

Micro Black Hole Candidates and the Planck Scale

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Abstract

One of the main challenges in modern physics has been to understand the Planck scale and to detect the Planck scale even indirectly. Micro black holes are well known from the literature to be closely connected to the Planck mass. To gain further insight in the Planck scale a precise study and comparison of already suggested candidates of micro black holes could be useful. As we will demonstrate the analysis done in the past have not gone too much in depth and has not been very precise, something that has led to even incorrect conclusions and misconceptions around the Planck mass and micro black holes.

A series of mass candidates all close or equal to the Planck mass have been suggested in the literature for micro black holes. All of these have masses close to the Planck mass. However, as we will discuss in detail here none of the suggested candidates can match more than maximum a few aspects of the Planck scale. Why is this, and are there one micro black hole mass candidate that stands out above the others? Or are there perhaps a series of different types of micro black holes? We will also suggest a new micro black hole candidate that to our own surprise matches all the aspects of the Planck scale, but that require one to re-consider Lorentz relativistic mass that has been ignored by standard gravity theory.

Key Words: Planck scale, micro black hole candidates.

1 Micro black holes seems to be linked to the Planck scale

Max Planck [1, 2] in 1899 and 1906 suggested there where a unique length $l_p = \sqrt{\frac{G\hbar}{c^3}}$, time $t_p = \sqrt{\frac{G\hbar}{c^5}}$, mass $m_p = \sqrt{\frac{\hbar c}{G}}$ and temperature $T_p = \sqrt{\frac{\hbar c^5}{Gk_b^2}}$, today known as the Planck units. They were found by assuming there where three important universal constants, the Newton gravitational constant G , the Planck constant \hbar and the speed of light c and then combining this with dimensional analysis.

Einstein [3] suggested already in one of his most famous papers on general relativity theory in 1916 that a quantum gravity would likely be the next big step in gravitational theory. Already in 1918 Eddington [4] suggested that the Planck length likely would play an important role in such a theory, but without suggesting how. Today most physicists working on quantum gravity theories thinks the Planck scale will play an important role in a unified quantum gravity theory, see for example [5–7].

Max Planck had said little about what the Planck mass represented in the physical world or why it was so special. Loyd Motz [8, 9] in 1962, when working at the Rutherford laboratories, was likely the first to suggest there could be a Planck mass particle. He was well aware that this particle had much larger mass ($m_p \approx 2.17 \times 10^{-8}$ kg) than any observed particle. He therefore suggested it had been created in the big bang and that the Planck mass particles had radiated into all the particles we have today.

Soon it was also suggested micro black holes could be indirectly or directly linked to the Planck scale. This was suggested already in 1966 by Markov [10] and in 1971 by Hawking [11]. So, to analyse micro black holes in more depth at least for some aspects that have not been done before a could potentially add important insight to understand the Planck scale and even quantum gravitation.

Why have micro black holes not been detected? One possibility was they only existed just after the big bang, but many other speculative ideas exist. Jacobs and Seitzer [12] suggested in 1977 that micro black holes are forming today in the cores of condensed, stellar-mass objects, but this has not been observed directly or indirectly. Chapline [13], MacGibbon [14] and Obermair [15] have suggested black holes as candidates for dark matter, see also [16, 17].

A good start would be more direct detection of dark matter, but also dark matter has not yet been confirmed, and alternative theories not needing dark matter also exist, like Milgroms [18] Modified Newton Dynamics as well as other minimum acceleration models see [19–22]. Dvali [23], suggested that properties of the micro black holes could likely be detected in the Large Hadron Collider (LHC). Wondrak et. al [24] discusses various ways to detect signatures of micro black holes through particle colliders such as LHC, see also [25, 26]. Bleicher [27] suggested even the possibility one to create micro black holes in the Large Hadron Collider. Still no micro black hole has been detected or created in LHC so far. One possibility is that one needs stronger accelerators to detect and/or create micro black holes. Sokolov and Pshirkov [28] has suggested one needs a 100 TeV collider to create micro black holes, LHC is only approximately 10 TeV, in other words a 10 times as powerful particle accelerator.

Mack, Song and Vincent [29] have recently suggested how to detect signatures of micro black holes in the next generation of large-scale neutrino observatories, however they rely on theories of higher dimensions that in no way are fully accepted, but naturally still interesting. Ding and Hao [30] has suggested there could be bypassing micro black holes from the big bang and that one should try to detect them. Harms and Micu [31] has suggested that microscopic black holes could be a source of ultra-high energy γ -rays, and that if this was the case then detection of ultra-high gamma rays would be an indirect detection of micro black holes.

Potential signatures of micro black holes have been searched for many places. Ball lightning is a not fully understood phenomenon, so Burov and Sheleg [32] even put forward the speculative hypothesis that ball lightning can represent the “micro black hole” signature, but based on analysis of data from ball lightning against predictions from micro black holes they reject this hypothesis. So, it looks like at least one less place to look for micro black holes.

Despite more than 70 years since the idea of micro black holes were suggested and more than 100 years since the Planck scale was suggested little progress has been done in finding them, at least until perhaps very recently. One thing is clear further theoretical and experimental studies related to micro black holes are clearly warranted.

Bah et. al [33] suggest to look for special gravitational footprints related to micro black holes. In this paper we will refer point out that micro black holes possibly even recently have been detected from gravity observations.

Important in the analysis of micro black holes could pontifically be a series of additional properties linked to the Planck scale that have been derived in the years after Max Planck and after the first introduction of micro black holes. Harrison in 1970 [34] seems to be the first to describe the Planck mass density. Hawking [35] in 1978 is likely the first to mention Planck volume. Scarpetta [36] introduced Planck acceleration in 1984, see also Fala and Landsberg [37] that assume the Planck acceleration is the maximum acceleration possible. That the Planck acceleration is the maximum makes sense, as if the Planck acceleration act on a rest mass particle even for a duration as short as the Planck time, then the velocity goes from zero to that of the speed of light in the Planck time. So, if the Planck time is the shortest possible time, then the Planck acceleration must be the highest possible acceleration. Planck power was likely first introduced by Gerlach [38] in 1996. Christian [39]¹ introduced what we can call the Planck speed, $\frac{l_p}{t_p} = c$ in 2004. Table 1 show some Planck scale properties.

When it comes to the Planck density this is often expressed as simply $\frac{E_p}{l_p^3} = \frac{m_p c^2}{l_p^3}$. This will however give the density if a Planck mass (energy) was inside a cube (box) with sides equal to the Planck length. When working with black holes in the Schwarzschild metric it is however natural to work with spherical objects, so we will assume the micro black hole is inside a sphere. To have the Planck energy inside a sphere gives density $\frac{E_p}{\frac{4}{3}\pi l_p^3} = \frac{m_p c^2}{\frac{4}{3}\pi l_p^3}$.

We will in this paper investigate to what degree different micro black hole candidates that have been suggested in the literature matches the different Planck properties. Is there one mass candidate that is outstanding or is there something with the theory not yet understood?

2 Black holes a short historical background

The idea of a mass where gravity field so strong that not even light can escape goes all the way back to Michell [40] in 1784. He had calculated a radius of a hypothetical star based on Newtonian gravity that were identical to the Schwarzschild radius. Michell called these objects dark stars, as the gravity just inside this radius was so strong that not even light could escape, that is the escape velocity would be above c just inside this radius (horizon). The escape velocity in Newton [41] mechanics can be found by simply solving the following equation with respect to v

$$\frac{1}{2}mv^2 - G\frac{Mm}{R} = 0 \quad (1)$$

¹or we would suspect some even much earlier, but have not found a direct reference showing so, but then it is difficult to know all that has been published.

Planck property	Formula:
Planck length	$l_p = \sqrt{\frac{G\hbar}{c^3}}$
Planck time	$t_p = \sqrt{\frac{G\hbar}{c^5}}$
Planck mass	$m_p = \sqrt{\frac{\hbar c}{G}}$
Planck energy	$E_p = \sqrt{\frac{\hbar c^5}{G}}$
Planck temperature	$T_p = \sqrt{\frac{\hbar c^5}{Gk_B^2}}$
Planck surface area sphere	$A = 4\pi r^2 = 4\pi l_p^2$
Planck surface area cube	$A = 6l_p^2$
Planck volume sphere	$V_p = \frac{4}{3}\pi l_p^3$
Planck volume cube	$V_{p,c} = l_p^3$
Planck acceleration	$a_p = \frac{Gm_p}{l_p^2}$
Planck density	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$
Planck power	$\frac{m_p c^2}{t_p} = \frac{c^4}{G}$
Planck pressure	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$
Planck speed	$\frac{l_p}{t_p} = c$

Table 1: The table shows a series of properties linked to the Planck scale.

This gives

$$v = \sqrt{\frac{2GM}{R}} \quad (2)$$

which is the well known escape velocity, and normally therefore given the symbol v_e . This is exactly the same escape velocity one can derive from the Schwarzschild metric in general relativity theory, see Augousti and Radosz [42]. Next we can set $v = c$ and solve with respect to R and we get $R = \frac{2GM}{c^2}$ which is identical to the Schwarzschild radius. The escape velocity and the Schwarzschild radius derived from Einstein's [3] field equation are identical to that of the Newton theory. A black hole is basically just an object where the mass is inside this radius. This means in general not even light can escape. The term "black hole" was likely first suggested in 1967 by a student at a lecture by John Wheeler.

3 The Compton Wavelength and the Schwarzschild Relation

A series of researchers claims the Schwarzschild radius has to be equal or larger than the Compton [43] wavelength of the mass in question. The idea is that the mass of a particle maximum can be contained inside the Compton wavelength and some the reduced Compton wavelength due to quantum effects. For example Lake and Carr [44, 45] states

"The Compton wavelength gives the minimum radius within which the mass of a particle may be localized due to quantum effects, while the Schwarzschild radius gives the maximum radius within which the mass of a black hole may be localized due to classical gravity."

The same Carr [46] in 2017 seems to claim a black hole must be larger than the reduced Compton wavelength of the particle. However there seems to be no full agreement if the black hole Schwarzschild radius must be bigger or equal to the Compton wavelength or the reduced Compton wavelength. The difference is quite large as the Compton wavelength is 2π times the reduced Compton wavelength. We will in this paper try to come to a more precise answer also here.

The Planck mass has a Schwarzschild radius of

$$R_s = \frac{2Gm_p}{c^2} = 2l_p \quad (3)$$

while it has a reduced Compton wavelength of the Planck mass must be

$$\bar{\lambda} = \frac{\hbar}{m_p c} = l_p \quad (4)$$

The Compton wavelength is then $\lambda = 2\pi l_p$. That is a Planck mass as candidate for a micro black hole has a larger Schwarzschild radius than its reduced Compton wavelength and thereby fits the requirement that the Schwarzschild radius is bigger or equal to this. It is smaller than the Compton wavelength, so if it is a criterion that the Schwarzschild radius must be larger than the Compton wavelength, then the Planck mass does not fit this requirement. This means the Planck mass cannot be a black hole under the criteria that the Schwarzschild radius must be larger than the Compton wavelength, but it can be a black hole based on the criteria that the Schwarzschild radius must be larger or equal to the reduced Compton wavelength. One can naturally also wonder why the Schwarzschild radius of a Planck mass micro black hole should be exactly twice that of the reduced Compton wavelength of the Planck mass, is it a coincident, or do it mean something special? We will try to also answer such questions.

Lake and Carr [44] even suggest a modified reduced Compton wavelength given by $\lambda_C = \frac{\hbar}{2mc} = 2\bar{\lambda}$, this would therefore means the modified reduced Compton wavelength would match the Schwarzschild radius of the micro black hole with mass equal to the Planck mass. To modify the reduced Compton wavelength, we think is actually not the way to go. As we will show there is another and, in our view, more sound way to get a match between the reduced Compton wavelength and the radius of a micro black hole.

Lake [45] claims “*‘particles’ with rest masses $m > m_p$ have sub-Planckian Compton wavelengths, $\lambda < l_p$, but super-Planckian Schwarzschild radii, $R_S > l_p$, and may be interpreted as black holes*”. Only the latter is actually correct, a mass with higher mass than the Planck mass will always have a Schwarzschild radii, larger than the Planck length, but even this is unprecise as actually any mass above half the Planck mass has a Schwarzschild radii larger than the Planck length. Further several masses with mass below the Planck mass has Compton wavelength larger than the Planck length but have a reduced Compton wavelength smaller than the Planck length.

A series of papers gives the incorrect impression that the Schwarzschild radius is the same as the Planck length for a Planck mass size black hole, this we think is one of the reasons few if any have asked the question why they are close to each, but not the same. For example Baez [47] incorrectly states that the Schwarzschild radius is

$$R_s = \frac{GM}{c^2} \quad (5)$$

and since the reduced Compton wavelength is correctly given by

$$\bar{\lambda} = \frac{\hbar}{mc} \quad (6)$$

and since the Planck mass is given by $m_p = \sqrt{\frac{\hbar c}{G}}$, now inputting this mass into Baez incorrect Schwarzschild radius we get

$$R_s = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \sqrt{\frac{G\hbar}{c^3}} \quad (7)$$

and for the reduced Compton wavelength we get

$$\bar{\lambda} = \frac{\hbar}{mc} = \frac{\hbar}{c\sqrt{\frac{\hbar c}{G}}} = \sqrt{\frac{G\hbar}{c^3}} = l_p \quad (8)$$

And since the Planck length is given by $\sqrt{\frac{G\hbar}{c^3}}$ this indicates the reduced Compton wavelength and the Schwarzschild radius of a Planck mass is the same. Baez [47] has however used the incorrect formula for the Schwarzschild radius without pointing this out, it should be $\frac{2GM}{c^2}$ rather than $\frac{GM}{c^2}$. By correcting for this we get

$$R_s = \frac{2G\sqrt{\frac{\hbar c}{G}}}{c^2} = 2\sqrt{\frac{G\hbar}{c^3}} = 2l_p \quad (9)$$

which is twice that of the reduced Compton wavelength of the Planck mass. Also Adler [5] gives the incorrect impression that the reduced Compton wavelength of a Planck mass is equal to the Schwarzschild radius as he writes about the Planck mass; “*when the size approaches the Schwarzschild radius $l \approx GM/c^2$* ”. Well, it is not directly wrong as he uses an approximation sign, but the Schwarzschild radius is 2 times this. The impression one gets from several authors are that the reduced Compton wavelength from the Planck mass is identical to the Schwarzschild radius, but the Schwarzschild is twice of that. Much of the lack of precision in the micro black hole literature have led to the incorrect impression among many that the Planck mass perfectly fits the criteria of a micro black hole under general relativity theory, which is not the case as we will demonstrate in this paper.

There is only one mass where the reduced Compton wavelength is equal to the Schwarzschild radius, it can be found by solving the following formula with respect to x

$$\frac{\hbar}{xm_p c} = \frac{2Gxm_p}{c^2} \quad (10)$$

This gives $x = \frac{1}{\sqrt{2}}$, that is only a micro black hole with mass $m_b = \frac{1}{\sqrt{2}}m_p$ has a reduced Compton wavelength equal to the Schwarzschild radius, such a micro black hole was likely first suggested in 2016 by Haug² [49], but indirectly suggested already by Harrison [34] in 1970.

Alternatively, one could look at the Compton wavelength rather than the reduced Compton wavelength. Then the only mass that has a Schwarzschild radius equal to the Compton wavelength of the mass is given by solving the formula

$$\frac{h}{xm_p c} = \frac{\hbar 2\pi}{xm_p c} = \frac{2Gxm_p}{c^2} \quad (11)$$

This leads to a micro black hole mass of $m_b = \sqrt{\pi}m_p$. This candidate for a micro black holes was first suggested by Haug in 2016 [49] and discussed in more detail by Faraoni [50] in 2017.

So, to summarize this section the Planck mass as a candidate for a micro black hole gives a Schwarzschild radius twice to the reduced Compton wavelength of the Planck mass and a radius that is shorter than the Compton wavelength. The micro black hole candidate with mass $\frac{1}{\sqrt{2}}m_p$ has an identical Schwarzschild radius and a reduced Compton wavelength of $R_s = \bar{\lambda} = \sqrt{2}l_p$, while a micro black hole with mass $\sqrt{\pi}m_p$ has a Compton wavelength equal to the Schwarzschild radius $R_s = \lambda = 2\sqrt{\pi}l_p$, however this last micro black hole mas candidate will have a reduced Compton wavelength below the Planck length. Another mass candidate for a micro black hole is half the Planck mass, which was first suggested by Motz and Epstein [51] in 1979. Here the Schwarzschild radius is exactly equal to the Planck length, but it does not match the reduced Compton wavelength that now is $2l_p$ or the Compton wavelength that is $4\pi l_p$. So, an important question is if some of these mass candidates more likely to be a micro black hole than the other candidates, if so, why? This are important questions to answer that we think we have a good answer for.

4 The Planck Mass Micro Black hole and its fit to the Planck scale?

First back to the Planck mass as a candidate for the micro black hole. Here the micro black hole clearly matches the Planck mass per definition, that is one aspect of the Planck scale. However, we have already seen that the Schwarzschild radius is twice that of the Planck length, while the reduced Compton wavelength of this black hole is the Planck length. The time to cross the radius of the micro black hole for light is $2t_p$, further the gravitational acceleration at the black hole surface (Schwarzschild radius) is given by

$$g = \frac{Gm_p}{R_s^2} = \frac{1}{4} \frac{Gm_p}{l_p^2} \quad (12)$$

In other words its gravitational acceleration is $\frac{1}{4}$ of the Planck acceleration, so it also does not seem to match this property of the Planck scale. The energy density of this micro black hole is

$$\rho_E = \frac{m_p c^2}{\frac{4}{3}\pi R_s^3} = \frac{1}{8} \frac{m_p c^2}{\frac{4}{3}\pi l_p^3} \quad (13)$$

That is the energy density $\frac{1}{8}$ of the Planck density.

The escape velocity at the Schwarzschild radius is naturally c , actually per definition as R_s is where the escape velocity is c . An interesting question to ask is what the escape velocity at the reduced Compton wavelength is, which is the Planck length, it is

$$v_e = \sqrt{\frac{2Gm_p}{l_p}} = c\sqrt{2} \quad (14)$$

²See also [48]

That is the escape velocity at the Planck length is above c , which is impossible according to special and general relativity theory that this is derived from. One could and perhaps should argue that since l_p and the reduced Compton wavelength is inside the black hole it makes no sense to calculate the escape velocity inside the surface of the black hole. But before we exclude predictions at what happen at the Planck length radius let us also look at the predicted gravitational time dilation at the Planck length distance. In general relativity the gravitational time dilation is given by

$$T_h = T_L \sqrt{1 - \frac{v_{e,h}^2}{c^2}} \quad (15)$$

For general relativity the escape velocity is given by $v_e = \sqrt{2GM/R^2}$. This means we have the well-known formula

$$T_h = T_L \frac{\sqrt{1 - \frac{2GM}{R_h c^2}}}{\sqrt{1 - \frac{2GM}{R_L c^2}}} \quad (16)$$

We can call $\sqrt{1 - \frac{2GM}{Rc^2}}$ the gravitational time dilation factor. In the special case of a Planck mass size object the reduced Compton wavelength of this mass is the Planck length. That is $\bar{\lambda} = \frac{\hbar}{m_p c} = l_p$. It is often assumed a mass can be constrained inside its reduced Compton wavelength. Assume we now stand at the surface of this mass, that is at $R = l_p$ then we get a time dilation factor of

$$\sqrt{1 - \frac{2GM}{Rc^2}} = \sqrt{1 - \frac{2Gm_p}{l_p c^2}} = \sqrt{1 - 2} = \sqrt{-1} = i \quad (17)$$

In other words, the time dilation there is imaginary. Imaginary numbers in quantum mechanics are not that abnormal, but imaginary output in gravitational analysis like we have here is abnormal. How should this be interpreted. Actually in general relativity theory the simplest solution is to say that this is an irrelevant result as the Schwarzschild radius of the Planck mass is as we have shown before $R_s = \frac{2Gm_p}{c^2} = 2l_p$. It is often typically considered meaningless to try to explain what is happening inside a black hole. This makes some sense if one only looks at general relativity and not at the same time on alternative or extended theories, something we soon will do. Let us also look at what the time dilation factor is at the Schwarzschild radius, it is then given by

$$\sqrt{1 - \frac{2GM}{R_s c^2}} = \sqrt{1 - \frac{2Gm_p}{2l_p c^2}} = \sqrt{1 - 1} = \sqrt{0} = 0 \quad (18)$$

In other words, time ceases to exist at the Schwarzschild radius, a result we think makes full sense.

Our main point is simply that the Planck mass as a candidate for a micro black hole does not match any many properties of the Planck scale. It matches the Planck mass and then naturally the Planck energy as the Planck energy is simply the mass multiplied with c^2 . On the other hand, the surface acceleration, the time to cross the radius by a light beam and the density are for the Planck mass micro black hole all different than the properties of the Planck scale, still they are relatively close to the properties of the Planck scale, so is this a coincidence or does it mean something special?

5 The suggested masses for micro black holes and their match to the Planck scale

As already indicated a series of candidates in addition the Planck mass has been suggested as masses for a micro black hole. All these candidates have masses close to the Planck mass. Markov suggested the micro black hole was of the size of a Planck mass, Hawking indicated it was approximately equal to the Planck mass. Motz and Epstein in 1979 tried to get the micro black hole to match the Planck length rather than the Planck mass and then got a mass equal to half the Planck mass. Haug and Faraoni has suggested a mass candidate equal to $\sqrt{\pi}$ times the Planck mass, this gives an identical Schwarzschild radius and a Compton wavelength. One can also suggest a mass equal to $\frac{1}{\sqrt{\pi}}$ as this gives a match between the Schwarzschild radius and the reduced Compton wavelength. Still, none of these candidates matches more than maximum a few aspects of the Planck scale. For example, I could say I think it is an important property that the micro black hole surface acceleration should match the Planck

acceleration. The only mass that do this has a mass off $\frac{1}{4}$ of the Planck mass. Such a micro black hole would however have a Schwarzschild radius smaller than the Planck length, which is impossible if the Planck length is some limit on minimum size.

Table 1 shows the masses of the micro black hole candidates we have found suggested in the literature. The candidates have typically been selected to match a property of the Planck scale or to have the Schwarzschild radius match the Compton or reduced Compton wavelength. As we can see none of the suggested candidates, including a few new candidates suggested in this paper, such as the mass candidate to match the Planck density fits more than one to three aspects of the Planck scale. Properties of the micro black hole that match the Planck scale or other suggested important aspects are marked in bold.

So how should this be interpreted? Are there many types of micro black holes, or do micro black holes even changes their mass over time due to some radiation and thereby represent one or several of these micro black hole candidates? It is not obvious from the literature how any of these candidates are outstanding as the mass of a micro black hole relative to the others. All we can say is that a mass approximately equal to the Planck mass seems somehow to be related to the Planck scale. So why is this the case, could there be something we not yet understand here?

Candidate number: Reference first mentioned:	Fits at least one property of the Planck scale					No fits to the Planck scale				
	1	2	3	4	5	6	7	8	9	10
	[15]	[51]	[49] [50]	[48]	This paper	[10]	[48]	[48]	[48]	[34] and [49]
Micro black hole mass candidate:	m_p	$\frac{1}{2}m_p$	$\sqrt{\pi}m_p$	$\frac{1}{4}m_p$	$\frac{1}{\sqrt{8}}m_p$	$2m_p$	πm_p	$\frac{1}{\sqrt{\pi}}m_p$	$\sqrt{2}m_p$	$\frac{1}{\sqrt{2}}m_p$
Schwarzschild radius $R_s = \frac{2Gm}{c^2}$	$2l_p$	l_p	$2\sqrt{\pi}l_p$	$\frac{1}{2}l_p$	$\frac{1}{\sqrt{2}}l_p$	$4l_p$	$2\pi l_p$	$\frac{1}{\sqrt{2}}l_p$	$\sqrt{8}l_p$	$l_p\sqrt{2}$
Reduced Compton wavelength $\frac{h}{mc}$	l_p	$2l_p$	$\frac{l_p}{\sqrt{\pi}}$	$4l_p$	$l_p\sqrt{8}$	$\frac{l_p}{2}$	$\frac{l_p}{\pi}$	$l_p\sqrt{\pi}$	$\frac{l_p}{\sqrt{2}}$	$l_p\sqrt{2}$
Compton wavelength $\frac{h}{mc}$	$2\pi l_p$	$4\pi l_p$	$2\sqrt{\pi}l_p$	$8\pi l_p$	$\sqrt{32}\pi l_p$	πl_p	$2l_p$	$2\sqrt{\pi^3}l_p$	$\sqrt{2}\pi l_p$	$\sqrt{8}\pi l_p$
Schwarzschild time $\frac{R_s}{c}$	$2t_p$	t_p	$2\sqrt{\pi}t_p$	$\frac{1}{2}t_p$	$\frac{1}{\sqrt{2}}t_p$	$4t_p$	$2\pi t_p$	$\frac{2}{\sqrt{\pi}}t_p$	$\sqrt{8}t_p$	$\sqrt{2}t_p$
Escape velocity at $\bar{\lambda}$	$c\sqrt{2}$	$\frac{c}{\sqrt{2}}$	$c\sqrt{2\pi}$	$\frac{c}{\sqrt{8}}$	$c/2$	$c\sqrt{8}$	$c\pi\sqrt{2}$	$\frac{c\sqrt{2}}{\sqrt{\pi}}$	$2c$	c
Escape velocity at λ	$\frac{c}{\sqrt{\pi}}$	$\frac{c}{2\sqrt{\pi}}$	c	$\frac{c}{\sqrt{16\pi}}$	$\frac{c}{\sqrt{8\pi}}$	$\frac{2c}{\sqrt{\pi}}$	$c\sqrt{\pi}$	$\frac{c}{\pi}$	$\frac{c\sqrt{2}}{\sqrt{\pi}}$	$\frac{c}{\sqrt{2\pi}}$
Schwarzschild density $\frac{m}{\frac{4}{3}\pi R_s^3}$	$\frac{1}{8}\rho_p$	$\frac{1}{2}\rho_p$	$\frac{1}{8\pi}\rho_p$	$2\rho_p$	ρ_p	$\frac{1}{32}\rho_p$	$\frac{1}{8\pi^2}\rho_p$	$\frac{\pi}{8}\rho_p$	$\frac{1}{16}\rho_p$	$\frac{1}{4}\rho_p$
Planck energy mc^2	E_p	$\frac{E_p}{2}$	$\sqrt{\pi}E_p$	$\frac{E_p}{4}$	$\frac{E_p}{\sqrt{8}}$	$2E_p$	πE_p	$\frac{E_p}{\sqrt{\pi}}$	$\sqrt{2}E_p$	$\frac{E_p}{\sqrt{2}}$
Surface area acceleration $g = \frac{Gm}{R_s^2}$	$\frac{a_p}{4}$	$\frac{a_p}{2}$	$\frac{a_p}{4\sqrt{\pi}}$	a_p	$\frac{a_p}{\sqrt{2}}$	$\frac{a_p}{8}$	$\frac{a_p}{4\pi}$	$\frac{a_p\sqrt{\pi}}{4}$	$\frac{a_p}{\sqrt{32}}$	$\frac{a_p}{\sqrt{8}}$
Number of Planck scale match	3	2	1	1	1	0	0	0	0	0
Compton or reduced match	0	0	1	0	0	0	0	0	0	1
Conflict Planck scale limits $R_s < l_p$	no	no	no	yes	yes	no	no	no	no	no

Table 2: The table shows a series of suggested mass candidates that have been suggested to be a micro black hole. None of them matches more than a few properties of the Planck scale. Why is this? Are there a series of different micro black holes or is there something we do not understand? The next section will give a possible answer.

6 Lorentz Relativistic Mass and Micro Black Holes

Lorentz [52] suggested already in 1899 suggested that mass likely was relativistic. He suggested a transverse and a longitudinal mass, the formula for his longitudinal mass was $m\gamma$, where $\gamma = 1/\sqrt{1-v^2/c^2}$ (the standard Lorentz factor). His Longitudinal relativistic mass is what a series of university text books today call relativistic mass, see for example [53, 54]. Modern textbooks referring to relativistic mass do not seems to recognize the relativistic mass of the form $m\gamma$ that they describe actually was introduced by Lorentz and one get the impression it was introduced by Einstein, but this is not correct.

Einstein [55] in the end of his most famous 1905 paper on special relativity theory also suggest two relativistic mass formulas, $m\gamma^2$ and $m\gamma^3$ and none of these are used today. Max Planck [56] then in 1906 introduced relativistic momentum in the form we know it today. Sometime after the invention of Minkowski [57] space-time Einstein decided to abandon relativistic mass all together, and instead incorporate Planck's relativistic momentum into a fourth momentum approach. Several well-known physicists, like Adler [58], Taylor and Wheeler [59], Okun [60] and Hecht [61] have all been very negative to relativistic mass, and even ridiculed researchers using relativistic mass as not aware of that Einstein was negative to it.

Others like Rindler that clearly was supporting both special and general relativity theory seems to be more positive to relativistic mass, see [62, 63], the same with Jammer [64]. However, despite researchers positive and negative to relativistic mass there has been little actually studies trying to find out what incorporating relativistic mass leads to off predictions. However, this has recently changed. Haug [65] has recently shown that incorporating

relativistic mass seems to make wormholes mathematically forbidden. Further he [66] has shown that incorporating relativistic mass seems to give predictions of high Z supernovas that can be predicted very well without the need for dark energy. Also incorporating relativistic mass seems to give a simpler model of the universe than the Friedmann [67] model. None of this should naturally be taken for granted, but we think it is worth the research communities time to re-consider if it could be worth incorporating relativistic mass.

To ignore relativistic mass and only assume the mass always is the same no matter from what frame it is observed seems to also lead to non-logical consequences in interpretation. The relativistic momentum introduced by Planck and incorporated in standard physics to this day is given by

$$p = mv\gamma = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (19)$$

If the mass is relativistic then this simply means the Lorentz factor work on the mass, so the faster the mass moves the larger is this mass as observed from another frame and when multiplied by the velocity this gives a relativistic momentum. If one switches to the interpretation that was initiated by Einstein and today is the main view among physicists (in particular those working with gravity) then the mass is constant and always the rest-mass. In that case the Lorentz factor can then only work on the velocity itself to give us relativistic momentum. This because there is only mass and velocity and the Lorentz factor in the formula. Assume the Lorentz factor work on the velocity, then a velocity faster than $c/\sqrt{2} \approx 0.71c$ will give a relativistic velocity above c . This seems absurd. Similar we have for kinetic energy that is given by

$$E = mc^2\gamma - mc^2 \quad (20)$$

Here the speed of light is a constant. So, if the mass cannot change then increased kinetic energy for a mass moving faster and faster can only come from the Lorentz factor working on the speed of light. There is nothing else than the mass and the speed of light in the kinetic energy formula the Lorentz factor can work on. It makes no sense it is acting on the speed of light as it is a constant (in particular under Einstein's own theory) and this would also lead to a speed above c , the only other thing the Lorentz factor then can work on is the mass. The mass is moving, and it is affected. The moving mass is what we call kinetic energy.

Still the output value is naturally the same in the relativistic momentum as well as in the kinetic energy formula no matter how one interprets what the Lorentz factor works on. So, in special relativity theory it will in general not affect the predictions if one ignores or consider relativistic mass. It is first when one comes to gravity that ignoring relativistic mass will have big consequences on the predictions one get. In the Newtonian gravity formula $F = \frac{Mm}{R^2}$, as well as in general relativity theory and the many predictions that can be derived from these there are often just mass (or masses) and not any momentum or kinetic energy.

Here we incorporate relativistic mass and then take a close look at micro black holes in relation to the Planck scale. The escape velocity when considering Lorentz relativistic mass is found by solving the following equation with respect to v

$$mc^2\gamma - mc^2 - G\frac{Mm\gamma}{R^2} \quad (21)$$

That is we are standing on M and observe m , so m is moving relative to the observer frame at velocity v . The v is found in $\gamma = 1/\sqrt{1 - v^2/c^2}$. We end up with

$$v = \sqrt{1 - \frac{2GM}{Rc^2} + \frac{G^2M^2}{R^2c^2}} \quad (22)$$

Next we set $v = c$ and solve with respect to R , this gives

$$R = \frac{GM}{c^2} \quad (23)$$

That is the radius where the escape velocity is c is half of that of the Schwarzschild radius, we will use the symbol R_h for this radius. In other words, this is also the formula that gives the radius of a black hole with mass M . We now have what we need to check masses for micro black hole candidates in our theory. This means a Planck mass size micro black hole has a radius of

$$R_h = \frac{Gm_p}{c^2} = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \sqrt{\frac{G\hbar}{c^3}} = l_p \quad (24)$$

So already now we have matched two properties of the Planck scale, the Planck mass and the Planck length. What is now the surface gravitational acceleration at the black hole radius, it is

$$g = \frac{Gm_p}{R_h^2} = \frac{Gm_p}{l_p^2} = g_p \quad (25)$$

This is now identical to the Planck acceleration, so the Planck mass candidate of a micro black hole has now matched three properties of the Planck scale. Next lets look at the density of this micro black hole, it must be

$$\rho = \frac{m_p}{\frac{4}{3}\pi R^3} = \frac{m_p}{\frac{4}{3}\pi l_p^3} = \rho_p \quad (26)$$

That is it is equal to the Planck mass density, so we have now matched four properties of the Planck scale.

Next let us look at the time dilation factor for the Planck mass at the Planck length, which is now the surface of the black hole. The time dilation factor is here now given by

$$\sqrt{1 - \frac{2GM}{Rc^2} + \frac{G^2M^2}{R^2c^2}} = \sqrt{1 - \frac{2Gm_p}{l_p c^2} + \frac{G^2m_p^2}{l_p^2 c^2}} = 0 \quad (27)$$

This means time stand still when one gets to the Planck length, that is the surface of the micro black hole, but not before this. The time dilation is well behaved all the way down to and including the surface of the micro black hole. This also means when considering Lorentz relativistic mass, we have a time dilation factor that makes sense all the way down and including the Planck length. That is again we see everything makes logical sense at the Planck scale when considering Lorentz relativistic mass, but not when ignoring it. Again, for the Planck mass size black hole when not incorporating relativistic mass gives an imaginary time dilation at the Planck length. The time dilation factor is only well behaved down to twice the Planck length for a Planck mass under general relativity theory.

Table 3 below compares the properties of the Planck mass micro black hole as predicted from general relativity theory and as predicted when we incorporate relativistic mass. When we take into account Lorentz relativistic mass then the Planck mass micro black hole matches all the aspects of the Planck scale and also other things such as time dilation all the way down and including the Planck length makes logical sense.

	General relativity	Relativistic mass incorporated in gravity
Mass micro black hole candiate:	m_p	m_p
Escape radius where $v = c$	$2l_p$	l_p
Reduced Compton wavelength $\frac{\hbar}{mc}$	l_p	l_p
Compton wavelength $\frac{\hbar}{m_0c}$	$2\pi l_p$	$2\pi l_p$
Schwarzschild time R_s/c	$2t_p$	t_p
Escape velocity at $\bar{\lambda}$	$c\sqrt{2}$	c
Escape velocity at λ	$c/\sqrt{\pi}$	$c/\sqrt{2\pi}$
Density $\frac{m}{\frac{4}{3}\pi R^3}$	$\frac{1}{8}\rho_p$	ρ_p
Planck energy mc^2	E_p	E_p
Surface area acceleration $\kappa = \frac{Gm}{R_s^2}$	$\frac{a_p}{4}$	a_p
Time dilation at the Planck length	Imaginary	Time stand still
Number of Planck scale match	3	8 (all)
Reduced Compton wavelength match	no	yes

Table 3: The table shows a series of suggested mass candidates that have been suggested to be a micro black hole. None of them matches more than a few properties of the Planck scale. Why is this? Are there a series of different micro black holes or is there something we do not understand? The next section will give a possible answer.

We therefore think one perhaps prematurely have ignored relativistic mass. Incorporating relativistic mass seems to give one clear mass candidate for micro black holes, namely exactly the Planck mass.

7 But where can these micro black holes be hiding?

The main point of this article is to highlight that general relativity theory leads to that micro black hole mass candidates only can match a few aspects of the Planck scale, but that when incorporating relativistic mass then the Planck mass suddenly fits all the aspects of the Planck scale. Purely mathematically we can easily demonstrate why this is the case, still is there any deeper physical explanation behind this?

From section XX it is clear that many different suggestions have been given by many different researchers of how micro black holes are created and where they perhaps can be found. Still no experiment has detected micro black holes. The research community should therefore be open to also new speculative suggestions about where the micro black holes can be. Assume relativistic mass must be taken into account in our models. Then we have seen the Planck mass suddenly fit all the properties of the Planck scale. Still what dose these properties present in the physical world? We will come with the controversial suggestion and say that micro black holes are simply photon-photon collisions, mostly internally in elementary particles.

The idea is simple, inside an elementary particle there are photons moving back and forth over the reduced Compton wavelength of the particle. Assume minimum two such photons, or we should say building blocks of photons. At the Compton periodicity these photons collide. The photons stand still for a very short instant at the very moment of collision, before so dissolving into photons again that again move the reduced Compton wavelength and collide. Even in standard physics photon-photon collisions are assumed to be able to create matter, see [68]. This mass created by the building blocks of photons we will claim is the Planck mass, however the duration of this collision and thereby the existence of the Planck mass only last the Planck time. Let us take an electron the reduced Compton frequency based on that the photons are moving back and forth at the speed of light over the reduced Compton wavelength is given by $f_e = \frac{c}{\lambda_e}$, where λ_e is the reduced Compton wavelength of the electron. Next, we assume there is a photon-photon collision at each of these events in the reduced Compton frequency that creates a Planck mass, still these Planck masses only last for the Planck time, so the mass of an electron must be

$$m_e = f_e m_p t_p \approx 9.11 \times 10^{-31} \text{ kg} \quad (28)$$

This is basically the well-known kilogram mass of the electron. The Planck mass is enormous, but because it in this model lasts such a short duration, the Planck time, then even a very large number of Planck masses only makes up the electron mass in this case. Every mass can be represented simply as the number of Planck mass events in the reduced Compton frequency of the particle in question, for example also for a proton we can find the reduced Compton wavelength despite it is not a elementary particle, see for example Levitt [69] and Haug [22]. Here we have assumed the duration is the Planck time and that the photon-photon collision gives a mass equal to the Planck mass. We can start out with a model where both this mass and time is unknown. Based on calibrating a new quantum gravity model named collision-space-time to gravitational observations we get that the duration of the Planck mass is the Planck time and that this mass is the Planck mass.

The Planck mass if it only lasts the Planck time is given by

$$m_p t_p \approx 1.17 \times 10^{-51} \text{ kg} \cdot \text{s} \quad (29)$$

that is the Planck mass only last the Planck time, but if the observational time window is one second this will correspond to only 1.17×10^{-51} kilogram in this one second observational time window. This could explain why one never have detected something with mathematical characteristics of the micro black hole, as one has been searching for a very high mass ($m_p \approx 2.17 \times 10^{-8} \text{ kg}$) or alternatively very high energies; $E = m_p c^2 \approx 1954056587 \text{ Joule}$ or $1.12 \times 10^{16} \text{ TeV}$, while LHC is only about 10 TeV, so this is far above anything detected for example in LHC. With our new view one should look for an incredible small mass or incredible small energy. Interesting $1.17 \times 10^{-51} \text{ kg}$ is very close to what has been suggested as the hypothetical photon mass based on a survey of existing and proposed classical and quantum approaches, see Spavieri et. al [70].

Be aware that all other masses than the Planck mass will be observational time independent if the observational time window is considerably above the reduced Compton time, see [22, 71] for a discussion about this point. The reduced Compton time of the electron is $t_c = \frac{\lambda_e}{c} \approx 1,28 \times 10^{-21}$ seconds, which is far below the observational time window that can be observed with the best atomic or optical clocks today, which is about 10^{-19} seconds precision, see [72].

This ultimately means a micro black hole potentially only is a photon-photon collision, that all matter potentially is built from. It is likely not a hole, but a collision of two photons. Then suddenly also many of the Planck scale properties makes fully sense, for example the Planck mass acceleration lasting even only the Planck time will accelerate a rest-mass particle from rest to the speed of light. No mass can move with the speed of light. But the two photons colliding are at rest relative to each other when colliding, but if the collision last only the Planck time before they separate again then they indeed accelerate from zero to the speed of light.

From the sections above we also know that the relativistic escape velocity is equal to c at the Planck length radius of a Planck mass micro black hole when we take into account relativistic mass, but $c\sqrt{2}$ at this radius in general relativity theory. An exact escape velocity of c at the Planck length means only light can escape and no mass. This makes indeed sense for a photon-photon collision.

For very many years one have been searching for finding the micro black hole, that we know likely is linked to the Planck scale, further one has been trying to detect indirect the Planck scale. Non of this have been achieved. In 1984 Cahill [73, 74] suggested that the gravity constant could be written on the form $G = \frac{hc}{m_p^2}$. This is nothing more than the Planck mass formula of Max Planck solved with respect to G . Cahill thought the Planck mass was a more fundamental entity than G . However already in 1987 Cohen [75] pointed out that this lead to a circular problem that not seemed able to solve, namely that one needed to know G to find m_p (by dimensional analysis), so one could not find m_p independent of G . This has been a view held by the physics community until recently, see for example [76].

Recently it has however been demonstrated that all the Planck units easily can be found without any knowledge of G , h or even c from series of simple gravity observations, see [77–81]. This fits in with a recent view that detection of gravity itself is detection off the Planck scale. Each Planck event is however incredible small, not very large as thought (in terms of energy or mass). This again because the Planck mass event only last the Planck time. The only way to detect Planck mass events and thereby so called micro black holes that likely are nothing more than photon-photon collisions is to detect a massive amount of them at the same time, because they individuall are too small for direct measurment. Still we are able to extract indirectly even a single such event as demonstrated in the papers just refered to. This is exactly why we only can measure effects from gravity from macroscopic gravitational masses, a single Planck mass event is so small. We do not ask the readers to take any of this for granted, just to be curious enough to study the papers we here have referred to and think carefully about if this is a way to go?

8 Conclusion

We have demonstrated that no mass candidate for a micro black hole under general relativity theory can match more than a few aspects of the Planck scale. On the other hand, by incorporating Lorentz relativistic mass in gravity theory then the Planck mass candidate to a micro black hole is matching. We are suggesting these micro black holes could be photon-photon collisions internally in matter.

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