Inter-temporal Dynamics of Cost Asymmetry

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Abstract: Despite its large body and fundamental importance, the literature on asymmetric cost behavior has evolved empirically and lacks theoretical foundation. To address this gap in the literature, we introduce a novel theoretical framework for studying the dynamics of cost behavior over time in a competitive environment. Our theoretical analysis delves into the primitives of asymmetric cost behavior and explores its inter-temporal dynamics. The analysis yields the result that cost behavior is asymmetric by default, transpiring in the pattern of cost stickiness following resource expansion while taking the form of cost anti-stickiness subsequent to resource contraction. The analysis further suggests that the magnitude of cost asymmetry, either stickiness or anti-stickiness, is decreasing in demand uncertainty and competition intensity, increasing in adjustment costs and time discounting, and greater for small demand shocks than for large demand shocks.

Keywords: cost behavior; cost asymmetry; cost stickiness; cost anti-stickiness.

1. Introduction

The topic of cost behavior lies at the heart of the academic literature in managerial accounting and attracts wide research interest due to its critical implications for firms' profitability. Modern cost behavior research deviates from the traditional view that costs are proportionally adjusted to changes in demand. This paradigm shift is manifested in the influential study of Anderson, Banker, and Janakiraman (2003), showing empirical evidence of the tendency of costs to increase more when revenues rise than to decrease when revenues fall by an equivalent amount. The observed asymmetric cost response to positive versus negative changes in demand, known as cost stickiness, implies that firms choose different resource levels for the same demand level depending on whether demand increased or decreased from the prior period. This surprising finding has triggered an extensive empirical research into the fundamental determinants of the cost stickiness pattern and the reasons for its varying degrees across different firms.[1] The explanations provided in the literature for cost stickiness include asymmetric upward versus downward resource adjustment costs (e.g., Banker, Byzalov, and Chen 2013), asymmetric persistence of positive versus negative demand shocks (e.g., Banker and Byzalov 2014), managerial incentives and preferences such as empire building motivation (e.g., Anderson, Banker, and Janakiraman 2003; Chen, Lu, and Sougiannis 2012), managerial beliefs and behavioral biases such as optimism or overconfidence (e.g., Qin, Mohan, and Kuang 2015; Chen, Kama, and Lehavy 2019), and flaws in empirical measurement (e.g., Banker and Byzalov 2014; Riegler and Weiskirchner-Merten, 2021). The literature also identifies factors that mitigate the pattern of cost stickiness and even trigger the opposite pattern of cost anti-stickiness, as introduced by Weiss (2010). Among them, unutilized resources (e.g., Chen, Kama and Lehavy 2019), strong corporate governance (e.g., Chen, Lu, and Sougiannis 2012), and managerial incentives to meet earnings targets (e.g., Dierynck, Landsman, and Renders 2012; Kama and Weiss 2013). This line of literature, in its vast majority, has evolved empirically, and it thus lacks a theoretical foundation.[2] Recognizing this gap in the literature, we offer a novel theoretical framework to analyze the rationale behind cost asymmetry and its inter-temporal dynamics.

Unlike the empirical literature, which suggests both rational and irrational explanations for asymmetric cost behavior, but faces difficulties in disentangling between them, we establish cost asymmetry on a purely rational ground. Our analysis is built on a multi-period setting with multiple identical firms that operate in the same product market, producing and selling the same product. The firms in our model repeatedly compete à la Cournot over an infinite number of successive periods. The Cournot competition game between the firms in each period is subject to changing demand over time, and it departs

[1] See Banker and Byzalov (2014) for a survey of the cost stickiness literature.
[2] See Labro (2015) on the lack of theoretical underpinning of the cost stickiness pattern, which hardens the work of empiricists.

from the classical Cournot setting by the inclusion of resource adjustment costs. To capture the intertemporal dynamics of the demand for the products of the firms, we assume an exogenous random demand that follows a Markovian stochastic process, where the demand of each period is realized at the beginning of the period and its realization is drawn from a distribution whose mean is the demand of the previous period. We preclude from the model any frictions already known in the literature to cause cost stickiness or cost anti-stickiness. Specifically, the adjustment cost per unit of resource is assumed to be symmetric for upward and downward adjustments. The exogenous random demand in each period is assumed to be distributed symmetrically, upward and downward, around the demand of the previous period. Since unutilized resources are known as a determinant of cost anti-stickiness, we also restrict the analysis to fully utilized resources by assuming the adjustment cost per unit of resource adjustment is lower than the ongoing cost per unit of retaining resources. We further assume that the managers of the firms are risk-neutral and rational, devoid of any behavioral biases and motivated to maximize the economic value of their firm.

As a natural starting point of reference, we consider first the benchmark case where resource adjustments are costless. In the absence of resource adjustment costs, intertemporal concerns are excluded from the managers' strategic considerations when making resource adjustments in response to demand shocks, and thus the competition game in each period is independent of other periods. The benchmark case therefore yields the classic Cournot equilibrium in each period. Accordingly, the optimal choice of resource adjustment of each firm in each period is proportional to the demand shock realization. Hence, when resource adjustment costs are absent, firms adjust resources upward in response to a positive demand shock and symmetrically adjust resources downward in response to a negative demand shock of an identical magnitude. This symmetric cost behavior, however, does not carry over to the case where resource adjustments are costly.

To demonstrate that the existence of resource adjustment costs generates asymmetric cost behavior, even when assuming an otherwise frictionless setting, we move on to analyzing the case where resource adjustments are costly. When expanding production upon a positive demand shock, each additional unit of production is more expensive since it involves both the resource adjustment cost and the production cost. Consequently, production expands in response to an increase in demand, but reaches a lower level compared to the benchmark case (under-production). Conversely, when reducing production in response to a negative demand shock, the removal of each unit entails a smaller cost saving since the firm avoids the production cost but incurs the resource adjustment cost. As a result, the production contracts in response to a decrease in demand, but to a higher level relative to the benchmark case (over-production). Hence, adjustment costs restrain both upward and downward resource adjustments relative to the benchmark case. So, the equilibrium resource adjustment in the presence of resource adjustment costs is no longer proportional to the demand shock. The analysis reveals that firms are more likely to adjust their resources and adjust them more significantly in response to a demand shock that follows the course of the previous shock (positive or negative) than they do in response to a demand shock reversed to the previous shock. Subsequent to an upward resource adjustment, a consecutive expansion shifts the production upward from under production in the past to a similar under production in the present, thereby generating full adjustment (equal to the benchmark). Resource contraction following prior resource expansion, however, shifts the production downward from under production in the past to over production in the present, thus triggering a smaller adjustment. Therefore, a prior resource expansion stimulates cost stickiness. Similarly, following a downward resource adjustment, a consecutive negative demand shock shifts the production downward from over production in the past to a similar over production in the present, while resource expansion causes a smaller adjustment that shifts the production upward from over production in the past to under production in the present. Hence, a prior resource contraction induces cost anti-stickiness.

We therefore show that resource adjustment costs generate asymmetric cost behavior, even when assuming otherwise frictionless setting. Both cost stickiness and cost anti-stickiness are expected to rationally and naturally appear in equilibrium. Our analysis further explores the dynamic over time of cost behavior, suggesting that the direction of cost asymmetry is determined by the direction of the prior resource adjustment, and showing that cost asymmetry emerges in the form of cost stickiness following resource expansion and transpires in the form of cost anti-stickiness after resource contraction. In addition to exploring the determinants of the direction of cost asymmetry, the analysis also sheds light on the economic forces that affect the magnitude of cost asymmetry. We show that the magnitude of cost asymmetry, either stickiness or anti-stickiness, increases in the resource adjustment cost, increases in the time discounting, decreases in the number of competing firms, and decreases in the demand uncertainty. A higher resource adjustment cost has a more significant restraining impact on both upward and downward resource adjustments, and accordingly the resulting cost asymmetry is increasing in the resource adjustment cost. An increase in the number of competing firms has the opposite effect of decreasing the cost asymmetry because the response of firms to demand shocks is less significant under enhanced competition, and so is the proportional restraint in resource adjustments due to adjustment costs. A decrease in the time discounting also has a mitigating effect on the restraint in resource adjustment and on the resulting cost asymmetry, because resource adjustment in response to the demand shock in each period is likely to continue serving the firm in the future, so managers are more willing to adjust resources when the time discounting is lower. The effect of demand uncertainty on cost asymmetry is subtler. As uncertainty increases, the potential demand shocks become larger, in absolute terms, and so does the size of the corresponding resource adjustments. Conversely, the magnitude of the restraint in resource adjustment remains constant, regardless of the demand shock. Therefore, it becomes smaller relative to the adjustment size as demand uncertainty increases, mitigating the cost asymmetry.

The paper proceeds as follows. In Section 2, we present the model underlying the analysis, which is by design devoid of any friction already known in the literature to cause cost asymmetry. In Section 3, we derive the equilibrium outcomes that the model yields and analyze their economic properties and implications. Section 4 summarizes and offers concluding remarks. The proofs of all the results appear in the appendix.

2. *Model*

We consider a multi-period setting with n identical firms. The firms operate in the product market, producing and selling the same product, for an infinite number of periods. The special case of $n = 1$ captures a monopoly market, whereas the case of $n \geq 2$ depicts an oligopoly, ranging from a duopoly when $n = 2$ to a perfect competitive market when n approaches infinity. In each period, the n firms compete à la Cournot, choosing simultaneously and independently the quantity they produce and sell in response to an exogenous demand shock. As our focus is the analysis of the outcomes of the competition in a representing period t, we assume an infinite number of past periods and an infinite number of future periods, and thus allow the period index t to be any integer number.

The number of product units that firm $i \in \{1,2, \dots, n\}$ chooses to produce and sell in period $t \in \mathbb{Z}$ is denoted $q_{i,t}$. For simplicity, and without loss of generality, we assume that the production of each unit of product requires one unit of resources, and thus $q_{i,t}$ represents the resource quantity of firm i in period t. Accordingly, $q_{i,t} - q_{i,t-1}$ represents the resource adjustment of firm i in period t. Assuming a standard linear demand function for the product, we represent the retail price for each unit of product in period t by $p_t = d_t - \sum_{j=1}^{n} q_{j,t}$, where d_t is the demand for the product in period t. The uncertain demand for the product in period t is represented by the random variable $\tilde{d}_t$, whose realization d_t becomes publicly known at the beginning of period t. We assume that the dynamics over time of the demand follows a Markovian stochastic process. Specifically, the random variable $\tilde{d}_t$ is assumed to be uniformly distributed around a mean that equals to the demand d_{t-1} of the previous period and with a variance of σ^2. Hence, $\tilde{d}_t \sim U\left[d_{t-1} - \sqrt{3}\sigma, d_{t-1} + \sqrt{3}\sigma\right]$.[3]

[3] By the properties of the uniform distribution, the mean of the random variable $\tilde{d}_t \sim U\left[d_{t-1} - \sqrt{3}\sigma, d_{t-1} + \sqrt{3}\sigma\right]$ equals $((d_{t-1} - \sqrt{3}\sigma) + (d_{t-1} + \sqrt{3}\sigma))/2 = d_{t-1}$, and its variance equals $((d_{t-1} - \sqrt{3}\sigma) - (d_{t-1} + \sqrt{3}\sigma))^2/12 = \sigma^2$. The uniform distribution of the demand simplifies the analysis significantly, but it might generate over time a negative demand realization. To approximately preclude scenarios where the demand realization gets negative, we assume that the demand mean exceeds the demand variance by a large amount, so that the probability of a negative demand is negligible.

The marginal cost of operating one unit of resources, per one period, is given by the positive scalar $c > 0$. Consistent with prior studies (e.g., Banker and Hughes, 1994; Göx 2002), resource adjustment is assumed to be costly. Our assumption is that any resource adjustment of one unit, upwards or downwards, entails a positive one-time adjustment cost of $k > 0$. The parameter k reflects, for example, the one-time cost per unit of hiring (firing) labor or installing (liquidating) machinery, which is associated with an upward (downward) resource adjustment. We assume that $k < c$, implying that it is always worthwhile for firms to avoid unutilized resources.

We denote the profit of firm i in period t by $\pi_{i,t}$, and assume that the profit $\pi_{i,t}$ is distributed as a dividend to the shareholders at the end of period t. The profit of firm i in period t is given by $\pi_{i,t} = (p_t - c) \cdot q_{i,t} - k \cdot |q_{i,t} - q_{i,t-1}|$ or, equivalently, can be also represented as $\pi_{i,t} = \left(d_t - \sum_{j=1}^{n} q_{j,t} - c\right) \cdot q_{i,t} - k \cdot |q_{i,t} - q_{i,t-1}|$. We assume that the managers of the firms are rational and risk neutral, and they all aim at maximizing the expected economic value of their firm. Hence, in making the production quantity decision in each period t, the manager of each firm i strives to maximize the present value $\sum_{s=t}^{\infty} \frac{\pi_{i,s}}{(1+r)^{s-t}}$ of the accumulated profits of firm i in period t and in the successive periods discounted by a discount rate of $r > 0$. The existence of resource adjustment costs generates a linkage between the profit of the firms in the current period to their production quantity in the previous period, and it is thus the source of the intertemporal dynamics of the firms' strategic behavior.

We preclude from the model any frictions already known in the literature to cause cost stickiness or cost anti-stickiness. Specifically, the cost k per unit of resource adjustment is assumed to be symmetric for upward and downward adjustments. The exogenous demand shock in each period t is depicted in the model by the realization of the random variable $\tilde{d}_t - \tilde{d}_{t-1} \sim U[-\sqrt{3}\sigma, \sqrt{3}\sigma]$, which is assumed to be symmetrically distributed around zero. Since unutilized resources are known as a determinant of cost anti-stickiness, we also restrict the analysis to fully utilized resources by assuming the adjustment cost k per unit of resource adjustment is lower than the marginal production cost c. We further assume that the managers of the firms are all rational, devoid of any behavioral biases, and motivated to maximize the economic value of their firm.

Equilibrium in the competing game between the n firms in each period t consists of the production quantities of the firms in period t. Accordingly, equilibrium in the competition game between the n firms in each period t is formally defined as a vector $(q_{1,t}, q_{2,t}, \dots, q_{n,t})$ of their production quantities. We look for Nash equilibrium, in which the managers of all firms make optimal decisions that maximize the economic value of their firms given all their available information, as well as their rational expectations regarding the strategic behavior of their rivals, utilizing Bayes' rule to make inferences and update their beliefs. In equilibrium, for any $i \in \{1,2,\ldots,n\}$, $q_{i,t}$ is the optimal production quantity that maximizes the expected economic value of firm i in period t, given the production quantity $q_{i,t-1}$ of firm i in the previous period $t-1$, conditional on the realization d_t of the demand shock $\tilde{d}_t$ in period t, and based on the rational expectations about the production quantity $q_{j,t}$ of any other firm j $(j \neq i)$ in period t.

Cost stickiness (anti-stickiness) transpires when costs tend, on average, to increase more (less) when the demand rises than to decrease when the demand falls by an equivalent amount.[4] To analyze the direction and magnitude of the cost asymmetry that the model yields in equilibrium, we formulate a measure of cost asymmetry of firm $i \in \{1,2,\ldots,n\}$ in period $t \in \mathbb{Z}$ and denote it $M_{i,t}$. In defining the cost asymmetry measure, we use the notation $t(-1)$ to denote the last period of conducting resource adjustment prior to period t, and we similarly use the notation $t(-2)$ to denote the last period of conducting resource adjustment prior to period $t(-1)$. Utilizing this notation, the cost asymmetry measure $M_{i,t}$ is defined as $M_{i,t} \equiv \frac{E_{\tilde{d}_t}[q_{i,t}-q_{i,t(-1)}|\tilde{d}_t-d_{t(-1)}>0]}{E_{\tilde{d}_t}[\tilde{d}_t-d_{t(-1)}|\tilde{d}_t-d_{t(-1)}>0]} - \frac{E_{\tilde{d}_t}[q_{i,t}-q_{i,t(-1)}|\tilde{d}_t-d_{t(-1)}<0]}{E_{\tilde{d}_t}[\tilde{d}_t-d_{t(-1)}|\tilde{d}_t-d_{t(-1)}<0]}$.[5] The measure $M_{i,t}$ is the difference between two terms. The former is the ratio of the average resource adjustment of firm i in period t to the corresponding average demand shock, conditional on a positive demand shock. The latter is the ratio of the average resource adjustment of firm i in period t to the corresponding average demand shock, conditional on a negative demand shock. A positive (negative) value of this measure reflects cost stickiness (anti-stickiness), because it indicates that the upward resource adjustment upon a given demand increase is, on average, greater (smaller) than the downward resource adjustment upon an equivalent demand decrease. A value of zero stands for symmetric cost behavior. While the sign of the cost asymmetry measure indicates whether the cost behavior is sticky or anti-sticky, its absolute value captures the magnitude of cost asymmetry.[6]

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[4] In the extant literature revenues and demand are used interchangeably when discussing cost stickiness. We note that demand is an exogenous variable, whereas revenues are endogenously determined based on current and expected demand, available resources and adjustment costs. Therefore, while revenues serve in empirical studies as a proxy for demand, on a theoretical ground the driving force behind resource adjustment decisions of managers is change in demand.

[5] As we show below, in equilibrium, firms may not adjust resources if the demand shock is sufficiently small and the adjustment is costly. Therefore, the amount of resource in period $t-1$ is the consequent of demand in period $t(-1)$, which is the most recent period in which resource adjustment took place.

[6] The measure of cost asymmetry can be alternatively defined in terms of the change in costs, $cq_{i,t} - cq_{i,t(-1)}$, instead of the change in quantities, $q_{i,t} - q_{i,t(-1)}$, but this alternative measure equals $cM_{i,t}$ and thus it is proportional to $M_{i,t}$, so that the two alternative definitions are equivalent. Empirical measures of cost asymmetry are defined in terms of costs, since data on quantities is not available to researchers. They usually rely on the total reported costs, although only the change in the ongoing production costs is relevant for depicting resource adjustments, due to the difficulty in separating between ongoing production costs and one-time adjustment costs. Thus, the commonly used empirical measure of changes in costs as a proxy for resource adjustments ascribe a cost of $c+k$ per unit of upward adjustment and a lower cost of $c-k$ per unit of downward adjustment, generating a systematic empirical bias toward cost stickiness in estimating cost asymmetry.

3. *Equilibrium Analysis*

3.1. Benchmark

As a natural starting point of reference, it is convenient to consider first the benchmark case of $k = 0$ where resource adjustments are costless. The assumption $k = 0$ excludes intertemporal concerns from the firms' strategic considerations and thereby eliminates the linkage between their strategic decisions over time. In the absence of resource adjustment costs, the choice of the production quantity of each firm in each period is independent of the production quantity in the previous period $t-1$ and has no effect on future profits. The benchmark case of $k = 0$ therefore yields the classic Cournot equilibrium in each period t. This argument is formally stated in Observation 1, using the superscript $k = 0$ in presenting the equilibrium outcomes of the benchmark case.

OBSERVATION 1. *In the benchmark case of $k = 0$, there exists a unique equilibrium $\left(q_{1,t}^{k=0}, q_{2,t}^{k=0}, \dots, q_{n,t}^{k=0}\right)$ in each period $t \in \mathbb{Z}$, which is given by $\forall i \in \{1,2,\dots,n\}$: $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$. In equilibrium, the resource adjustment of each firm i in each period t is $q_{i,t}^{k=0} - q_{i,t-1}^{k=0} = \frac{d_t - d_{t-1}}{n+1}$. The cost asymmetry measure satisfies $M_{i,t}^{k=0} = 0$ independently of i and t .*

As shown in Observation 1, in the benchmark equilibrium, the production quantity of each firm i in each period t upon the demand realization d_t is $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$, as in the classical equilibrium that a one-shot Cournot competition game yields. Hence, in the absence of resource adjustment costs, the competition game in each period is independent of the competition games in other periods. Accordingly, the optimal choice of resource adjustment $q_{i,t}^{k=0} - q_{i,t-1}^{k=0}$ of each firm i in each period t is given by $q_{i,t}^{k=0} - q_{i,t-1}^{k=0} = \frac{d_t - d_{t-1}}{n+1}$ and is proportional to the demand shock realization $d_t - d_{t-1}$. Hence, when resource adjustment costs are absent, firms adjust resources upward in response to a positive demand shock and symmetrically adjust resources downward in response to a negative demand shock of an identical magnitude. Figure 1 graphically illustrates this symmetric cost behavior. The blue diagonal line describes the resource quantity $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$ of each firm i in period t as a function of the demand d_t in period t. The resulting resource adjustment of each firm in period t is illustrated in the figure under two scenarios with respect to the demand shock in period t. The first scenario (illustrated in green) pertains to a positive demand shock $d_t - d_{t-1} > 0$ in period t, which causes an upward resource adjustment of $\frac{d_t - d_{t-1}}{n+1}$ units, marked by the green arrow, from the prior quantity of $q_{i,t-1}^{k=0} = \frac{d_{t-1} - c}{n+1}$ to the current higher quantity of $q_{i,t}^{k=0} =$

$\frac{d_t - c}{n+1}$. The second scenario (illustrated in orange) pertains to a negative demand shock $d_t - d_{t-1} < 0$ of the same amount in period t, which causes a downward resource adjustment of $\frac{|d_t - d_{t-1}|}{n+1}$ units, marked by the orange arrow, from the prior quantity of $q_{i,t-1}^{k=0} = \frac{d_{t-1} - c}{n+1}$ to the current lower quantity of $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$. This symmetric resource adjustment in response to positive and negative demand shocks of the same amount is reflected in the cost asymmetry measure $M_{i,t}^{k=0}$, which is always zero, as indicated by Observation 1. The symmetric cost behavior, however, does not carry over to the case where resource adjustments are costly, as will be established in the next sections.

[Figure 1]

3.2. Equilibrium under extreme time discounting

To demonstrate our argument that the existence of resource adjustment costs generates both cost stickiness and cost anti-stickiness, even when assuming an otherwise frictionless setting, we move on to analyzing the case of $k > 0$ where resource adjustments are costly. To facilitate the exposition, we build and present the analysis in two steps. We first consider in this section the case where the discount rate r approaches infinity. In the next section, we relax this assumption and analyze the unrestricted model. The case where the discount rate r approaches infinity captures situations of an extreme time discounting, where decision making aims only at maximizing the profit of the current period while ignoring the expected profits in future periods. The analysis of this case highlights that resource adjustment costs trigger an intertemporal linkage between the managers' strategic decisions even when they consider only the current period. Proposition 2 explores the equilibrium outcomes under such extreme time discounting, using the superscript $r = \infty$ in presenting them.

PROPOSITION 2. *In the case of $r = \infty$, there exists a unique equilibrium $\left(q_{1,t}^{r=\infty}, q_{2,t}^{r=\infty}, \ldots, q_{n,t}^{r=\infty}\right)$ in each period $t \in \mathbb{Z}$. The equilibrium in period t takes the following form:*

$$\text{If } d_{t(-1)} - d_{t(-2)} > 0, \text{ then } \forall i \in \{1,2,\ldots,n\}\text{: } q_{i,t}^{r=\infty} = \begin{cases} \frac{d_t - c - k}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ q_{i,t-1}^{r=\infty} & if\ -2k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t - c + k}{n+1} & if\ d_t - d_{t(-1)} < -2k \end{cases}.$$

$$\text{If } d_{t(-1)} - d_{t(-2)} < 0, \text{ then } \forall i \in \{1,2,\ldots,n\}\text{: } q_{i,t}^{r=\infty} = \begin{cases} \frac{d_t - c - k}{n+1} & if\ d_t - d_{t(-1)} > 2k \\ q_{i,t-1}^{r=\infty} & if\ 0 \le d_t - d_{t(-1)} \le 2k \\ \frac{d_t - c + k}{n+1} & if\ d_t - d_{t(-1)} < 0 \end{cases}.$$

Proposition 2 implies that the presence of resource adjustment costs results in under-production in response to positive demand shocks and over-production in response to negative demand shocks, as compared to the benchmark case. Hence, resource adjustment costs work to restrain both upward and downward resource adjustments relative to the benchmark case. This restraint stems from the effect of adjustment costs on the marginal production cost. Specifically, when expanding production, each additional unit of product is more expensive to produce since it entails both the adjustment cost k and the operating cost c per one unit of resource, so the cumulative marginal cost is $c + k$. Consequently, a positive demand shock increases production, but to a level lower by the amount of $\frac{k}{n+1}$ relative to the benchmark case (under-production). Conversely, when scaling down production, the elimination of each unit of product entails a smaller cost saving since the firm avoids the production cost c but incurs the adjustment cost k, hence costs decline by $c - k$ per unit. As a result, the production contracts upon a negative demand shock, but to a level higher by the amount of $\frac{k}{n+1}$ relative to the benchmark case (over-production).

Note that the resource level of each firm i in each period t depends not only on the latest demand shock $d_t - d_{t(-1)}$ but also on the sign of the previous demand shock $d_{t(-1)} - d_{t(-2)}$ that triggered a resource adjustment. The sign of the previous demand shock determines whether the firm begins the current period in a state of under-production or over-production, thus influences the response to the current shock. Moreover, unlike the benchmark case, where all demand shocks trigger resource adjustment, in the presence of resource adjustment costs firms sometimes strategically choose to refrain from adjusting resources in response to demand shocks. Such is the case for a sufficiently small negative shock following prior expansion. The rationale is that ensuing the prior positive shock each firm operates in under-production anyhow. Therefore, a small decrease in demand does not merit further resource cut down considering the attached adjustment cost. The same applies for a sufficiently small positive shock following a prior contraction. In this case each firm already operates in over-production, so a small increase in demand does not trigger a costly resource addition.

[Figure 2]

These results of Proposition 2 are graphically illustrated in Figure 2. The green and the orange curves describe the resource quantity $q_{i,t}^{r=\infty}$ of each firm i in period t as a function of the demand d_t in period t, under two scenarios with respect to the last prior demand shock $d_{t(-1)} - d_{t(-2)}$ that triggered a resource adjustment. The green curve pertains to the scenario where the prior demand shock was positive, and the orange curve pertains to the scenario where it was negative. For comparison, the blue diagonal dashed line describes the benchmark resource quantity as a function of the demand d_t. The green curve demonstrates that, following a prior resource expansion in period $t(-1)$, each firm keeps resource quantity at its previous level for sufficiently small negative demand shocks in the range $(-2k, 0)$ in period t. Larger negative shocks decrease resource quantity to the level of $\frac{d_t-c+k}{n+1}$, which reflects over-production relative to the lower benchmark quantity of $\frac{d_t-c}{n+1}$. Positive shocks, however, always trigger an upwards adjustment to the level of $\frac{d_t-c-k}{n+1}$, which reflects under-production relative to the benchmark quantity. Therefore, following an upward resource adjustment, a successive upward adjustment is more likely to occur than a subsequent downward adjustment. The orange curve depicts a mirror image. Following a resource contraction in period $t(-1)$, each firm keeps resource quantity at its level in period $t-1$ for sufficiently small positive demand shocks in the range $(0,2k)$ in period t. Upon larger positive demand shocks, the resource quantity increases to the level of $\frac{d_t-c-k}{n+1}$, which reflects under-production relative to the benchmark quantity. Nevertheless, all negative shocks trigger a downward adjustment to the level of $\frac{d_t-c+k}{n+1}$, reflecting over-production relative to the benchmark. Thus, following a downward resource adjustment, a successive downward adjustment is more likely to occur than a subsequent upward adjustment. The green and the orange curves coincide into the same curve for sufficiently large demand shocks, but they diverge from each other for small demand shocks that belong to the range $(-2k, 2k)$. This implies that firms may choose in equilibrium different resource levels in response to the same demand level d_t and to the same demand shock $d_t - d_{t(-1)}$, depending on whether the previous demand shock $d_{t(-1)} - d_{t(-2)}$ was positive or negative. While Proposition 2 and Figure 2 present the equilibrium outcomes in terms of the relationship between resource quantity and the corresponding demand shock, the following Corollary recasts the equilibrium results in terms of the relationship between resource adjustment (the amount of resources added or removed in the current period) and the corresponding demand shock.

COROLLARY TO PROPOSITION 2. *In the case of $r = \infty$, the equilibrium in period t satisfies for any $i \in \{1,2, \dots, n\}$:*

$$If\ d_{t(-1)} - d_{t(-2)} > 0,\ then\ q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} = \begin{cases} \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ 0 & if\ -2k \leq d_t - d_{t(-1)} \leq 0 \\ \frac{d_t-d_{t(-1)}+2k}{n+1} & if\ d_t - d_{t(-1)} < -2k \end{cases}.$$

$$If\ d_{t(-1)} - d_{t(-2)} < 0,\ then\ q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} = \begin{cases} \frac{d_t-d_{t(-1)}-2k}{n+1} & if\ d_t - d_{t(-1)} > 2k \\ 0 & if\ 0 \leq d_t - d_{t(-1)} \leq 2k \\ \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} < 0 \end{cases}.$$

The corollary indicates that subsequent to a positive demand shock $d_{t(-1)} - d_{t(-2)}$ that caused a prior upward resource adjustment in period $t(-1)$, a successive positive demand shock in period t generates an upward resource adjustment which is greater than the downward adjustment that an equivalent negative demand shock generates. This implies that a prior upward resource adjustment stimulates cost stickiness. The opposite occurs following a negative demand shock that caused a prior downward resource adjustment in period $t(-1)$. Hence, a prior downward resource adjustment induces cost anti-stickiness. These results are graphically illustrated in Figure 3, where Figure 3a pertains to the case of $d_{t(-1)} - d_{t(-2)} > 0$ and Figure 3b relates to the case of $d_{t(-1)} - d_{t(-2)} < 0$.

We start by considering the illustration in Figure 3a, which pertains to the case where the most recent resource adjustment was an upward adjustment triggered by a positive demand shock $d_{t(-1)} - d_{t(-2)}$. Following the upward adjustment in period $t(-1)$, production level was set to $q_{i,t(-1)}^{r=\infty} = q_{i,t-1}^{r=\infty} = \frac{d_{t(-1)}-c-k}{n+1}$, lower than the benchmark production for a similar demand $d_{t(-1)}$ by $\frac{k}{n+1}$. A successive positive demand shock $d_t - d_{t(-1)}$ triggers an upward adjustment to a level of $q_{i,t}^{r=\infty} = \frac{d_t-c-k}{n+1}$, which is also smaller by $\frac{k}{n+1}$ than the benchmark quantity. Since each firm shifts from under-production to under-production and the shortfall is constant, the resource adjustment is equal to $q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} = \frac{d_t-d_{t(-1)}}{n+1}$, similar to the adjustment that would occur following a similar sequence of positive demand shocks in the benchmark case. This adjustment is as illustrated in Figure 3a by the green arrow. However, a negative demand shock following a prior expansion triggers a downward resource adjustment only if it is sufficiently large. In such case, each firm reduces resources to a quantity of $q_{i,t}^{r=\infty} = \frac{d_t-c+k}{n+1}$, which is greater by $\frac{k}{n+1}$ than the benchmark quantity. Here, the firm starts with under-production of $\frac{k}{n+1}$ and decreases resources to a level that reflects over-production of $\frac{k}{n+1}$ relative to the benchmark. Therefore, the downward resource adjustment equals to $q_{i,t}^{r=\infty} - q_{i,t-s}^{r=\infty} = \frac{d_t-d_{t(-1)}+2k}{n+1} = -\frac{|d_t-d_{t(-1)}|-2k}{n+1}$, as illustrated in Figure 3a by the orange arrow, and its absolute value is smaller by $\frac{2k}{n+1}$ compared to the adjustment that would have been made upon a similar sequence of shocks in the benchmark case. Thus, subsequent to an upward resource adjustment, a successive positive demand shock generates greater resource adjustment than an equivalent negative demand shock, and accordingly cost stickiness arises.

The opposite pattern emerges when the most recent resource adjustment was a downward adjustment triggered by a negative demand shock $d_{t(-1)} - d_{t(-2)}$, as Figure 3b demonstrates. In this case, production level was set in period $t(-1)$ to $q_{i,t(-1)}^{r=\infty} = q_{i,t-1}^{r=\infty} = \frac{d_{t(-1)}-c+k}{n+1}$, higher than the benchmark production for a similar demand $d_{t(-1)}$ by $\frac{k}{n+1}$. A successive negative demand shock $d_t - d_{t(-1)}$ triggers a downward adjustment to a level of $q_{i,t}^{r=\infty} = \frac{d_t-c+k}{n+1}$, which is also greater by $\frac{k}{n+1}$ than the benchmark quantity. Since each firm shifts from over-production to over-production and the surplus is constant, the resource adjustment is equal to $q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} = \frac{d_t-d_{t(-1)}}{n+1}$, marked in Figure 3b by the orange arrow. It is equal to the adjustment that would occur for similar consecutive negative demand shocks in the benchmark case. However, a positive demand shock following a prior contraction triggers an upward resource adjustment only if it is sufficiently large. In such case, each firm increases resources to a quantity of $q_{i,t}^{r=\infty} = \frac{d_t-c-k}{n+1}$, which is smaller by $\frac{k}{n+1}$ than the benchmark quantity. Here, the firm starts with over-production of $\frac{k}{n+1}$ and increases resources to a level that reflects under-production of $\frac{k}{n+1}$ relative to the benchmark. Therefore, the upward resource adjustment equals to $q_{i,t}^{r=\infty} - q_{i,t-s}^{r=\infty} = \frac{d_t-d_{t(-1)}-2k}{n+1}$, as illustrated in Figure 3b by the green arrow, and it is smaller by $\frac{2k}{n+1}$ compared to the adjustment that would have been made upon a similar sequence of shocks in the benchmark case. Thus, following a downward resource adjustment, a successive positive demand shock generates smaller resource adjustment than an equivalent negative demand shock, and accordingly cost anti-stickiness appears.

[Figure 3]

Cumulatively, the Corollary to Proposition 2 implies that the presence of resource adjustment cost triggers asymmetric cost behavior, which can take the form of cost stickiness or cost anti-stickiness, depending on the demand intertemporal dynamics. The asymmetric cost behavior stems from the dissimilar response to different sequences of demand shocks when resource adjustments are costly. Firms are more likely to adjust their resources, and adjust them more significantly, in response to a demand shock that follows the course of the last shock (positive or negative) than they do in response to a shock that reverses the last shock. Hence, cost asymmetry takes the shape of cost stickiness following resource expansion and transpires in the form of cost anti-stickiness after resource contraction. Proposition 3 formally establishes this argument in terms of the cost asymmetry measure $M_{i,t}^{r=\infty}$.

PROPOSITION 3. *In the case of $r = \infty$, the cost asymmetry measure $M_{i,t}^{r=\infty}$ is independent of i and t. It is given by* $M_{i,t}^{r=\infty} = \begin{cases} +M^{r=\infty} & if\ d_{t(-1)} - d_{t(-2)} > 0 \\ -M^{r=\infty} & if\ d_{t(-1)} - d_{t(-2)} < 0 \end{cases}$, *where* $M^{r=\infty} = \begin{cases} \frac{4k(\sqrt{3}\sigma - k)}{3\sigma^2(n+1)} & if\ 2k < \sqrt{3}\sigma \\ \frac{1}{n+1} & otherwise \end{cases}$ *and* $M^{r=\infty} > 0$.

Proposition 3 indicates that, in the presence of adjustment costs, the cost asymmetry measure always deviates from the benchmark of zero. This implies that the existence of resource adjustment costs is sufficient to generate asymmetric cost behavior, even when assuming otherwise frictionless setting. Proposition 3 further indicates that the sign of the cost asymmetry measure is determined by the direction of the last resource adjustment, which took place in period $t(-1)$. If resources were adjusted upward (downward) in period $t(-1)$ in response to a positive (negative) demand shock $d_{t(-1)} - d_{t(-2)}$, then the cost asymmetry measure in period t takes a positive (negative) value. Thus, both cost stickiness and cost anti-stickiness are expected to rationally and naturally appear in equilibrium. Cost stickiness emerges following resource expansion, and cost anti-stickiness transpires after resource contraction.[7] While cost behavior is asymmetric by default, its magnitude depends on the characteristics of the economy and thus varies across industries. Proposition 4 presents the sensitivity of the magnitude of the cost asymmetry measure, as captured by $\left|M_{i,t}^{r=\infty}\right| = M^{r=\infty}$, to the modeling parameters.

PROPOSITION 4. *In the case of $r = \infty$, as long as $2k < \sqrt{3}\sigma$, the magnitude of cost asymmetry, as captured by the measure $M^{r=\infty}$, is increasing in the adjustment cost k, decreasing in the number n of competing firms, and decreasing in the demand uncertainty σ. For $2k \geq \sqrt{3}\sigma$, the measure $M^{r=\infty}$ is decreasing in n, and independent of k and σ.*

Proposition 4 shows that, as long as $2k < \sqrt{3}\sigma$, the magnitude of cost asymmetry, either stickiness or anti-stickiness, increases in the resource adjustment cost, decreases in the number of competing firms, and decreases in the demand uncertainty. As detailed above, resource adjustment costs generate cost asymmetry due to their restraining effect on resource adjustments. As resource adjustment costs increase, their restraining impact on both upward and downward resource adjustments becomes more significant, and accordingly the resulting cost asymmetry increases. An increase in the number of competing firms has the opposite effect of decreasing the cost asymmetry. This is because enhanced competition decreases the response of firms to

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[7] If demand shocks are uncorrelated between industries, we would expect the entire economy to exhibit, on average, cost symmetry because some industries demonstrate cost stickiness while others demonstrate cost anti-stickiness. However, since demand in most industries is positively correlated with macro-economic activity, we expect to observe a similar pattern of cost asymmetry across industries.

demand shocks and thus decreases proportionally the restraint in resource adjustments due to adjustment costs and the consequent cost asymmetry. The effect of demand uncertainty is subtler. As uncertainty increases, the potential demand shocks become larger, in absolute terms, and so does the extent of the consequent resource adjustments. However, the distortion caused by adjustment costs remains constant. Therefore, even though the absolute size of the restraint in resource adjustments due to adjustment costs is independent of demand uncertainty, it becomes smaller relative to the adjustment size when demand uncertainty increases, making the asymmetry between upward and downward adjustments less significant. In the rare circumstances where the adjustment cost k per one unit of resource adjustment is extremely large such that twofold the adjustment cost is at least the maximum possible demand shock (i.e., $2k \geq \sqrt{3}\sigma$), the magnitude of cost asymmetry reaches its peak level of $\frac{1}{n+1}$ and it no longer depends on the parameters k and σ. In such cases the adjustment cost effectively prevents any resource adjustment upon a demand shock reversing the previous shock, while a demand shock prolonging the previous shock triggers a full adjustment (equal to the benchmark), hence the magnitude of cost asymmetry is the greatest possible.

3.3. Equilibrium in the unrestricted model

The unrestricted model captures situations where the managers of the firms care not only about the profit of the current period but also about the expected profits in future periods. Thus, when contemplating resource adjustment in response to a demand shock, the managers consider both its current and future consequences. The future implication of resource adjustment in the current period depends on whether the current demand shock persists in the next period or reverts. The Markovian stochastic process of the demand implies that a positive (negative) demand shock in the current period also increases (decreases) the expected demand in the future. So, resource adjustment in response to the current demand shock is likely to continue serving the firm in the future. Therefore, managers are more willing to adjust resource in the unrestricted case relative to the case of $r = \infty$, where the managers care only about the profit of the current period. This equilibrium outcome is formally presented in Proposition 5.

PROPOSITION 5. *In the unrestricted model, there exists a unique equilibrium* $(q_{1,t}, q_{2,t}, \dots, q_{n,t})$ *in each period* t *of the following form, where* λ *is a scalar:*

$$\text{If } d_{t(-1)} - d_{t(-2)} > 0, \text{ then } \forall i \in \{1,2,\dots,n\}\text{: } q_{i,t} = \begin{cases} \frac{d_t - c - \lambda k}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ q_{i,t-1} & if\ -2\lambda k \leq d_t - d_{t(-1)} \leq 0 \\ \frac{d_t - c + \lambda k}{n+1} & if\ d_t - d_{t(-1)} < -2\lambda k \end{cases}.$$

If $d_{t(-1)} - d_{t(-2)} < 0$*, then* $\forall i \in \{1,2,\dots,n\}$: $q_{i,t} = \begin{cases} \frac{d_t - c - \lambda k}{n+1} & if\ d_t - d_{t(-1)} > 2\lambda k \\ q_{i,t-1} & if\ 0 \le d_t - d_{t(-1)} \le 2\lambda k \\ \frac{d_t - c + \lambda k}{n+1} & if\ d_t - d_{t(-1)} < 0 \end{cases}$.

In this unique equilibrium, the scalar λ *is given by* $\lambda = \begin{cases} \frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k} & if\ \frac{1+2r}{1+r}k < \sqrt{3}\sigma \\ \frac{\sqrt{3}\sigma}{2k(1+2r)} + \frac{r}{1+r} & otherwise \end{cases}$. *The scalar* λ *satisfies* $0 < \lambda < 1$*, and it is decreasing in the adjustment cost* k*, increasing in the demand uncertainty* σ*, and increasing in the discount rate* r*, converging* 1 *to when* r *approaches* ∞*.*

Proposition 5 shows that the equilibrium for the case where $r = \infty$ can be generalized to the unrestricted model. Indeed, when the time discounting rate r approaches ∞, λ converges to 1, and the equilibrium presented in Proposition 5 converges to the equilibrium of Proposition 2. However, while establishing the existence and uniqueness of equilibrium in the unrestricted model that takes this generalized form, we do not eliminate the potential existence of equilibria that have another structure due to tractability constraints. Proposition 5 indicates that in the unrestricted model each firm under-produces in response to positive demand shocks and over-produces in response to negative demand shocks, as compared to the benchmark case of $k = 0$ where resource adjustments are costless. This restrained resource adjustment strategy is similar to the equilibrium strategy obtained in the case of $r = \infty$, where the managers care only about the profit of the current period in making their resource adjustment decision. However, the magnitude of the restrain in adjusting resources in the unrestricted model is smaller by a factor $\lambda \in (0,1)$ relative to the case of $r = \infty$. Intuitively, while the benefit from adjusting resources in response to the current demand shock is limited to the current period in the case of $r = \infty$, the forward-looking managers in the unrestricted model get an incremental future benefit from the current resource adjustment, which enhances their motivation to adjust resources. Their incremental future benefit from the current resource adjustment is increasing in the adjustment cost k and decreasing in both the discount rate r and the demand uncertainty σ. Accordingly, the restraint factor λ is decreasing in k and increasing in both r and σ.

[Figure 4]

The results of Proposition 5 are graphically illustrated in Figure 4. The illustration in Figure 4 is similar to that of Figure 2, demonstrating the restrain in the resource adjustment relative to the benchmark. This restrain in adjusting resources is however smaller by the factor λ relative to the case of $r = \infty$ as illustrated in Figure 2. The green and the orange curves describe the resource quantity $q_{i,t}$ of each firm i in period t as a function of the demand d_t in period t, for the case of $d_{t(-1)} - d_{t(-2)} > 0$

and the case of $d_{t(-1)} - d_{t(-2)} < 0$, respectively. For comparison, the corresponding functions in the case of $r = \infty$ are illustrated by the dashed green and orange curves, whereas the blue diagonal dashed line describes the benchmark resource quantity $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$ as a function of the demand d_t. Unlike the benchmark strategy, where all demand shocks trigger resource adjustment, in the unrestricted model firms refrain from adjusting resources when the prior demand shock is followed by an opposite and sufficiently small demand shock (whose magnitude is less than $2\lambda k$), as illustrated by the horizontal sections in the green and orange curves. The range of demand shocks that do not trigger resource adjustment is however narrower in the unrestricted model as compared to the case of $r = \infty$. Upon a sufficiently large positive (negative) demand shock, the firm adjusts resources upward (downward) to a lesser extent than it would in the benchmark case of $k = 0$ but to a greater extent than it would in the case of $r = \infty$. The green and the orange curves coincide into the same curve for sufficiently large demand shocks, but unlike the benchmark case the two curves diverge from each other for small demand shocks that belong to the range $(-2\lambda k, 2\lambda k)$. This range is however narrower than the corresponding divergent range $(-2k, 2k)$ in the case of $r = \infty$. Hence, unlike the benchmark case, the equilibrium resource adjustments in the unrestricted model are not proportional to the corresponding demand shocks. But, as formally stated in the following corollary to Proposition 5, this disproportion is of a lower magnitude relative to the case of extreme time discounting.

COROLLARY TO PROPOSITION 5. *In the unrestricted model, using the notation* $\lambda = \frac{\sqrt{3}\sigma(1+r)}{k+\sqrt{3}\sigma(1+r)}$ *, the equilibrium in period* t *satisfies for any* $i \in \{1,2,\dots,n\}$*:*

$$\text{If } d_{t(-1)} - d_{t(-2)} > 0, \text{ then } q_{i,t} - q_{i,t(-1)} = \begin{cases} \frac{d_t - d_{t(-1)}}{n+1} & \text{if } d_t - d_{t(-1)} > 0 \\ 0 & \text{if } -2\lambda k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t - d_{t(-1)} + 2\lambda k}{n+1} & \text{if } d_t - d_{t(-1)} < -2\lambda k \end{cases}.$$

$$\text{If } d_{t(-1)} - d_{t(-2)} < 0, \text{ then } q_{i,t} - q_{i,t(-1)} = \begin{cases} \frac{d_t - d_{t(-1)} - 2\lambda k}{n+1} & \text{if } d_t - d_{t(-1)} > 2\lambda k \\ 0 & \text{if } 0 \le d_t - d_{t(-1)} \le 2\lambda k \\ \frac{d_t - d_{t(-1)}}{n+1} & \text{if } d_t - d_{t(-1)} < 0 \end{cases}.$$

As in the case of extreme time discounting, cost asymmetry emerges in the unrestricted model because of the dissimilar response to a demand shock that continues the course of the last shock versus a demand shock that reverses it. Cost asymmetry takes the shape of cost stickiness subsequent to resource expansion while transpiring in the form of cost anti-stickiness following resource contraction. This is the same cost asymmetry pattern that emerges in the case of $r = \infty$ but its magnitude is smaller by a factor of $\lambda \in (0,1)$.

[Figure 5]

The results of the corollary are graphically illustrated in Figure 5, where Figure 5a pertains to the case of $d_{t(-1)} - d_{t(-2)} > 0$ and Figure 5b relates to the case of $d_{t(-1)} - d_{t(-2)} < 0$. Figure 5 demonstrates how the presence of resource adjustment costs triggers asymmetric cost behavior, which takes the form of cost stickiness following past resource expansions (the case of $d_{t(-1)} - d_{t(-2)} > 0$) and the opposite form of cost anti-stickiness subsequent to past resource contractions (the case of $d_{t(-1)} - d_{t(-2)} < 0$). The asymmetric cost behavior follows the same pattern as illustrated in Figure 3 for the case of $r = \infty$, but its magnitude is smaller. This is reflected in the cost asymmetry measure $M_{i,t}$ that the unrestricted model yields in equilibrium, as presented in Proposition 6.

PROPOSITION 6. *In the unrestricted model, the cost asymmetry measure $M_{i,t}$ is independent of i and t. It is given by* $M_{i,t} = \begin{cases} +M \text{ if } d_{t(-1)} - d_{t(-2)} > 0 \\ -M \text{ if } d_{t(-1)} - d_{t(-2)} < 0, \end{cases}$ *where* $M = \begin{cases} \frac{4\lambda k(\sqrt{3}\sigma - \lambda k)}{3\sigma^2(n+1)} \text{ if } \frac{1+2r}{1+r}k < \sqrt{3}\sigma \\ \frac{1}{n+1} \quad \text{otherwise} \end{cases}$ *and* $0 < M \leq M^{r=\infty}$.

Proposition 6 indicates that the cost asymmetry measure always deviates from the benchmark of zero, implying that the existence of resource adjustment costs is sufficient to generate asymmetric cost behavior, even when assuming otherwise frictionless setting. This result stands in contrast to the conventional perception in the existing literature that some friction is necessary in order to shift cost behavior from its perceived fundamental symmetric pattern. Proposition 6 suggests that cost behavior is asymmetric by default, and further points to the direction of the most recent resource adjustment as the determinant of the sign of the cost asymmetry measure. Our result is consistent with the empirical evidence of Banker, Byzalov, Ciftci and Mashruwala (2014), who report that the direction of cost asymmetry depends on prior change in sales, but explain it by the effects of past change in sales on managers' expectations for future change in sales and on the amount of resource slack carried from previous period. Our theoretical analysis reveals that cost asymmetry is more fundamental, as it exists independently of these effects. In our model past changes in demand do not affect managers' expectations for the direction of future demand shock (either rational or irrational), nor does the model encompass resource slack. While the sign of the cost asymmetry measure is the same as in the case of $r = \infty$, its absolute value is lower, as reflected by the inequality $M < M^{r=\infty}$ presented in Proposition 6. Proposition 7 establishes the sensitivity of the magnitude of the cost asymmetry measure, as captured by $|M_{i,t}| = M$, to the modeling parameters.

PROPOSITION 7. *In the unrestricted model, as long as* $\frac{1+2r}{1+r}k < \sqrt{3}\sigma$*, the magnitude of cost asymmetry, as captured by the measure* M*, is increasing in the adjustment cost* k*, decreasing in the number* n *of competing firms, decreasing in the demand uncertainty* σ*, and increasing in the discount rate* r. For $\frac{1+2r}{1+r}k \geq \sqrt{3}\sigma$*, the measure* M *is decreasing in* n*, and independent of* k*,* σ *and* r*.*

Proposition 7 indicates that cost asymmetry in the unrestricted model is affected by the adjustment cost k, the number n of competing firms and demand uncertainty σ in the same manner as in the case of $r = \infty$. Additionally, in the unrestricted model where the managers care about future profits, the discount rate r is also a determinant of the extent of cost asymmetry. Resource adjustment in response to the current demand shock not only benefits the firm in the current period, it is also likely to continue serving the firm in the future because of the intertemporal dynamics of the demand. For this reason, forward looking managers care less for the adjustment costs, and they distort less their adjustment decision, thus mitigating cost asymmetry. The value of future benefits stemming from current resource adjustment is decreasing in r. Therefore, as r increases, cost asymmetry is enhanced, reaching its peak in the case of $r = \infty$.

The results of Proposition 7 offer useful guidance to the empirical literature by providing new empirical predictions, as well as alternative explanations to existing empirical observations. Consistent with Proposition 7, empirical studies indeed document that the degree of cost stickiness is increasing in the magnitude of resource adjustment costs (e.g., Anderson, Banker and Janakiraman, 2003; Banker, Byzalov, and Chen, 2013; Banker and Byzalov, 2014). The literature, however, attributes this empirical finding to asymmetry, upward and downward, in either adjustment costs or persistence of demand shocks. We show that the positive relation between cost asymmetry and adjustment costs applies not only to cost stickiness but also to cost anti-stickiness, further indicating that this is true even in a setting with symmetric upward and downward adjustment costs and under symmetric distribution of the demand shocks around zero. The empirical findings on other determinants of cost asymmetry are limited and inconclusive. Addressing the effect of competition, Li and Zheng (2017) find that cost stickiness is increasing in product market competition, but Li and Lou (2021) report that product market competition reduces cost stickiness in emerging markets. The effect of demand uncertainty on cost asymmetry was hardly explored, except from a handful of empirical papers that address political uncertainty and yield mixed results. Lee, Pittman and Saffar (2020) show that cost stickiness rises during election periods, while Jin and Wu (2021) find that cost stickiness decreases with economic policy uncertainty. The mixed evidence is consistent with our theoretical result that these determinants can enhance either cost stickiness or cost anti-stickiness, depending on the context. Lastly, the literature insofar does not examine the effect of the interest rate on cost asymmetry. To complete the analysis, we examine how the magnitude of the demand shock affects cost asymmetry. Proposition 8 shows the difference in cost asymmetry for small versus large demand shocks.

PROPOSITION 8. *In the unrestricted model, when* $\frac{1+2r}{1+r}k < \sqrt{3}\sigma$, $\left|M_{i,t}^{L}\right| > \left|M_{i,t}^{H}\right| > 0$ *for any* $i \in \{1,2,\dots,n\}$ *and* $t \in \mathbb{Z}$, *where* $M_{i,t}^{L} \equiv \frac{E_{\tilde{d}_t}[q_{i,t}-q_{i,t(-1)}|0<\tilde{d}_t-d_{t(-1)}\leq 2\lambda k]}{E_{\tilde{d}_t}[\tilde{d}_t-d_{t(-1)}|0<\tilde{d}_t-d_{t(-1)}\leq 2\lambda k]} - \frac{E_{\tilde{d}_t}[q_{i,t}-q_{i,t(-1)}|-2\lambda k\leq\tilde{d}_t-d_{t(-1)}<0]}{E_{\tilde{d}_t}[\tilde{d}_t-d_{t(-1)}|-2\lambda k\leq\tilde{d}_t-d_{t(-1)}<0]}$ *and* $M_{i,t}^{H} \equiv \frac{E_{\tilde{d}_t}[q_{i,t}-q_{i,t(-1)}|\tilde{d}_t-d_{t(-1)}>2\lambda k]}{E_{\tilde{d}_t}[\tilde{d}_t-d_{t(-1)}|\tilde{d}_t-d_{t(-1)}>2\lambda k]} - \frac{E_{\tilde{d}_t}[q_{i,t}-q_{i,t(-1)}|\tilde{d}_t-d_{t(-1)}<-2\lambda k]}{E_{\tilde{d}_t}[\tilde{d}_t-d_{t(-1)}|\tilde{d}_t-d_{t(-1)}<-2\lambda k]}$.

Proposition 8 splits our measure $M_{i,t}$ of cost asymmetry into two separate measures: the measure $M_{i,t}^{L}$ that applies to small demand shocks in the range $(-2\lambda k, 2\lambda k)$, and the measure $M_{i,t}^{H}$ that applies to large demand shocks whose absolute value is greater than $2\lambda k$.[8] Using these two separate measures, Proposition 8 suggests that cost asymmetry arises for both small and large demand shocks, but it is of a greater magnitude for small demand shocks than for large demand shocks, as capture by the inequality $\left|M_{i,t}^{L}\right| > \left|M_{i,t}^{H}\right| > 0$. This is because small demand shocks trigger a full resource adjustment in case of a prolonging shock but no adjustment in case of a reversing shock, whereas large demand shocks always trigger a resource adjustment even in response to a reversing shock, albeit of a relatively lower magnitude as compared to the resource adjustment in response to a prolonging shock. Our result that the magnitude of cost asymmetry is inversely correlated with the size of the demand shock is consistent with the empirical evidence in Ciftci and Zoubi (2019). Their empirical findings suggest that cost asymmetry is greater for small changes in sales than for large changes in sales. As a reasoning for their empirical observation, Ciftci and Zoubi (2019) argue that following a prior increase in sales, managers are likely to consider small decreases in sales as temporary and large decreases in sales as permanent. In addition, following a prior decrease in sales, slack resources retained from the prior period have a greater impact on cost behavior for small changes in sales than for large ones. Notwithstanding these arguments, our analysis demonstrates that stronger cost asymmetry for modest demand changes is expected to occur irrespective of managerial expectations or slack resources.

4. Summary and Conclusions

Challenged by the empirical finding of the cost stickiness pattern by Anderson, Banker, and Janakiraman (2003), extensive empirical research has been conducted to explore the fundamental determinants of the cost stickiness phenomenon and to explain its observed varying degrees in different

[8] Large demand shocks whose absolute value is greater than $2\lambda k$ belong to the intervals $(-\sqrt{3}\sigma, -2\lambda k)$ and $(2\lambda k, \sqrt{3}\sigma)$. Hence, the measure $M_{i,t}^{H}$ is well defined only under the assumption $2\lambda k < \sqrt{3}\sigma$, which is equivalent to $\frac{1+2r}{1+r}k < \sqrt{3}\sigma$.

firms. The literature also indicates circumstances that work to counterbalance the pattern of cost stickiness and may even lead to the opposite pattern of cost anti-stickiness. The research on the asymmetric cost behavior, despite its large body and fundamental importance, has evolved empirically and lacks theoretical guidance. Our study addresses this gap in the literature by offering a theoretical framework for investigating various aspects of the phenomenon of cost asymmetry, which has been so far studied in the literature almost solely on the basis of empirical observations.

Our theoretical analysis delves into the primitives of asymmetric cost behavior and explores its inter-temporal dynamics in a competitive environment. The analysis shows that cost behavior is asymmetric by its fundamental nature in the sense that both cost stickiness and cost anti-stickiness are expected to rationally and naturally emerge in equilibrium even in a frictionless setting, which is devoid of any frictions known in the literature to trigger cost asymmetry. We further explore the dynamic over time of cost behavior, showing that cost asymmetry transpires in the pattern of cost stickiness following resource expansion while taking the form of cost anti-stickiness subsequent to resource contraction. While the direction of cost asymmetry is primarily determined by the direction of the prior resource adjustment, the magnitude of cost asymmetry, either stickiness or anti-stickiness, is shown to be decreasing in demand uncertainty and competition intensity, increasing in adjustment costs and time discounting, and it is greater in response to smaller demand shocks. These insights offer useful guidance to the empirical literature by providing new empirical predictions, as well as alternative explanations to existing empirical observations.

Appendix – Proofs

Prof of Observation 1. In the benchmark case of $k=0$, the profit of firm i in period t is given by $\pi_{i,t}^{k=0} = \left(p_t^{k=0} - c\right) \cdot q_{i,t}^{k=0}$ or, equivalently, can be also represented as $\pi_{i,t}^{k=0} = \left(d_t - \sum_{j=1}^{n} q_{j,t}^{k=0} - c\right) \cdot q_{i,t}$. Hence, the production quantity of firm i in period t affects only the profit of firm i in period t and has no effect on the profits of firm i in the successive periods. Therefore, the optimal production quantity $q_{i,t}^{k=0}$ that maximizes the expected economic value $\sum_{s=t}^{\infty} \frac{\pi_{i,s}^{k=0}}{(1+r)^{s-t}}$ of firm i in period t is the one that maximizes the profit $\pi_{i,t}^{k=0}$ of firm i in period t.

The first order condition is as follows:

$$\frac{d\pi_{i,t}^{k=0}}{dq_{i,t}^{k=0}} = d_t - 2q_{i,t}^{k=0} - \sum_{j \neq i} q_{j,t}^{k=0} - c = d_t - q_{i,t}^{k=0} - \sum_{j=1}^{n} q_{j,t}^{k=0} - c = 0 \tag{1.1}$$

The second order condition is as follows:

$$\frac{d^2\pi_{i,t}^{k=0}}{d{q_{i,t}^{k=0}}^2} = -2 < 0 \tag{1.2}$$

It follows from equation (1.1) that $q_{i,t}^{k=0} = d_t - \sum_{j=1}^{n} q_{j,t}^{k=0} - c$, implying $q_{1,t}^{k=0} = q_{2,t}^{k=0} = \cdots = q_{n,t}^{k=0}$. Therefore, equation (1.1) is equivalent to $d_t - (n+1)q_{i,t}^{k=0} - c = 0$, and thus $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$. We conclude that, in the case of $k=0$, the equilibrium $\left(q_{1,t}^{k=0}, q_{2,t}^{k=0}, \dots, q_{n,t}^{k=0}\right)$ in the competition game between the firms in each period $t \in \mathbb{N}$ is given by $\forall i \in \{1,2,\dots,n\}$: $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$. So, for each firm i in each period t, the resource adjustment is $q_{i,t}^{k=0} - q_{i,t-1}^{k=0} = \frac{d_t - c}{n+1} - \frac{d_{t-1} - c}{n+1} = \frac{d_t - d_{t-1}}{n+1}$.

For each firm i in each period t, the expected positive resource adjustment $q_{i,t}^{k=0} - q_{i,t-1}^{k=0} > 0$ conditional on a positive demand shock $d_t - d_{t-1} > 0$ is $E_{\tilde{d}_t}\left[q_{i,t}^{k=0} - q_{i,t-1}^{k=0} \middle| \tilde{d}_t - d_{t-1} > 0\right] = \int_0^{\sqrt{3}\sigma} \frac{\Delta}{n+1} \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$ and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{k=0} - q_{i,t-1}^{k=0} \middle| \tilde{d}_t - d_{t-1} > 0\right] = \frac{\sqrt{3}\sigma}{2(n+1)} \tag{1.3}$$

For each firm i in each period t, the expected negative resource adjustment $q_{i,t}^{k=0} - q_{i,t-1}^{k=0} < 0$ conditional on a positive demand shock $d_t - d_{t-1} < 0$ is $E_{\tilde{d}_t}\left[q_{i,t}^{k=0} - q_{i,t-1}^{k=0} \middle| \tilde{d}_t - d_{t-1} < 0\right] = \int_{-\sqrt{3}\sigma}^{0} \frac{\Delta}{n+1} \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$ and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{k=0} - q_{i,t-1}^{k=0} \middle| \tilde{d}_t - d_{t-1} < 0\right] = -\frac{\sqrt{3}\sigma}{2(n+1)} \tag{1.4}$$

For each firm i in each period t, the expected demand shock $d_t - d_{t-1}$ conditional on a positive demand shock $d_t - d_{t-1} > 0$ is $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t-1} \middle| \tilde{d}_t - d_{t-1} > 0\right] = \int_0^{\sqrt{3}\sigma} \Delta \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$ and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t-1} \middle| \tilde{d}_t - d_{t-1} > 0\right] = \frac{\sqrt{3}\sigma}{2} \tag{1.5}$$

For each firm i in each period t, the expected demand shock $d_t - d_{t-1}$ conditional on a negative demand shock $d_t - d_{t-1} < 0$ is $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t-1} \middle| \tilde{d}_t - d_{t-1} < 0\right] = \int_{-\sqrt{3}\sigma}^{0} \Delta \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$ and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t-1} \middle| \tilde{d}_t - d_{t-1} < 0\right] = -\frac{\sqrt{3}\sigma}{2} \tag{1.6}$$

By equations (1.3)-(1.6), $M_{i,t}^{k=0} = \frac{\frac{\sqrt{3}\sigma}{2(n+1)}}{\frac{\sqrt{3}\sigma}{2}} - \frac{-\frac{\sqrt{3}\sigma}{2(n+1)}}{-\frac{\sqrt{3}\sigma}{2}} = 0$ independently of i and t. □

Prof of Proposition 2 and its corollary. In the case of $r = \infty$, the economic value $\sum_{s=t}^{\infty} \frac{\pi_{i,s}^{r=\infty}}{(1+r)^{s-t}}$ of firm i in period t equals the profit $\pi_{i,t}^{r=\infty}$ of firm i in period t. Therefore, the optimal production quantity $q_{i,t}^{r=\infty}$ that maximizes the economic value $\sum_{s=t}^{\infty} \frac{\pi_{i,s}^{r=\infty}}{(1+r)^{s-t}}$ of firm i in period t is the one that maximizes the profit $\pi_{i,t}^{r=\infty} = \left(d_t - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c\right) \cdot q_{i,t}^{r=\infty} - k \cdot \left|q_{i,t}^{r=\infty} - q_{i,t-1}^{r=\infty}\right|$ of firm i in period t.

We first look for a local maximum of the function $\pi_{i,t}^{r=\infty}$ in the interval $\left(q_{i,t-1}^{r=\infty}, \infty\right)$. For any $q_{i,t}^{r=\infty} \in \left(q_{i,t-1}^{r=\infty}, \infty\right)$, the function $\pi_{i,t}^{r=\infty}$ becomes $\pi_{i,t}^{r=\infty} = \left(d_t - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c\right) \cdot q_{i,t}^{r=\infty} - k \cdot \left(q_{i,t}^{r=\infty} - q_{i,t-1}^{r=\infty}\right)$.

The first order condition is as follows:

$$\frac{d\pi_{i,t}^{r=\infty}}{dq_{i,t}^{r=\infty}} = d_t - 2q_{i,t}^{r=\infty} - \sum_{j \neq i} q_{j,t}^{r=\infty} - c - k = d_t - q_{i,t}^{r=\infty} - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c - k = 0 \tag{2.1}$$

The second order condition is as follows:

$$\frac{d^2\pi_{i,t}^{r=\infty}}{d{q_{i,t}^{r=\infty}}^2} = -2 < 0 \tag{2.2}$$

It follows from equation (2.1) that $q_{i,t}^{r=\infty} = d_t - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c - k$, implying $q_{1,t}^{r=\infty} = q_{2,t}^{r=\infty} = \cdots = q_{n,t}^{r=\infty}$. Therefore, equation (2.1) is equivalent to $d_t - (n+1)q_{i,t}^{r=\infty} - c - k = 0$, and thus $q_{i,t}^{r=\infty} = \frac{d_t - c - k}{n+1}$. The solution $q_{i,t}^{r=\infty} = \frac{d_t - c - k}{n+1}$ belongs to the interval $\left(q_{i,t-1}^{r=\infty}, \infty\right)$ if and only if $q_{i,t}^{r=\infty} = \frac{d_t - c - k}{n+1} > q_{i,t-1}^{r=\infty}$ or equivalently $d_t > (n+1)q_{i,t-1}^{r=\infty} + c + k$.

We next look for a local maximum of the function $\pi_{i,t}^{r=\infty}$ in the interval $\left(-\infty, q_{i,t-1}^{r=\infty}\right)$. For any $q_{i,t}^{r=\infty} \in \left(-\infty, q_{i,t-1}^{r=\infty}\right)$, the function $\pi_{i,t}^{r=\infty}$ becomes $\pi_{i,t}^{r=\infty} = \left(d_t - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c\right) \cdot q_{i,t}^{r=\infty} + k \cdot \left(q_{i,t}^{r=\infty} - q_{i,t-1}^{r=\infty}\right)$.

The first order condition is as follows:

$$\frac{d\pi_{i,t}^{r=\infty}}{dq_{i,t}^{r=\infty}} = d_t - 2q_{i,t}^{r=\infty} - \sum_{j\neq i} q_{j,t}^{r=\infty} - c + k = d_t - q_{i,t}^{r=\infty} - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c + k = 0 \qquad (2.3)$$

The second order condition is as follows:

$$\frac{d^2\pi_{i,t}^{r=\infty}}{{dq_{i,t}^{r=\infty}}^2} = -2 < 0 \qquad (2.4)$$

It follows from equation (2.3) that $q_{i,t}^{r=\infty} = d_t - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c + k$, implying $q_{1,t}^{r=\infty} = q_{2,t}^{r=\infty} = \cdots = q_{n,t}^{r=\infty}$. Therefore, equation (2.3) is equivalent to $d_t - (n+1)q_{i,t}^{r=\infty} - c + k = 0$, and thus $q_{i,t}^{r=\infty} = \frac{d_t-c+k}{n+1}$. The solution $q_{i,t}^{r=\infty} = \frac{d_t-c+k}{n+1}$ belongs to the interval $\left(-\infty, q_{i,t-1}^{r=\infty}\right)$ if and only if $q_{i,t}^{r=\infty} = \frac{d_t-c+k}{n+1} < q_{i,t-1}^{r=\infty}$ or equivalently $d_t < (n+1)q_{i,t-1}^{r=\infty} + c - k$.

So, the maximum of the function $\pi_{i,t}^{r=\infty}$ is obtained at $q_{i,t}^{r=\infty} = \frac{d_t-c-k}{n+1}$ when $d_t > (n+1)q_{i,t-1}^{r=\infty} + c + k$, and it is obtained at $q_{i,t}^{r=\infty} = \frac{d_t-c+k}{n+1}$ when $d_t < (n+1)q_{i,t-1}^{r=\infty} + c - k$.

We now look for the maximum of the function $\pi_{i,t}^{r=\infty}$ under the assumption $(n+1)q_{i,t-1}^{r=\infty} + c - k \leq d_t \leq (n+1)q_{i,t-1}^{r=\infty} + c + k$. If $q_{i,t}^{r=\infty} < q_{i,t-1}^{r=\infty}$, then $\pi_{i,t}^{r=\infty} = \left(d_t - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c\right) \cdot q_{i,t}^{r=\infty} + k \cdot \left(q_{i,t}^{r=\infty} - q_{i,t-1}^{r=\infty}\right)$, and thus $\frac{d\pi_{i,t}^{r=\infty}}{dq_{i,t}^{r=\infty}} = d_t - (n+1)q_{i,t}^{r=\infty} - c + k$. Using the assumption $d_t \geq (n+1)q_{i,t-1}^{r=\infty} + c - k$, it follows that $\frac{d\pi_{i,t}^{r=\infty}}{dq_{i,t}^{r=\infty}} \geq (n+1) \cdot \left(q_{i,t-1}^{r=\infty} - q_{i,t}^{r=\infty}\right) > 0$. If $q_{i,t}^{r=\infty} > q_{i,t-1}^{r=\infty}$, then $\pi_{i,t}^{r=\infty} = \left(d_t - \sum_{j=1}^{n} q_{j,t}^{r=\infty} - c\right) \cdot q_{i,t}^{r=\infty} - k \cdot \left(q_{i,t}^{r=\infty} - q_{i,t-1}^{r=\infty}\right)$, and thus $\frac{d\pi_{i,t}^{r=\infty}}{dq_{i,t}^{r=\infty}} = d_t - (n+1)q_{i,t}^{r=\infty} - c - k$. Using the assumption $d_t \leq (n+1)q_{i,t-1}^{r=\infty} + c + k$, it follows that $\frac{d\pi_{i,t}^{r=\infty}}{dq_{i,t}^{r=\infty}} \leq (n+1) \cdot \left(q_{i,t-1}^{r=\infty} - q_{i,t}^{r=\infty}\right) < 0$. Thus, when $(n+1)q_{i,t-1}^{r=\infty} + c - k \leq d_t \leq (n+1)q_{i,t-1}^{r=\infty} + c + k$, the optimal solution is $q_{i,t}^{r=\infty} = q_{i,t-1}^{r=\infty}$.

Therefore, in the case of $r = \infty$, the equilibrium in each period $t \in \mathbb{N}$ is given by $\forall i \in \{1,2,\ldots,n\}$: $q_{i,t}^{r=\infty} = \begin{cases} \frac{d_t-c-k}{n+1} \; if \; d_t > (n+1)q_{i,t-1}^{r=\infty} + c + k \\ q_{i,t-1}^{r=\infty} \; if \; (n+1)q_{i,t-1}^{r=\infty} + c - k \leq d_t \leq (n+1)q_{i,t-1}^{r=\infty} + c + k \\ \frac{d_t-c+k}{n+1} \; if \; d_t < (n+1)q_{i,t-1}^{r=\infty} + c - k \end{cases}$. This implies that $q_{i,t-1}^{r=\infty} = q_{i,t(-1)}^{r=\infty}$ can be either or $\frac{d_{t(-1)}-c-k}{n+1}$ or $\frac{d_{t(-1)}-c+k}{n+1}$.

If $q_{i,t-1}^{r=\infty} = q_{i,t(-1)}^{r=\infty} = \frac{d_{t(-1)}-c-k}{n+1}$, then the inequity $d_t > (n+1)q_{i,t-1}^{r=\infty} + c + k$ is equivalent to $d_t - d_{t(-1)} > 0$, and the inequity $d_t < (n+1)q_{i,t-1}^{r=\infty} + c - k$ is equivalent to $d_t - d_{t(-1)} < -2k$. So, $q_{i,t}^{r=\infty} = \begin{cases} \frac{d_t-c-k}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ q_{i,t-1}^{r=\infty} & if -2k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t-c+k}{n+1} & if\ d_t - d_{t(-1)} < -2k \end{cases}$ and $q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} = \begin{cases} \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ 0 & if -2k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t-d_{t(-1)}+2k}{n+1} & if\ d_t - d_{t(-1)} < -2k \end{cases}$.

If $q_{i,t-1}^{r=\infty} = q_{i,t(-1)}^{r=\infty} = \frac{d_{t(-1)}-c+k}{n+1}$, then the inequity $d_t > (n+1)q_{i,t-1}^{r=\infty} + c + k$ is equivalent to $d_t - d_{t(-1)} > 2k$, and the inequity $d_t < (n+1)q_{i,t-1}^{r=\infty} + c - k$ is equivalent to $d_t - d_{t(-1)} < 0$. So, $q_{i,t}^{r=\infty} = \begin{cases} \frac{d_t-c-k}{n+1} & if\ d_t - d_{t(-1)} > 2k \\ q_{i,t-1}^{r=\infty} & if\ 0 \le d_t - d_{t(-1)} \le 2k \\ \frac{d_t-c+k}{n+1} & if\ d_t - d_{t(-1)} < 0 \end{cases}$ and $q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} = \begin{cases} \frac{d_t-d_{t(-1)}-2k}{n+1} & if\ d_t - d_{t(-1)} > 2k \\ 0 & if\ 0 \le d_t - d_{t(-1)} \le 2k \\ \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} < 0 \end{cases}$.

Since resource adjustment has occurred in period $t(-1)$, we get that $q_{i,t(-1)}^{r=\infty} = \frac{d_{t(-1)}-c-k}{n+1}$ is equivalent to $d_{t(-1)} - d_{t(-2)} > 0$, and $q_{i,t(-1)}^{r=\infty} = \frac{d_{t(-1)}-c+k}{n+1}$ is equivalent to $d_{t(-1)} - d_{t(-2)} < 0$. □

Prof of Proposition 3. For firm i in period t, $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} > 0\right]$ equals $\int_0^{\sqrt{3}\sigma} \Delta \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} > 0\right] = \frac{\sqrt{3}\sigma}{2} \tag{3.1}$$

For firm i in period t, $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} < 0\right]$ equals $\int_{-\sqrt{3}\sigma}^{0} \Delta \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} < 0\right] = -\frac{\sqrt{3}\sigma}{2} \tag{3.2}$$

We consider separately two cases: the case of $d_{t(-1)} - d_{t(-2)} > 0$, and the case of $d_{t(-1)} - d_{t(-2)} < 0$.

<u>Case 1</u>: $d_{t(-1)} - d_{t(-2)} > 0$

By Proposition 2, for $d_{t(-1)} - d_{t(-2)} > 0$, $q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} = \begin{cases} \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ 0 & if -2k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t-d_{t(-1)}+2k}{n+1} & if\ d_t - d_{t(-1)} < -2k \end{cases}$.

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} \middle| \tilde{d}_t - d_{t(-1)} > 0\right]$ equals $\int_0^{\sqrt{3}\sigma} \frac{\Delta}{n+1} \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} \middle| \tilde{d}_t - d_{t(-1)} > 0\right] = \frac{\sqrt{3}\sigma}{2(n+1)} \tag{3.3}$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} \middle| \tilde{d}_t - d_{t(-1)} < 0\right]$ equals $\int_{-\sqrt{3}\sigma}^{-2k} \frac{\Delta+2k}{n+1} \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$ when $2k < \sqrt{3}\sigma$ and zero otherwise, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} \middle| \tilde{d}_t - d_{t(-1)} < 0\right] = \begin{cases} -\frac{\sqrt{3}\sigma}{2(n+1)} + \frac{2k(\sqrt{3}\sigma - k)}{\sqrt{3}\sigma(n+1)} & if\ 2k < \sqrt{3}\sigma \\ 0 & otherwise \end{cases} \quad (3.4)$$

By equations (3.1)-(3.4), $$M_{i,t}^{r=\infty} = \begin{cases} \frac{\frac{\sqrt{3}\sigma}{2(n+1)}}{\frac{\sqrt{3}\sigma}{2}} - \frac{-\frac{\sqrt{3}\sigma}{2(n+1)} + \frac{2k(\sqrt{3}\sigma-k)}{\sqrt{3}\sigma(n+1)}}{-\frac{\sqrt{3}\sigma}{2}} = \frac{4k(\sqrt{3}\sigma - k)}{3\sigma^2(n+1)} & if\ 2k < \sqrt{3}\sigma \\ \frac{\frac{\sqrt{3}\sigma}{2(n+1)}}{\frac{\sqrt{3}\sigma}{2}} - \frac{0}{-\frac{\sqrt{3}\sigma}{2}} = \frac{1}{n+1} & otherwise \end{cases} = +M^{r=\infty}.$$

<u>Case 2</u>: $d_{t(-1)} - d_{t(-2)} < 0$

By Proposition 2, for $d_{t(-1)} - d_{t(-2)} < 0$, $$q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} = \begin{cases} \frac{d_t - d_{t(-1)} - 2k}{n+1} & if\ d_t - d_{t(-1)} > 2k \\ 0 & if\ 0 \le d_t - d_{t(-1)} \le 2k \\ \frac{d_t - d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} < 0 \end{cases}.$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} \middle| \tilde{d}_t - d_{t(-1)} > 0\right]$ equals $\int_{2k}^{\sqrt{3}\sigma} \frac{\Delta-2k}{n+1} \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$ when $2k < \sqrt{3}\sigma$ and zero otherwise, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} \middle| \tilde{d}_t - d_{t(-1)} > 0\right] = \begin{cases} \frac{\sqrt{3}\sigma}{2(n+1)} - \frac{2k(\sqrt{3}\sigma - k)}{\sqrt{3}\sigma(n+1)} & if\ 2k < \sqrt{3}\sigma \\ 0 & otherwise \end{cases} \quad (3.5)$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} \middle| \tilde{d}_t - d_{t(-1)} < 0\right]$ equals $\int_{-\sqrt{3}\sigma}^{0} \frac{\Delta}{n+1} \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty} - q_{i,t(-1)}^{r=\infty} \middle| \tilde{d}_t - d_{t(-1)} < 0\right] = -\frac{\sqrt{3}\sigma}{2(n+1)} \quad (3.6)$$

By eq. (3.1),(3.2),(3.5),(3.6), $$M_{i,t}^{r=\infty} = \begin{cases} \frac{\frac{\sqrt{3}\sigma}{2(n+1)} - \frac{2k(\sqrt{3}\sigma-k)}{\sqrt{3}\sigma(n+1)}}{\frac{\sqrt{3}\sigma}{2}} - \frac{-\frac{\sqrt{3}\sigma}{2(n+1)}}{-\frac{\sqrt{3}\sigma}{2}} = -\frac{4k(\sqrt{3}\sigma - k)}{3\sigma^2(n+1)} & if\ 2k < \sqrt{3}\sigma \\ \frac{0}{\frac{\sqrt{3}\sigma}{2}} - \frac{-\frac{\sqrt{3}\sigma}{2(n+1)}}{-\frac{\sqrt{3}\sigma}{2}} = -\frac{1}{n+1} & otherwise \end{cases} = -M^{r=\infty}.$$

It follows from $2k < \sqrt{3}\sigma$ that $\sqrt{3}\sigma - k > 0$, and thus $M^{r=\infty} > 0$. □

Prof of Proposition 4. For $2k < \sqrt{3}\sigma$, the derivatives of $M^{r=\infty}$ with respect to the parameters k, n and σ are: $\frac{dM^{r=\infty}}{dk} = \frac{4(\sqrt{3}\sigma - 2k)}{3\sigma^2(n+1)} > 0$, $\frac{dM^{r=\infty}}{dn} = -\frac{4k(\sqrt{3}\sigma - k)}{3\sigma^2(n+1)^2} < 0$, $\frac{dM^{r=\infty}}{d\sigma} = -\frac{4k(\sqrt{3}\sigma - 2k)}{3\sigma^3(n+1)} < 0$. For $2k \ge \sqrt{3}\sigma$, the derivatives are: $\frac{dM^{r=\infty}}{dk} = 0$, $\frac{dM^{r=\infty}}{dn} = -\frac{1}{(n+1)^2} < 0$, $\frac{dM^{r=\infty}}{d\sigma} = 0$. □

Prof of Proposition 5 and its corollary. We show that there exists a scalar $\lambda \in (0,1)$, such that the following production quantities constitute equilibrium in the unrestricted model:

$$q_{i,t} = \begin{cases} q_{i,t}^1 & if\ d_t > (n+1)q_{i,t(-1)} + c + \lambda k \\ q_{i,t-1} & if\ (n+1)q_{i,t(-1)} + c - \lambda k \le d_t \le (n+1)q_{i,t(-1)} + c + \lambda k \\ q_{i,t}^2 & if\ d_t < (n+1)q_{i,t(-1)} + c - \lambda k \end{cases}$$, where

$q_{i,t}^1 = \frac{d_t - c - \lambda k}{n+1}$ and $q_{i,t}^2 = \frac{d_t - c + \lambda k}{n+1}$.

Note that $d_t > (n+1)q_{i,t(-1)} + c + \lambda k$ implies $q_{i,t}^1 > q_{i,t(-1)}$, and $d_t < (n+1)q_{it(-1)} + c - \lambda k$ implies $q_{i,t}^2 < q_{i,t(-1)}$. Therefore, the corresponding profits are as follows:

$$\pi_{i,t} = \begin{cases} \pi_{i,t}^1 & if\ d_t > (n+1)q_{i,t(-1)} + c + \lambda k \\ \pi_{i,t-1} & if\ (n+1)q_{i,t(-1)} + c - \lambda k \le d_t \le (n+1)q_{i,t(-1)} + c + \lambda k \\ \pi_{i,t}^2 & if\ d_t < (n+1)q_{i,t(-1)} + c - \lambda k \end{cases}$$, where

$\pi_{i,t}^1 = \left(d_t - \sum_{j=1}^n q_{j,t}^1 - c\right)q_{i,t}^1 - k\left(q_{i,t}^1 - q_{i,t(-1)}\right)$ and

$\pi_{i,t}^2 = \left(d_t - \sum_{j=1}^n q_{j,t}^2 - c\right)q_{i,t}^2 + k\left(q_{i,t}^2 - q_{i,t(-1)}\right)$.

We next consider three cases separately: the case of $d_t > (n+1)q_{i,t(-1)} + c + \lambda k$, the case of $d_t < (n+1)q_{i,t(-1)} + c - \lambda k$, and the case of $(n+1)q_{i,t(-1)} + c - \lambda k \le d_t \le (n+1)q_{i,t(-1)} + c + \lambda k$.

<u>Case 1</u>: $d_t > (n+1)q_{i,t(-1)} + c + \lambda k$

In this case, conditional on the demand realization d_t, the production quantity of each firm i in period t is $q_{i,t} = \frac{d_t - c - \lambda k}{n+1}$ and the profit is $\pi_{i,t} = \left(d_t - \sum_{j=1}^n q_{j,t} - c\right)q_{i,t} - k\left(q_{i,t} - q_{i,t(-1)}\right)$. Thus, the portion of $\pi_{i,t}$ that depends upon $q_{i,t}$ is given by the following function:

$$f_0(q_{i,t}) = \left(d_t - \sum_{j=1}^n q_{j,t} - c - k\right)q_{i,t} \tag{5.1}$$

Also,

$$\frac{E[\pi_{i,t+1}]}{1+r} = \frac{1}{1+r}\int_{d_t-\sqrt{3}\sigma}^{d_t-\min(2\lambda k,\sqrt{3}\sigma)} \left(\left(d_{t+1} - \sum_{j=1}^n q_{j,t+1}^2 - c\right)q_{i,t+1}^2 + k\left(q_{i,t+1}^2 - q_{i,t}\right)\right)\frac{1}{2\sqrt{3}\sigma}dd_{t+1} +$$
$$\frac{1}{1+r}\int_{d_t}^{d_t+\sqrt{3}\sigma} \left(\left(d_{t+1} - \sum_{j=1}^n q_{j,t+1}^1 - c\right)q_{i,t+1}^1 - k\left(q_{i,t+1}^1 - q_{i,t}\right)\right)\frac{1}{2\sqrt{3}\sigma}dd_{t+1} +$$
$$\frac{1}{1+r}\int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t} \left(d_{t+1} - \sum_{j=1}^n q_{j,t} - c\right)q_{i,t}\frac{1}{2\sqrt{3}\sigma}dd_{t+1}.$$

The portion of $\frac{E[\pi_{i,t+1}]}{1+r}$ that depends upon $q_{i,t}$ is given by the following function:

$$f_1(q_{i,t}) = -\frac{1}{1+r}\int_{d_t-\sqrt{3}\sigma}^{d_t-\min(2\lambda k,\sqrt{3}\sigma)} kq_{i,t}\frac{1}{2\sqrt{3}\sigma}dd_{t+1} + \frac{1}{1+r}\int_{d_t}^{d_t+\sqrt{3}\sigma} kq_{i,t}\frac{1}{2\sqrt{3}\sigma}dd_{t+1} +$$
$$\frac{1}{1+r}\int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t} \left(d_{t+1} - \sum_{j=1}^n q_{j,t} - c\right)q_{i,t}\frac{1}{2\sqrt{3}\sigma}dd_{t+1}.$$

Equivalently,

$$f_1(q_{i,t}) = \frac{\min(2\lambda k, \sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\left(d_t - \sum_{j=1}^{n} q_{j,t} - c + k - 0.5\min(2\lambda k, \sqrt{3}\sigma)\right)q_{i,t} \tag{5.2}$$

For any $m \geq 2$, such that $q_{i,t+m-1} = \cdots = q_{i,t+1} = q_{i,t}$, $\frac{E[\pi_{i,t+m}]}{(1+r)^m}$ is given by

$$\frac{1}{(1+r)^m(2\sqrt{3}\sigma)^{m-1}}\int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t}\int_{d_{t+1}-\min(2\lambda k,\sqrt{3}\sigma)}^{d_{t+1}}\cdots\int_{d_{t+m-2}-\min(2\lambda k,\sqrt{3}\sigma)}^{d_{t+m-2}}\omega\, dd_{t+m-1}\ldots dd_{t+2}dd_{t+1},$$

where

$$\omega = \int_{d_{t+m-1}-\sqrt{3}\sigma}^{d_{t+m-1}-\min(2\lambda k,\sqrt{3}\sigma)}\left(\left(d_{t+m} - \sum_{j=1}^{n} q_{j,t+m}^2 - c\right)q_{i,t+m}^2 + k\cdot\left(q_{i,t+m}^2 - q_{i,t}\right)\right)\frac{1}{2\sqrt{3}\sigma}dd_{t+m} +$$
$$\int_{d_{t+m-1}}^{d_{t+m-1}+\sqrt{3}\sigma}\left(\left(d_{t+m} - \sum_{j=1}^{n} q_{j,t+m}^1 - c\right)q_{i,t+m}^1 - k\left(q_{i,t+m}^1 - q_{i,t}\right)\right)\frac{1}{2\sqrt{3}\sigma}dd_{t+m} +$$
$$\int_{d_{t+m-1}-\min(2\lambda k,\sqrt{3}\sigma)}^{d_{t+m-1}}\left(d_{t+m} - \sum_{j=1}^{n} q_{j,t} - c\right)q_{i,t}\frac{1}{2\sqrt{3}\sigma}dd_{t+m}.$$

The portion of $\frac{E[\pi_{i,t+m}]}{(1+r)^m}$ that depends upon $q_{i,t}$ is given by the following function:

$$f_m(q_{i,t}) =$$
$$\frac{1}{(1+r)^m(2\sqrt{3}\sigma)^{m-1}}\int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t}\int_{d_{t+1}-\min(2\lambda k,\sqrt{3}\sigma)}^{d_{t+1}}\cdots\int_{d_{t+m-2}-\min(2\lambda k,\sqrt{3}\sigma)}^{d_{t+m-2}}\varphi\, dd_{t+m-1}\ldots dd_{t+2}dd_{t+1},$$

where $\varphi = \frac{\min(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma}\left(d_{t+m-1} - \sum_{j=1}^{n} q_{j,t} - c + k - 0.5\min(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}$.

After algebraic rearrangements, we get

$$f_m(q_{i,t}) = \frac{\min(2\lambda k,\sqrt{3}\sigma)}{\left(2\sqrt{3}\sigma(1+r)\right)^m}I_m(t) + \left(\frac{\min(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m\left(-\sum_{j=1}^{n} q_{j,t} - c + k - 0.5\min(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t},$$

where

$$I_m(t) = \int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t}\int_{d_{t+1}-\min(2\lambda k,\sqrt{3}\sigma)}^{d_{t+1}}\cdots\int_{d_{t+m-2}-\min(2\lambda k,\sqrt{3}\sigma)}^{d_{t+m-2}} d_{t+m-1}q_{i,t}\,dd_{t+m-1}\ldots dd_{t+2}dd_{t+1}.$$

We prove by induction on m that $I_m(t) = \left(\min(2\lambda k,\sqrt{3}\sigma)\right)^{m-1}\left(d_t - 0.5(m-1)\min(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}$. Starting with $m = 2$, we have $I_2(t) = \int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t} d_{t+1}q_{i,t}dd_{t+1} = \min(2\lambda k,\sqrt{3}\sigma)\big(d_t - 0.5\min(2\lambda k,\sqrt{3}\sigma)\big)q_{i,t}$. Utilizing now the induction assumption $I_{m-1}(t+1) = \left(\min(2\lambda k,\sqrt{3}\sigma)\right)^{m-2}\cdot\left(d_{t+1} - 0.5(m-2)\min(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}$, we get

$$I_m(t) = \int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t} I_{m-1}(t+1)dd_{t+1}$$
$$= \int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t}\left(\min(2\lambda k,\sqrt{3}\sigma)\right)^{m-2}\left(d_{t+1} - 0.5(m-2)\min(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}dd_{t+1}$$
$$= \left(\min(2\lambda k,\sqrt{3}\sigma)\right)^{m-2}\int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t} d_{t+1}q_{i,t}dd_{t+1}$$
$$-\left(\min(2\lambda k,\sqrt{3}\sigma)\right)^{m-2}\int_{d_t-\min(2\lambda k,\sqrt{3}\sigma)}^{d_t} 0.5(m-2)\left(\min(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}dd_{t+1}$$
$$= \left(\min(2\lambda k,\sqrt{3}\sigma)\right)^{m-1}\left(d_t - 0.5\min(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}$$

$$-\left(\min\left(2\lambda k,\sqrt{3}\sigma\right)\right)^{m-1}0.5(m-2)\left(\min\,(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}$$

$$=\left(\min\,(2\lambda k,\sqrt{3}\sigma)\right)^{m-1}\left(d_t-0.5(m-1)\min\,(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}.$$

Thus, for any $m\geq 1$, the portion of $\frac{E[\pi_{i,t+m}]}{(1+r)^m}$ that depends upon $q_{i,t}$ is given by the following function:

$$f_m(q_{i,t})=\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m\left(d_t-\sum_{j=1}^n q_{j,t}-c+k-0.5m\cdot\min\,(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t} \tag{5.3}$$

Thus, using the notation $S_1=\sum_{m=0}^{\infty}\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m$ and $S_2=\sum_{m=0}^{\infty}m\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m$, the portion of $E\left[\sum_{m=0}^{\infty}\frac{\pi_{i,t+m}}{(1+r)^m}\right]$ that depends upon $q_{i,t}$ is $f(q_{i,t})=\sum_{m=0}^{\infty}f_m(q_{i,t})$, and by equations (5.1)-(5.3) it is given by the following function:

$$f(q_{i,t})=S_1\left(d_t-\sum_{j=1}^n q_{j,t}-c+k\right)q_{i,t}-S_2\left(0.5\min\,(2\lambda k,\sqrt{3}\sigma)\right)q_{i,t}-2kq_{i,t} \tag{5.4}$$

The series $S_1=\sum_{m=0}^{\infty}\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m$ is a geometric series, which converges to $S_1=\frac{1}{1-\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}}=\frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min\,(2\lambda k,\sqrt{3}\sigma)}$ because $\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}<1$. The series $S_2=\sum_{m=0}^{\infty}m\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m$ equals $S_2=\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\sum_{m=0}^{\infty}m\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^{m-1}$. Since the series $\sum_{m=0}^{\infty}m\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^{m-1}$ is the derivatives series of $\sum_{m=0}^{\infty}\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m$, it converges to $\frac{1}{\left(1-\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^2}=\frac{12\sigma^2(1+r)^2}{\left(2\sqrt{3}\sigma(1+r)-\min\,(2\lambda k,\sqrt{3}\sigma)\right)^2}$. Hence, the series $S_2=\sum_{m=0}^{\infty}m\left(\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m$ converges to $S_2=\frac{\min\,(2\lambda k,\sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\cdot\frac{12\sigma^2(1+r)^2}{\left(2\sqrt{3}\sigma(1+r)-\min\,(2\lambda k,\sqrt{3}\sigma)\right)^2}$ or, equivalently, $S_2=\frac{2\sqrt{3}\sigma(1+r)\min\,(2\lambda k,\sqrt{3}\sigma)}{\left(2\sqrt{3}\sigma(1+r)-\min\,(2\lambda k,\sqrt{3}\sigma)\right)^2}$.

Substituting $S_1=\frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min\,(2\lambda k,\sqrt{3}\sigma)}$ and $S_2=\frac{2\sqrt{3}\sigma(1+r)\min\,(2\lambda k,\sqrt{3}\sigma)}{\left(2\sqrt{3}\sigma(1+r)-\min\,(2\lambda k,\sqrt{3}\sigma)\right)^2}$ in equation (5.4), and rearranging, we get the following equation:

$$f(q_{i,t})=\frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min\,(2\lambda k,\sqrt{3}\sigma)}\cdot$$

$$\left(d_t-\sum_{j=1}^n q_{j,t}-c-k-\frac{0.5\left(\min\left(2\lambda k,\sqrt{3}\sigma\right)\right)^2}{2\sqrt{3}\sigma(1+r)-\min\left(2\lambda k,\sqrt{3}\sigma\right)}+\frac{\min\,(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}\right)q_{i,t} \tag{5.5}$$

The first order condition is as follows:

$$f'(q_{i,t})=\frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min\,(2\lambda k,\sqrt{3}\sigma)}\cdot$$

$$\left(d_t-q_{i,t}-\sum_{j=1}^n q_{j,t}-c-k-\frac{0.5\left(\min\left(2\lambda k,\sqrt{3}\sigma\right)\right)^2}{2\sqrt{3}\sigma(1+r)-\min\left(2\lambda k,\sqrt{3}\sigma\right)}+\frac{\min\,(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}\right)=0 \tag{5.6}$$

The second order condition is as follows:

$$f''(q_{i,t}) = -2\frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)} < 0 \tag{5.7}$$

It follows from equation (5.6) $q_{i,t} = d_t - \sum_{j=1}^{n} q_{j,t} - c - k - \frac{0.5\left(\min\left(2\lambda k,\sqrt{3}\sigma\right)\right)^2}{2\sqrt{3}\sigma(1+r)-\min\left(2\lambda k,\sqrt{3}\sigma\right)} + \frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}$, implying $q_{1,t} = q_{2,t} = \cdots = q_{n,t}$. Therefore, equation (5.6) is equivalent to $d_t - (n+1)q_{i,t} - c - k - \frac{0.5\left(\min\left(2\lambda k,\sqrt{3}\sigma\right)\right)^2}{2\sqrt{3}\sigma(1+r)-\min\left(2\lambda k,\sqrt{3}\sigma\right)} + \frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)} = 0$, and thus $q_{i,t} = \frac{d_t - c - k - \frac{0.5\left(\min\left(2\lambda k,\sqrt{3}\sigma\right)\right)^2}{2\sqrt{3}\sigma(1+r)-\min\left(2\lambda k,\sqrt{3}\sigma\right)} + \frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}}{n+1}$. We now can extract the scalar λ from the equation $q_{i,t} = \frac{d_t - c - k - \frac{0.5\left(\min\left(2\lambda k,\sqrt{3}\sigma\right)\right)^2}{2\sqrt{3}\sigma(1+r)-\min\left(2\lambda k,\sqrt{3}\sigma\right)} + \frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}}{n+1} = \frac{d_t - c - \lambda k}{n+1}$. It thus follows that the scalar λ is the solution of the following equation:

$$h(\lambda) = \lambda k - k - \frac{0.5\left(\min\left(2\lambda k,\sqrt{3}\sigma\right)\right)^2}{2\sqrt{3}\sigma(1+r)-\min\left(2\lambda k,\sqrt{3}\sigma\right)} + \frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)} = 0 \tag{5.8}$$

If $2\lambda k < \sqrt{3}\sigma$, then by applying algebraic rearrangement we get that equation (5.8) is equivalent to $2k\left(1+\frac{\mathrm{k}}{\sqrt{3}\sigma(1+r)}\right)\lambda^2 - \left(3k+\sqrt{3}\sigma(1+r)\right)\lambda + \sqrt{3}\sigma(1+r) = 0$. There are two solutions λ to this quadratic equation, which are given by $\frac{3k+\sqrt{3}\sigma(1+r)\pm\sqrt{\left(3k+\sqrt{3}\sigma(1+r)\right)^2 - 8k\left(\sqrt{3}\sigma(1+r)+k\right)}}{4k\left(1+\frac{\mathrm{k}}{\sqrt{3}\sigma(1+r)}\right)}$. After algebraic rearrangements, we get the following two solutions: $\lambda = \frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k}$ and $\lambda = \frac{\sqrt{3}\sigma(1+r)}{2k}$. The former solution satisfies the assumption $2\lambda k < \sqrt{3}\sigma$ if and only if $\frac{1+2r}{1+r}k < \sqrt{3}\sigma$. The later solution does not satisfy this assumption, and it thus eliminated. If $2\lambda k \geq \sqrt{3}\sigma$, then equation (5.8) is equivalent to $\lambda k - k - \frac{0.5\sqrt{3}\sigma}{1+2r} + \frac{k}{1+r} = 0$. The unique solution of this equation is $\lambda = \frac{\sqrt{3}\sigma}{2k(1+2r)} + \frac{r}{1+r}$. This solution satisfies the assumption $2\lambda k \geq \sqrt{3}\sigma$ if and only if $\frac{1+2r}{1+r}k \geq \sqrt{3}\sigma$. We therefore conclude that there exists a unique solution λ to equation (5.8), which is given by $\lambda = \begin{cases} \frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k} & if\ \frac{1+2r}{1+r}k < \sqrt{3}\sigma \\ \frac{\sqrt{3}\sigma}{2k(1+2r)} + \frac{r}{1+r} & otherwise \end{cases}$.

It follows from $0 < \frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k} < 1$ and $0 < \frac{\sqrt{3}\sigma}{2k(1+2r)} + \frac{r}{1+r} < \frac{1}{1+2r} + \frac{r}{1+r} < \frac{1}{1+r} + \frac{r}{1+r} = 1$ that $\lambda \in (0,1)$. The derivatives of λ with respect to the parameters k, σ and r are as follows:

$\frac{d\lambda}{dk} = \begin{cases} -\frac{\sqrt{3}\sigma(1+r)}{\left(\sqrt{3}\sigma(1+r)+k\right)^2} < 0 & if\ \frac{1+2r}{1+r}k < \sqrt{3}\sigma \\ -\frac{\sqrt{3}\sigma}{2k^2(1+2r)} < 0 & otherwise \end{cases}$, $\frac{d\lambda}{d\sigma} = \begin{cases} \frac{\sqrt{3}(1+r)k}{\left(\sqrt{3}\sigma(1+r)+k\right)^2} > 0 & if\ \frac{1+2r}{1+r}k < \sqrt{3}\sigma \\ \frac{\sqrt{3}}{2k(1+2r)} > 0 & otherwise \end{cases}$, and

$\frac{d\lambda}{dr} = \begin{cases} \frac{\sqrt{3}\sigma k}{\left(\sqrt{3}\sigma(1+r)+k\right)^2} > 0 & if\ \frac{1+2r}{1+r}k < \sqrt{3}\sigma \\ -\frac{\sqrt{3}\sigma}{2k(1+2r)^2} + \frac{1}{(1+r)^2} > -\frac{1}{(1+2r)^2} + \frac{1}{(1+r)^2} > 0 & otherwise \end{cases}$. Hence, λ is decreasing in k, increasing in σ, and increasing r. Also, $\lim_{r\to\infty} \frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k} = 1$ and $\lim_{r\to\infty} \left(\frac{\sqrt{3}\sigma}{2k(1+2r)} + \frac{r}{1+r}\right) = 1$, so $\lim_{r\to\infty} \lambda = 1$.

Case 2: $d_t < (n+1)q_{i,t(-1)} + c - \lambda k$

The analysis of case2 is based on similar arguments as in case 1, and thus is given in a brief format. In this case, conditional on the demand realization d_t,the production quantity of each firm i in period t is $q_{i,t} = \frac{d_t - c + \lambda k}{n+1}$ and the profit is $\pi_{i,t} = \left(d_t - \sum_{j=1}^{n} q_{j,t} - c\right) q_{i,t} + k\left(q_{i,t} - q_{i,t(-1)}\right)$.

The portion of $\pi_{i,t}$ that depends upon $q_{i,t}$ is given by the following function:

$$g_0(q_{i,t}) = \left(d_t - \sum_{j=1}^{n} q_{j,t} - c + k\right) q_{i,t} \tag{5.9}$$

The portion of $\frac{E[\pi_{i,t+1}]}{1+r}$ that depends upon $q_{i,t}$ is given by the following function:

$$g_1(q_{i,t}) = \frac{\min(2\lambda k, \sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)} \left(d_t - \sum_{j=1}^{n} q_{j,t} - c - k + 0.5\min(2\lambda k, \sqrt{3}\sigma)\right) q_{i,t} \tag{5.10}$$

For any $m \geq 1$, the portion of $\frac{E[\pi_{i,t+m}]}{(1+r)^m}$ that depends upon $q_{i,t}$ is given by the following function:

$$g_m(q_{i,t}) = \left(\frac{\min(2\lambda k, \sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m \left(d_t - \sum_{j=1}^{n} q_{j,t} - c - k + 0.5m \cdot \min(2\lambda k, \sqrt{3}\sigma)\right) q_{i,t} \tag{5.11}$$

Thus, using the notation $S_1 = \sum_{m=0}^{\infty} \left(\frac{\min(2\lambda k, \sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m$ and $S_2 = \sum_{m=0}^{\infty} m \left(\frac{\min(2\lambda k, \sqrt{3}\sigma)}{2\sqrt{3}\sigma(1+r)}\right)^m$, the portion of $E\left[\sum_{m=0}^{\infty} \frac{\pi_{i,t+m}}{(1+r)^m}\right]$ that depends upon $q_{i,t}$ is $g(q_{i,t}) = \sum_{m=0}^{\infty} g_m(q_{i,t})$, and by equations (5.9)-(5.11) it is given by the following function:

$$g(q_{i,t}) = S_1\left(d_t - \sum_{j=1}^{n} q_{j,t} - c - k\right) q_{i,t} + S_2\left(0.5\min(2\lambda k, \sqrt{3}\sigma)\right) q_{i,t} + 2k q_{i,t} \tag{5.12}$$

Substituting $S_1 = \frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r) - \min(2\lambda k, \sqrt{3}\sigma)}$ and $S_2 = \frac{2\sqrt{3}\sigma(1+r)\min(2\lambda k, \sqrt{3}\sigma)}{\left(2\sqrt{3}\sigma(1+r) - \min(2\lambda k, \sqrt{3}\sigma)\right)^2}$ in equation (5.12), and rearranging, we get the following equation:

$$g(q_{i,t}) = \frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r) - \min(2\lambda k, \sqrt{3}\sigma)} \cdot \left(d_t - \sum_{j=1}^{n} q_{j,t} - c + k + \frac{0.5\left(\min(2\lambda k, \sqrt{3}\sigma)\right)^2}{2\sqrt{3}\sigma(1+r) - \min(2\lambda k, \sqrt{3}\sigma)} - \frac{\min(2\lambda k, \sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}\right) q_{i,t} \tag{5.13}$$

The first order condition is as follows:

$$g'(q_{i,t}) = \frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r) - \min(2\lambda k, \sqrt{3}\sigma)} \cdot \left(d_t - q_{i,t} - \sum_{j=1}^{n} q_{j,t} - c + k + \frac{0.5\left(\min(2\lambda k, \sqrt{3}\sigma)\right)^2}{2\sqrt{3}\sigma(1+r) - \min(2\lambda k, \sqrt{3}\sigma)} - \frac{\min(2\lambda k, \sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}\right) = 0 \tag{5.14}$$

The second order condition is as follows:

$$g''(q_{i,t}) = -2\frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)} < 0 \tag{5.15}$$

It follows from equation (5.14) $q_{i,t} = d_t - \sum_{j=1}^{n} q_{j,t} - c + k + \frac{0.5(\min(2\lambda k,\sqrt{3}\sigma))^2}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)} - \frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}$, implying $q_{1,t} = q_{2,t} = \cdots = q_{n,t}$. Therefore, equation (5.14) is equivalent to $d_t - (n+1)q_{i,t} - c - k - \frac{0.5(\min(2\lambda k,\sqrt{3}\sigma))^2}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)} + \frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)} = 0$, and thus $q_{i,t} = \frac{d_t-c+k+\frac{0.5(\min(2\lambda k,\sqrt{3}\sigma))^2}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)}-\frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}}{n+1}$. We now can extract the scalar λ from the equation $q_{i,t} = \frac{d_t-c+k+\frac{0.5(\min(2\lambda k,\sqrt{3}\sigma))^2}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)}-\frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)}}{n+1} = \frac{d_t-c+\lambda k}{n+1}$. It thus follows that the scalar λ is the solution of the following equation:

$$h(\lambda) = \lambda k - k - \frac{0.5(\min(2\lambda k,\sqrt{3}\sigma))^2}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)} + \frac{\min(2\lambda k,\sqrt{3}\sigma)k}{\sqrt{3}\sigma(1+r)} = 0 \tag{5.16}$$

Equation (5.16) is the same as equation (5.8), so they have the same its unique solution is λ.

<u>Case 3</u>: $(n+1)q_{i,t(-1)} + c - \lambda k \le d_t \le (n+1)q_{i,t(-1)} + c + \lambda k$

If $q_{i,t} < q_{i,t-1}$, then $\frac{\partial E\left[\sum_{m=0}^{\infty}\frac{\pi_{i,t+m}}{(1+r)^m}\right]}{\partial q_{i,t}} = \frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)}\left(d_t - (n+1)q_{i,t} - c + \lambda k\right)$. Using the assumption $d_t \ge (n+1)q_{i,t-1} + c - \lambda k$, it follows that $\frac{\partial E\left[\sum_{m=0}^{\infty}\frac{\pi_{i,t+m}}{(1+r)^m}\right]}{\partial q_{i,t}} \ge \frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)}(n+1)\cdot(q_{i,t-1}-q_{i,t}) > 0$. If $q_{i,t} > q_{i,t-1}$, then $\frac{\partial E\left[\sum_{m=0}^{\infty}\frac{\pi_{i,t+m}}{(1+r)^m}\right]}{\partial q_{i,t}} = \frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)}\left(d_t - (n+1)q_{i,t} - c - \lambda k\right)$. Using the assumption $d_t \le (n+1)q_{i,t-1} + c + \lambda k$, it follows that $\frac{\partial E\left[\sum_{m=0}^{\infty}\frac{\pi_{i,t+m}}{(1+r)^m}\right]}{\partial q_{i,t}} \le \frac{2\sqrt{3}\sigma(1+r)}{2\sqrt{3}\sigma(1+r)-\min(2\lambda k,\sqrt{3}\sigma)}(n+1)\cdot(q_{i,t-1}-q_{i,t}) < 0$. The optimal solution is therefore $q_{i,t} = q_{i,t-1}$.

Based on the analysis of the three cases, we conclude that, in the unrestricted, the equilibrium in each period $t \in \mathbb{N}$ is given by $\forall i \in \{1,2,\ldots,n\}$: $q_{i,t} = \begin{cases} \frac{d_t-c-\lambda k}{n+1} & if\ d_t > (n+1)q_{i,t-1} + c + \lambda k \\ \frac{d_t-c+\lambda k}{n+1} & if\ d_t < (n+1)q_{i,t-1} + c - \lambda k \\ q_{i,t-1} & otherwise \end{cases}$. This implies $q_{i,t-1} = q_{i,t(-1)}$ can be either or $\frac{d_{t(-1)}-c-\lambda k}{n+1}$ or $\frac{d_{t(-1)}-c+\lambda k}{n+1}$.

If $q_{i,t-1} = q_{i,t(-1)} = \frac{d_{t(-1)}-c-\lambda k}{n+1}$, then the inequity $d_t > (n+1)q_{i,t-1} + c + \lambda k$ is equivalent to $d_t - d_{t(-1)} > 0$, and the inequity $d_t < (n+1)q_{i,t-1} + c - \lambda k$ is equivalent to $d_t - d_{t(-1)} < -2k$. So, $q_{i,t} = \begin{cases} \frac{d_t-c-\lambda k}{n+1} if\ d_t - d_{t(-1)} > 0 \\ q_{i,t-1}^{r=\infty} \quad if -2\lambda k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t-c+\lambda k}{n+1} if\ d_t - d_{t(-1)} < -2\lambda k \end{cases}$ and $q_{i,t} - q_{i,t(-1)} = \begin{cases} \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ 0 & if -2\lambda k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t-d_{t(-1)}+2\lambda k}{n+1} & if\ d_t - d_{t(-1)} < -2\lambda k \end{cases}$.

If $q_{i,t-1} = q_{i,t(-1)} = \frac{d_{t(-1)}-c+\lambda k}{n+1}$, then the inequity $d_t > (n+1)q_{i,t-1} + c + \lambda k$ is equivalent to $d_t - d_{t(-1)} > 2k$, and the inequity $d_t < (n+1)q_{i,t-1} + c - \lambda k$ is equivalent to $d_t - d_{t(-1)} < 0$. So, $q_{i,t} = \begin{cases} \frac{d_t-c-\lambda k}{n+1}\ if\ d_t - d_{t(-1)} > 2\lambda k \\ q_{i,t-1}^{r=\infty} \quad if\ 0 \le d_t - d_{t(-1)} \le 2\lambda k \\ \frac{d_t-c+\lambda k}{n+1}\ if\ d_t - d_{t(-1)} < 0 \end{cases}$ and $q_{i,t} - q_{i,t(-1)} = \begin{cases} \frac{d_t-d_{t(-1)}-2\lambda k}{n+1} & if\ d_t - d_{t(-1)} > 2\lambda k \\ 0 & if\ 0 \le d_t - d_{t(-1)} \le 2\lambda k \\ \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} < 0 \end{cases}$.

Since resource adjustment has accrued in period $t(-1)$, we get that $q_{i,t(-1)} = \frac{d_{t(-1)}-c-\lambda k}{n+1}$ is equivalent to $d_{t(-1)} - d_{t(-2)} > 0$, and $q_{i,t(-1)} = \frac{d_{t(-1)}-c+\lambda k}{n+1}$ is equivalent to $d_{t(-1)} - d_{t(-2)} < 0$. □

Prof of Proposition 6. For firm i in period t, $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} > 0\right]$ equals $\int_0^{\sqrt{3}\sigma} \Delta \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} > 0\right] = \frac{\sqrt{3}\sigma}{2} \tag{6.1}$$

For firm i in period t, $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} < 0\right]$ equals $\int_{-\sqrt{3}\sigma}^{0} \Delta \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} < 0\right] = -\frac{\sqrt{3}\sigma}{2} \tag{6.2}$$

We consider separately two cases: the case of $d_{t(-1)} - d_{t(-2)} > 0$, and the case of $d_{t(-1)} - d_{t(-2)} < 0$.

<u>Case 1</u>: $d_{t(-1)} - d_{t(-2)} > 0$

By Proposition 5, for $d_{t(-1)} - d_{t(-2)} > 0$, $q_{i,t} - q_{i,t(-1)} = \begin{cases} \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ 0 & if -2\lambda k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t-d_{t(-1)}+2\lambda k}{n+1} & if\ d_t - d_{t(-1)} < -2\lambda k \end{cases}$.

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t} - q_{i,t(-1)} \middle| \tilde{d}_t - d_{t(-1)} > 0\right]$ equals $\int_0^{\sqrt{3}\sigma} \frac{\Delta}{n+1} \cdot \frac{1}{\sqrt{3}\sigma} dd_t$, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t} - q_{i,t(-1)} \middle| \tilde{d}_t - d_{t(-1)} > 0\right] = \frac{\sqrt{3}\sigma}{2(n+1)} \tag{6.3}$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty}-q_{i,t(-1)}^{r=\infty}\middle|\tilde{d}_t-d_{t(-1)}<0\right]$ equals $\int_{-\sqrt{3}\sigma}^{-2\lambda k}\frac{\Delta+2\lambda k}{n+1}\cdot\frac{1}{\sqrt{3}\sigma}d\Delta$ when $2\lambda k<\sqrt{3}\sigma$ and zero otherwise, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty}-q_{i,t(-1)}^{r=\infty}\middle|\tilde{d}_t-d_{t(-1)}<0\right]=\begin{cases}-\frac{\sqrt{3}\sigma}{2(n+1)}+\frac{2\lambda k(\sqrt{3}\sigma-\lambda k)}{\sqrt{3}\sigma(n+1)} & if\ 2\lambda k<\sqrt{3}\sigma\\ 0 & otherwise\end{cases} \quad (6.4)$$

By equations (6.1)-(6.4), $M_{i,t}^{r=\infty}=\begin{cases}\frac{\frac{\sqrt{3}\sigma}{2(n+1)}}{\frac{\sqrt{3}\sigma}{2}}-\frac{-\frac{\sqrt{3}\sigma}{2(n+1)}+\frac{2\lambda k(\sqrt{3}\sigma-\lambda k)}{\sqrt{3}\sigma(n+1)}}{-\frac{\sqrt{3}\sigma}{2}}=\frac{4\lambda k(\sqrt{3}\sigma-\lambda k)}{3\sigma^2(n+1)} & if\ 2\lambda k<\sqrt{3}\sigma\\ \frac{\frac{\sqrt{3}\sigma}{2(n+1)}}{\frac{\sqrt{3}\sigma}{2}}-\frac{0}{-\frac{\sqrt{3}\sigma}{2}}=\frac{1}{n+1} & otherwise\end{cases}=M>0.$

<u>Case 2</u>: $d_{t(-1)}-d_{t(-2)}<0$

By Proposition 5, for $d_{t(-1)}-d_{t(-2)}<0$, $q_{i,t}-q_{i,t(-1)}=\begin{cases}\frac{d_t-d_{t(-1)}-2\lambda k}{n+1} & if\ d_t-d_{t(-1)}>2\lambda k\\ 0 & if\ 0\le d_t-d_{t(-1)}\le 2\lambda k\\ \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t-d_{t(-1)}<0\end{cases}.$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty}-q_{i,t(-1)}^{r=\infty}\middle|\tilde{d}_t-d_{t(-1)}>0\right]$ equals $\int_{2k}^{\sqrt{3}\sigma}\frac{\Delta-2\lambda k}{n+1}\cdot\frac{1}{\sqrt{3}\sigma}d\Delta$ when $2\lambda k<\sqrt{3}\sigma$ and zero otherwise, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty}-q_{i,t(-1)}^{r=\infty}\middle|\tilde{d}_t-d_{t-1}>0\right]=\begin{cases}\frac{\sqrt{3}\sigma}{2(n+1)}-\frac{2\lambda k(\sqrt{3}\sigma-\lambda k)}{\sqrt{3}\sigma(n+1)} & if\ 2\lambda k<\sqrt{3}\sigma\\ 0 & otherwise\end{cases} \quad (6.5)$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty}-q_{i,t(-1)}^{r=\infty}\middle|\tilde{d}_t-d_{t(-1)}<0\right]$ equals $\int_{-\sqrt{3}\sigma}^{0}\frac{\Delta}{n+1}\cdot\frac{1}{\sqrt{3}\sigma}d\Delta$, and after algebraic rearrangements it can be rewritten as follows:

$$E_{\tilde{d}_t}\left[q_{i,t}^{r=\infty}-q_{i,t(-1)}^{r=\infty}\middle|\tilde{d}_t-d_{t(-1)}<0\right]=-\frac{\sqrt{3}\sigma}{2(n+1)} \quad (6.6)$$

By (6.1),(6.2),(6.5),(6.6), $M_{i,t}=\begin{cases}\frac{\frac{\sqrt{3}\sigma}{2(n+1)}-\frac{2\lambda k(\sqrt{3}\sigma-\lambda k)}{\sqrt{3}\sigma(n+1)}}{\frac{\sqrt{3}\sigma}{2}}-\frac{-\frac{\sqrt{3}\sigma}{2(n+1)}}{-\frac{\sqrt{3}\sigma}{2}}=-\frac{4\lambda k(\sqrt{3}\sigma-\lambda k)}{3\sigma^2(n+1)} & if\ \frac{1+2r}{1+r}k<\sqrt{3}\sigma\\ \frac{0}{\frac{\sqrt{3}\sigma}{2}}-\frac{-\frac{\sqrt{3}\sigma}{2(n+1)}}{-\frac{\sqrt{3}\sigma}{2}}=-\frac{1}{n+1} & otherwise\end{cases}=-M.\square$

Prof of Proposition 7. Under the assumption $\frac{1+2r}{1+r}k<\sqrt{3}\sigma$, $M=\frac{4\lambda k(\sqrt{3}\sigma-\lambda k)}{3\sigma^2(n+1)}$ and $\lambda=\frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k}$. It follows from $\frac{1+2r}{1+r}k<\sqrt{3}\sigma$ that $2\lambda k=2\frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k}k<2\frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+\sqrt{3}\sigma\frac{1+r}{1+2r}}\sqrt{3}\sigma\frac{1+r}{1+2r}=\sqrt{3}\sigma$. By Proposition 5, $\frac{d\lambda}{dk}<0, \frac{d\lambda}{d\sigma}>0$ and $\frac{d\lambda}{dr}>0$. The derivatives of M with respect to the parameters k,n,σ,r:

$$\frac{dM}{dk}=\frac{4\left(\lambda+\frac{d\lambda}{dk}k\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^2(n+1)}=\frac{4\left(\lambda-\frac{\sqrt{3}\sigma(1+r)}{\left(\sqrt{3}\sigma(1+r)+k\right)^2}k\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^2(n+1)}=\frac{4\left(\lambda-\frac{1}{\sqrt{3}\sigma(1+r)+k}\lambda k\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^2(n+1)}$$

$$= \frac{4\lambda\left(1-\frac{k}{\sqrt{3}\sigma(1+r)+k}\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^2(n+1)} = \frac{4\lambda\frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k}(\sqrt{3}\sigma-2\lambda k)}{3\sigma^2(n+1)} = \frac{4\lambda^2(\sqrt{3}\sigma-2\lambda k)}{3\sigma^2(n+1)} > 0,$$

$$\frac{dM}{dn} = -\frac{4\lambda k(\sqrt{3}\sigma-\lambda k)}{3\sigma^2(n+1)^2} < 0,$$

$$\frac{dM}{d\sigma} = \frac{4\sigma\left(\lambda+\frac{d\lambda}{dk}k\right)(\sqrt{3}\sigma-2\lambda k)-8\lambda k(\sqrt{3}\sigma-\lambda k)}{3\sigma^3(n+1)} = \frac{4\sigma\lambda^2(\sqrt{3}\sigma-2\lambda k)-8\lambda k(\sqrt{3}\sigma-\lambda k)}{3\sigma^3(n+1)} = \frac{4\lambda(\sigma\lambda-2k)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^3(n+1)}$$

$$= \frac{4\lambda\left(\sigma\frac{\sqrt{3}\sigma(1+r)}{\sqrt{3}\sigma(1+r)+k}-2k\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^3(n+1)} > \frac{4\lambda\left(\frac{(1+r)\sigma}{\sqrt{3}\sigma(1+r)+k}\cdot\frac{1+2r}{1+r}k-2k\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^3(n+1)} = \frac{4\lambda k\left(\frac{(1+2r)\sigma}{\sqrt{3}\sigma(1+r)+k}-2\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^3(n+1)}$$

$$= \frac{4\lambda k\left((1+2r)\sigma-2\sqrt{3}\sigma(1+r)-2k\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^3(\sqrt{3}\sigma(1+r)+k)(n+1)} = -\frac{4\lambda k\left((2\sqrt{3}-1)\sigma+2(\sqrt{3}-1)\sigma+2k\right)(\sqrt{3}\sigma-2\lambda k)}{3\sigma^3(\sqrt{3}\sigma(1+r)+k)(n+1)} < 0,$$

$$\frac{dM}{dr} = \frac{dM}{d\lambda}\cdot\frac{d\lambda}{dr} = \frac{4k\sqrt{3}\sigma-8\lambda k^2}{3\sigma^2(n+1)}\cdot\frac{d\lambda}{dr} = \frac{4k(\sqrt{3}\sigma-2\lambda k)}{3\sigma^2(n+1)}\cdot\frac{d\lambda}{dr} > 0.$$

For $\frac{1+2r}{1+r}k > \sqrt{3}\sigma$, the derivatives are: $\frac{dM}{dk} = 0$, $\frac{dM}{dn} = -\frac{1}{(n+1)^2} < 0$, $\frac{dM}{d\sigma} = 0$, $\frac{dM}{dr} = 0$. □

Prof of Proposition 8. For firm i in period t, $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| 0 < \tilde{d}_t - d_{t(-1)} \le 2\lambda k\right]$ equals $\int_0^{2\lambda k} \Delta \cdot \frac{1}{2\lambda k} d\Delta$ or equivalently

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| 0 < \tilde{d}_t - d_{t(-1)} \le 2\lambda k\right] = \lambda k \tag{8.1}$$

For firm i in period t, $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| -2\lambda k \le \tilde{d}_t - d_{t(-1)} < 0\right]$ equals $\int_{-2\lambda k}^{0} \Delta \cdot \frac{1}{2\lambda k} d\Delta$ or equivalently

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| -2\lambda k \le \tilde{d}_t - d_{t(-1)} < 0\right] = -\lambda k \tag{8.2}$$

For firm i in period t, $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} > 2\lambda k\right]$ equals $\int_{2\lambda k}^{\sqrt{3}\sigma} \Delta \cdot \frac{1}{\sqrt{3}\sigma-2\lambda k} d\Delta$ or equivalently

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} > 2\lambda k\right] = \frac{\sqrt{3}\sigma+2\lambda k}{2} \tag{8.3}$$

For firm i in period t, $E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| \tilde{d}_t - d_{t(-1)} < -2\lambda k\right]$ equals $\int_{-\sqrt{3}\sigma}^{-2\lambda k} \Delta \cdot \frac{1}{\sqrt{3}\sigma} d\Delta$ or equivalently

$$E_{\tilde{d}_t}\left[\tilde{d}_t - d_{t(-1)} \middle| -2\lambda k \le \tilde{d}_t - d_{t(-1)} < 0\right] = -\frac{\sqrt{3}\sigma+2\lambda k}{2} \tag{8.4}$$

We consider separately two cases: the case of $d_{t(-1)} - d_{t(-2)} > 0$, and the case of $d_{t(-1)} - d_{t(-2)} < 0$.

Case 1: $d_{t(-1)} - d_{t(-2)} > 0$

By Proposition 5, in the case $d_{t(-1)} - d_{t(-2)} > 0$, the resource adjustment of firm i in period t is given by

$$q_{i,t} - q_{i,t(-1)} = \begin{cases} \frac{d_t - d_{t(-1)}}{n+1} & if\ d_t - d_{t(-1)} > 0 \\ 0 & if -2\lambda k \le d_t - d_{t(-1)} \le 0 \\ \frac{d_t - d_{t(-1)} + 2\lambda k}{n+1} & if\ d_t - d_{t(-1)} < -2\lambda k \end{cases}.$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t} - q_{i,t(-1)} \middle| 0 < \tilde{d}_t - d_{t(-1)} \le 2\lambda k\right]$ equals $\int_0^{2\lambda k} \frac{\Delta}{n+1} \cdot \frac{1}{2\lambda k} d\Delta$ or equivalently

$$E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|0<\tilde{d}_t-d_{t(-1)}\leq 2\lambda k\right]=\frac{\lambda k}{n+1} \tag{8.5}$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|-2\lambda k\leq \tilde{d}_t-d_{t(-1)}<0\right]$ equals $\int_{-2\lambda k}^{0} 0\cdot\frac{1}{2\lambda k}d\Delta$, so:

$$E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|-2\lambda k\leq \tilde{d}_t-d_{t(-1)}<0\right]=0 \tag{8.6}$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|\tilde{d}_t-d_{t(-1)}>2\lambda k\right]$ equals $\int_{2\lambda k}^{\sqrt{3}\sigma}\frac{\Delta}{n+1}\cdot\frac{1}{\sqrt{3}\sigma-2\lambda k}d\Delta$ or equivalently

$$E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|\tilde{d}_t-d_{t(-1)}>2\lambda k\right]=\frac{\sqrt{3}\sigma+2\lambda k}{2(n+1)} \tag{8.7}$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|\tilde{d}_t-d_{t(-1)}<-2\lambda k\right]$ equals $\int_{-\sqrt{3}\sigma}^{-2\lambda k}\frac{\Delta+2\lambda k}{n+1}\cdot\frac{1}{\sqrt{3}\sigma-2\lambda k}d\Delta$ or

$$E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|\tilde{d}_t-d_{t(-1)}<0\right]=-\frac{\sqrt{3}\sigma+2\lambda k}{2(n+1)}+\frac{2\lambda k}{n+1} \tag{8.8}$$

Hence, $M_{i,t}^{L}=\frac{1}{n+1}-0=\frac{1}{n+1}$ and $M_{i,t}^{H}=\frac{1}{n+1}-\left(\frac{1}{n+1}-\frac{4\lambda k}{(n+1)(\sqrt{3}\sigma+2\lambda k)}\right)=\frac{4\lambda k}{(n+1)(\sqrt{3}\sigma+2\lambda k)}$.

<u>Case 2</u>: $d_{t(-1)}-d_{t(-2)}<0$

By Proposition 5, in the case $d_{t(-1)}-d_{t(-2)}<0$, the resource adjustment of firm i in period t is given by

$$q_{i,t}-q_{i,t(-1)}=\begin{cases}\frac{d_t-d_{t(-1)}-2\lambda k}{n+1} & if\ d_t-d_{t(-1)}>2\lambda k\\ 0 & if\ 0\leq d_t-d_{t(-1)}\leq 2\lambda k\\ \frac{d_t-d_{t(-1)}}{n+1} & if\ d_t-d_{t(-1)}<0\end{cases}.$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|0<\tilde{d}_t-d_{t(-1)}\leq 2\lambda k\right]$ equals $\int_{0}^{2\lambda k} 0\cdot\frac{1}{2\lambda k}d\Delta$, so

$$E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|0<\tilde{d}_t-d_{t(-1)}\leq 2\lambda k\right]=0 \tag{8.9}$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|-2\lambda k\leq \tilde{d}_t-d_{t(-1)}<0\right]$ equals $\int_{-2\lambda k}^{0}\frac{\Delta}{n+1}\cdot\frac{1}{2\lambda k}d\Delta$ or

$$E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|-2\lambda k\leq \tilde{d}_t-d_{t(-1)}<0\right]=-\frac{\lambda k}{n+1} \tag{8.10}$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|\tilde{d}_t-d_{t(-1)}>2\lambda k\right]$ equals $\int_{2\lambda k}^{\sqrt{3}\sigma}\frac{\Delta-2\lambda k}{n+1}\cdot\frac{1}{\sqrt{3}\sigma-2\lambda k}d\Delta$ or

$$E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|\tilde{d}_t-d_{t(-1)}>2\lambda k\right]=\frac{\sqrt{3}\sigma+2\lambda k}{2(n+1)}-\frac{2\lambda k}{n+1} \tag{8.11}$$

For firm i in period t, $E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|\tilde{d}_t-d_{t(-1)}<-2\lambda k\right]$ equals $\int_{-\sqrt{3}\sigma}^{-2\lambda k}\frac{\Delta}{n+1}\cdot\frac{1}{\sqrt{3}\sigma-2\lambda k}d\Delta$ or

$$E_{\tilde{d}_t}\left[q_{i,t}-q_{i,t(-1)}\middle|\tilde{d}_t-d_{t(-1)}<-2\lambda k\right]=-\frac{\sqrt{3}\sigma+2\lambda k}{2(n+1)} \tag{8.12}$$

Hence, $M_{i,t}^{L}=0-\frac{1}{n+1}=-\frac{1}{n+1}$ and $M_{i,t}^{H}=\frac{1}{n+1}-\frac{4\lambda k}{(n+1)(\sqrt{3}\sigma+2\lambda k)}-\frac{1}{n+1}=-\frac{4\lambda k}{(n+1)(\sqrt{3}\sigma+2\lambda k)}$.

In both case 1 and case 2, for $2\lambda k<\sqrt{3}\sigma$, we get $\left|M_{i,t}^{L}\right|=\frac{1}{n+1}>\frac{4\lambda k}{(n+1)(\sqrt{3}\sigma+2\lambda k)}=\left|M_{i,t}^{H}\right|>0$. □

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Figures

Figure 1

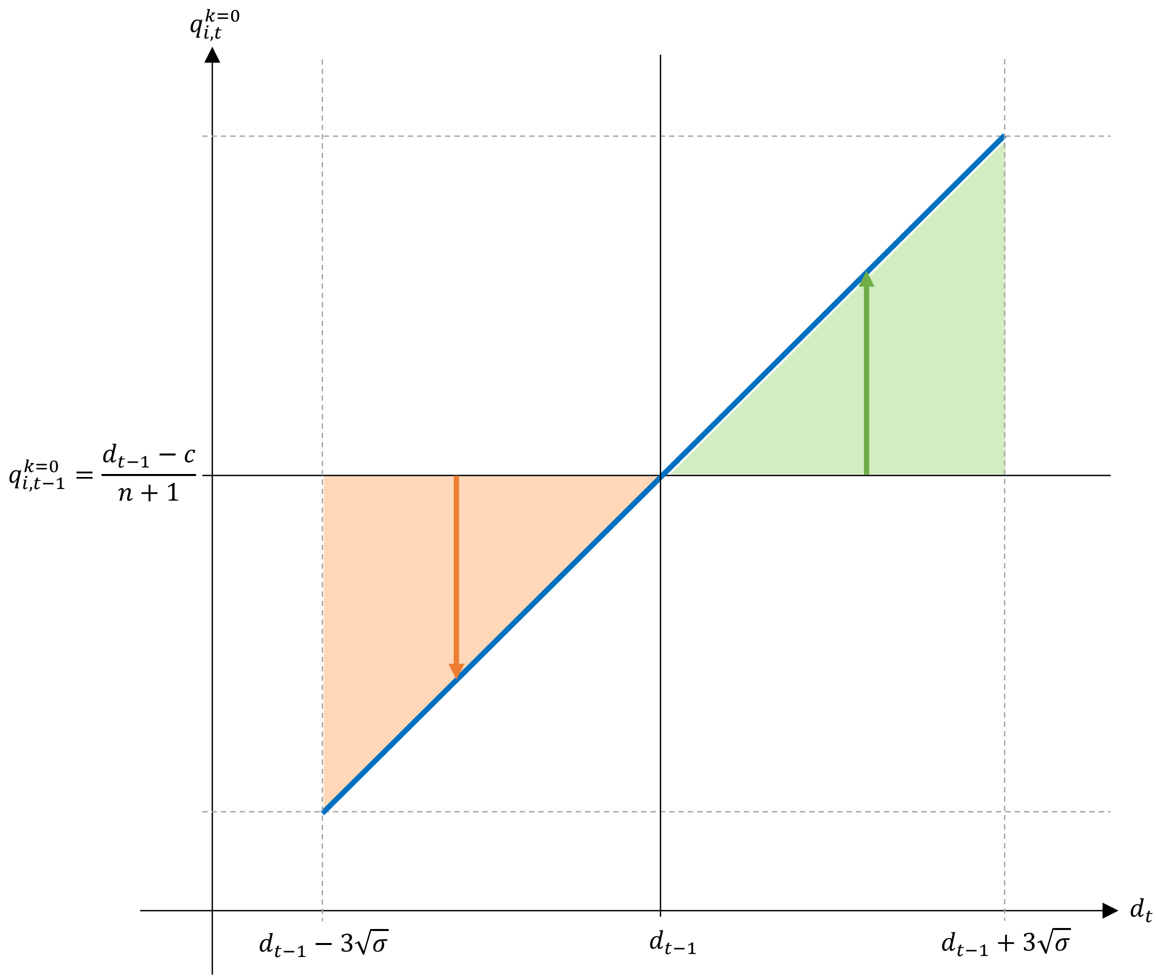

Figure 1 pertains to the benchmark case of $k = 0$ and illustrates the resource adjustment of each firm as a function of the demand shock in period t. The horizontal axis represents the demand d_t in period t, whereas the vertical axis represents the resource quantity $q_{i,t}^{k=0}$ of each firm i in period t. The blue diagonal line describes the resource quantity $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$ of each firm i in period t as a function of the demand d_t in period t. A positive demand shock $d_t - d_{t-1} > 0$ in period t causes an upward resource adjustment of $\frac{d_t - d_{t-1}}{n+1}$ units, marked by the green arrow, from the prior quantity of $q_{i,t-1}^{k=0} = \frac{d_{t-1} - c}{n+1}$ to the current higher quantity of $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$. A negative demand shock $d_t - d_{t-1} < 0$ in period t causes a symmetric downward resource adjustment of $\frac{|d_t - d_{t-1}|}{n+1}$ units, marked by the orange arrow, from the prior quantity of $q_{i,t-1}^{k=0} = \frac{d_{t-1} - c}{n+1}$ to the current lower quantity of $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$.

Figure 2

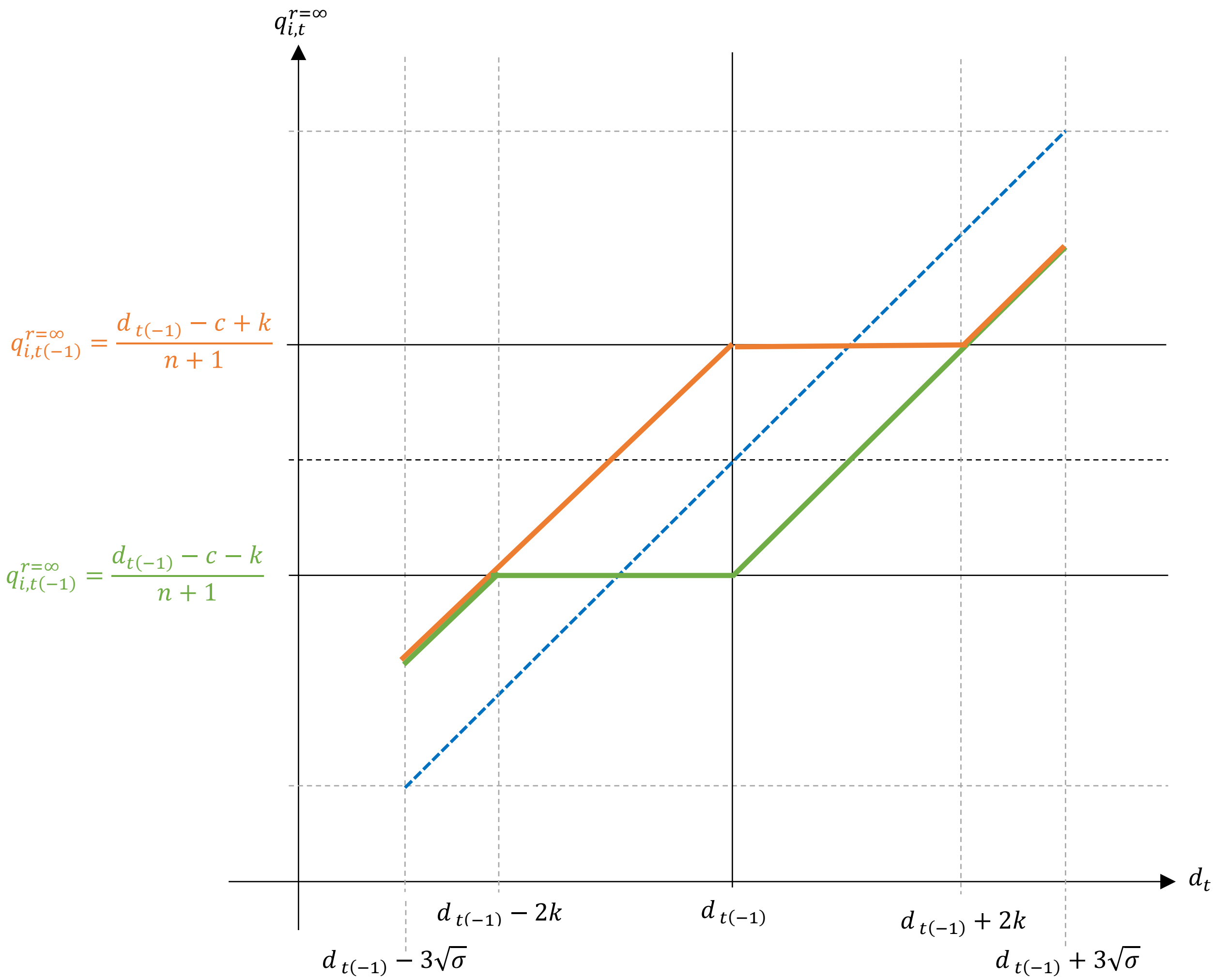

Figure 2 pertains to the case of $r = \infty$ and illustrates the resource quantity of each firm as a function of the demand shock in period t, given that the two most recent adjustments took place in periods $t(-2)$ and $t(-1)$. The horizontal axis represents the demand d_t in period t, whereas the vertical axis represents the resource quantity $q_{i,t}^{r=\infty}$ of each firm i in period t. The green and the orange curves describe resource quantity $q_{i,t}^{r=\infty}$ of each firm i in period t as a function of the demand d_t in period t, under two scenarios with respect to the prior demand shock $d_{t(-1)} - d_{t(-2)}$. The green curve pertains to the scenario where $d_{t(-1)} - d_{t(-2)} > 0$. The orange curve pertains to the scenario where $d_{t(-1)} - d_{t(-2)} < 0$. The blue diagonal dashed line pertains to the benchmark case of $k = 0$ and describes the benchmark resource quantity $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$ of each firm i in period t as a function of the demand d_t in period t.

Figure 3a

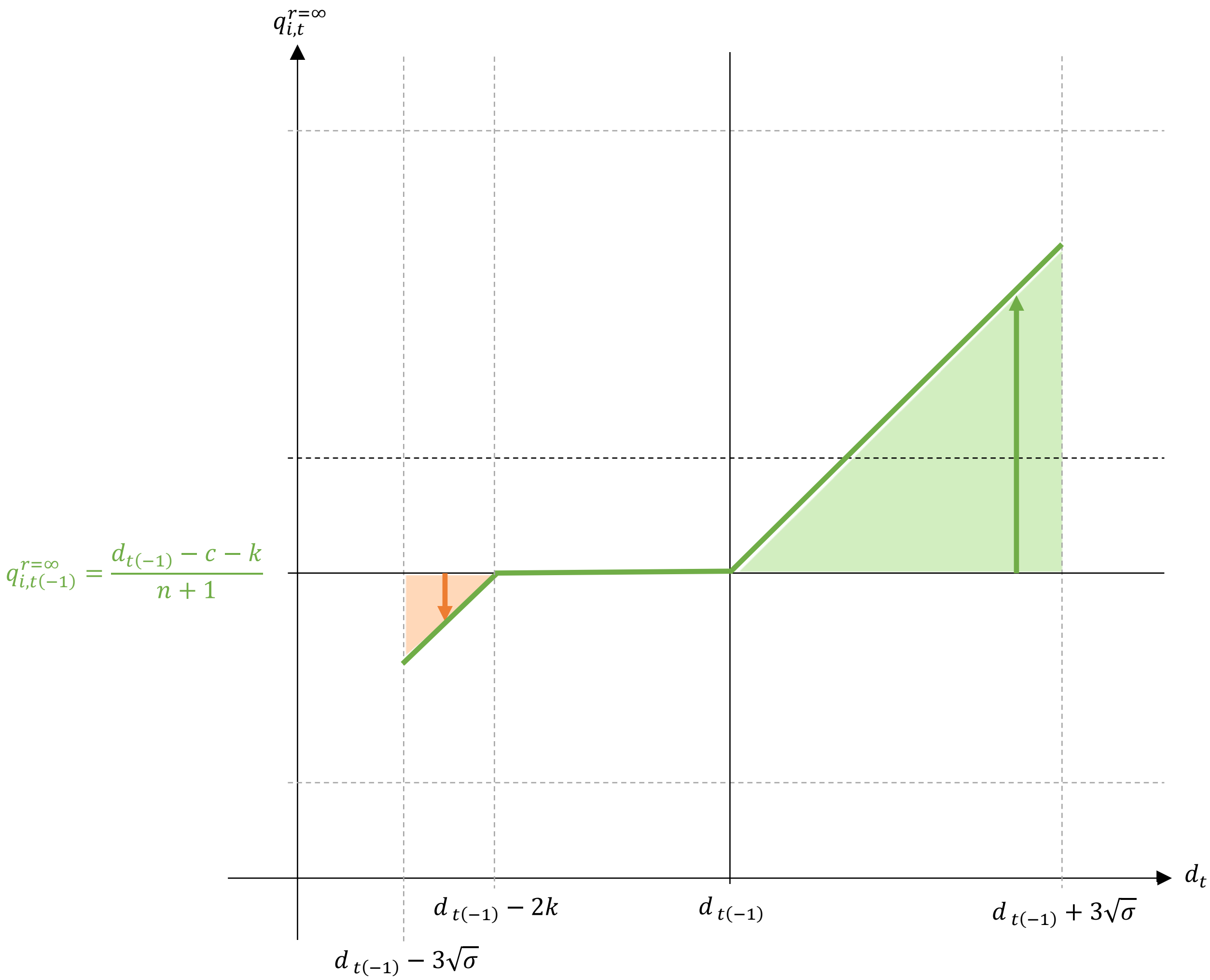

Figure 3a pertains to the case of $r = \infty$ and illustrates the resource adjustment of each firm as a function of the demand shock in period t under the assumption that the most recent resource adjustment was an upward adjustment conducted in period $t(-1)$. The horizontal axis represents the demand d_t in period t, whereas the vertical axis represents the resource quantity $q_{i,t}^{r=\infty}$ of each firm i in period t. The green curve describes the resource quantity $q_{i,t}^{r=\infty}$ of each firm i in period t as a function of the demand d_t in period t. A positive demand shock $d_t - d_{t(-1)} > 0$ in period t causes an upward resource adjustment of $\frac{d_t - d_{t(-1)}}{n+1}$ units, marked by the green arrow, from the prior quantity of $q_{i,t(-1)}^{r=\infty} = \frac{d_{t(-1)} - c - k}{n+1}$ to the current higher quantity of $q_{i,t}^{r=\infty} = \frac{d_t - c - k}{n+1}$. A sufficiently large negative demand shock $d_t - d_{t(-1)} < -2k$ in period t causes a restrained downward resource adjustment of $-\frac{d_t - d_{t(-1)} + 2k}{n+1} = \frac{|d_t - d_{t(-1)}| - 2k}{n+1}$ units, marked by the orange arrow, from the prior quantity of $q_{i,t(-1)}^{r=\infty} = \frac{d_{t(-1)} - c - k}{n+1}$ to the current lower quantity of $q_{i,t}^{r=\infty} = \frac{d_t - c + k}{n+1}$. A negative demand shock of a smaller magnitude does not cause any resource adjustment.

Figure 3b

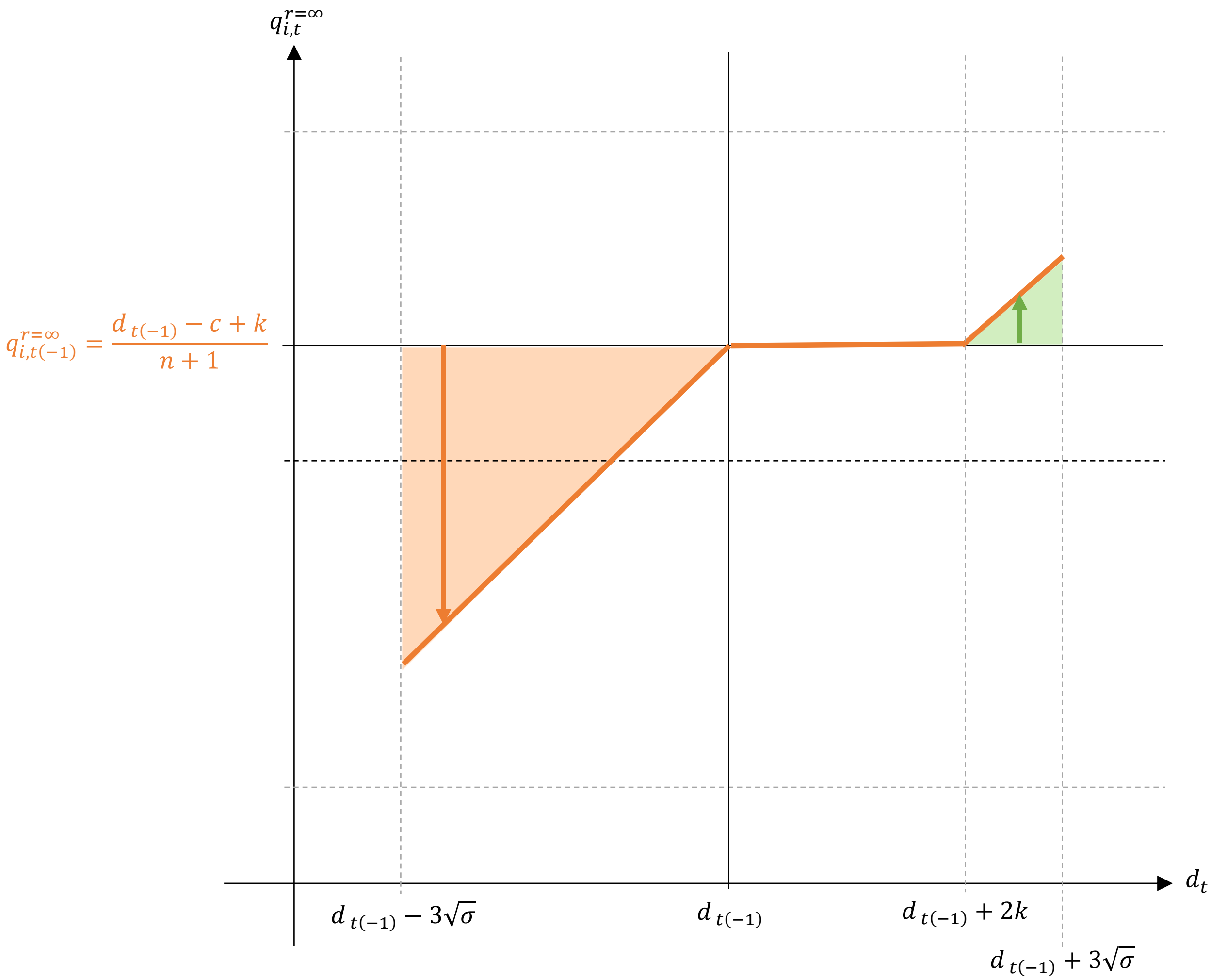

Figure 3b pertains to the case of $r = \infty$ and illustrates the resource adjustment of each firm as a function of the demand shock in period t under the assumption that the most recent resource adjustment was a downward adjustment conducted in period $t(-1)$. The horizontal axis represents the demand d_t in period t, whereas the vertical axis represents the resource quantity $q_{i,t}^{r=\infty}$ of each firm i in period t. The orange curve describes the resource quantity $q_{i,t}^{r=\infty}$ of each firm i in period t as a function of the demand d_t in period t. A negative demand shock $d_t - d_{t(-1)} < 0$ in period t causes a downward resource adjustment of $\frac{|d_t - d_{t(-1)}|}{n+1}$ units, marked by the orange arrow, from the prior quantity of $q_{i,t(-1)}^{r=\infty} = \frac{d_{t(-1)} - c + k}{n+1}$ to the current lower quantity of $q_{i,t}^{r=\infty} = \frac{d_t - c + k}{n+1}$. A sufficiently large positive demand shock $d_t - d_{t(-1)} > 2k$ in period t causes a restrained upward resource adjustment of $\frac{d_t - d_{t(-1)} - 2k}{n+1}$ units, marked by the green arrow, from the prior quantity of $q_{i,t(-1)}^{r=\infty} = \frac{d_{t(-1)} - c + k}{n+1}$ to the current higher quantity of $q_{i,t}^{r=\infty} = \frac{d_t - c - k}{n+1}$. A positive demand shock of a smaller magnitude does not cause any resource adjustment.

Figure 4

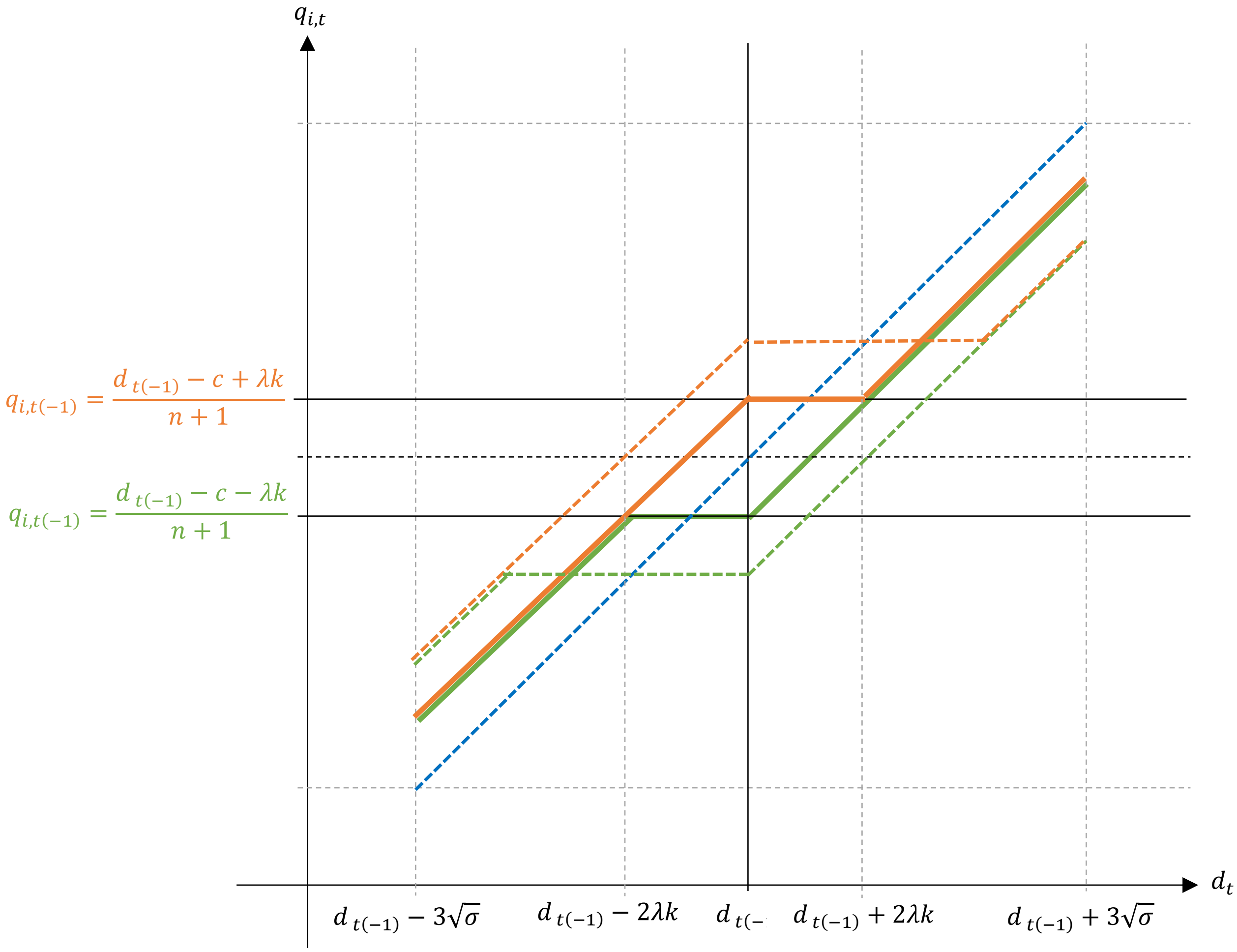

Figure 4 pertains to the unrestricted model and illustrates the resource quantity of each firm as a function of the demand shock in period t, given that the two most recent adjustments took place in periods $t(-2)$ and $t(-1)$. The horizontal axis represents the demand d_t in period t, whereas the vertical axis represents the resource quantity $q_{i,t}$ of each firm i in period t. The green and the orange curves describe resource quantity $q_{i,t}$ of each firm i in period t as a function of the demand d_t in period t, under two scenarios with respect to the prior demand shock $d_{t(-1)} - d_{t(-2)}$. The green curve pertains to the scenario where $d_{t(-1)} - d_{t(-2)} > 0$. The orange curve pertains to the scenario where $d_{t(-1)} - d_{t(-2)} < 0$. The dashed green and the orange curves are the corresponding curves in the case of $r = \infty$. The blue diagonal dashed line pertains to the benchmark case of $k = 0$ and describes the benchmark resource quantity $q_{i,t}^{k=0} = \frac{d_t - c}{n+1}$ of each firm i in period t as a function of the demand d_t in period t.

Figure 5a

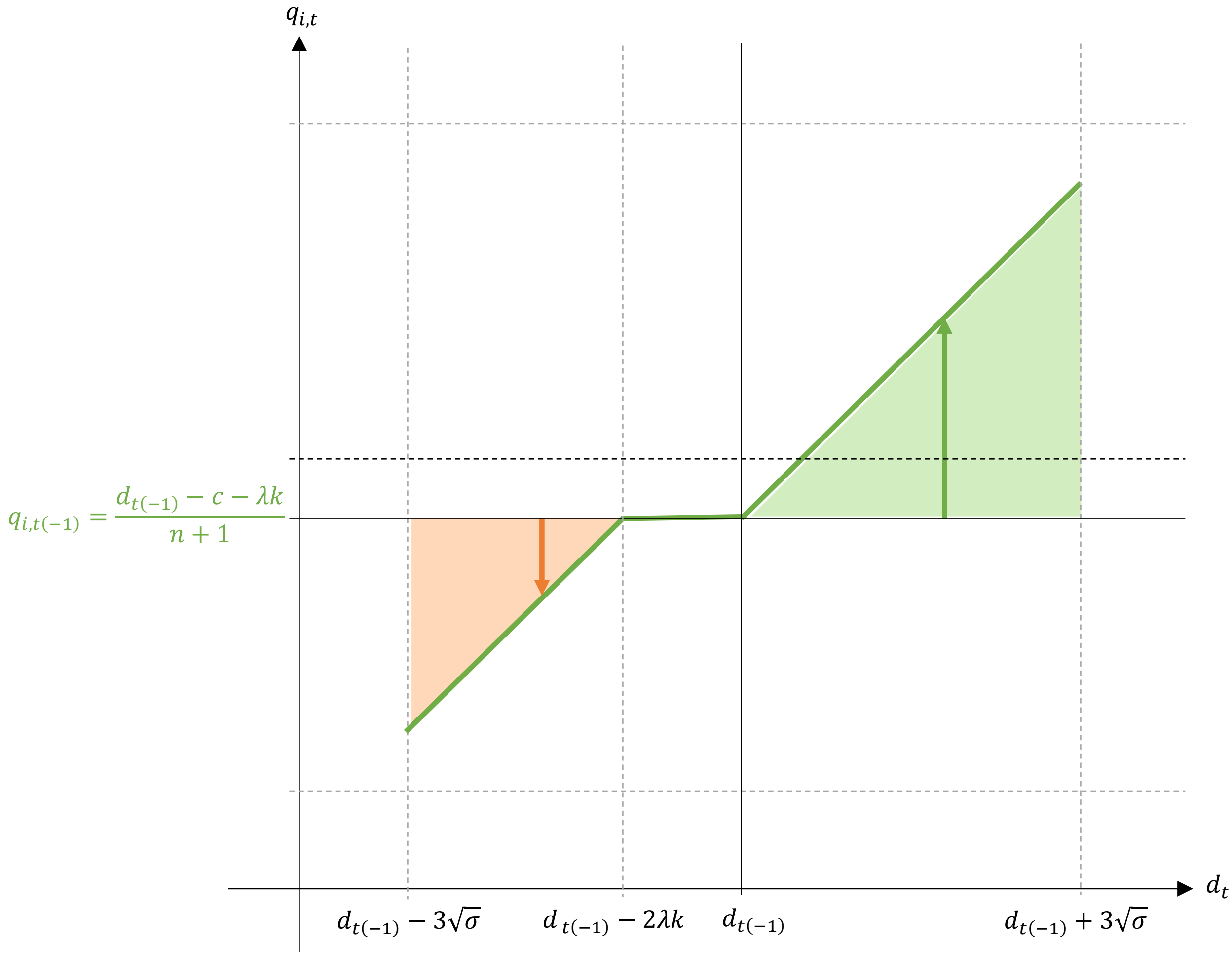

Figure 5a pertains to the unrestricted model and illustrates the resource adjustment of each firm as a function of the demand shock in period t under the assumption that the most recent resource adjustment was an upward adjustment conducted in period $t(-1)$. The horizontal axis represents the demand d_t in period t, whereas the vertical axis represents the production quantity $q_{i,t}$ of each firm i in period t. The green curve describes the resource quantity $q_{i,t}$ of each firm i in period t as a function of the demand d_t in period t. A positive demand shock $d_t - d_{t(-1)} > 0$ in period t causes an upward resource adjustment of $\frac{d_t - d_{t(-1)}}{n+1}$ units, marked by the green arrow, from the prior quantity of $q_{i,t(-1)} = \frac{d_{t(-1)} - c - \lambda k}{n+1}$ to the current higher quantity of $q_{i,t} = \frac{d_t - c - \lambda k}{n+1}$. A sufficiently large negative demand shock $d_t - d_{t(-1)} < 2\lambda k$ in period t causes a restrained downward resource adjustment of $-\frac{d_t - d_{t(-1)} + 2\lambda k}{n+1} = \frac{|d_t - d_{t(-1)}| - 2\lambda k}{n+1}$ units, marked by the orange arrow, from the prior quantity of $q_{i,t(-1)} = \frac{d_{t(-1)} - c - \lambda k}{n+1}$ to the current lower quantity of $q_{i,t} = \frac{d_t - c + \lambda k}{n+1}$. A negative demand shock of a smaller magnitude does not cause any resource adjustment.

Figure 5b

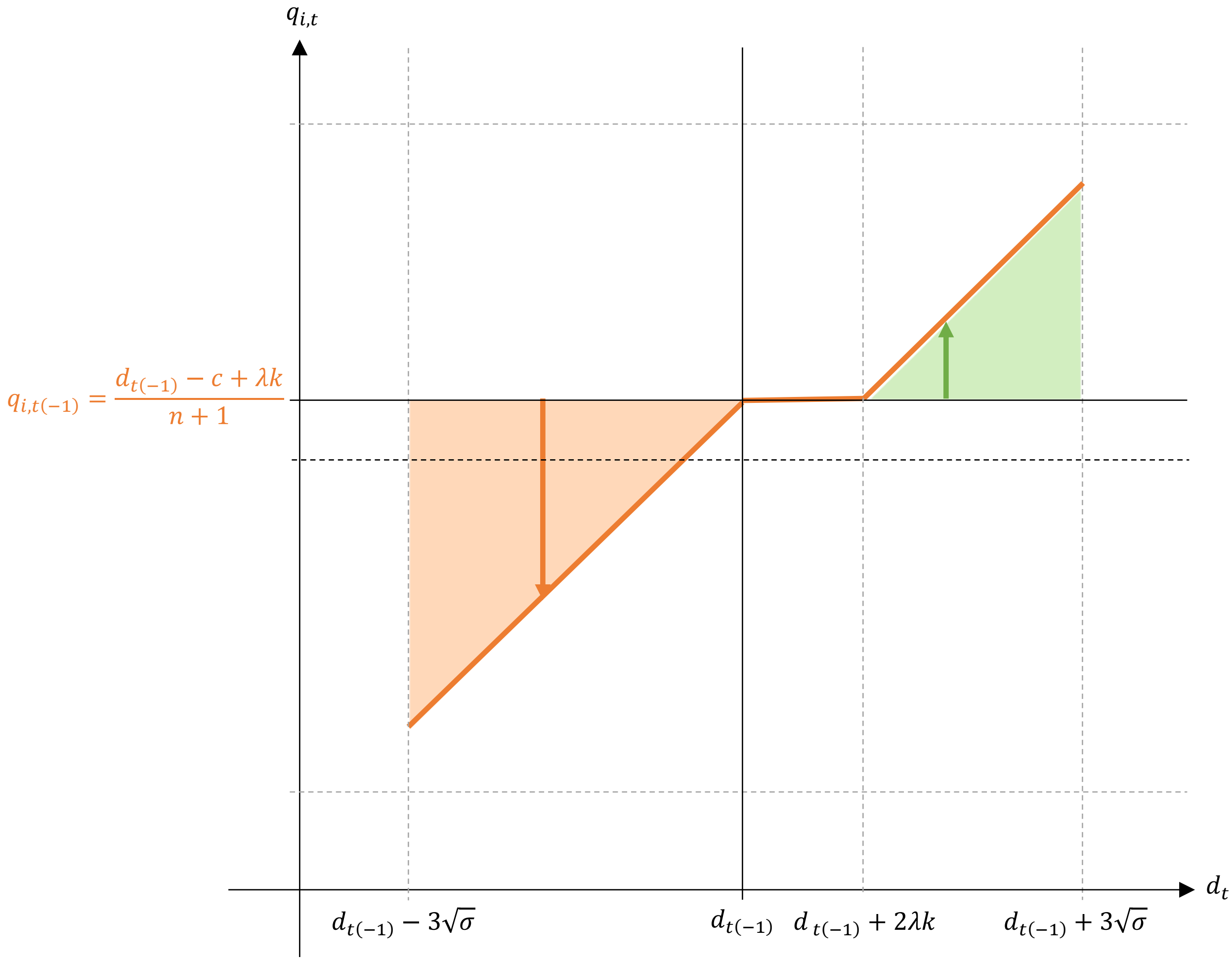

Figure 5b pertains to the unrestricted model and illustrates the resource adjustment of each firm as a function of the demand shock in period t under the assumption that the most recent resource adjustment was a downward adjustment conducted in period $t(-1)$. The horizontal axis represents the demand d_t in period t, whereas the vertical axis represents the production quantity $q_{i,t}$ of each firm i in period t. The orange curve describes the production quantity $q_{i,t}$ of each firm i in period t as a function of the demand d_t in period t. A negative demand shock $d_t - d_{t(-1)} < 0$ in period t causes a downward resource adjustment of $\frac{|d_t - d_{t(-1)}|}{n+1}$ units, marked by the orange arrow, from the prior production quantity of $q_{i,t(-1)} = \frac{d_{t(-1)} - c + \lambda k}{n+1}$ to the current lower production quantity of $q_{i,t} = \frac{d_t - c + \lambda k}{n+1}$. A sufficiently large positive demand shock $d_t - d_{t(-1)} > 2\lambda k$ in period t causes a restrained upward resource adjustment of $\frac{d_t - d_{t(-1)} - 2\lambda k}{n+1}$ units, marked by the green arrow, from the prior production quantity of $q_{i,t(-1)} = \frac{d_{t(-1)} - c + \lambda k}{n+1}$ to the current higher production quantity of $q_{i,t} = \frac{d_t - c - \lambda k}{n+1}$. A positive demand shock of a smaller magnitude does not cause any resource adjustment.