$2\rho_p$	$\boldsymbol{\rho_p}$	$\frac{1}{32}\rho_p$	$\frac{1}{8\pi^2}\rho_p$	$\frac{\pi}{8}\rho_p$	$\frac{1}{16}\rho_p$	$\frac{1}{4}\rho_p$
Planck energy mc^2	$\boldsymbol{E_p}$	$\frac{E_p}{2}$	$\sqrt{\pi}E_p$	$\frac{E_p}{4}$	$\frac{E_p}{\sqrt{8}}$	$2E_p$	πE_p	$\frac{E_p}{\sqrt{\pi}}$	$\sqrt{2}E_p$	$\frac{E_p}{\sqrt{2}}$
Surface area acceleration $g = \frac{Gm}{R_s^2}$	$\frac{a_p}{4}$	$\frac{a_p}{2}$	$\frac{a_p}{4\sqrt{\pi}}$	$\boldsymbol{a_p}$	$\frac{a_p}{\sqrt{2}}$	$\frac{a_p}{8}$	$\frac{a_p}{4\pi}$	$\frac{a_p\sqrt{\pi}}{4}$	$\frac{a_p}{\sqrt{32}}$	$\frac{a_p}{\sqrt{8}}$
Number of Planck scale matches	3	2	1	1	1	0	0	0	0	0
Compton or reduced matches	0	0	1	0	0	0	0	0	0	1
Conflict Planck scale limits $R_s < l_p$	*no*	no	no	yes	yes	no	no	no	no	no

Table 2: The table shows a series of suggested mass candidates that have been suggested to be a micro black hole. None of them matches more than a few properties of the Planck scale as predicted and analysed in the Schwarzschild metric . Why is this? Are there a series of different micro black holes or is there something we do not understand? The next section will give a possible answer.

6 Reissner-Nordström micro black holes

The Reissner-Nordström [58, 59] metric was derived for spherical stationary objects with charge, is an exact solution to Einstein's [3] field equation and is given by:

$$ds^2 = \left(1 - \frac{2GM}{c^2R} + \frac{R_Q^2}{R^2}\right)c^2dt^2 - \left(1 - \frac{2GM}{c^2R} + \frac{R_Q^2}{R^2}\right)^{-1}dR^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{20}$$

The Schwarzschild metric is a special case of this metric when $R_Q = 0$, that is when the black hole has no charge. When working with SI units, as we will do, the following definition of R_Q^2 applies (see, for example, [60, 61]):

$$R_Q^2 = k_e qq\frac{G}{c^4} = \frac{q^2}{4\pi\epsilon_0}\frac{G}{c^4} \tag{21}$$

Here, $k_e = \frac{1}{4\pi\epsilon_0}$ represents the Coulomb constant, q denotes the charge, and G stands for Newton's gravitational constant. In the special case where the mass is a Planck mass with a Planck charge q_p, the metric becomes:

$$ds^2 = \left(1 - \frac{2Gm_p}{c^2R} + \frac{q_p^2G}{4\pi\epsilon_0c^4R^2}\right)c^2dt^2 - \left(1 - \frac{2Gm_p}{c^2R} + \frac{q_p^2G}{4\pi\epsilon_0c^4R^2}\right)^{-1}dR^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{22}$$

where q_p is the Planck charge given by (see [62, 63]):

$$q_p = \sqrt{4\pi\epsilon_0\hbar c} = \frac{e}{\alpha} = \sqrt{\frac{\hbar}{c}10^7} \tag{23}$$

The event horizons of a Reissner-Nordström black hole are given by (see for example [61, 64, 65])

$$R_\pm = \frac{GM}{c^2} \pm \sqrt{\frac{G^2M^2}{c^4} - R_Q^2} \tag{24}$$

, that is Reissner-Nordström black holes are predicted to have two horizons, however in the special case when $R_Q^2 = k_eqq\frac{G}{c^4} = \frac{G^2M^2}{c^4}$, then it clearly has only one event horizon, that is $R = \frac{GM}{c^2}$. This is well known as the extremal solution of the Reissner-Nordström metric (see [66]). When we work with Planck charge then R_Q^2 is given by

$$R_Q^2 = k_eq_pq_p\frac{G}{c^4} \tag{25}$$

In the special case of the electrostatic force between two Planck charges then this is identical to the gravity force between two Planck masses and we have (see also [67])

$$k_e \frac{q_p q_p}{R^2} = G \frac{m_p m_p}{R^2} \tag{26}$$

So this mean we must have

$$R_{\pm} = \frac{GM}{c^2} \pm \sqrt{\frac{G^2 m_p^2}{c^4} - k_e q_p q_p \frac{G}{c^4}} = \frac{GM}{c^2} \tag{27}$$

.

That is a Planck mass with Planck charge in the Reissner-Nordström metric will form a (micro) black hole with radius

$$R = \frac{Gm_p}{c^2} = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \sqrt{\frac{G\hbar}{c^3}} = l_p \tag{28}$$

. This is in contrast to the Schwarzschild metric, where the Planck mass micro black hole has an event horizon of $R_s = \frac{2Gm_p}{c^2} = 2l_p$. We can now calculate a series of properties of the Reissner-Nordström micro black hole. The many different properties of such a micro black hole are given in Table 3. As we can see, the Reissner-Nordström micro black hole matches all the properties of the Planck scale, while the Schwarzschild Planck mass micro black hole only matches a few properties of the Planck scale. We have also demonstrated in the previous sections that this is the case for any type of Schwarzschild-type micro black hole mass candidate; they can at most match a few properties of the Planck scale.

	Schwarzschild metric	Reissner-Nordström extremal solution
Mass micro black hole candiate:	$\boldsymbol{m_p}$	$\boldsymbol{m_p}$
Escape radius where $v = c$	$2l_p$	$\boldsymbol{l_p}$
Reduced Compton wavelength $\frac{\hbar}{mc}$	$\boldsymbol{l_p}$	$\boldsymbol{l_p}$
Time to travel radius	$2t_p$	$\boldsymbol{t_p}$
Escape velocity at $\bar{\lambda}$	$c\sqrt{2}$	$\boldsymbol{c}$
Escape velocity at λ	$c/\sqrt{\pi}$	$c/\sqrt{2\pi}$
Density $\frac{m}{\frac{4}{3}\pi R^3}$	$\frac{1}{8}\rho_p$	$\boldsymbol{\rho_p}$
Planck energy mc^2	$\boldsymbol{E_p}$	$\boldsymbol{E_p}$
Surface area acceleration $\kappa = \frac{Gm}{R_s^2}$	$\frac{a_p}{4}$	$\boldsymbol{a_p}$
Charge	0	$\boldsymbol{q_p}$
Angular momentum	0	0
Number of Planck scale match	3	9
Reduced Compton wavelength match	*no*	yes

Table 3: The table shows predictions for a Planck mass micro black hole under the standard Schwarzschild metric (weak field) and alternatively as predicted from Reissner-Nordström extremal solution. Under the standard Schwarzschild, we see that the Planck mass micro black hole matches only a few aspects of the Planck scale, while the predictions from the Reissner-Nordström extremal solution matches all the aspects of the Planck scale.

7 Rotating micro black holes

The Kerr [68] metric[3] is given by (see, for example, [70, 71]):

$$ds^2 = \frac{\Delta}{\Sigma}\left(cdt - a\sin^2\theta d\phi\right)^2 - \frac{\Sigma}{\Delta}dR^2 - \Sigma d\theta^2 - \frac{\sin^2\theta}{\Sigma}\left(adt - (R^2 + a^2)d\phi\right)^2, \tag{29}$$

where $\Delta = 1 - \frac{R_s}{R} + \frac{a^2}{R^2}$ and $\Sigma = 1 + \frac{a^2\cos^2\theta}{R^2}$, and $R_s = \frac{2GM}{c^2}$. Furthermore, the Kerr parameter $a = \frac{J}{Mc}$, where J is the angular momentum of the mass M. Often, the Kerr parameter is described as the angular momentum per mass. This is in geometric units when $G = c = 1$. However, here we use SI units. The Kerr metric can be used for rotating black holes without charge. The horizons in this metric are given by:

$$R_H = \frac{GM}{c^2} \pm \sqrt{\frac{G^2M^2}{c^4} - a^2}. \tag{30}$$

[3] When expressed in Boyer-Lindquist [69] coordinates.

For a Planck mass particle, the Planck angular momentum is given by $J = Mvr = m_p c l_p$. Therefore, the horizon of a Planck mass black hole with radius l_p in the Kerr metric must be

$$R_H = \frac{Gm_p}{c^2} \pm \sqrt{\frac{G^2 m_p^2}{c^4} - \frac{(m_p c l_p)^2}{m_p^2 c^2}} = \frac{Gm_p}{c^2} = l_p. \tag{31}$$

This means that it corresponds to the extremal solution of the Kerr metric. The rotating Planck mass black holes will match all the properties of the Planck scale except for the Planck charge, in contrast to the Schwarzschild metric, where micro black holes only match a few aspects of the Planck scale.

The most general metric for spherical black holes is the Kerr-Newman [72, 73] metric. We will not present the full metric here, but the horizons are given by:

$$R_H = \frac{GM}{c^2} \pm \sqrt{\frac{G^2 M^2}{c^4} - a^2 - Q^2}. \tag{32}$$

To ensure that the micro black hole matches the Planck scale for the Planck mass, we must have:

$$R_H = \frac{Gm_p}{c^2} \pm \sqrt{\frac{G^2 m_p^2}{c^4} - a^2 - Q^2} = l_p. \tag{33}$$

This condition holds when $a^2 + Q^2 = \frac{G^2 m_p^2}{c^4}$. In the special case where $a = 0$ and $Q = \frac{Gm_p}{c^2}$, it simplifies to the Reissner-Nordström micro black hole we discussed earlier. This then naturally represents a non-rotating black hole with Planck charge. If $Q = 0$ and $a = \frac{Gm_p}{c^2}$, it corresponds to the Kerr solution for a micro black hole that we just discussed. It represents a rotating black hole with Planck angular momentum but without charge. What is important is that the Reissner-Nordström metric, the Kerr metric, and the Kerr-Newman metric all have a very good match with many properties of the Planck scale for a micro black hole, while the Schwarzschild metric can only match a few properties. This could be important for quantum gravity research as well as a natural way to better understand micro black holes.

8 The Bagge an Phipps relativistic Newtonian model

Bagge [74] and Phipps [75] have suggested the following relativistic modification of the Newton gravitational force formula

$$F = G\frac{Mm\gamma}{R^2} \tag{34}$$

where γ is the Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$. To find the escape velocity we then need to solve the following formula with respect to v

$$mc^2\gamma - mc^2 - G\frac{Mm\gamma}{R} = 0 \tag{35}$$

This gives (see [76]):

$$v = \sqrt{\frac{2GM}{R} - \frac{G^2 M^2}{R^2 c^2}} \tag{36}$$

Next we set $v = c$ and solve with respect to R to obtain:

$$R = \frac{GM}{c^2} \tag{37}$$

. Interestingly, this corresponds precisely to the horizon of the Reissner-Nordström extremal solution. In the relativistic Newtonian solution, we also observe that the orbital velocity equals the escape velocity when at the event horizon. This suggests that photons (light) could potentially orbit the black hole (dark body) at the horizon, implying the presence of a rotating black hole.

Further it is already well known that the escape velocity and the radius where the escape velocity is equal to c are also identical in standard Newton gravity and in the Schwarzschild metric. Hawking [77] emphasized the similarity between Newton dark bodies and black holes, not only mathematically but also to a large degree in interpretation.

On the other hand, researchers such as Loinger [78] claim that the dark body predicted in Newtonian theory has nothing to do with the black holes predicted by Einstein's general relativity theory. We will not conclude one way or the other if these relations are coincidences or if they possibly means there are more connections between Newton theory and general relativity theory than presently understood, but we think it is a question that should be investigated further before one can make too firm conclusions.

Table 4 below compares the properties of the Planck mass size micro black hole as predicted from general relativity in the Schwarzschild metric, the Reissner-Nordström metric extremal solution, and also in the Bagge Phipps relativistic modified Newton model. As we can see the Reissner-Nordström metric and the Bagge and Phipps model are both able to match the Planck scale while the Schwarzschild metric Planck mass black hole is only able to match a few properties of the Planck scale.

	Schwarzschild metric	Relativistic modified Newton model	Reissner-Nordström extremal solution	Kerr extremal solution
Mass micro black hole candiate:	$\boldsymbol{m_p}$	$\boldsymbol{m_p}$	$\boldsymbol{m_p}$	$\boldsymbol{m_p}$
Escape radius where $v = c$	$2l_p$	$\boldsymbol{l_p}$	$\boldsymbol{l_p}$	
Reduced Compton wavelength $\frac{\hbar}{mc}$	$\boldsymbol{l_p}$	$\boldsymbol{l_p}$	$\boldsymbol{l_p}$	$\boldsymbol{l_p}$
Compton wavelength $\frac{h}{mc}$	$2\pi l_p$	$2\pi l_p$	$2\pi l_p$	$2\pi l_p$
Time to travel radius	$2t_p$	$\boldsymbol{t_p}$	$\boldsymbol{t_p}$	$\boldsymbol{t_p}$
Escape velocity at $\bar{\lambda}$	$c\sqrt{2}$	$\boldsymbol{c}$	$\boldsymbol{c}$	$\boldsymbol{c}$
Escape velocity at λ	$c/\sqrt{\pi}$	$c/\sqrt{2\pi}$	$c/\sqrt{2\pi}$	$c/\sqrt{2\pi}$
Density $\frac{m}{\frac{4}{3}\pi R^3}$	$\frac{1}{8}\rho_p$	$\boldsymbol{\rho_p}$	$\boldsymbol{\rho_p}$	$\boldsymbol{\rho_p}$
Planck energy mc^2	$\boldsymbol{E_p}$	$\boldsymbol{E_p}$	$\boldsymbol{E_p}$	$\boldsymbol{E_p}$
Surface area acceleration $\kappa = \frac{Gm}{R_s^2}$	$\frac{a_p}{4}$	$\boldsymbol{a_p}$	$\boldsymbol{a_p}$	$\boldsymbol{a_p}$
Charge	0	?	$\boldsymbol{q_p}$	0
Angular momentum	0	$\boldsymbol{m_p c l_p}$ (?) see text	0	$\boldsymbol{m_p c l_p}$
Number of Planck scale match	3	8 or 9	9	9
Reduced Compton wavelength match	*no*	yes	yes	yes

Table 4: The table shows predictions for a Planck mass micro black hole .

The Bagge and Phipps relativistic modified Newton model was dismissed quite early on due to its incorrect prediction of Mercury's precession, as pointed out by Peters [79], something admitted by Phipps [75]. However, recently Corda [80] has suggested that taking into account both the mass of Mercury and special relativistic effects, this modified Newton theory gives a correct prediction of Mercury's precession. Vossoss, Vossos, and Massouros [81] have suggested that a new central scalar gravitational potential, according to special relativity and Newtonian physics, explains the precession of Mercury's perihelion. Therefore, it might be too early to reject the relativistic modified Newton theory based on claims that it does not accurately predict Mercury's precession. This seems to require further investigation before firm conclusions can be made.

In the Newton relativistic modified formula we have the relativistic mass $m\gamma$, which is not consider acceptable by great part of the general relativity community, even though it is still an ongoing discussion. First of all we can possibly totally avoid relativistic mass by treating the mass as rest-mass energy as have been suggested by Adler and Bazin [82] in 1965 as well again by Wang [83] in 2020. This would likely mean it is the energy embedded in mass that is relevant to gravity, something that likely can be concluded after developing a proper quantum gravity theory. However it is easy to see this method would remove the need for relativistic mass.

We will however shortly discuss the historical and still ongoing debate on relativistic mass below:

Lorentz [84] already suggested in 1899 that mass was likely relativistic. He suggested a transverse and a longitudinal mass; the formula for his longitudinal mass was $m\gamma$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$ (the standard Lorentz factor). His longitudinal relativistic mass is what a series of university text books today call relativistic mass; see, for example [85, 86]. Modern textbooks referring to relativistic mass do not seem to recognize that the relativistic mass of the form $m\gamma$ that they describe was actually introduced by Lorentz and one gets the impression it was introduced by Einstein, but this is not correct.

Einstein [87], at the end of his most famous 1905 paper on special relativity theory, also suggested two relativistic mass formulas, $m\gamma^2$ and $m\gamma^3$ and neither of these is used today. Max Planck [88] in 1906 then introduced relativistic momentum in the form we know it today. Sometime after the invention of Minkowski [89] space-time, Einstein decided to abandon relativistic mass altogether and instead incorporated Planck's relativistic momentum into a fourth momentum approach. Several well-known physicists, like Adler [90], Taylor and Wheeler [91], Okun [92], and Hecht [93] have all been very negative towards relativistic mass, and even ridiculed researchers using relativistic mass as being unaware that Einstein was negative towards it.

Others, like Rindler, that clearly were supporting both special and general relativity theory, seem to be more positive towards relativistic mass; see [94, 95]. The same is true of Jammer [96]. However, despite researchers being either positive or negative towards relativistic mass, there have been few actual studies trying to find out what incorporating relativistic mass in predictions will lead to.

Sadnin [97] defended relativistic mass in a 1991 paper and went through many of the arguments against relativistic mass put forward by Okun and Taylor and Wheeler. He concluded that their arguments against it are rather weak and stated, "*Relativistic mass paints a picture of nature that is beautiful in its simplicity. We should continue to use relativistic mass along with consistent interpretations of Newton's second law and* $E = mc^2$ *in introductory courses. Insisting on its removal as a useful tool from all textbooks, as Okun does, is a form of unnecessary censorship.*

Atkin [98] in 2015 defends relativistic mass and states in his conclusion, "The term mass should mean relativistic mass rather than the currently popular meaning of rest mass." However, Sullivan [99] comments on the paper and refers to the critics of relativistic mass given by Okun, Adler, and Hecht, but also to the defense of relativistic mass by Sadnin [97]. Sullivan states, "*Of course, there is nothing wrong with introducing relativistic mass, and some physicists have made a case for its retention [7, 8]; ultimately, it comes down to an individual pedagogical decision."* Dai and Dai [100], in a recent 2018 paper, rely on relativistic mass when deriving mass-energy equivalence and the mass-velocity relation without relying on light. They state, "In relativity, the inertial mass of a moving particle is not a constant quantity but a function of the particle's velocity."

Field [101] in a 2020 paper criticizes several of the arguments made by Okun, Adler, and Wheeler on relativistic mass. For example, Field points out that "*Okun is erroneously conflating the 'rest mass of a photon' and the entirely different 'mass equivalent of the energy of a photon'.*"

Oas [102] in 2008 went through over 600 books and papers in physics and showed that a majority of books and papers mentioning relativistic mass are positive about it. However, many of these books and papers only touch upon the topic, so the fact that a large number of physicists (if not the majority) are still positive about relativistic mass is not a good argument in itself.

Of the recent books that really go into a detailed discussion of relativistic mass, Jammer's seems to be neutral on the topic and only concludes that it is a long ongoing debate with many interesting arguments pro and con. Petkov [103] in his 2009 book "*Relativity and The Nature of Spacetime*" criticizes the critics of relativistic mass. He discusses relativistic mass in quite some detail and concludes, "*So, if we cannot talk about relativistic mass, by the same argument we should only talk about proper time, which is invariant, and deny the name 'time' to the coordinate time.*" as he claims that some of the same arguments used against relativistic mass can be used against relativistic time."

We will here not here conclude if relativistic mass is a valid concept or not, or if it is even needed in relativistic modified Newton theory, we are just mentioning this to show that there is an ongoing interesting debate here. That the Reissner-Nordström metric gives the same predictions as relativistic modified Newton for micro black holes in terms of event horizon and also that standard Newton without relativistic effects gives the same prediction of event horizon of the Schwarzschild metric should be investigated further to see if it is a mere coincidence or if there are connections here that can help us understand gravity even better. Only time can tell.

9 Conclusion

We have demonstrated that no mass candidate for a micro black hole discussed in literature can match more than a few aspects of the Planck scale when predicted through the Schwarzschild metric. On the other hand, in the extremal solution of the Reissner-Nordström metric the Planck mass candidate for a micro black hole matches all properties of the Planck scale. Also in the relativistic Newton modified gravity model by Bagge and Phipps one gets a perfect match.

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