Service and Cost Benefits through Clicks-and-Mortar Integration: Implications for the Centralization/Decentralization Debate

Elliot Bendoly *
Department of Decision & Information Analysis
Goizueta Business School, Emory University
Atlanta, GA 30322

Doug Blocher
Department of Operations and Decision Technologies
Kelley School of Business, Indiana University
Bloomington, IN 47405

Kurt M. Bretthauer
Department of Operations and Decision Technologies
Kelley School of Business, Indiana University
Bloomington, IN 47405

M. A. Venkataramanan
Department of Operations and Decision Technologies
Kelley School of Business, Indiana University
Bloomington, IN 47405

June 16, 2004

Subject Areas: e-business, Multi-echelon Inventory Management, Optimization.

* Corresponding Author

[Accepted for publication in EJOR]
Introduction

As information technology continues to grow and evolve, organizations are faced with many new challenges and opportunities. In this paper, we focus on the development of innovative supply chain inventory management strategies for firms entering or engaged in on-line retail sales. We address aspects of supply chain management for traditional “Brick-and-Mortar” firms that have added or intend to add web-based sales to their existing retail sales channel. The concept of a “Clicks-and-Mortar” operation implies a capability to operate in the traditional in-store retail market as well as in the emerging on-line market. This type of capability is especially relevant now that the number of pure play “Clicks” firms has diminished considerably. To operate in both the in-store and on-line channels, “Brick-and-Mortar” firms with an established distribution system are faced with options ranging from using their current distribution infrastructure as is, to creating an independent delivery network to meet on-line demand. Developing an integrated system that handles both customer-direct delivery and conventional retailing operations is of interest for those firms trying to keep overall distribution investments and costs as low as possible, as well as providing an image of superior integrated service to the market as a whole (Bendoly, et al. 2005).

Inventory management is just one facet of the operation that is positioned to provide opportunities for such an integrated system. Inventory policies impact not only inventory and warehouse investments, but also operational costs and service levels. For example, Boyer et al. (2002) refer to Webvan, acknowledging that their inventory improvement from centralized distribution was “negated by losses in other areas – most notably vastly increased shipping costs.” The lack of order fulfillment capability has been cited (Mullaney 2001) as one of the reasons both large, established giants and the newer dot.com companies have failed to keep pace in the e-business environment. This paper investigates the total cost impact of different inventory allocation policies for firms that desire to compete in both in-store and on-line retail operations.

On-line sales conceivably can be met from any location in the distribution network that has stock, including the retail outlets. Therefore, Clicks-and-Mortar firms have the opportunity to realize benefits by combining, or pooling, inventory set aside for retail sales with inventory for on-line sales. Inventory pooling has the effect of increasing the service level and/or decreasing the total operating cost. In this paper, we show that the impact on cost and service of such inventory pooling is a function of the following: the location of the inventory in the supply chain, the fraction of a firm’s demand occurring on-line, and the amount of demand per retail store. Because the on-line customer typically is not concerned with the origin of the order shipment, firms can adopt policies ranging from pooling all on-line inventory at a central dedicated depot, to pushing its entire on-line inventory out to some or all of its many retail outlets where the on-line and in-store inventory can be pooled. Throughout the remainder of the paper, we will refer to the strategy of carrying all on-line inventories at a central depot as “centralization,” and the strategy of carrying all on-line inventories at the retail outlets as “decentralization.” We assume all inventory carried at a retail store is pooled, whether it is for on-line or in-store sales.

Hill et al. (2002) agree that the combined Clicks-and-Mortar firm “appears to be emerging as the dominant business model” and they point out that there is no dominant distribution method being employed to handle these two channels. An article based on an Andersen Consulting report (Cooke, 2000) indicates that six of the retail firms they were studying employed a strategy whereby a “professional shopper” walked the isles of the store to first pick and then later pack each internet order. The Cooke article indicates that 2 of the 8 traditional retailers with on-line shopping are
picking the orders from their retail stores. Much of what has been reported about the on-line grocery industry is consistent with having an employee pick and pack an order from a retail outlet’s shelves (Montgomery, 2002; Associated Press, 2003).

Another distribution method used by Barnes and Noble and barnesandnoble.com (Brady 2000) has predominantly been one of segregating the in-store stock and customer-direct delivery stock, and carrying all inventory for on-lines sales at a central site. This same distribution method is employed by J.C. Penney which has segregated its retail and direct shipping operations using “entirely separate warehouse systems” (Hill et al. 2002). What’s even more interesting is that they report 60% of the internet customers pick up their goods at a J.C. Penney retail store even though “those goods are still shipped from the Internet division warehouse rather than being pulled from the shelves of the stores. Many other firms, such as Wal-Mart, Macy’s, and Bloomingdale’s, have taken this segregation strategy a step further and have outsourced all their internet orders to third party firms” (Hill et al. 2002). Their article indicates that there is no universally accepted way of handling these orders, and in fact, leads us to believe that different firms are experimenting with different systems without any real knowledge of which method is the best.

We consider a 2-echelon fixed period order-up-to inventory system. The goal is to minimize total cost while maintaining a pre-specified service level. The task then is to determine when and if decentralization of on-line inventory is ideal; and, if it is warranted, to determine to what degree the inventory should be decentralized to all stores. The total system costs include the fixed costs of setting up operations to meet on-line demand, the variable shipping and handling costs of supplying the on-line demand, and the inventory holding cost for all inventory in the system.

While it might be easiest to interpret this distribution network as one where there is a central warehouse supplying retail outlets and on-line customers, this research applies to some other 2-echelon systems as well. Suppose a large firm has essentially a system where a central warehouse supplies regional distribution facilities and then these regional distribution centers (RDCs) supply retail outlets. If one views the RDCs as analogous to a retail outlet where the customer demand to the RDC comes daily from the actual retail stores (as is the case with many large retailers, drug chains, and grocery firms), then the decision we discuss is whether the central warehouse should meet the on-line demand or the RDC should. Still another interpretation for a large firm with RDCs, is that the RDC is actually viewed as the central warehouse and thus this large firm would essentially make the decision about on-line orders region by region. So, while we will continue to discuss the concepts in terms of a central warehouse and retail store satellite, one can think of other 2-echelon systems with analogous decisions.

The findings of this research indicate that the fraction of a firm’s demand satisfied on-line is an important factor in the decision of whether to centralize or decentralize on-line inventory. For the setting studied, the results further support using one of the following two allocation policies: complete centralization of on-line inventory, or complete decentralization of such inventory. Complete centralization is preferable when on-line sales are a large proportion of the firm’s total sales, while complete decentralization is preferable for small proportions. As will be discussed later in the paper, this result remains true even when the fixed costs of setting up and maintaining on-line fulfillment capabilities are ignored. More importantly, the findings further show that there exists a threshold electronic market percentage, dependent upon such factors as service levels, total sales per retail store, and the relevant cost parameters, below which decentralization is
favored. Thus, the results provide valuable insights for managers and form a foundation for future research in this important emerging environment.

**Literature Review**

Within supply chain management research, distribution system design and allocation of inventory have received a great deal of attention. The environment considered here is that of a 2-echelon fixed period order-up-to inventory model. These ordering policies are characterized by the placement of an order that raises total incoming plus in-house inventory levels to some fixed quantity on a periodic basis. Such models have been widely applied in the past. For example, both Clark (1972) and Schwarz (1981) provide coverage of early research regarding these models and others.

When demand is stochastic, as is assumed in this paper, the task of determining the cost minimizing order-up-to level is quite challenging. Analytical approaches to the consideration of such systems have typically relied upon assumptions that allow for approximations of system characteristics. One of the most prominent of these assumptions has been the restriction that no inventory is held for a significant amount of time at the central warehouse. In such a system, the role of the central warehouse is assumed to be one of a break-bulk or coordination point (Eppen and Schrage 1981). We make a similar assumption in this paper.

The work of Eppen and Schrage (1981) has served as a benchmark for the application of this setting. These authors provide a general set of equations for use in approximating the cost minimizing order-up-to levels for 2-echelon systems faced with normal demand distributions at each of the lower echelon satellites. The flexibility and encompassing nature of their derivations has made their work commonly cited in the inventory control literature. In an extension to Eppen and Schrage, Federgruen and Zipkin (1984) study the application of a dynamic programming approach to similar problems of relatively restricted network size. Major deviations from Eppen and Schrage’s assumptions come in the form of non-stationary demand, non-normal demand distributions, and unique holding / backorder cost ratios across satellites.

Erkip, Hausman and Nahmias (1990) take a different approach by pursuing explicitly described optimal policies for specific scenarios. In particular, they provide closed form approximations for scenarios in which demand at each satellite is correlated both over time and between these facilities. Schwarz (1989) also considers the lead time circumstances under which direct shipment from the upstream supplier to the individual satellites might be preferred over the use of the consolidating central warehouse form of Eppen and Schrage’s system.

In an attempt to demonstrate the benefits of slight modifications in inventory management policies, Johnson and Silver’s (1987) approach considers the complete redistribution of inventories at satellites one period before new replenishment orders are placed. The intent of such a move is to reduce overall risk of stockout before replenishment, and is essentially a prelude to transshipment studies to follow later. Kumar, Schwarz and Ward (1995) consider scenarios in which restock deliveries arrive at different times for alternate satellites. They show how policies allowing allocation adjustments as these orders arrive can reduce inventory imbalances and benefit total system costs. The assumption that no inventory is held at the central warehouse is again justified for certain cases. More recent extensions along these same lines are presented in studies...
by Van der Heijen (1997) and Barnes-Schuster and Bassok (1997) in which more complex forward-looking restock policies are tested. Adding another level of complexity, Chiang and Gutierrez’s (1996) work provides an overdue look at the interaction of multiple modes of distribution.

The present work also focuses on the use of multiple modes of distribution. Specifically, these include the use of on-line versus in-store channels, and local versus central deliveries for on-line purchases. In part due to the contemporary nature of supply chain issues in on-line environments, this is an important area for research. Recent literature on inventory pooling in multi-facility order-up-to networks provides yet another benchmark for this research. Inter-echelon transshipment settings have been particularly common with regards to this topic, with recent examples including the works of Taragas (1999) and Kumar, Schwarz and Ward (1995). We contribute to this research stream as well by developing a foundation for the consideration of pooling effects on on-line allocation policies not only within a single-echelon, but also between adjacent echelons. To our knowledge, the present work is the first in the stochastic multi-echelon fixed order-period literature to consider inventory allocation policies given the multi-mode demand and service characteristics of Clicks-and-Mortar systems.

Model Structure

The primary interest of this research is to determine whether on-line fulfillment in a 2-echelon system should be handled by a centralized facility, the decentralized store satellites, or both. At the store satellites, we assume inventory intended for both in-store and on-line sales is combined. Two competing pooling effects are relevant to this decision. When a central facility fulfills all on-line orders, the implication is that all on-line inventory is stored in a single place, therefore providing pooling benefits from the perspective of the on-line channel. On the other hand, when store satellites handle this on-line fulfillment, the on-line stock can be pooled with stock for in-store sales so that both channels realize inventory-pooling benefits.

Two common network configurations for combining fulfillment of on-line demand with standard in-store demand are shown in Figure 1. In Figure 1, the J satellites refer to J retail stores. Scenario 1 has a separate dedicated fulfillment depot for on-line demand that is outside the normal distribution system. According to Hill et al. (2002) this approach is common for the many companies that outsource their on-line fulfillment to another company, as well as a few other firms such as JC Penney. Scenario 2 has a dedicated on-line depot as well, but one that is integrated within the company’s distribution system and gets its product from the company’s central warehouse. The CVS warehouse in Indianapolis, Indiana is set up in this way, where the on-line depot is supplied from the warehouse like any other retail store, even though the on-line depot is adjacent to the warehouse.

As one looks at these two scenarios, one might also argue that there is a third scenario where the central warehouse could simply fulfill the on-line orders. Although we believe this type of configuration is rare due to the pick and pack operations that would be required in the middle of a central warehouse that is primarily handling bulk packages, we will show later how models for scenarios 1 and 2 can be used as approximations for this third scenario.
We focus on the impact that the two competing pooling effects have on total cost and inventory allocation decisions in a combined on-line, in-store distribution network. For scenario 1, we introduce Model 1, where goods flow from a supplier to either a central restock warehouse or an on-line depot, depending on the nature of the demand. When either the central warehouse or the on-line depot places an order with the supplier, the order is received after a lead time of $L$ periods. When any of the $J$ satellite stores places an order with the central warehouse, it takes $\ell$ periods of lead time to receive the order. $L_{sat,E}$ and $L_{dep,E}$ are the lead times associated with shipping from a store to on-line customers and from the dedicated on-line depot to on-line customers. For scenario 2, we introduce Model 2, where goods flow from a supplier to a central restock warehouse and then on to one of the $J$ satellite stores or to the on-line depot, again, depending on the nature of the demand. When the central warehouse places an order with a supplier, the order is received after a lead time of $L$ periods. When the satellite stores or the on-line depot place an order on the central restock warehouse, the order is received after $\ell$ periods.

In both Models 1 and 2, the central warehouse serves as a break-bulk or coordination point for items to be sent to the stores. Consistent with the literature on the two-echelon inventory problem and with what is becoming the more common management practice of cross docking, we assume that the central warehouse holds no inventory. The central warehouse is responsible for restocking the stores, and the on-line depot in the case of the second model, but does not fulfill any of the demand directly.

The average shipping distance from the dedicated on-line depot to on-line customers will be greater than the average shipping distance from geographically dispersed stores. Due to bulk service contracts, however, it is reasonable to assume that these lead times are similar in many settings. In addition, shipping cost tables from UPS and FedEx suggest that the cost differential can assure the same lead times, where the cost is higher for longer distance shipments to arrive with the same lead time to the customer, irrespective of the origin of shipment. We measure the service level based on whether or not the retail store or the on-line depot has the item in stock at the time of the order. If a customer asks for a product in a retail store and the item is not available, a backorder is incurred. Similarly, if a customer places an on-line order and the depot does not have the item at the time the order is received at the depot, then a backorder is incurred.
Each of the $J$ satellite stores is faced with independent per-period mean demand of $\mu_j$. The on-line depot is faced with a per-period mean demand of $\mu_{dep}$. Thus, per-period mean demand for the total system is $\mu_{Tot} = \Sigma \mu_j + \mu_{dep}$, where the $\mu_j$ are summed over all $J$ satellite stores. A fraction, $p_E$, of the total system demand consists of orders placed on-line, while the remaining demand $(1- p_E)$ occurs as a result of in-store purchases.

Each of the $J$ satellite stores is capable of holding inventory and is responsible for meeting its own in-store demand. No transshipments are allowed between the satellite stores or between the satellite stores and the on-line depot. The on-line demand can be met from any of the satellites or the on-line depot, where the fraction of on-line demand met by the on-line depot is denoted $r_{dep}$. Figure 2 illustrates the two extreme situations for scenario 1 where the on-line depot fulfills all of the on-line demand, $r_{dep}=1$, and where all on-line demand is met from the retail satellites, $r_{dep}=0$.

Demand is assumed to be normally distributed based on a binomial random variable representing customer purchases. The approximation of normality through a binomial random variable helps facilitate system comparisons. Specifically, the assumption used here is that there is a total market population of $N$ individuals, each of which has a $p_N$ probability of making a purchase in a given day (regardless of whether that purchase is made on-line or in a store). The standard deviation of demand at each of the $J$ satellite stores is $\sigma_j$ and at the on-line depot is $\sigma_{dep}$. A summary of the demand information is shown in Table 1.

<table>
<thead>
<tr>
<th>In-store + On-line sales at the average store satellite</th>
<th>On-line sales at the on-line depot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean demand per period</td>
<td></td>
</tr>
<tr>
<td>$\mu_{sat} = \mu_{Tot} \cdot \frac{(1- p_E + p_E \cdot (1- r_{dep}))}{J}$</td>
<td>$\mu_{dep} = \mu_{Tot} \cdot p_E \cdot r_{dep}$</td>
</tr>
<tr>
<td>$= N \cdot p_N \cdot (1- p_E \cdot r_{dep})/J$</td>
<td>$= N \cdot p_N \cdot p_E \cdot r_{dep}$</td>
</tr>
<tr>
<td>Stand. Dev. of demand per period</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{sat} = \sqrt{\mu_{sat} \cdot (1- p_N)}$</td>
<td>$\sigma_{dep} = \sqrt{\mu_{dep} \cdot (1- p_N)}$</td>
</tr>
</tbody>
</table>

Table 1: A summary of the means and standard deviations of demand
It should be noted that although \( \mu_{\text{sat}} \) and \( \sigma_{\text{sat}} \) are used to depict average mean and standard deviation levels across all satellites in Table 1, the actual means and standard deviations of each of the heterogeneous satellites (\( \mu_i \) and \( \sigma_i \)) used in the modeling and in the computational study are distributed around these overall averages. Inventory control for both models is assumed to follow a fixed-period order-up-to policy. In such a policy, some quantity is ordered every \( m \) periods to restock the facilities. When this quantity arrives at the central warehouse, it is allocated to each satellite store in such a way as to ensure that the expected probability of a stock-out \( \ell + m - 1 \) periods from then will be equal at each satellite. This policy is based on the simplifying assumption that the most significant stock-out levels will occur one period before satellite replenishment (e.g., Eppen and Schrage 1981). Since the service level is determined by availability at the time an order is placed, \( L_{\text{sat},E} \) and \( L_{\text{dep},E} \) do not play a role in the inventory cost or in the order up to quantity. On the other hand, \( L_{\text{sat},E} \) and \( L_{\text{dep},E} \) could have a major impact on the transportation cost. Therefore, they will therefore be subsumed into the per unit variable costs (\( V_{\text{sat},E} \) and \( V_{\text{dep},E} \)) to be discussed. Further, the inventory system must maintain a minimum service level defined as the maximum number of allowable backorders in any one period, \( \beta \). We use a single backorder limit for every part of the system although the formulations could easily be modified to have a different \( \beta \) for each component. A summary of the key system parameter notation is shown in Table 2.

<table>
<thead>
<tr>
<th>System Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J )</td>
</tr>
<tr>
<td>( L )</td>
</tr>
<tr>
<td>( \ell )</td>
</tr>
<tr>
<td>( L_{\text{sat},E} )</td>
</tr>
<tr>
<td>( L_{\text{dep},E} )</td>
</tr>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>( p_E )</td>
</tr>
<tr>
<td>( \beta )</td>
</tr>
<tr>
<td>( \mu_i )</td>
</tr>
<tr>
<td>( \mu_{\text{sat}} )</td>
</tr>
<tr>
<td>( \mu_{\text{dep}} )</td>
</tr>
<tr>
<td>( \sigma_i )</td>
</tr>
<tr>
<td>( \sigma_{\text{sat}} )</td>
</tr>
<tr>
<td>( \sigma_{\text{dep}} )</td>
</tr>
<tr>
<td>( \mu_{\text{Tot}} )</td>
</tr>
<tr>
<td>( N )</td>
</tr>
<tr>
<td>( p_N )</td>
</tr>
</tbody>
</table>

*Table 2: System Parameter Notation*

System performance is measured by total system costs. The relevant costs include the following: (a) the variable costs of dealing with on-line demand at the central warehouse, \( V_{\text{cw},E} \), which includes the handling costs of on-line demand at the central warehouse and the shipping costs of on-line demand from the supplier to the central warehouse to the store satellites, (b) the fixed and variable costs associated with the satellite stores handling on-line demand, \( F_{\text{sat},E} \) and \( V_{\text{sat},E} \) respectively, where \( V_{\text{sat},E} \) includes the handling costs of on-line demand at the store satellites and shipping costs of on-line demand from the store satellites to the on-line customers,
(c) the fixed and variable costs of dealing with on-line demand at the on-line depot, $F_{\text{dep},E}$ and $V_{\text{dep},E}$ respectively, where $V_{\text{dep},E}$ includes handling costs at the depot and shipping costs from the supplier to the on-line customer, and (d) the cost of holding one unit of inventory for one period anywhere in the system, $C_{\text{inv}}$. We do not include the fixed and variable costs of handling in-store demand at either the central warehouse or the retail satellites as these costs do not change with changes in how we handle on-line demand. It is important to note that the fixed costs here are the fixed cost of operating some part of the system, not the fixed investment cost of establishing the system in the first place. Fixed investment decisions should be included in the capital budgeting process where net present value methods are used over the life of the venture. While coverage of this kind of investment model is beyond the scope of this research, the model can be used to evaluate alternative configurations of the fulfillment system to understand the proper costs associated with the best type of on-line distribution.

We assume that there is always some level of in-store demand and for any given level of on-line demand $(0.02 \leq p_E \leq 0.98)$ these in-store demand costs do not change based on which part of the system handles the on-line demand. Unsatisfied demand is backordered and dealt with when stock becomes available. There can be backorders at the satellite stores for both in-store and on-line demand, and backorders at the depot for on-line demand. These system costs are summarized in Table 3.

<table>
<thead>
<tr>
<th>Cost Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{inv}}$</td>
<td>System inventory holding costs per unit per period</td>
</tr>
<tr>
<td>$V_{\text{cw},E}$</td>
<td>Variable cost per unit of the central warehouse dealing with on-line demand</td>
</tr>
<tr>
<td>$F_{\text{dep},E}$</td>
<td>Fixed operating cost of the on-line depot dealing with on-line demand</td>
</tr>
<tr>
<td>$V_{\text{dep},E}$</td>
<td>Variable cost per unit of the on-line depot dealing with on-line demand</td>
</tr>
<tr>
<td>$F_{\text{sat},E}$</td>
<td>Fixed operating cost of the satellites dealing with on-line demand</td>
</tr>
<tr>
<td>$V_{\text{sat},E}$</td>
<td>Variable cost per unit of the satellites dealing with on-line demand</td>
</tr>
</tbody>
</table>

Table 3: Cost Parameters Utilized in Mathematical Modeling

Model 1 assumes that the on-line direct deliveries that come from the on-line depot are kept separate from the portion of the system that replenishes the store satellites for both in-store and on-line fulfillment. This allows us to model the inventory and backorder levels without much trouble because that portion of the system involving the store satellites can be modeled with the equations of Eppen and Schrage (1981). The on-line demand coming from the on-line depot is a simple one-stage inventory model. Model 1 combines the two-stage store satellite system with the one stage on-line depot system.

Model 2 uses the results of Eppen and Schrage (1981) for the whole system. The assumption that the lead time to each of the satellite stores and to the on-line depot is the same $\ell$ periods is one that is necessary to use these results. In the case where the on-line depot is adjacent to the central restock warehouse, this model may overstate the shipment lead time and thus overstate the amount of inventory necessary to cover the demand uncertainty over the lead time. Therefore, we believe the results are useful despite this assumption.

The mathematical formulations of Model 1 and Model 2 are shown next. Additional notation needed for the modeling is presented in Table 4.
Main Decision Variables

\[ y_{cw,1} = \text{Order-up-to quantity for the central warehouse in Model 1} \]
\[ y_{dep,1} = \text{Order-up-to quantity for the on-line depot in Model 1} \]
\[ y_{cw,2} = \text{Order-up-to quantity for the central warehouse in Model 2} \]
\[ r_{dep} = \text{Fraction of on-line orders fulfilled by the on-line depot} \]
\[ X_{dep,E} = \text{Binary (0,1) variable indicating whether the on-line depot ships to customers} \]
\[ X_{sat,E} = \text{Binary (0,1) variable indicating whether retail store satellites ship directly to on-line customers} \]

Secondary Variables/Quantities that are a Function of Main Decision Variables

\[ I_{dep,1} = \text{Expected units of inventory per period at on-line depot in Model 1} \]
\[ I_{sat,1} = \text{Expected units of inventory per period summed over all retail store satellites in Model 1} \]
\[ I_{dep,2} = \text{Expected units of inventory per period at on-line depot in Model 2} \]
\[ I_{sat,2} = \text{Expected units of inventory per period summed over all retail store satellites in Model 2} \]
\[ B_{dep,1} = \text{Expected units backordered per period at on-line depot in Model 1} \]
\[ B_{sat,1} = \text{Expected units backordered per period summed over all retail store satellites in Model 1} \]
\[ B_{dep,2} = \text{Expected units backordered per period at on-line depot in Model 2} \]
\[ B_{sat,2} = \text{Expected units backordered per period summed over all retail store satellites in Model 2} \]
\[ Q_{sat,E} = \text{Quantity of on-line demand shipped from central warehouse to retail store satellites to on-line customers} \]
\[ Q_{dep,E} = \text{Quantity shipped from on-line depot to on-line customers} \]

Table 4: Additional Modeling Notation

Model 1 - Two-System Model

In Model 1, the retail satellite stores and the central on-line depot operate as independent subsystems. While Model 1 has the advantage of independent operations, the drawback is that shipments out of the supplier are not combined system-wide. Because of the two separate subsystems, the form of the optimization model used to determine the system's order-up-to levels, \( y_{cw,1} \) and \( y_{dep,1} \), and inventory allocation policy \( r_{dep} \) can be written as follows:

\[
\begin{align*}
\text{Min} & \quad C_{\text{inv}} \cdot [I_{dep,1}(y_{dep,1}, r_{dep}) + I_{sat,1}(y_{cw,1}, r_{dep})] + F_{dep,E} \cdot X_{dep,E} + J \cdot F_{sat,E} \cdot X_{sat,E} \\
& \quad + (V_{cw,E} + V_{sat,E}) \cdot Q_{dep,E}(r_{dep}) + V_{dep,E} \cdot Q_{dep,E}(r_{dep}) \\
\text{s.t.} & \quad B_{sat,1}(y_{cw,1}, r_{dep}) \leq \beta \\
& \quad B_{dep,1}(y_{dep,1}, r_{dep}) \leq \beta \\
& \quad M \cdot X_{dep,E} \geq Q_{dep,E}(r_{dep}) \\
& \quad M \cdot X_{sat,E} \geq Q_{sat,E}(r_{dep}) \\
& \quad 0 \leq r_{dep} \leq 1 \\
& \quad y_{cw,1}, y_{dep,1} \geq 0 \\
& \quad X_{dep,E}, X_{sat,E} \in \{0,1\}
\end{align*}
\]
In constraints (1b) and (1c), $B_{sat,1}(ycw,1,r_{dep})$ represents the expected units backordered per period summed over all the satellites, $B_{dep,1}(y_{dep},1,r_{dep})$ represents the expected units backordered per period as a result of direct to customer on-line orders handled at the on-line depot, and $\beta$ represents an upper limit on these backorder quantities. In constraints (1d) and (1e), $M$ is a large constant.

For the retail outlet sub-system, the per-period estimates of Eppen and Schrage can be used to yield the following equations:

$$I_{sat,1}(ycw,1, r_{dep}) = (\ell + (m - 1)/2) \cdot \sum_{j=1}^{J} \mu_{j} + z_{cw,1} \cdot \sum_{j=1}^{J} \hat{\sigma}_{j} + B_{sat,1}(ycw,1, r_{dep})$$  \hfill (1i)

$$B_{sat,1}(ycw,1, r_{dep}) = R(z_{cw,1}) \cdot \frac{\sum_{j=1}^{J} \hat{\sigma}_{j}}{m}$$  \hfill (1j)

The function $R(\bullet)$ represents the right-tailed linear loss function, while $z_{cw,j}$ and $\hat{\sigma}_{j}$ are defined as follows:

$$z_{cw,1} = [y_{cw,1} - (L + \ell + m) \cdot \sum_{j=1}^{J} \mu_{j}] / \sum_{j=1}^{J} \hat{\sigma}_{j}$$  \hfill (1k)

$$\hat{\sigma}_{j} = \sigma_{j} \cdot \sqrt{L \cdot \sum_{j=1}^{J} \sigma_{j}^2 / (\sum_{j=1}^{J} \sigma_{j})^2 + \ell + m}$$  \hfill (1l)

$$I_{dep,1}(y_{dep},1, r_{dep}) = y_{dep,1} - \mu_{dep} \cdot [(L + m) - (m - 1)/2] + B_{dep,1}(y_{dep},1, r_{dep})$$  \hfill (1m)

$$B_{dep,1}(y_{dep},1, r_{dep}) = \sigma_{dep} \cdot \sqrt{m + L} \cdot R\left(\frac{y_{dep,1} - (L + m) \cdot \mu_{dep}}{\sigma_{dep} \cdot \sqrt{m + L}}\right) / m$$  \hfill (1n)

Since all demand is ultimately fulfilled, either immediately or via backorders, the equations for $Q_{dep,E}(r_{dep})$ and $Q_{sat,E}(r_{dep})$ used in the shipping and handling cost calculations are straightforward:

$$Q_{dep,E}(r_{dep}) = \mu_{dep}(r_{dep}) = \mu_{Tot} \cdot P_{E} \cdot r_{dep}$$  \hfill (1o)

$$Q_{sat,E}(r_{dep}) = \mu_{Tot} \cdot P_{E} \cdot [1-r_{dep}]$$  \hfill (1p)

**Model 2 - Equal Lead Time Model**

In Model 2, we consider a setting with an on-line depot modeled as an additional satellite that is replenished from the central warehouse. As long as the lead time to this on-line depot is assumed to be the same as for the regular retail satellites, Eppen and Schrage’s two-echelon model can be used for an approximation of the average inventory level and the backorder level for the store satellites and the on-line depot.

The form of the optimization model used to determine the order-up-to level ($y_{cw,2}$) and inventory allocation policy ($r_{dep}$) for Model 2 can be written as follows:
\[ \text{Min} \quad C_{\text{inv}} \left[ I_{\text{dep},2}(y_{\text{cw},2}, r_{\text{dep}}) + I_{\text{sat},2}(y_{\text{cw},2}, r_{\text{dep}}) \right] + F_{\text{dep},E} \cdot X_{\text{dep},E} + J \cdot F_{\text{sat},E} \cdot X_{\text{sat},E} \\
+ (V_{\text{cw},E} + V_{\text{sat},E}) \cdot Q_{\text{sat},E}(r_{\text{dep}}) + V_{\text{dep},E} \cdot Q_{\text{dep},E}(r_{\text{dep}}) \] (2a)

s.t. \quad B_{\text{sat},2}(y_{\text{cw},2}, r_{\text{dep}}) \leq \beta \quad (2b)

\[ B_{\text{dep},2}(y_{\text{cw},2}, r_{\text{dep}}) \leq \beta \] (2c)

\[ M \cdot X_{\text{dep},E} \geq Q_{\text{dep},E}(r_{\text{dep}}) \quad (2d) \]

\[ M \cdot X_{\text{sat},E} \geq Q_{\text{sat},E}(r_{\text{dep}}) \quad (2e) \]

\[ 0 \leq r_{\text{dep}} \leq 1 \quad (2f) \]

\[ y_{\text{cw},2} \geq 0 \quad (2g) \]

\[ X_{\text{dep},E}, X_{\text{sat},E} \in \{0, 1\} \quad (2h) \]

The estimates for the per-period inventory and backorder quantities are:

\[ I_{\text{dep},2} + I_{\text{sat},2} = (\ell + (m - 1) / 2) \cdot (\sum_{j=1}^{J} \mu_j + \mu_{\text{dep}}) + z_{\text{cw},2} \cdot (\sum_{j=1}^{J} \sigma_j + \sigma_{\text{dep}}) \]

\[ + B_{\text{sat},2}(y_{\text{cw},2}, r_{\text{dep}}) + B_{\text{dep},2}(y_{\text{cw},2}, r_{\text{dep}}) \] (2i)

\[ B_{\text{sat},2} = R(z_{\text{cw},2}) \cdot \sum_{j=1}^{J} \sigma_j / m \] (2j)

\[ B_{\text{dep},2} = R(z_{\text{cw},2}) \cdot \sigma_{\text{dep}} / m \] (2k)

where \( z_{\text{cw},2} \) and \( \sigma_{\text{dep}} \) in general are defined as:

\[ z_{\text{cw},2} = \left[ y_{\text{cw},2} - (L + \ell + m) \cdot (\sum_{j=1}^{J} \mu_j + \mu_{\text{dep}}) \right] / (\sum_{j=1}^{J} \sigma_j + \sigma_{\text{dep}}) \] (2l)

\[ \sigma_j = \sigma_j \cdot \sqrt{L \cdot (\sum_{j=1}^{J} \sigma_j^2 + \sigma_{\text{dep}}^2) / (\sum_{j=1}^{J} \sigma_j + \sigma_{\text{dep}})^2 + \ell + m} \] (2m)

The equations for \( Q_{\text{dep},E}(r_{\text{dep}}) \) and \( Q_{\text{sat},E}(r_{\text{dep}}) \) are equivalent to those defined for Model 1.

**Analysis and Results**

To reiterate, the purpose of this investigation is to determine situations in which the complete or partial decentralized storage of on-line inventory may provide total cost benefits that outweigh those of a completely centralized policy. Therefore, while the order-up-to level decision remains a primary decision variable, this investigation also requires the consideration of how on-line demand fulfillment is allocated to either the integrated retail satellites or the on-line depot. Also of interest is the impact that the fraction \( (p_E) \) of the firm’s total demand occurring on-line can have on the allocation policies as well as the impact of non-identical demands across the retail stores.
Model 1 - Two-System Model

To approximately solve the optimization problem for Model 1 given by equations (1a)-(1p), we apply the following approach to all possible combinations of values of $X_{\text{dep},E}$ and $X_{\text{sat},E}$. It is an approximate solution to the model because we fix $r_{\text{dep}}$ at a range of values and then solve each of the resulting subproblems for the order-up-to quantity. Note that with $X_{\text{dep},E}$, $X_{\text{sat},E}$, and $r_{\text{dep}}$ fixed, Model 1 decomposes into two single variable subproblems, one involving $y_{\text{dep},1}$ and the other $y_{\text{cw},1}$. Each subproblem has one backorder constraint and one non-negativity condition on the order-up-to quantity. Because inventory is increasing in the order-up-to quantity and backorders are decreasing in the order-up-to quantity, the single backorder constraint in each subproblem will be tight at optimality. Therefore, we solve for the root of this one nonlinear equation to obtain the order-up-to quantity. The solution over this range of $r_{\text{dep}}$ values is the one yielding the minimum total cost.

We solved Model 1 with various parameter values, and they all provide the same general conclusions as discussed below. For illustration, we set the backorder constraints to 15% and purchasing probability sufficiently small to maintain coefficients of variation approximately equal to 1 throughout (e.g., $p_N = 1E-5$). The minimum total cost levels are illustrated in Figures 3 - 6.

Figure 3 provides a set of plots of total cost as a function of $p_E$ for various values of the fixed cost of e-fulfillment at the satellites ($F_{\text{sat},E}$). Other problem parameters were set at $J=20$, $C_{\text{inv}}=0.2$, $V_{\text{cw},E} = V_{\text{sat},E} = 0.5$, $V_{\text{dep},E} = 1$, and $F_{\text{dep},E} = 1$. Recall that solution values were obtained for $p_E$ values between 2% and 98%, which is why the lines in the figures stop before reaching 0% on the left and 100% on the right.

As anticipated, when the fixed cost of operating the satellites for shipping on-line deliveries is high ($F_{\text{sat},E} = .06$), centralization ($r_{\text{dep}} = 1$) dominates over all proportions of on-line demand ($p_E$),
as can be seen in the rightmost plot of Figure 3. When there is no fixed operating cost to having
the satellites \( F_{\text{sat},E} = 0.0 \) but there is a fixed cost for operating the central on-line depot \( F_{\text{dep},E} = 1 \),
then decentralization \( \nu_{\text{dep}} = 0 \) dominates, as can be seen in the leftmost plot of Figure 3.
However, as shown in the center plot of Figure 3, at some intermediate satellite fixed-cost level, a
dominance cross-over point is observed. Under this intermediate scenario, a completely
centralized on-line depot system dominates only at high \( p_E \) levels (i.e., when a large portion of the
firm’s market is electronic). Below certain threshold \( p_E \) levels (illustrated by the dotted line in
Figure 3), complete decentralization to the retail satellites is preferred.

The discussion above shows that there is some relative level of the two operating fixed
costs \( (F_{\text{sat},E}, F_{\text{dep},E}) \) which leads to conclusions about centralization or decentralization. For the
rest of the discussion about the model, the explanations about the preferences of a centralized
system or a decentralized one are much easier when the fixed costs are removed. This is done to
illustrate that even though the fixed costs do play a part in determining the correct system
configuration, the fixed costs may not be the only reason for the dominance cross-over point. In
fact, we will show that the general tendencies of models to exhibit dominance cross-over points
(i.e., \( p_E \) thresholds) are not affected by the inclusion of the fixed costs – the fixed costs simply shift
the location of the dominance cross-over point. Hence, the fixed costs of operating the on-line
depot and the retail satellites are set to zero \( (F_{\text{dep},E} = 0.0 \text{ and } F_{\text{sat},E} = 0.0) \) for the rest of the
discussion in order to study the implications of the other relevant costs.

In Figure 4, we consider three cases for the value of the unit variable shipping and
handling cost from the on-line depot, \( V_{\text{dep},E} \): (1) \( V_{\text{dep},E} < V_{\text{cw},E} + V_{\text{sat},E} \), (2) \( V_{\text{dep},E} = V_{\text{cw},E} +
V_{\text{sat},E} \), and (3) \( V_{\text{dep},E} > V_{\text{cw},E} + V_{\text{sat},E} \). For example, case (3) could be reasonable due to
economies of scale in bulk shipping from the central warehouse to the satellites and the proximity
of the satellites to on-line patrons. This effect would be expected to be higher in the case of large
sized items. Figure 4 provides a set of plots of total cost as a function of \( p_E \) for various values of the variable cost parameter \( V_{\text{dep},E} \) illustrating the above three cases. Other problem parameters
were set at \( J = 20 \), \( C_{\text{inv}} = 0.2 \), \( V_{\text{cw},E} = V_{\text{sat},E} = 0.5 \), \( F_{\text{sat},E} = F_{\text{dep},E} = 0 \). Again, note that all fixed costs
of on-line fulfillment are set to zero.
As expected, when variable costs at the online depot are relatively low ($V_{dep,E} = 0.1$), the centralized strategy yielded the lowest total costs across nearly all $p_E$ scenarios. When variable costs at the online depot are relatively high ($V_{dep,E} = 2.5$), complete decentralization provided the lowest total costs. But for case (2), where $V_{dep,E} = 1 = V_{cw,E} + V_{sat,E}$, the best strategy depends largely on the value of $p_E$. As in Figure 3, at low $p_E$ decentralization dominates, and at high $p_E$ centralization dominates.

In Figure 5, we consider how various values of inventory carrying costs impact the recommended policy. In this case the results continue to show dominance either by the complete centralized or complete decentralized policies. As in Figures 3 and 4, at low $p_E$ values the decentralized policy dominates, at high $p_E$ values the centralized policy dominates. Problem parameters were set at $J=20$, $V_{cw,E} = V_{sat,E} = 0.5$, $V_{dep,E} = 1$, $F_{sat,E} = F_{dep,E} = 0$. 

It is interesting to note that, as in the previous cases, there is no preference for mixed allocation policies (part centralized and part decentralized, i.e., we never have $0 < r_{dep} < 1$). While this policy is somewhat intuitive for the cases where the fixed costs dominate, it is not so intuitive where only the two competing inventory pooling effects and variable costs are present.

Finally, one might expect that as the number of satellites is increased, the best system would tend to be one of centralized fulfillment from the online depot. This generally is true as one can see from the dark single dashed line representing the threshold in each of the three panels of Figure 6. Problem parameters were set at $C_{inv}=0.1$, $V_{cw,E} = V_{sat,E} = 0.5$, $V_{dep,E} = 1.25$, $F_{sat,E} = 0$. 

Figure 4: Minimal Costs for Various Shipping & Handling Scenarios (Model 1)

Figure 5: Minimal Costs for Various Inventory Holding Cost Scenarios (Model 1)
$F_{\text{dep},E} = 0$. (Ignore for now the double lines identified as A and B.) Note that as one proceeds from a small number of stores to a large number of stores, the threshold $p_E$ value moves from a relatively high percentage to a relatively low percentage. Thus, for a small number of stores, most values of $p_E$ leads to a decentralized system, and for a large number of stores, most values of $p_E$ leads to a centralized system. Still, there is a threshold for each case and it should be noted that mixed allocation policies are always dominated even as the number of satellites is changed.

Figure 6: Minimal Inventory Cost for Various Facility Scenarios (Model 1)

We use Figure 6 and Table 5 below to illustrate one further issue – what happens to the threshold $p_E$ value when the variable costs are changed. If one removes all of the variable costs ($V_{cw,E} = V_{sat,E} = V_{\text{dep},E} = 0$) as compared to the base case ($V_{cw,E} = V_{sat,E} = 0.5$, $V_{\text{dep},E} = 1.25$), then one is left with Case A, where inventory cost alone is present in the model. Under this extreme case, the threshold value moves lower to the double dashed line labeled A. Without the model, it is not apparent what the best policy would be when the inventory cost is the only factor considered since there are two competing pooling effects, one due to centralization of the on-line inventory at the depot and the other due to the pooling of the on-line and in-store inventory at the satellite.

Next, for Case B, when the variable costs are negligible for on-line orders fulfilled from the satellite, as could happen if customers picked up their on-line orders at a local retail satellite. If the on-line fulfillment costs from the satellite are negligible ($V_{\text{sat},E} = 0.0$) as compared to the base case ($V_{\text{sat},E} = 0.5$), one would expect that the preferred strategy would be more one of a decentralization strategy as compared to the base case. This is indeed what the model predicts as the threshold value moves from the base case threshold (single dark dashed line) to the right double dashed line labeled B, showing that for more values of $p_E$, the decentralized system is preferred.

Table 5 shows this same information in a slightly different format. It shows that depending on the number of satellites and different values of the costs, the threshold $p_E$ value can be as high as 97.8% and as low as 4.7%. For a given number of satellites, the table shows the range of
resulting threshold values as the variable on-line depot costs are increased from 0 to 1.25. Thus, as centralized fulfillment costs for the on-line customers are increased, the threshold level for moving to a centralized system increases as well. This could be an explanation for what happened to Webvan, for as Boyer et al. (2002) point out, WebVan’s competitive strategy of centralized distribution to increase inventory turns resulted in higher transportation costs from this central facility, thereby negating the inventory pooling benefits at the central fulfillment site.

<table>
<thead>
<tr>
<th># Satellites (J)</th>
<th>Case A: Inventory Cost Only</th>
<th>Case B: Negligible Satellite Costs for On-line Customer Delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C_{in} = 0.1 ) ( V_{cw,E} = V_{sat,E} = V_{dep,E} = 0.0 )</td>
<td>( C_{in} = 0.1, V_{cw,E} = 0.5, V_{dep,E} = 1.25 ) ( F_{sat,E} = F_{dep,E} = 0 )</td>
</tr>
<tr>
<td>10</td>
<td>Threshold ( p_E = 34.4% )</td>
<td>Threshold ( p_E = 97.8% )</td>
</tr>
<tr>
<td>20</td>
<td>Threshold ( p_E = 22.3% )</td>
<td>Threshold ( p_E = 96.5% )</td>
</tr>
<tr>
<td>100</td>
<td>Threshold ( p_E = 4.7% )</td>
<td>Threshold ( p_E = 18.2% )</td>
</tr>
</tbody>
</table>

Table 5: Threshold \( p_E \) Values for Special Cases

Given these interpretations, perhaps of greatest managerial interest are the threshold \( p_E \) levels above and below which these different allocation strategies are preferred, and at which points these often dramatic changes in costs dynamics can be realized. The extent to which decentralization can be beneficial system-wide is certainly dependent on fixed and variable costs but also dependent on the extent to which safety stock requirements at local stores can be reduced through channel pooling, as compared to the pooling benefits at the on-line depot. Therefore, a method to predict whether centralization or decentralization is preferred might consider (1) backorder limits and (2) demand per local facility as main factors impacting these threshold levels. With this idea in mind, these thresholds are presented in Figure 7. The backorders limits are given as percentages and are set at the same value at both the on-line depot and the satellites.
Centralized vs. Decentralized On-Line Storage (Model 1)

Using Figure 7, a firm can plot their own $p_E$ and $\mu_{Tot/J}$ for a given service level and determine if they fall above the curve where complete centralization is optimal ($r_{dep} = 1$), or below the curve where complete decentralization is preferred ($r_{dep} = 0$). As suggested by the discussion of factors that effect the location of policy thresholds, these curves will shift with changes in cost parameters. As the fixed and variable costs increase for the on-line depot, the curve shifts up. As the fixed and variable costs increase for the retail satellites, the curve shifts down. Since the specific location of these curves are dependent on fixed and variable costs, firms can take advantage of expected trends in such costs and be proactive in determining their inventory allocation strategy.

So far the analyses have focused on cost tradeoffs in fulfillment strategy in order to determine the threshold $p_E$ values at which centralization or decentralization is preferred from the overall cost perspective. Therefore, the demand at each site was assumed the same. To study the impact of non-identical demand on the threshold $p_E$ values, the mean demand across satellites was varied from the case where all satellite mean demand fell within ±10% of the overall mean demand to the case where satellite mean demand fell within ±90% of the mean demand. Thus, individual satellite mean demand was varied within some proportion factor $k$, $k=[0.1,0.9]$, of the overall mean demand (i.e., $\mu_i = [\mu_{sat}-k*\mu_{sat}, \mu_{sat}+k*\mu_{sat}]$). The results of these heterogeneous demand trials illustrates that as the variation in mean demand across satellites is increased, the threshold $p_E$ values also increases. This behavior is depicted in Figure 8.

![Figure 8](image)

**Figure 8. Relationship between $p_E$ Thresholds and variations in mean demand among satellites**

The threshold $p_E$ values tend to increase with the increase in variation in the demand among the satellites. The change gets smaller as the number of stores increases. In general, the results show that the analyses are applicable under varying demand conditions.
Model 2 - Equal Lead Time Model

As stated before, an alternate approach is to model the system in Figure 1 as one in which both the on-line depot and the various retail outlets are characterized by lead times of \( \ell \) from a single central warehouse. As in Model 1, we solve the optimization problem given by (2a)-(2m) by fixing \( r_{dep} \) at a range of values, solving for the corresponding \( y_{Tot,2} \) values, and choosing the solution as the one that yields the minimal total cost.

Once again, for the environments considered in this paper, either complete centralization (\( r_{dep} = 1 \)) or complete decentralization (\( r_{dep} = 0 \)) was always best in terms of minimizing total costs subject to the backorder constraint. The result is a threshold curve of the form proposed in Figure 9. Note the general similarity in trends depicted here for Model 2 with those presented earlier for Model 1. While we will not show all of the same graphs, all of the results for Model 2 are similar to those of Model 1.

![Graphs showing threshold levels for centralized vs. decentralized on-line storage](image)

Figure 9: Threshold \( p_E \) Levels for the Optimality of Centralized vs. Decentralized On-Line Storage (Model 2)

An organization may want to use slightly different approaches for setting upper limits on backorders. For example, they could set an upper limit on total backorders, rather than one on on-line depot backorders and another on retail store backorders. Based on this approach, an alternate formulation can easily be developed. This alternate version includes just one constraint on total backorders, rather than two separate backorder constraints. We solved a set of test problems using this approach and obtained the same general results as just discussed.

A third scenario firms might consider is where the central warehouse doubles as an on-line depot. Therefore, the next section addresses this setting. Again, as stated earlier, we believe this scenario is one that is rarer than scenarios 1 and 2 – central warehousing facilities and operations are generally not set up for picking and packing unit deliveries.
Scenario 3: Consolidating Replenishment and On-line Delivery from the Central Warehouse

In scenario 3, where the central warehouse ships on-line orders, the associated model for deriving order up to levels and minimum costs is difficult to formulate and solve. Therefore, in this section we check whether Model 1 and/or Model 2 can be used to approximate the system described in Figure 10 below. We compare the Model 1 and 2 results with results from a simulation of the 2-echelon system in Figure 10.

An experimental design for such a simulation involves considering a range of levels for the decision variables, as well as a range of levels for certain system parameters of key interest (specifically $p_E$ and $J$ in this case). For Model 1, in order to determine reasonable levels of the order up to levels to use in the simulation, these values are based directly off of those that would be estimated as optimal (by the optimization problem (1a)-(1p)) under specific backorder requirements. We use lead times $\ell = 1$ day and $L = 1$ day, and purchasing probability sufficiently small to maintain coefficients of variation approximately equal to 1 throughout (e.g., $p_N = 1E-5$). Of course, the analysis can be performed for any values of these parameters.

A complete description of the experimental design used is provided in Table 6.

<table>
<thead>
<tr>
<th>Design Factor</th>
<th>Factor Levels</th>
<th>Interpretations of Level Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage electronic market (pe)</td>
<td>0.05, 0.50, 0.95 (3 Levels)</td>
<td>“Mostly In-Store” to “Mostly On-line”</td>
</tr>
<tr>
<td>Percentage of on-line inventory centralized (rdep)</td>
<td>0, 0.5, 1.0 (3 Levels)</td>
<td>“All decentralized” to “All centralized”</td>
</tr>
<tr>
<td>Number of satellites (J)</td>
<td>2, 4, 16 (3 Levels)</td>
<td>2 satellites to 16 satellites</td>
</tr>
<tr>
<td>Backorder levels (as a %)</td>
<td>5%, 15%, 30% (3 Levels)</td>
<td>5% to 30% at satellites &amp; central warehouse</td>
</tr>
</tbody>
</table>

Table 6: Experimental Design for Comparing Model 1 with Simulation
For the range of factors in Table 6, the simulated backorder and inventory position estimates were shown to be statistically indistinguishable from those estimated by the approximations of equations (1a)–(1p) at the 10% level (the null hypothesis states that the simulated inventory or backorder level is equal to the estimated value). Table 7 provides specific p-values for t-test comparisons conducted within each level of the four experimental design factors. For a given factor level, each test was carried out over all level combinations of the other factors, resulting in 3x3x3x(5 runs per cell)=135 comparisons per cell. These results suggest that Model 1 provides a reasonable model of the 2-echelon system for the factor levels considered.

Table 7. Comparisons of mathematically estimated and simulated performance measures for Model 1

<table>
<thead>
<tr>
<th>Experimental Factor Levels</th>
<th>Within-level paired t-Test p-Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I_{sat,1}</td>
</tr>
<tr>
<td>P_E = 0.05</td>
<td>0.877</td>
</tr>
<tr>
<td></td>
<td>0.735</td>
</tr>
<tr>
<td></td>
<td>0.869</td>
</tr>
<tr>
<td>r_{dep} = 0</td>
<td>0.896</td>
</tr>
<tr>
<td></td>
<td>0.433</td>
</tr>
<tr>
<td></td>
<td>0.896</td>
</tr>
<tr>
<td>r_{dep} = 0.5</td>
<td>0.894</td>
</tr>
<tr>
<td></td>
<td>0.832</td>
</tr>
<tr>
<td></td>
<td>0.755</td>
</tr>
<tr>
<td>r_{dep} = 1</td>
<td>0.826</td>
</tr>
<tr>
<td></td>
<td>0.852</td>
</tr>
<tr>
<td></td>
<td>0.861</td>
</tr>
</tbody>
</table>

For Model 2, estimates of inventory and backorders were again compared with the simulation results. As anticipated, the addition of the non-zero lead time between the central warehouse and the on-line depot created a gap between the estimates and the simulation results. This gap can be seen by the many p-values below 0.10 in the first three columns of p-values in Table 8, indicating a significant difference between the estimated and simulated values for these cells. In general, however, this difference could be roughly “approximated out” through the subtraction of one period worth of inventory in transit to the centralized on-line depot ($\mu_{dep}$) for the replenishment lag $m$. The last three columns of Table 8 provide the p-values for comparing the “corrected” estimates with the simulated values. As would be expected, the correction for the additional lead time component provides much more comparable results to those simulated, therefore validating the need for its use in Model 2. There remain, however, three cells in the last three columns where there is a significant difference between the corrected estimates and the simulated values (p-value < 0.10). The results show that Model 1 and corrected Model 2 can be used by decision makers to reasonably approximate scenario 3 as given in Figure 10.
Table 8. Comparisons of mathematically estimated and simulated performance measures for Model 2

Discussion and Concluding Remarks

In this paper we have addressed the issue of where to carry inventory in the supply chain for a firm that has web-based sales or is adding web-based sales to their existing retail sales channel. The present work is unique in its consideration of inventory pooling mechanisms made possible through on-line inventory centralization at a central facility versus pooling on-line and in-store inventory at the retail stores. When local channel pooling is possible, we show that complete decentralization of on-line inventory can provide system-wide total cost benefits under certain market scenarios.

Perhaps the most relevant finding of the current work is that there exist threshold on-line demand levels, as percentages of a firm’s total demand, which represent a cutoff point between the two best on-line inventory allocation strategies. Below these threshold levels of on-line demand, complete decentralization of on-line inventory to the retail stores is preferable from a system-wide cost minimizing perspective, while above these thresholds complete centralization of on-line inventory to a central facility is preferable. Furthermore, these threshold levels themselves can be roughly approximated given knowledge of the ratio of total system demand per facility, and managerially established backorder constraints at those facilities.

Although valuable, these findings represent only a limited view of the considerations involved in Clicks-and-Mortar operating strategies. From an inventory management perspective, we illustrate that transportation and handling costs related to customer-direct distribution, either centrally or locally, as well as the fixed costs can have a major impact on the allocation policy selected. These strategies may also include the need for additional cooperation in order to maintain strong relationships with satellite partners (given a real or perceived threat of business losses for those partners to on-line sales). On the other hand, the localization of on-line inventories also implies an ability to rely on neighboring facilities for customer-direct delivery service when inventories are low. These issues represent an important direction for future research.
References

Associated Press, Safeway adds Seattle to online grocery shopping service, USA Today, July 17, 2003.


Brady, D., How Barnes & Noble Misread the Web, Business Week, 3667, 2000, pp. 63


Montgomery, T., New deliveries on the block: Online grocery services are back, this time with a new business model, The Orange County Register, Feb. 9, 2002.

Mullaney, T.J., Gone But Not Forgotten: The dot-com dropout class of 2000 tells a lot about what won't work online, Business Week, 3716, 2001, EB14-EB16


Schwarz, L., A Model for Assessing the Value of Warehouse Risk-Pooling, Management Science,
1989, 35(7), pp. 828-843
