Baseband Distortion Modeling for a Parametric Loudspeaker System Using Volterra Kernels

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Abstract—A directional sound beam can be generated by a parametric loudspeaker system through the nonlinear interaction between finite-amplitude ultrasonic waves in air. However, this nonlinear interaction also produces harmonic components in addition to the desired audible sound. In order to investigate this nonlinear phenomenon, a baseband distortion model is developed from nonlinear system identification using Volterra kernels along with results obtained from both numerical simulations and actual measurements that take into account the emitter’s response. A nonlinear model with reduced complexity to the 2nd-order Volterra kernel is found to agree with the mathematical model. Based on this model, we can predict the total harmonic distortion in the far field and perform compensation to remove it.

I. INTRODUCTION

The original prototype of parametric loudspeaker in air was firstly studied experimentally by Bennett and Blackstock in the 1970s [1]. The nonlinear interaction between two finite-amplitude ultrasonic waves, which are referred as primary waves, can give rise to an end-fire array of virtual sources along the propagation path. Thus, a difference frequency sound beam, which retains the high directivity of the ultrasonic waves, is generated. Due to this nonlinear mechanism, an audible sound modulating an ultrasonic carrier at the input of the parametric loudspeaker system can be reproduced at the output of the system. However, the frequency band of the demodulated secondary sound is extended from the original baseband with the generation of harmonics in the self-demodulation process and thus the desired baseband sound is distorted. Considerable studies have been carried out on how to mitigate the distortion brought about by the conventional double-sideband amplitude modulation (DSBAM) with improved modulation techniques, which mainly consist of single-sideband amplitude modulation (SSBAM) [2], square-root amplitude modulation (SRAM) [3], and modified amplitude modulation (MAM) [4]. The advantages and disadvantages of these modulation techniques have been studied theoretically and experimentally in [5] with a conventional piezoelectric ultrasonic emitter array.

The Volterra filter is widely used in modeling the nonlinear mechanism of a nonlinear system, rendering the system input-output equation as a polynomial series. For example, the nonlinear process in horns and ducts has been studied by Klippel using a Volterra series expansion up to the nth-order kernels [6]. The nth-order Volterra filter has the advantage of modeling the nonlinear system with a straightforward filter structure, which is capable of approximating the nonlinear system with finite number of coefficients [7]. With reference to the previous theoretical study of parametric loudspeaker systems, the distortion is largely attributed to the second harmonic in the demodulated signal, especially when the DSBAM technique is used [8]. Therefore, in this paper, a baseband distortion model is developed using an adaptive Volterra filter consisting of the 1st- and 2nd-order kernels, which is able to predict the sound pressures of the reproduced baseband term (fundamental) and the nonlinear quadratic term (second harmonic) in the sound field.

The rest of this paper is organized as follows. Section 2 gives a brief overview of the parametric loudspeaker system. Section 3 describes the adaptive structure and algorithms for computing the coefficients of the 1st- and 2nd-order Volterra kernels. In Section 4, simulations with the Volterra kernels are conducted to match the theoretical output of the parametric loudspeaker system based on a mathematical model. Measurements are carried out to train the adaptive filter for modeling the actual system and the modeling performance is evaluated in Section 5. Finally, Section 6 concludes this paper.

II. PARAMETRIC LOUDSPEAKER SYSTEM

The acoustic parametric effect occurs when parametric loudspeaker systems operate in the nonlinear region of finite-amplitude ultrasounds propagation in air. The consequence of this nonlinear phenomenon is the generation of an audible sound beam retaining the high directivity of the ultrasounds.

Instead of transmitting multiple ultrasonic frequencies to generate audible signals in air, a parametric loudspeaker system is able to reproduce the desired audible sound by modulating an ultrasonic carrier using different modulation techniques at the system input stage. As shown in Fig. 1, the input baseband audible sound modulates the ultrasonic carrier to form a modulated signal, which is channeled by a power amplifier to drive the ultrasonic emitter to operate at a suitable pressure level for achieving the acoustic parametric effect in air. Due to the nonlinear interaction between the primary waves, the desired signal is reproduced from the end-fire virtual source array but suffers from harmonic distortion. Generally, the system output end is referred as a certain listening zone away from the ultrasonic emitter. In this paper,
the nonlinear distortion present in the reproduced baseband signal is examined for the DSBAM technique and the output end is observed at the Rayleigh distance on the propagation axis.

III. ADAPTIVE VOLterra FILTER

The polynomial-series-based Volterra filter is very popular in modeling a causal and time-invariant nonlinear system with the following series expansion [9]

\[ y(n) = h_0 + \sum_{i=1}^{\infty} \sum_{m_1=0}^{m_1(i)} \cdots \sum_{m_i=0}^{m_i(i)} h_i(m_1, m_2, \ldots, m_i) \cdot \prod_{j=1}^{i} x(n - m_j) \]

where \( x(n) \) and \( y(n) \) represent the input and output signals, respectively. \( h_i(m_1, m_2, \ldots, m_i) \) is defined as the \( i \)th-order Volterra kernel, and \( h_0 \) can generally be ignored [9]. In this paper, we focus on using the Volterra filter to model the linear and quadratic nonlinear output of the parametric loudspeaker system. Thus, a truncated series expansion up to the 2nd-order kernel is utilized and can be expressed as

\[ y(n) = \sum_{m_1=0}^{N_1} h_1(m_1) x(n - m_1) + \sum_{m_1=0}^{N_1} \sum_{m_2=0}^{N_2} h_2(m_1, m_2) x(n - m_1) x(n - m_2), \]

where \( N_1 \) and \( N_2 \) are the memory lengths of the kernels \( H_1[n] \) and \( H_2[n] \), respectively. The Volterra system identification process is implemented by using a cascaded adaptive structure combining the 1st- and 2nd-order kernels, as illustrated in Fig. 2.

In Fig.2, the parametric loudspeaker system takes the input signal \( x(n) \) as the modulating signal. Through the nonlinear interaction taking place the primary waves transmitted by the ultrasonic emitter, the self-demodulated secondary sound \( d(n) \) is generated, consisting of the reproduced baseband signal and higher-order harmonics. Adaptations for the 1st- and 2nd-order kernels are carried out simultaneously during the identification process. The 1st-order kernel serves to model the linear component in \( d(n) \) with error \( e_1(n) \) feeding back to adapt the 1st-order kernel. In addition, the residual signal \( e_2(n) \) is also sent to the 2nd-order kernel in the second-stage adaptation, in which the coefficients \( h_2(m_1, m_2) \) are used to model the quadratic component present in \( e_2(n) \) by feeding back error signal \( e_2(n) \) to update \( H_2[n] \).

In our proposed structure, the normalized least-mean-square (NLMS) algorithm is employed as the adaptive algorithm, which is based on the steepest decent approach to minimize the squared error signal at each iteration. With the cascaded adaptive structure, the coefficients of the 1st- and 2nd-order kernels are adapted using the NLMS algorithm as follows

\[ h_1(i; n + 1) = h_1(i; n) + \mu_1(n)e_1(n)x(n - i), \quad i = 0, 1, \ldots, N_1 - 1, \]

\[ h_2(j, k; n + 1) = h_2(j, k; n) + \mu_2(n)e_2(n)(x(n - j)x(n - k)), \quad j, k = 0, 1, \ldots, N_1 - 1, \]

where

\[ \mu_1(n) = \frac{\mu_0}{X_1(n)X_1(n)^T}, \]

\[ X_1(n) = [x(n) \ldots x(n - i) \ldots x(n - N_1 + 1)]^T \]

and

\[ \mu_2(n) = \frac{\mu_0}{X_2(n)X_2(n)^T}, \]

\[ X_2(n) = [x^2(n) \ldots x(n - j)x(n - k) \ldots x^2(n - N_1 + 1)]^T \]

are the time-varying step size and input vector for the 1st- and 2nd-order kernels, respectively. \( \mu_0 \) is the auxiliary step size, being in the range of \( 0 < \mu_0 < 2 \).
IV. NUMERICAL SIMULATION

It is well known that the Khokhlov–Zabolotskaya–Kuznetsov (KZK) nonlinear parabolic wave equation [10], which accounts for the combined effects of diffraction, absorption and nonlinearity, is capable of giving an accurate description of the propagation characteristics of finite-amplitude sound beams. For axisymmetric sound beams propagating in the z direction, the KZK equation can be written as

\[
\frac{\partial^2 p}{\partial z \partial \tau} = \frac{c_n}{2} \left( \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right) + \frac{D}{2c_n} \frac{\partial^3 p}{\partial \tau^3} + \frac{\beta}{2 \rho_c c_n} \frac{\partial^5 p^2}{\partial \tau^5},
\]

(9)

where \( p \) is the sound pressure, \( \tau = t - z / c_n \) is the retarded time, \( r \) is the radial distance from \( z \) axis, \( c_n \) is the small signal sound speed, \( \rho_c \) is the ambient density of the medium, \( D \) is the sound diffusivity and \( \beta \) is the nonlinear coefficient.

As there is no explicit analytic solution of the KZK equation, a numerical solution in time domain is developed by Lee et al. on the basis of a transformed expression of (10) as [11]

\[
\frac{\partial P}{\partial \sigma} = \frac{1}{4(1 + \sigma)^2} \int_{-\infty}^{\infty} \left( \frac{\partial^2 P}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial P}{\partial \rho} \right) d\rho + A \frac{\partial^2 P}{\partial \tau^2} + \frac{NP}{1 + \sigma} \frac{\partial P}{\partial \tau},
\]

(10)

For simplicity, we only focus on two important parameters \( A \) and \( N \), the detailed variable transformations can be referred to [11]. \( A = \alpha_0 \zeta_0 \) is the absorption parameter and \( N = \zeta_0 / \zeta \) is the nonlinearity parameter, where \( \alpha_0 \) is the attenuation coefficient, \( \zeta_0 \) is the Rayleigh distance, and \( \zeta \) represents the plane wave shock formation distance.

Based on the numerical solution of (10), simulation is conducted to model the linear and quadratic nonlinear output of the parametric loudspeaker system with band-limited white noise as the system input \( x(n) \). The frequency of the white noise input ranges from 0 to 5 kHz and the amplitude is uniformly distributed between -1 to 1. A 40 kHz ultrasonic carrier is modulated by the input with \( m = 0.8 \) to form the DSBAM primary waves. The desired signal \( d(n) \) is computed numerically from the KZK equation for the adaptation process shown in Fig. 2, in which the memory length of the 1st- and 2nd-order kernels is 100 and the auxiliary step size \( \mu_k \) is 0.5. It is observed in Fig. 3 that the mean-square values of \( e_1(n) \) and \( e_2(n) \) after 3.2x10^5 iterations reduce steadily down to around -13 dB and -18 dB, respectively.

The steady-state coefficients of the 2nd-order kernel are plotted in Fig. 4. We notice that the dominant coefficient for the 2nd-order kernel is located at \( h_2(0,0) \) while the other coefficients have only minimal values. This observation implies that the quadratic nonlinear effect in the parametric loudspeaker system is largely memoryless and can be approximated by using a square function.

V. EXPERIMENT

In this section, the parametric loudspeaker system is modeled with the previous Volterra filter by using the desired signal from actual measurements that take into account of the emitter’s response, and the Volterra modeling performance is evaluated based on the sound pressure level (SPL) and the total harmonic distortion (THD) measured in the experiments.

A. Nonlinear Modeling

Measurements were carried out for the actual secondary sound in an anechoic chamber with dimensions of 6 m long, 4 m wide, and 3.5 m high. Fig. 5 shows the block diagram of the experimental setup: A floating-point DSP platform implements the DSBAM signal and sends the modulated signal to a class-D power amplifier with low phase shift and output noise. An ultrasonic emitter consisting of 253 piezoelectric transducer units, which resonates at 40 kHz, is driven at a suitable voltage level to achieve the nonlinear effect in air. The demodulated secondary sound is captured by a B&K Type 4134 1/2-inch microphone attached to a Type 3110 module for a duration of 5 seconds per time capture at an axial distance of 4 m away from the emitter source. The captured signal is lowpass filtered at 20 kHz cut-off frequency to remove any ultrasonic signal, before computing errors for the adaptive Volterra cascade structure.

In the experiments, input white noise with the same characteristics as the one discussed in the previous section is fed to the parametric loudspeaker system to train the adaptive
Volterra filter for modeling the nonlinear system. With the adaptive structure shown in Fig. 2 and the same adaptive parameters for the 1st- and 2nd-order kernels as those in the simulation, the nonlinear model is trained for 1x10^5 iterations. In Fig. 6, we can see that the mean square values of \( e_i(n) \) and \( e_2(n) \) are stabilized at around −12 dB and −15 dB, respectively. Fig. 7 shows the final coefficients of the 2nd-order kernel using \( d(n) \) from the actual measurements that take the emitter’s response into account. It is noted that the surface formed by the coefficients of the 2nd-order kernel appears to agree well with that obtained in the simulation, which indicates that the quadratic nonlinearity largely resides on \( h_2(0,0) \) and the frequency response of the ultrasonic emitter does not affect the location of the dominant coefficient in the nonlinear modeling.

### B. Modeling Performance

In this subsection, the performance of the nonlinear system modeling is examined by passing five single tones from 1 kHz to 5 kHz through the parametric loudspeaker system and the Volterra filter with the steady-state coefficients obtained from the previous section. SPL and THD are calculated and compared between the Volterra filter output using the final coefficients from the model training and the secondary sound from the actual measurement, respectively.

Fig. 8 shows the SPL of the reproduced baseband signal and the second harmonic from the measurement and the Volterra modeling, respectively. It is seen that the modeling system can match the measurement results closely with an error of less than 1 dB.

THD defines the amount of distortion that exists in the secondary sound, which is given as

\[
\text{THD} = \frac{T_2^2 + T_3^2 + \cdots + T_n^2}{T_1^2 + T_2^2 + T_3^2 + \cdots + T_n^2} \times 100\% , \tag{11}
\]
where $T_i$ represents the amplitude of the $i$-th harmonic in the secondary sound. The THD level can be approximated by measuring up to the 2nd harmonic as

$$\text{THD}_{\text{approx}} = \frac{T_2}{\sqrt{T_1^2 + T_2^2}} \times 100\%. \quad (12)$$

Fig. 9 shows the THD calculated from the measurements based on (11) and (12), as well as from the Volterra modeling up to 2nd-order kernel. It can be found that the measured THD level calculated from (2) is almost identical to that from (1), which verifies that the baseband distortion is largely attributed to the contribution of the second harmonic. In addition, the THD predicted by the Volterra modeling can match the measured THD with only slight errors. It should be noted that the narrow bandwidth of the ultrasonic emitter makes the frequency response in Fig. 8 become flatter than the 12 dB/octave slope predicted by the mathematical model, and the THD level in Fig. 9 decrease as the modulating frequency increases.

VI. CONCLUSIONS

In this paper, we have developed a baseband distortion model based on the 1st- and 2nd-order Volterra kernels for studying the nonlinear effect in the demodulated secondary sound of a parametric loudspeaker system. The well-known NLMS algorithm was used in the nonlinear system identification with a cascaded adaptive Volterra filter structure. It has been found that the Volterra kernels are suitable in modeling the nonlinear distortion present in the reproduced baseband signal from the numerical simulations and actual measurements. The comparison between the two sets of kernel coefficients derived from the theoretical equation and experimental measurements shows good agreement and indicates that the quadratic nonlinear term is largely memoryless and can be approximated by a square function. The effectiveness of using the Volterra model has also been shown to produce good agreement between modeling prediction and measurement results in terms of SPL and THD. Therefore, compensation techniques based on the Volterra model can be more efficiently implemented to reduce the overall distortion for practical use of parametric loudspeaker systems.

REFERENCES


