Multistate System in Human Reliability Analysis

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Abstract — Application of mathematical model of Multistate System for human reliability analysis is considered in the paper. This paper describes new method for estimation of changes of more than one component states and they influence to Multistate System reliability by Dynamic Reliability Indices. The Multistate System failure is considered depending on decrease of some system component efficiency and the Multistate System repair is declared depending on replacement of some failed components. The mathematical approach of Logical Differential Calculus is used for analysis of the Multistate System reliability change that is caused by modifications of some system components states.

Keywords — Multistate System, Dynamic Reliability Indices, Structure Function.

I. INTRODUCTION

Hardware reliability has been increased by technological advances in large complex systems such as nuclear power plants, aircraft, and offshore oil plants. However, there has not been a vast improvement in human reliability for operating and managing those complex systems. It is well recognized that humans play an important role in the safe operation of complex industrial establishments. Examples are Three-Mile Island Unit 2 (TMI-2), Chernobyl, Challenger, Bhopal, Piper Alpha accidents that involved human deficiencies in various areas of management, operations, maintenance and training. Reduction of human error is one of the major interests for the enhancement of system safety and availability [1].

The human reliability analysis (HRA) in the context of a Reliability Analysis is an attempt to model and predict the impact of such interactions on the system’s safety and reliability. The term "human reliability" is usually defined as the probability that a person will correctly perform some system-required activity during a given time period (if time is a limiting factor) without performing any extraneous activity that can degrade the system [2]. HRA methods are based on the integration of two complementary approaches: (a) probabilistic modeling and decision theory, and (b) contextual human factors psychology. A probabilistic model, in general, can be applied to represent decision making as a part in an event sequence depending causally on some parameters. The uncertainties in the evolution of the events are expressed by probabilities. The decision analytic view in the modeling makes it possible to take the objectives of the decision makers into account in the analysis [1–3].

In this paper we propose to use Multistate System (MSS) methods reliability analysis for HRA. A MSS has many level of safety (more than only two) [4].

For estimation of MSS we have proposed measure that is named as Dynamic Reliability Indices (DRIs). Firstly these indices were declared in paper [4]. These indices are probabilities of a MSS safety level change that is caused by the change of a component state.

In this paper we evolve application of MSS reliability analysis methods for representation and estimation of system with “human component”. Fault Decision Tree (FDT) [1], Fuzzy Decision Tree [5] and DRIs [4, 6] estimation methods are used for this problem.

II. BASIC CONCEPTION

A. Mathematical Description of MSS

The MSS consists of n components. It is determined by a structure function and components probabilities unambiguously. The structure function is used for mathematical description of the MSS. This function declares a system reliability depending on its components states [4, 6, 7]:

\[
\phi(x_1, \ldots, x_n) = \phi(x): \{0, \ldots, m-1\}^n \rightarrow \{0, \ldots, m-1\} \tag{1}
\]

where \(\phi(x)\) is system reliability (system state), \(x_i\) is components state \((i = 1, \ldots, n)\), \(n\) is number of system components, \(m\) is discrete levels of reliability for system and its components \((m = 0, \ldots, m-1)\); zero correspond to complete failure of system or its components and \((m-1)\) is perfect functioning.

The component probability characterizes every system component state \(x_i\) form zero to \((m-1)\):

\[
p_{ij} = \Pr[x_i = s_j], \tag{2}
\]

where \(i = 1, \ldots, n\) and \(s_j = 0, \ldots, m-1\).

These assumptions for structure function (1) are used [4, 6]: (a) it is the Multiple-Valued Logic (MVL) function; (b) the...
structure function is monotone and \( \phi(s) = s \) \((s \in \{0, \ldots, m-1\})\); (c) all components are \( x \)-independent and are relevant to the system.

The assumption (1) allows applying MVL mathematical tools for reliability analysis of the MSS. In particular, the Logic Differential Calculus is used for analysis of dynamic properties of MVL function [4]. Therefore, this approach can be applied for analysis of dynamic behaviour of MSS that is determined by structure function (1). We propose to apply Direct Partial Logic Derivative that reflects the change in the value of the structure function when the values of variables change (Fig.1).

A Direct Partial Logic Derivative detect these influences

![Fig.1. Direct Partial Logic Derivative of structure function](image)

B. Direct Partial Logic Derivative for structure function of MSS

There are two types of Direct Partial Logic Derivatives in Logic Differential Calculus of MVL (Fig.2): with respect to one variable and with respect to variables vector. The first type permits to examine the influence of one variable change to modification of MVL function value. The second type of derivative reveals MVL function value changes depending on changes of some function variables. Definitions of these derivations for structure function of a MSS are in [4, 8].

![Fig.2. Direct Partial Logic Derivatives with respect to variable x_i and variables vector x^{(p)} for structure function of MSS](image)

A Direct Partial Logic Derivative \( \partial \phi(j \rightarrow h)/\partial x_i(a \rightarrow b) \) of a structure function \( \phi(x) \) of \( n \) variables with respect to variable \( x_i \) reflects the fact of changing of function from \( j \) to \( h \) when the value of variable \( x_i \) is changing from \( a \) to \( b \) [4, 8]:

\[
\frac{\partial \phi(j \rightarrow h)}{\partial x_i(a \rightarrow b)} = \begin{cases} 
  m - 1, & \text{if } \phi(a, x) = j & \text{& } \phi(b, x) = h \\
  0, & \text{in the other case}
\end{cases},
\]

where \( \phi(a, x) = \phi(x_1, \ldots, x_n, a, x_1, x_2, \ldots, x_n), \phi(b, x) = \phi(x_1, \ldots, x_n, b, x_1, x_2, \ldots, x_n), a_i, b_i \in \{0, \ldots, m-1\} \).

A Direct Partial Logic Derivatives of a structure function \( \phi(x) \) of \( n \) variables with respect to variables vector \( x^{(p)} = (x_{i1}, x_{i2}, \ldots, x_{in}) \) reflects the fact of changing of function from \( j \) to \( h \) when the value of every variable of vector \( x^{(p)} \) is changing from \( a \) to \( b \) [8, 9]:

\[
\frac{\partial \phi(j \rightarrow h)}{\partial x^{(p)}(a^{(p)} \rightarrow b^{(p)})} = \begin{cases} 
  m - 1, & \text{if } \phi(a^{(p)}, x) = j & \text{& } \phi(b^{(p)}, x) = h \\
  0, & \text{in the other case}
\end{cases}
\]

In (3) a change of value of \( i \)-th variable \( x_i \) form \( a \) to \( b \) agrees with a change of MSS component efficiency form \( a \) to \( b \). So, changes of some components states correspond with change of a variables vector \( x^{(p)} = (x_{i1}, x_{i2}, \ldots, x_{in}) \). Every variable values of this vector changes form \( a \) to \( b \) (Fig.3).

For example, a MSS reliability decrease from value “two” to value “one” is caused by deterioration of efficiency of the first component from value “three” to “two” and breakdown of the fifth component. This behaviour of MSS is described by (4) as:

\[
\frac{\partial \phi(2 \rightarrow 1)}{\partial x^{(2)}(a^{(2)} \rightarrow b^{(2)})} = \frac{\partial \phi(2 \rightarrow 1)}{\partial x(2 \rightarrow 1)x_i(1 \rightarrow 0)}.
\]

\[
x^{(p)} : \begin{array}{c}
  a_i \rightarrow b_i \\
  x_{i1} \rightarrow a_i \rightarrow b_i \\
  \ldots \\
  x_{in} \rightarrow a_i \rightarrow b_i
\end{array}
\]

C. Dynamic Reliability Indices (DRIS) of MSS Reliability Analysis

DRIs include two groups of probabilistic indices that examine two broad states of a MSS [4, 7, 8]. The broad states of system are system failure and system repair for repairable system. We analyse system failure caused by a human errors in this paper only.

In accordance with these paper the MSS failure depending on some components states deteriorations is represented as the changing of the function value \( \phi(x) \) from \( j \) into zero and as decrease of a system components efficiencies vector \( x^{(p)} \) from \( a^{(p)} \) to \( b^{(p)} \): \( \partial \phi(j \rightarrow 0)/\partial x^{(p)}(a^{(p)} \rightarrow b^{(p)}) \), \( a_i, b_i \in \{0, \ldots, m-1\} \) and \( a_i > b_i \). Because the structure
function is monotone (assumption (b)) the MSS failure is declared by a change function \( \phi(x) \) from “1” into zero and decreases of every of \( p \) system components availability from \( a_i \) to \((a_i - 1)\). The Direct Partial Logic Derivate \( \partial \phi(1 \rightarrow 0) / \partial x^{(0)}(a^{(0)} \rightarrow a^{(0)}) \) describes this system behaviour, where \( a^{(0)} = (a_1, ..., a_n) = ((a_1 - 1), ..., (a_n - 1)) \) and \( a_i = 1, (m,n) \).

A. Component Dynamic Reliability Indices for MSS reliability

One of DRIs groups is Component Dynamic Reliability Indices (CDRIs).

Definition 1. CDRIs for a MSS failure are probability of this MSS failure that is caused by \( p \) system components efficiencies decrease [8].

These indices are calculated as:

\[
P_i(x^{(0)}) = \left( \rho_j / \rho_i \right) \prod_{j=1}^{p} p_{j,i}, \quad (5)
\]

where \( \rho_j \) is number of system states when the value of variables vector \( x^{(0)} \) of MSS structure function changes from \( a^{(0)} \) to \( a^{(0)} \) and forces the system failure (5):

\[
\rho_j = \partial \phi(1 \rightarrow 0) / \partial x^{(0)}(a^{(0)} \rightarrow a^{(0)}) \neq 0, \quad (6)
\]

\( \rho_i \) is number of system states “1”, that \( \phi(x)=1 \) and \( x_j = a_j \) for \( j = 1, ..., p; \ p, i \), is component state probability (2).

Dynamic Integrated Reliability Indices for MSS reliability

Another group of DRIs is Dynamic Integrated Reliability Indices (DIRIs). The assumption (c) for structure function of MSS that all components are independent and relevant to the system takes account for DIRI’s definition.

Definition 2. DIRIs are probabilities of MSS failure and repair that are caused by changes of states of any \( p \) system components [8]:

\[
P_i = \sum P_i(x^{(0)}) \prod_{j=1}^{p} \left( 1 - P_j(x^{(0)}) \right), \quad (7)
\]

where \( P_i(x^{(0)}) \) is probabilities that are determined in (5) and;

\( x^{(0)} \) is the variable vector of \( p \) variables for which \( x^{(0)} = x^{(0)} \); \( z \) is number of combinations of \( n \) things taken \( p \), that determines number of variables vector \( x^{(0)} \) for the structure function of MSS which consist of \( n \) components:

\[
\binom{n}{p} = \frac{n!}{(n-p)! \cdot p!}
\]

IV. APPLICATION OF DRIS IN HUMAN RELIABILITY ANALYSIS

In paper [15] considered system 2-out-of-3-G (Fig.4) with hardware failure events \( C_i \) and consecutive human errors events \( H_i \) (1 = 1, 2, 3). The system fails if two or more trains (events \( C_i \) and \( H_i \)) fail:

\[
C_1C_2 + C_1C_3 + C_2C_3 + C_1H_2 + C_1H_3 + C_2H_3 + H_1C_2 + H_1C_3 + H_2C_3 + H_1H_2 + H_1H_3 + H_2H_3
\]

or in terminology of this paper the structure function of this system is:

\[
\phi(x) = x_1x_4 x_2x_5 \vee x_1x_4 x_3x_6 \vee x_2x_3 x_3x_6. \quad (8)
\]

Fig. 4. System block diagram

Therefore this system includes six components with three levels of availability and three of them is “human component”. Components probabilities are in Table 1.

<table>
<thead>
<tr>
<th>Table 1 Component state probability (m=3).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

Consider this system failure depending on human error events. These events agree with variables \( x_4, x_5 \) and \( x_6 \) in structure function (8). CDRIs for these variables (events) are determined by (5) and are:

\[
P_i(x_4) = 0.131024, \
P_i(x_5) = 0.179310, \
P_i(x_6) = 0.068376. 
\]

Therefore the maximal probability of the system failure correspond to event \( H_2 \) and it is \( P_i(x_4) = 0.179310. \) DIRI for this system failure depending of influence of on events as human error is calculated in conformity with (7) and is \( P_i = 0.1642488. \)

Consider this system failure depending on breakdown or availability decrease of two system and three components. CDRIs for the system in Fig.4 are in Table 2 for two cases: firstly, for two system component breakdown and secondly, for three system component failure. Note, probabilities of system failure depending on human errors (components \( x_4, x_5, x_6 \) are larger. It is determined by probabilities values for human errors (Table 1).

In Table 2 there are not only CDRIs for estimation of probability of human errors. CDRIs for hardware failure are in this Table too and these measures are determined by (5) too. Application of mathematical approach of the MSS reliability analysis based on the structure function system representation by DRIs allows to estimate influence of two types events both human errors and another components failure.

Consequently proposed approach of the MSS reliability analysis is universal and can be used for examination of reliability changes for the system with different types components including component as human element.
Table 2: CDRIs of the system (Fig.4) failure

<table>
<thead>
<tr>
<th>$p = 2$</th>
<th>$p = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>$x_i$</td>
</tr>
<tr>
<td>1 2</td>
<td>0.072800</td>
</tr>
<tr>
<td>1 3</td>
<td>0.072800</td>
</tr>
<tr>
<td>2 3</td>
<td>0.067600</td>
</tr>
<tr>
<td>1 4</td>
<td>0.056000</td>
</tr>
<tr>
<td>2 4</td>
<td>0.052000</td>
</tr>
<tr>
<td>3 4</td>
<td>0.052000</td>
</tr>
<tr>
<td>1 5</td>
<td>0.075600</td>
</tr>
<tr>
<td>2 5</td>
<td>0.070200</td>
</tr>
<tr>
<td>3 5</td>
<td>0.070200</td>
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<tr>
<td>4 5</td>
<td>0.054000</td>
</tr>
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<td>0.089600</td>
</tr>
<tr>
<td>2 6</td>
<td>0.083200</td>
</tr>
<tr>
<td>3 6</td>
<td>0.083200</td>
</tr>
<tr>
<td>4 6</td>
<td>0.064000</td>
</tr>
<tr>
<td>5 6</td>
<td>0.086400</td>
</tr>
</tbody>
</table>

The main area of applications in mind for the results has been the unavailability analysis of redundant standby safety systems that are periodically tested, calibrated or maintained. The probabilities obtained here are relevant input to system models. The current formalism applies also for calculating the probability of a plant transient (initiating event) if such is caused by repeated operator errors.

References


