Scaling Factor Design Issues in Variable Step Size Incremental Resistance MPPT in PV systems

Emad M. Ahmed¹, Masahito Shoyama²
¹Graduate School of Information Science and Electrical Engineering, Kyushu University, Fukuoka, Japan
²Graduate School of Information Science and Electrical Engineering, Kyushu University, Fukuoka, Japan
¹El-Bakoury@ieee.org, ²Shoyama@ees.kyushu-u.ac.jp

Abstract—Conventional maximum power point tracker (MPPT) uses fixed step size perturbation in order to track the maximum power point (MPP), therefore the tracking efficiency has decreased due to steady state oscillations around the MPP. Recently, many techniques have been developed proposing new approaches in the field of variable step size MPPT to overcome the shortcoming of the fixed step size techniques. These approaches update the controlling parameters adaptively using a convenient scaling factor (N) to pick the MPP. However most of them need some prior analysis in order to design the appropriate value of N, which control the hall performance of the variable step size tracker. In this paper, a new approach has proposed to design N. this approach has been develop based on the overall small signal model of the PV system around the MPP. Therefore using linear control techniques the bounded value of the scaling factor has been calculated. The validity and the feasibility of the proposed analysis have been verified using MATLAB SIMULINK toolbox, and also experimental using digital signal processor (DSP).

Index Terms— Maximum power point tracking (MPPT), incremental resistance (INR), scaling factor, Fixed step size, small signal stability, variable step size.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>V&lt;sub&gt;PV&lt;/sub&gt;</td>
<td>Photovoltaic module output voltage (V)</td>
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<td>I&lt;sub&gt;PV&lt;/sub&gt;</td>
<td>Photovoltaic module output current (A)</td>
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<td>D</td>
<td>Converter duty cycle at the MPP</td>
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<td>N</td>
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<td>V&lt;sub&gt;mpp&lt;/sub&gt;</td>
<td>PV module voltage at the MPP</td>
</tr>
<tr>
<td>I&lt;sub&gt;pp&lt;/sub&gt;</td>
<td>PV module current at the MPP</td>
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<td>R&lt;sub&gt;pp&lt;/sub&gt;</td>
<td>PV module equivalent resistance at MPP</td>
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<td>I&lt;sub&gt;ss&lt;/sub&gt;</td>
<td>Steady state inductor current</td>
</tr>
<tr>
<td>C&lt;sub&gt;in&lt;/sub&gt;</td>
<td>Converter Input capacitor</td>
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<tr>
<td>L</td>
<td>Boost converter inductor</td>
</tr>
<tr>
<td>C&lt;sub&gt;out&lt;/sub&gt;</td>
<td>converter output capacitor</td>
</tr>
<tr>
<td>R&lt;sub&gt;L&lt;/sub&gt;</td>
<td>Load resistance</td>
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<tr>
<td>V&lt;sub&gt;o&lt;/sub&gt;</td>
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<td>T</td>
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<td>Δe</td>
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I.INTRODUCTION

The ever-increasing demand for low-cost and growing concern about environmental issues has generated enormous interest in utilization of non-conventional energy sources such as the solar energy [1], [2].

A major challenge in the use of PV is posted by its nonlinear current-voltage (I-V) characteristics, which result in unique MPP on its power voltage curve (P-V). The matter is further complicated due to the dependence of these characteristics on solar irradiation and temperature. Therefore, it is essential to track continuously the MPP in order to maximize the output power from the PV system.

The subject of MPPT has been lectured in different ways in the literature: examples of fuzzy logic, extremum seeking, neural network, pilot cells have been proposed in [3] – [5]. Moreover, perturb and observe (P&O), and the incremental conductance INC MPPT techniques are widely used, particularly for their lower cost implementation when compared with the other techniques [6]. The main shortcoming of P&O method technique is that, at the steady state, the operating point oscillates around the MPP resulting in waste some amount of the available energy. Although several improvements of the P&O algorithms have been proposed in order to reduce the number of oscillations around the MPP in steady state, but they slow down the response speed of the algorithm to the atmospheric changing conditions and reduce the algorithm efficiency during rapidly changing atmospheric conditions [6]. However the INC MPPT algorithm had been designed based on the incremental and the instantaneous conductance value of the PV array as: the derivative of the PV module power $P_{pp}$ is positive before reaching the MPP, zero at the MPP, and negative after passing the MPP as shown in Fig. 1. The main advantage of the INC MPPT embedded in its ability to track efficiently the MPP without any tendency to deviate from the MPP due to rapidly changing atmospheric conditions as it happens in P&O technique [7].

![Fig. 1 Schematic diagram of INC-MPPT equipped with DC-DC Boost Converter](image-url)
The INC MPPT algorithm usually uses a fixed step size perturbation to track the maximum power point (MPP). Thus the tracking speed and accuracy are highly depending on the fixed step size perturbation \( \Delta D \). The power drawn from the PV array with a larger step size contributes to faster dynamics but excessive steady state oscillations around the MPP, which resulting in a comparatively low efficiency. However the situation is reversed with a smaller step size [8]. Accordingly, in order to overcome the aforementioned challenge a variable step size MPPT has to be adopted to address the tradeoff between the dynamics and the steady state oscillations. In the variable step size MPPT schemes, the automatic tuning equation of the variable step size were proposed to be a function in the PV power derivative with respect to the PV voltage as shown in Fig. 1. The updating variable step size equation had been written as (1) [8].

\[
D(k) = D(k-1) \pm N \times \frac{dp_{PV}}{dv_{PV}}
\]

\[
= D(k-1) \pm N \times \frac{p_{PV}(k) - p_{PV}(k-1)}{v_{PV}(k) - v_{PV}(k-1)}
\]

where \( D(k), D(k-1) \) are the converter duty cycle at instant \( k, k-1 \), respectively. Obviously from (1), the scaling factor \( N \) adjusts the input signal to a proper magnitude prior to determine the subsequent step size. Therefore it is very crucial to design the most appropriate value for the scaling factor \( N \), which ensures a better dynamic and steady state performance. Otherwise, bad designing for the scaling factor might lead to undesired performances such as steady state oscillations, and slow down the dynamic response.

The only method which proposed in the literature about designing the scaling factor is in [8]. Although this method can be considered as a simple method for designing the scaling factor but it requires some prior analysis of the system with fixed step size operation. Therefore this method had neither realized the effect of increasing or decreasing the scaling factor on system stability and the dynamic performance nor formalized a closed form or a straightforward approach for designing its value. Furthermore, bad design of the scaling factor will turn system operation from variable step size mode to fixed step size mode, which consequently reduces system efficiency.

![Fig. 2 Schematic diagram of INR-MPPT equipped with DC-DC Boost Converter](image)

In this paper, in order to design the appropriate value of the scaling factor \( N \) and study the overall system stability, The INR MPPT scheme for the PV system has been introduced in part II. System modeling and analysis, which contains converter small signal model, INR MPPT error linearization, and the complete small signal model of the PV system has been presented in part III. Part IV introduces MATLAB digital simulation results of the proposed small signal strategy with different values of the scaling factors. Moreover part V presents experimental results conducted in the laboratory with the PV solar panel and the digital signal processor DSP. Finally part VI introduces the conclusion of the main points in this paper.

II. INR-MPPT SCHEME FOR PV SYSTEM

The schematic diagram of the INR MPPT is shown in Fig. 2. The primary rules for INR MPPT algorithm can be deduced by duality from the INC MPPT such as follows: the power curve of the PV module shows that the derivative of the PV module power \( P_{PV} \) is positive before reaching the MPP, zero at the MPP, and negative after passing the MPP as shown in Fig. 3. The derivative of \( P_{PV} \) is given as in (2), and the resultant equation for the actuating error \( e \) is as in (3).

\[
dp_{PV} = \frac{dv_{PV} \times i_{PV}}{di_{PV}} = \frac{dv_{PV}}{di_{PV}} \times i_{PV} + v_{PV}
\]

\[
e = \frac{dv_{PV}}{di_{PV}} + \frac{v_{PV}}{i_{PV}}
\]

Therefore tracking the maximum power point MPP requires the following updating rule as in (3)

1) \( D(k) = D(k-1) + N \times |e(k)| e(k) > 0 \)

2) \( D(k) = D(k-1) e(k) = 0 \)

3) \( D(k) = D(k-1) - N \times |e(k)| e(k) < 0 \)

By examining (4), this equation can be implemented by a simple digital integrator with the error signal \( e \) considered as its input, and a scaling factor \( N \) as the integrator gain as shown in Fig. 2. The function of the scaling factor gain \( N \) is to adapt the error signal \( e \) to a proper range before the integral compensator. Since the
error signal \( e \) becomes smaller as the operating point approaches the MPP, therefore an adaptive and smooth tracking can be achieved [9], [10]. In order to design the appropriate scaling factor value \( N \), which ensures a good dynamic performance and stability, a small signal model has to be developed for the overall system linearized closely the MPP.

III. SYSTEM MODELING AND ANALYSIS

The straightforward approach for designing the appropriate scaling factor and studying system stability is developing the linearized model of the overall system around the MPP. The linearization process of the overall system can be divided into two main categories: the first one is developing converter small signal model with the PV model, and the second one is linearizing the INR actuating error \( e \) around the MPP. Every category will be explained in the following:

A. Converter small signal model.

In order to derive the small signal model of the boost converter equipped with the PV module around the MPP, the PV module has been replaced with the equivalent resistance \( R_{mpp} \) [6]. The state space averaging model of the boost converter equipped with the PV module at the MPP can be deduced using Fig. 4a and Fig. 4b [11].

\[
p_2 = R_{mpp} C_{in} L + R_{L} L C_{out}
\]

\[
p_3 = (1 - D)^2 R_{L} R_{mpp} C_{in}
\]

\[
p_4 = I_{L} R_{L} R_{mpp} (1 - D) + V_o
\]

Thus the boost converter equipped with the PV module has modeled around the MPP with a third order system containing the effect of the PV parameters \( (R_{mpp}) \) and the converter parameters \( (C_{in}, C_{out}, L, R_L) \) as shown in (5).

B. Incremental Resistance error Linearization.

For simplicity, the schematic block diagram of the PV system equipped with the digital INR MPPT is shown in Fig. 5. The feed forward loop contains the small signal model of the boost converter \( G_{vd}(s) \) and the discrete controller which consists of the discrete integrator and the zero order holder blocks.

\[
\text{Loop gain} = G(s) \times G_{vd}(s) \times \frac{-1}{R_{mpp}}
\]

where \( G(s) \) represent the continuous transfer function of the discrete controller \( G(z) \).

In order to drive the continuous transfer function \( G(s) \), the error signal \( e \) should be linearized around the MPP. Therefore a straight line has been drawn tangentially to the \( I-V \) curve. This line pass through \( (V_{mpp}, I_{mpp}) \), and has a slope \(-R_{mpp}\) as in Fig. 6. Thus the straight line equation can be written here as in (7)

\[
v_{py} = 2V_{mpp} - I_{py} \times R_{mpp}
\]

Using (7), the actuating error \( e \) has been linearized by adapting Taylor series expansion and retaining only the linear terms as in [13]

Thus the linearized actuating error variation around the MPP is rewritten again as

\[
\Rightarrow \Delta e = \left( \frac{2R_{mpp}}{I_{mpp}} \right) \Delta i_{py}
\]
Therefore using (8), the transfer function between \( \Delta i_{PV} \) and \( \Delta d \) can be written in the discrete form as shown in Fig. 5 as 

\[
\Delta d(z) = \frac{2R_{app}}{I_{app}} N \frac{e^{-sT}}{z-1} \tag{9}
\]

Lastly, the continuous time expression of the discrete controller is derived by substituting \( z = e^{sT} \) in (9), and then multiplying \( (1-e^{-sT})/sT \) in order to represent the effect of zero order hold (ZOH) \([9]\). The final expression is

\[
\Delta d(s) = \frac{2R_{app}}{I_{app}} N \frac{e^{-sT}}{sT} \tag{10}
\]

As the sampling time is very small, the time delay term \( e^{-sT} \) can be approximated using Taylor series expansion as \((1-sT)\) and neglecting the higher order terms. Therefore (10) can be re-written as

\[
\frac{\Delta d(s)}{\Delta i_{PV}(s)} = \frac{2R_{app}}{I_{app}} N (1-sT) \tag{11}
\]

The overall small signal model of the linearized INR MPPT can be drawn as in Fig. 7. Using automatic control theory, the characteristic equation of this system can be derived as

\[
\text{Cheq}=1+\frac{2N \times R_{app}}{I_{app}} \frac{(1-sT) (z_1+s)}{sT (p_1 s^3 + p_2 s^2 + p_3 s + p_4)} \tag{12}
\]

where \( \text{Cheq} \) represents system characteristic equation. It is clear from (12), system dynamic stability is highly depending on the scaling factor \( N \), and the sampling time \( T \): decreasing the sampling time \( T \) tends to increase system loop gain, which leads to duty cycle overcompensation and hence decreases system stability.

\[
\Delta d = \frac{-2R_{app}}{I_{app}} \Delta e + N \frac{(1-sT)}{sT} \frac{- (z_1+s)}{p_1 s^3 + p_2 s^2 + p_3 s + p_4} \Delta i_{PV}
\]

(\( \Delta d \) limited)

However increasing the sampling time \( T \) decreases system loop gain, which results in decelerating tracker performance. While the effect of the scaling factor \( N \) has not been studied in the literature. The root-locus plot of the overall system presented by (12) with system parameters defined in table 1 is shown in Fig. 8. It is clear from this plot, increasing the scaling factor \( N \) increases the loop dc gain, which reduces system stability. In order to ensure a stable operation, the scaling factor should be within \((0 < N \leq 0.038)\).

\[\text{Fig. 8 Scaling factor root-Locus plot of the overall system model at MPP (} N_{\text{critical}} = 0.038\)\]

IV. SIMULATION RESULTS

To validate the above analysis, a MATLAB SIMULINK model established for the PV module equipped with a DC-DC boost converter and a resistive load as shown in Fig. 2. The small signal model has been established for the PV system at the standard conditions (1000 W/m², 25 °C). The sampling time \( T \) has been selected to be 0.01s [6]. All system parameters are obtained from [12]. By drawing the root-locus of this system with the defined parameters, the boundary values of the scaling factor \( N \) has determined as \((0 < N \leq 0.038)\). In order to investigate the correctness of the proposed small signal model for the INR MPPT, the performance of the PV system has been checked with different values of the scaling factor \( N \).

The simulations are configured under exactly the same conditions. Since selecting the scaling factor \( N \) outside the stable boundary will lead to undesirable conditions, therefore a limiter has been added to limit the divergence in the duty cycle variation (\( \Delta d \)). The upper and lower values of this limiter has been selected as \( \pm \Delta d = \pm 0.05 \) as shown in Fig. 9

(\( \Delta d \) limited)

Three scaling factors have been selected: two values in the stability zone (\( N = 0.004, N = 0.01 \)) and the other in the marginally or instability zone (\( N = 0.04 \)).

Fig. 10a shows the PV module output power when there is a sudden change in the irradiation density from 1000 W/m² to 600 W/m² at instant 1 sec and back again from 600 W/m² to 1000 W/m² at instant 2 sec. The corresponding converter duty cycle is shown in Fig. 10b. It is clear the steady state response is free from any oscillation around the MPP and has a fast dynamic response (~ 0.1 sec). Fig. 10b shows the converter duty cycle during tracking the
peak power. On the other hand Fig. 10c and Fig. 10e show the dynamic response of the PV system at $N=0.01$, and 0.04, respectively. It is apparent from Fig. 10c, as the scaling factor increases and being very close to the boundary values, the dynamic response of the PV module starts to exhibit steady state oscillations around the MPP, and consequently waste small amount of the available power. However the dynamic response does not have a big change from the previous state (at $N=0.004$). This is also can be noticed in the converter duty cycle in Fig. 10d. As the scaling factor increases and being outside the stability zone, the dynamic response has changed to work as a fixed step size operation which characterized by lower efficiency than the variable step size. Thus the variation in the duty cycle changed to be with the fixed step size (0.05). These can be shown in Fig. 10e and Fig. 10f.

V. EXPERIMENTAL RESULTS

The investigation of the small signal stability proposed analysis is also evaluated by experiment. A PV power simulator Agilent E4360 has been used as the PV module. The PV simulator is programmed to simulate the PV curve for the simulated model [12]. The INR MPPT algorithm has been configured on a fixed point 12 bits digital signal processor TMS320F2808 using MATLAB SIMULINK toolboxes. The target has been programmed using code composer 3.3. A duty cycle variation limiter with $\pm \Delta D_{max}=\pm 0.05$ has been introduced.

The start waveforms of the output power have shown in Fig. 11. Figure 11 shows the dynamic response of the PV system employed by the INR MPPT with different scaling factors. The dynamic response of the fixed step size INR MPPT is shown in Fig. 11a. Although the tracker has a fast transient response (10 cycles – 0.1s) but the tracker cannot pick up the total maximum power ($P_{max}=51$ W) due to its operation with fixed step size. The average power extracted is equal 44.6 W. Fig. 11b, Fig. 11c, and Fig. 11d show the dynamic response of the PV module with different scaling factors 0.002, 0.01, and 0.04, respectively. It is seen that, a lower scaling factor ensures a better steady state performance without exhibiting any steady state oscillations around the MPP as shown in Fig. 11b. However a lower scaling factor slows down the transient response to 29 cycles (0.29 s).

Furthermore, by increasing the value of the scaling factor the transient response starts to have a fast dynamic response (10 cycles- 0.1s) and a higher efficiency than the fixed step size operation as shown in Fig. 11c, and Fig. 11d. On the other hand the steady state performance starts to exhibit steady state oscillations around the MPP, which in consequent reduces the average extracted power as shown. Moreover, it is obvious, increasing the scaling factor beyond the stability zone, which indicated in Fig. 8, will lead to variable step overcompensation. Therefore the variable step size operation has the tendency to operate as a fixed step size as shown in Fig.11d.

Fig. 10 PV array output power and the corresponding duty cycle with
(a), (b) $N=0.004 << 0.038$, (c), (d) $N=0.01 < 0.038$
(e), (f) $N=0.04 > 0.038$
While the variable step size introduces a better dynamic performance than the fixed step size, these are appearing in the higher power generated: at 1000 W/m², the generated power with variable step size is 48.8 W and with the fixed step size is 44.6 W. Also at 600 W/m², the generated power with variable step size is 28.8 W and with the fixed step size is 27.6 W.

VI. CONCLUSION

A new stability analysis for the INR MPPT has been provided in this paper. The proposed analysis depends on developing the overall small signal model of the PV system around the MPP. The small signal model of the PV system consists of the PV module, the boost converter, and the discrete INR MPPT. Using the root locus technique or any other stability analysis technique, the appropriate value of the scaling factor, which ensures a good dynamic performance, can be designed. The correctness of the suggested stability analysis has been verified by simulation using MATLAB SIMULINK toolbox and experimentally using digital signal processor DSPTMS320F2808 as well.

Fig. 12a shows the dynamic response of the PV module with fixed step size INR MPPT. The dynamic performance of the PV system exhibits large oscillations around the MPP. While the variable step size introduces a better dynamic performance than the fixed step size, these are appearing in the higher power generated: at 1000 W/m², the generated power with variable step size is 48.8 W and with the fixed step size is 44.6 W. Also at 600 W/m², the generated power with variable step size is 28.8 W and with the fixed step size is 27.6 W.

REFERENCES