Loading Factors and Equilibria in Insurance Markets

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Abstract: This paper examines the effect of introducing positive loading factors into insurance premia, in insurance markets consisting of groups of individuals in different risk categories. It is shown that that such loading factors may have far-reaching effects on insurance market equilibria.

INTRODUCTION

The existence and nature of equilibria reached in insurance markets consisting of groups of individuals in different risk categories has attracted much attention in the literature.[1] At the same time, there has been interest in the literature in the functional forms of the loading factor component in insurance premia.[2] Surprisingly, the relation between insurance loading factors and the type of equilibria in insurance markets has not been examined in the literature. It is the purpose of this paper show that loading factors may have far-reaching effects on insurance market equilibria.

The point is made by constructing a simplified model of an insurance market. The equilibrium in this market is then examined both in the absence and in the presence of a simple and very commonly used type of loading factor. Specifically, it is assumed that there are two types of risk-averse consumers—high and low-risk—and that these consumers can either fully insure their risk or remain fully exposed to it. Insurance coverage is indivisible. The market is competitive. Under these assumptions and in the absence of loading factors, the market will be characterized

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by one of two equilibria. The first is a pooling equilibrium, in which all consumers buy insurance. Because the price of insurance is competitively determined and equal for both types of individuals, this type of equilibrium implies that the low-risk individuals are subsidizing the price paid for insurance by the high-risk individuals. The second equilibrium is a separating equilibrium in which the high-risk individuals buy insurance whereas the low-risks do not. This occurs if the low-risk individuals are not sufficiently risk-averse to be induced to pay for insurance whose price partially depends on the riskiness of the high-risk individuals. In the absence of loading factors there will be no equilibrium in which no insurance is bought.

Once loading factors are introduced, however, potential market outcomes are significantly altered. In addition to the two types of equilibria described above, an equilibrium in which no insurance is bought becomes possible. This occurs if the loading factor exceeds the risk premium that the high-risk individuals are prepared to pay. Using simulations, and a specific numerical example, we show that large groups of potential buyers may refrain from buying insurance as a result of small loading factors.

THE MODEL

Consider a population in which every individual has initial wealth W and utility function $U(Z)$, where Z is final wealth. Individuals are risk-averse, i.e., $U'(Z) > 0$ and $U''(Z) < 0$, and they maximize expected utility. A proportion α of this population consists of high-risk (h) individuals, who stand to incur a loss equal to X with a probability π_h. The remaining individuals, who make up $1 - \alpha$ of the population, are low-risk (l), and these individuals stand to lose X with a probability π_h, where $\pi_h > \pi_l$. Individuals' risks are independent of each other. If an individual does not incur a loss, his wealth remains unchanged, $Z = W$.

An individual can either remain exposed to the risk or buy full insurance coverage against it.[3] However, while each individual knows his own type, this information is not available to insurance companies, because the cost of risk classification is prohibitively high. Insurance companies charge premia that reflect both expected claims and positive loading. The loading covers commissions, administrative costs, and settlement expenses, as well as normal profit.[4] Thus, given the expected value of a claim, A, the premium charged is $f(A)$ such that $f'(A) > 1$. In this paper we assume that $f(A) = (1 + \lambda)A$ where $\lambda (\geq 0)$ is a constant *loading factor*[5,6], whose value is determined by competitive market forces,[7] and investigate the relation between market equilibrium and λ. As mentioned above, we find that there are three

possible outcomes, depending on the values of α and λ. In the event that $\lambda = 0$, however, the three outcomes collapse into two.

NO LOADING FACTOR: $\lambda = 0$

A Separating Equilibrium

A *separating* equilibrium is one in which different individuals, in this case both the high and low-risks, behave differently. Thus, there are two potential separating equilibria. In the first, high-risk individuals insure and low-risk individuals do not. In the second, low-risk individuals insure and high-risk individuals do not. However, as is intuitive and shown below, only the first of these equilibria is viable. To see this, note that if only high-risks insure, the insurance premium is

$$P_s^h = \pi_h X.$$

Similarly, if only low-risks insure, the insurance premium is

$$P_s^l = \pi_l X.$$

Clearly, an equilibrium in which only low-risks insure is not viable, since if the low-risks are prepared to insure at a premium equal to P_s^l, then, *a fortiori*, so will the high-risks. Given a premium equal to P_{sl}^h, the benefit of insurance to high-risk individuals is

$$B_{sl}^h = U(W - \pi_h X) - (1 - \pi_h)U(W) - \pi_h U(W - X).$$

Also, given a premium equal to P_s^h the benefit of insurance to low-risk individuals is

$$B_{sl}^l = U(W - \pi_h X) - (1 - \pi_h)U(W) - \pi_l U(W - X).$$

Hence, since

$$B_{sl}^h - B_{sl}^l = (\pi_h - \pi_l)[U(W) - U(W - X)] > 0,$$

a separating equilibrium, in which the low-risks insure but the high-risks do not, is not possible. Moreover, if only the high-risks insure, and the insurance premium is, therefore, $\pi_h X$, the net benefit of insurance to a high-risk individual is

$$B_{sh}^h \; = \; U(W - \pi_h X) - (1 - \pi_h)U(W) - \pi_h U(W - X),$$

which is positive by the concavity of U. Thus, high-risks insure even if low-risks do not. Note that $B_{sh}^h > 0$ for all values of α.

A Pooling Equilibrium

A *pooling* equilibrium is one in which all individuals behave in the same way. Thus, there are, potentially, two such equilibria: first, an equilibrium in which both high and low-risk individuals purchase the insurance offered, and second, an equilibrium in which both high and low-risk individuals *do not* purchase the insurance offered. In a pooling equilibrium in which all individuals insure, the expected claim per policy is given by $A = (\alpha \pi_h + (1 - \alpha)\pi_l)$, implying that the insurance premium is

$$P_p \; = \; A \; = \; (\alpha \pi_h + (1 - \alpha)\pi_l)X.$$

In deciding whether or not to purchase insurance, each individual considers his expected utility with and without insurance. In the absence of insurance, the final wealth of a low-risk individual is W with probability $1 - \pi_l$, and $W - X$ with probability π_l. If he does buy insurance, his final wealth is $W - P_p$ with certainty. In a pooling equilibrium, therefore, the net benefit of buying insurance for an individual in risk category i, $(i = l, h)$ is B_p^i, where

$$B_p^i \; = \; U(W - P_p) - (1 - \pi_i)U(W) - \pi_i U(W - X).$$

An individual in group i buys insurance if and only if $B_p^i \geq 0$. Hence, since

$$B_p^h - B_p^l \; = \; (\pi_h - \pi_l)[U(W) - U(W - X)] > 0,$$

this pooling equilibrium exists if and only if

$$B_p^l = U[W - (\alpha\pi_h + (1-\alpha)\pi_l)X] - (1-\pi_i)(U(W)) - \pi_l U(W-X) \geq 0 .$$

It transpires that whether or not a pooling equilibrium exists depends on α and on the utility function of the individuals making up the population.[8] Note that if $\alpha = 0$, then

$$B_p^l = U[W - \pi_l X] - (1-\pi_l)U[W] - \pi_l U[W-X] > 0 ,$$

because U is concave. Thus, for a sufficiently small α, a pooling equilibrium will exist. Clearly,

$$\frac{\partial B_p^l}{\partial \alpha} = -(\pi_h - \pi_l)XU'(W - P_p) < 0 ,$$

i.e., an increase in α reduces B_p^l. An increase in the number of high-risk individuals raises the premium charged in a pooling equilibrium. This makes the buying of insurance less attractive to low-risk individuals. There may therefore exist a value of α, $\alpha*$ (defined by $B_p^l = 0$) such that a pooling equilibrium exists for $\alpha \leq \alpha*$ but only a separating equilibrium exists for $\alpha > \alpha*$. Of course, the value of $\alpha*$ is a function of W, X, π_h, and π_l, as well as the risk aversion of a representative individual. It is useful to illustrate this with a specific utility function and a simple numerical example. Let the utility function be

$$U = Z^{1-r}$$

where r is a constant and equal to relative risk aversion, $1 > r \geq 0$. Let $W = 10$, $X = 9$, $\pi_h = 0.2$, and $\pi_l = 0.1$. Then the premium charged in a pooling equilibrium in which everyone insures is

$$P_p = 0.9\alpha + 0.9 ,$$

so that

$$B_p^l = (10 - (0.9\alpha + 0.9))^{1-r} - 0.9(10)^{1-r} - 0.1 \geq 0 \,.$$

The value of α that just dissuades high-risk individuals from buying insurance is therefore given by

$$\alpha^* = -1.111 \quad 1\exp\left(\frac{-ln(9.0 + 0.1\exp(2.3026r)) + 2.3026r}{-1.0 + r}\right) + 10.111 \,.$$

The reader can gauge the impact of risk aversion on the possibility of a pooling equilibrium from Table 1 below. If an individual's relative risk aversion exceeds 0.854, then, given the above parameter values, a pooling equilibrium will emerge, *regardless of the value of* α. Thus, if $r \geq 0.854$, then even if there exist only a negligible number of low-risk individuals, so that the insurance premium reflects only high-risk individuals, the low-risk individuals will nonetheless buy insurance.

Table 1. The relation between r and α

r	α
0.854	1.000
0.800	0.905
0.700	0.743
0.600	0.597
0.500	0.468
0.400	0.352
0.300	0.248
0.200	0.156
0.100	0.075

As is clear from Table 1, as r declines, the tolerance of the low-risk individuals to α also declines. Thus, for example, while for $r = 0.7$, α must be less than 0.743 to ensure a pooling equilibrium, for $r = 0.2$, α must be less than 0.156 to ensure a pooling equilibrium.

A POSITIVE LOADING FACTOR: $\lambda > 0$

A Separating Equilibrium

Once again there is only one viable separating equilibrium—i.e., high-risk individuals insure and low-risk individuals do not. If only high-risks insure, the insurance premium is

$$P_s^h = (1 + l\lambda)\pi_h X.$$

Given a premium equal to P_s^h, the benefit of insurance to high-risk individuals is

$$B_s^h = U(W - \pi_h(1 + \lambda)X) - (1 - \pi_h)U(W) - \pi_h U(W - X).$$

However, even if $\lambda > 0$, then, the concavity of the utility function ensures that for some parameters combinations, $B_s^h > 0$. A separating equilibrium can emerge.[9]

However, since

$$\frac{\partial B_s^h}{\partial \lambda} = \pi_h X U'(W - \pi_h(1 + \lambda)X) < 0,$$

a sufficiently large λ will reduce all demand for insurance to zero. The reader may gauge the relation between λ and risk aversion by considering our above functional and numerical example. In this case

$$P_s^h = (1 + \lambda)(\pi_h X) = 1.8(1 + \lambda)$$

so that

$$B_s^h = (10 - 1.8(1 + \lambda))^{1-r} - 0.8(10)^{1-r} - 0.2 = 0.$$

The value of λ that just dissuades high-risk individuals from buying insurance is therefore given by

$$\lambda^* = -.555\,56\exp\left(\frac{-\,ln(8.0 + 0.2\exp(2.3026r)) + 2.3026r}{-1.0 + r}\right) + 4.5556 \quad .$$

To give the reader a sense of the magnitudes involved, λ is given in Table 2 below, for different values of r.[10]

Table 2. The relation between r and λ^*

r	λ
0.900	0.9067
0.800	0.7693
0.700	0.6420
0.600	0.5241
0.500	0.4157
0.400	0.3163
0.300	0.2256
0.200	0.1431
0.100	0.0688

As is clear from Table 2, as r declines, the tolerance of the high-risk individuals to λ also declines. Thus, for example, while for $r = 0.7$, λ must be less than 0.642 to ensure that this separating equilibrium is viable, for $r = 0.1$, λ must be less than 0.0688 to ensure a separating equilibrium.

Pooling Equilibria

As the loading factor, λ, becomes larger, buying insurance is less attractive for both categories. Thus, there are two possible pooling equilibria. In the first, no insurance is bought by either group, and in the second both groups by insurance.

No insurance bought. In this case, the parameters are such that even the high-risks do not buy insurance. Thus, if $\lambda > \lambda^*$, as defined above, a pooling equilibrium emerges, in which no insurance is bought.

Both groups buy insurance. In this case, the parameters are such that risk aversion is sufficiently high to cause even low-risks to buy insurance, despite paying a premium that is affected by the high-risks and despite the loading factor. To ensure that both groups buy insurance we require that the net benefit of the low-risks is non-negative—i.e., that

$$B_p^l = U[W - (1-\lambda)(\alpha\pi_h + (1-\alpha)\pi_l)X] - (1-\pi_l)U[W] - \pi_l U[W-X] \geq 0 ,$$

since the insurance premium in this case is

$$P_p = (1 + \lambda)[(\alpha\pi_h + (1 - \alpha)\pi_l)]X .$$

Once again, the critical values of the parameters α and λ are functions of W, X, π_h, and π_l, as well as the risk aversion of a representative individual. To get further insight, we revert to the above-specified utility function and numerical example. The premium in this case is

$$P_p = 0.9(1 + \lambda)(1 + \alpha) .[11]$$

To obtain a pooling equilibrium we therefore require that

$$(W - P_p)^{1-r} - (1 - \pi_l)(W)^{1-r} - \pi_l(W - X)^{1-r} \geq 0 ,$$

i.e., that

$$[10 - 0.9(1 + \lambda)(1.7)^{1-r} - .9 \times 10^{1-r}] \geq 0 .$$

The reader may find it of interest to examine the tradeoff between α and r in deterring the low-risks from insuring.

Table 3. λ^* for combinations of α and r

$\alpha \setminus r$	0.1	0.3	0.5	0.7	0.9
0.1	0.000	0.135	0.334	0.584	0.896
0.3	0.000	0.000	0.129	0.340	0.605
0.5	0.000	0.000	0.000	0.162	0.391
0.7	0.000	0.000	0.000	0.000	0.228
0.9	0.000	0.000	0.000	0.000	0.000

The above table suggests that even a small loading factor may have a large impact on the type of equilibrium reached: In order to generate a pooling equilibrium in which all individuals buy insurance, λ must be smaller than λ^*. Thus, for example, if 10 percent of the consumers are high-risks, insurance will not be bought by the other 90 percent , for $r = 0$, regardless of λ. Moreover, even if $r = 0.3$, no insurance will be bought by the low-risks, unless $\lambda < 0.136$. A loading factor of 15 percent , say, would

act to exclude 90 percent of the potential market. This should be compared with the earlier result, that in the absence of loading costs, all individuals will insure for these parameters' value, if $r > 0.075$.

CONCLUSION

This paper demonstrates that the incorporation of loading factors into the analysis of insurance market equilibria may yield interesting and potentially important results. The results presented here are based on a particular, though commonly used, functional form for the loading factor. They suggest that further investigation of the relationship between the type and existence of equilibria in insurance markets is in order.

NOTES

[1] The seminal works in this area are Rothschild and Stiglitz (1976) and Wilson (1977).

[2] See Pitkanen (1975) and Kahane (1979) for early discussions of this issue. For a more recent approach, see, for example, Taylor (1994).

[3] No partial insurance is permitted in this model.

[4] See Kahane (1979).

[5] See Pitkanen (1975). Gollier (1996) also refers to this form in discussing the standard in the literature.

[6] Proportional loading factors are often used by insurance companies for several reasons. First, this is a simple and straightforward pricing method. Second, it corresponds to the cost structure of the insurance company. Commissions, which are the main costs that the loading factor must cover, are generally determined as a fraction of the premium. Third, this pricing method enables the insurers to publish a price list of tariffs, in which the required premium is expressed as a percentage of the insured sum. Fourth, it enables insurance companies to decentralize the underwriting decisions. And fifth, it avoids the need to negotiate with the customer on each ordinary policy.

[7] Thus λ is such that insurance firms cover their costs and earn normal profits.

[8] Remember that all individuals have the same utility function.

[9] This will occur if the risk facing the low-risk is sufficiently low to cause them to reject insurance that is (partially) based on the risk facing the high-risk group.

[10] The value of r that dissuades high-risk individuals from buying insurance, even for $\lambda = 0$, = is 0.4739.

[11] Note that, in general, λ and α do not enter the insurance premium symmetrically. The fact that they do so here is a coincidence owing to the parameter values chosen.

REFERENCES

Gollier C. (1996) "Optimum Insurance of Approximate Losses," *The Journal of Risk and Insurance*, Vol. 63, No. 3, pp. 369–380.

Kahane Y. (1979) "The Theory of Insurance Risk Premium—A Re-examination in the Light of Recent Developments in Capital Market Theory." *The Astin Bulletin*, Vol. 10, No. 2, pp. 223–239.

Pikanen P. (1975) "Tariff Theory," *The Astin Bulletin*, Vol. 8, part 2, pp. 204–228.

Rothschild M., and J. Stiglitz (1976) "Equilibrium in Competitive Insurance Markets," *Quarterly Journal of Economics*, Vol. 90, No. 4, pp. 629–650.

Taylor G. (1994) "Fair Premium Rating Methods and the Relation Between Them," *The Journal of Risk and Insurance*, Vol. 61, No. 4, pp. 592–615.

Wilson C. (1977) "A Model of Insurance Markets with Incomplete Information," *Journal of Economic Theory*, Vol. 16, pp. 167–207.