

Origami-Inspired Active Structures: A Synthesis and Review

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Abstract. Origami, the ancient art of paper folding, has inspired the design of engineering devices and structures for decades. The underlying principles of origami are very general, which has led to applications ranging from cardboard containers to deployable space structures. More recently, researchers have become interested in the use of active materials (i.e., those that convert various forms of energy into mechanical work) to effect the desired folding behavior. When used in a suitable geometry, active materials allow engineers to create *self-folding* structures. Such structures are capable of performing folding and/or unfolding operations without being kinematically manipulated by external forces or moments. This is advantageous for many applications, including space systems, underwater robotics, small scale devices, and self-assembling systems. This article is a survey and analysis of prior work on active self-folding structures as well as methods and tools available for the design of folding structures in general and self-folding structures in particular. The goal is to provide researchers and practitioners with a systematic view of the state-of-the-art in this important and evolving area. Unifying structural principles for active self-folding structures are identified and used as a basis for a quantitative and qualitative comparison of numerous classes of active materials. Design considerations specific to folded structures are examined, including the issues of crease pattern identification and fold kinematics. Although few tools have been created with active materials in mind, many of them are useful in the overall design process for active self-folding structures. Finally, the article concludes with a discussion of open questions for the field of origami-inspired engineering.

Keywords: Origami, Active Materials, Origami Engineering, Morphing Structures, Smart Structures

1. Introduction

Traditionally, the term “origami” has been associated primarily with the art of folding paper. The term origami has the Japanese roots “*ori*” meaning “folded”, and “*kami*” meaning “paper” [1]. Its original purpose was not particularly utilitarian, but rather recreational and artistic [2]. Specifically, it was and remains the art of folding uncut sheets of paper into decorative and well-defined shapes, either abstract in form or representative of realistic objects (e.g., plants, animals). In origami, a goal shape is obtained from an initially planar sheet exclusively through folding operations. For an idealized sheet with no thickness, a *fold* is defined as any deformation of the sheet such that the in-surface distance between any two points in the sheet is preserved and self-intersection does not result [3]. Stretching and tearing are not permitted; bending is the essential deformation [3]. For a sheet with non-zero thickness, a *fold* is defined as any deformation of the sheet that preserves a continuous neutral surface (i.e., a surface that neither stretches nor contracts) and prevents self-intersection.

In the mid-1970s, mathematicians discovered that an endless number of shapes could in theory be created using traditional origami (initially planar shape, only *folds* allowed) [1]. These discoveries enabled new approaches for manufacturing, assembling, and morphing of devices and structures based on origami principles. This is evident in the increasing attention mathematicians, scientists, and engineers have given to origami theories and tools over the past four decades [1, 4, 5, 6].

Origami offers engineers novel ways to fabricate, assemble, store, and morph structures. Potential advantages include the capability to compactly store deployable structures (e.g., airbags [7, 8]), the potential for structures to be reconfigurable [9, 10, 11, 12], and a reduction in manufacturing complexity [13] (reduced part counts and improved assembly using collapsible/deployable parts). Furthermore, origami has been demonstrated to be applicable across scales via its applications ranging from DNA approaches at the nano-scale [14, 15, 16, 17] to deployable structures for space exploration at the very large scale [18, 19, 20, 21]. Other current applications of origami-inspired engineering include: micro-mirrors [22], various space structures (solar panels, solar sail, telescope lenses [23]), batteries and capacitors with improved properties [24, 25, 26, 27], robots [28, 29, 30], foldable wings and airplanes [31, 32, 33, 34, 35, 36], foldcore-based structures for improved mechanical properties and impact resistance [37, 38, 39, 40, 41, 42, 43], crash boxes and other energy absorption systems [44, 45, 46, 47], shelters [48, 49, 50], metamaterials [51, 52], microelectromechanical systems (MEMS) [53, 54, 55, 56, 57], biomedical devices [58, 59, 60, 61, 62, 63], and many others [64]. A representative cross-section of these engineering applications is shown in figure 1.

For certain origami-inspired engineering applications, it is impractical to externally execute the manipulations necessary to produce the folding operations. This is the case at very small or very large scales or in remote applications (e.g., underwater robotics, space structures, invasive biomedical devices). In these circumstances, *self-*

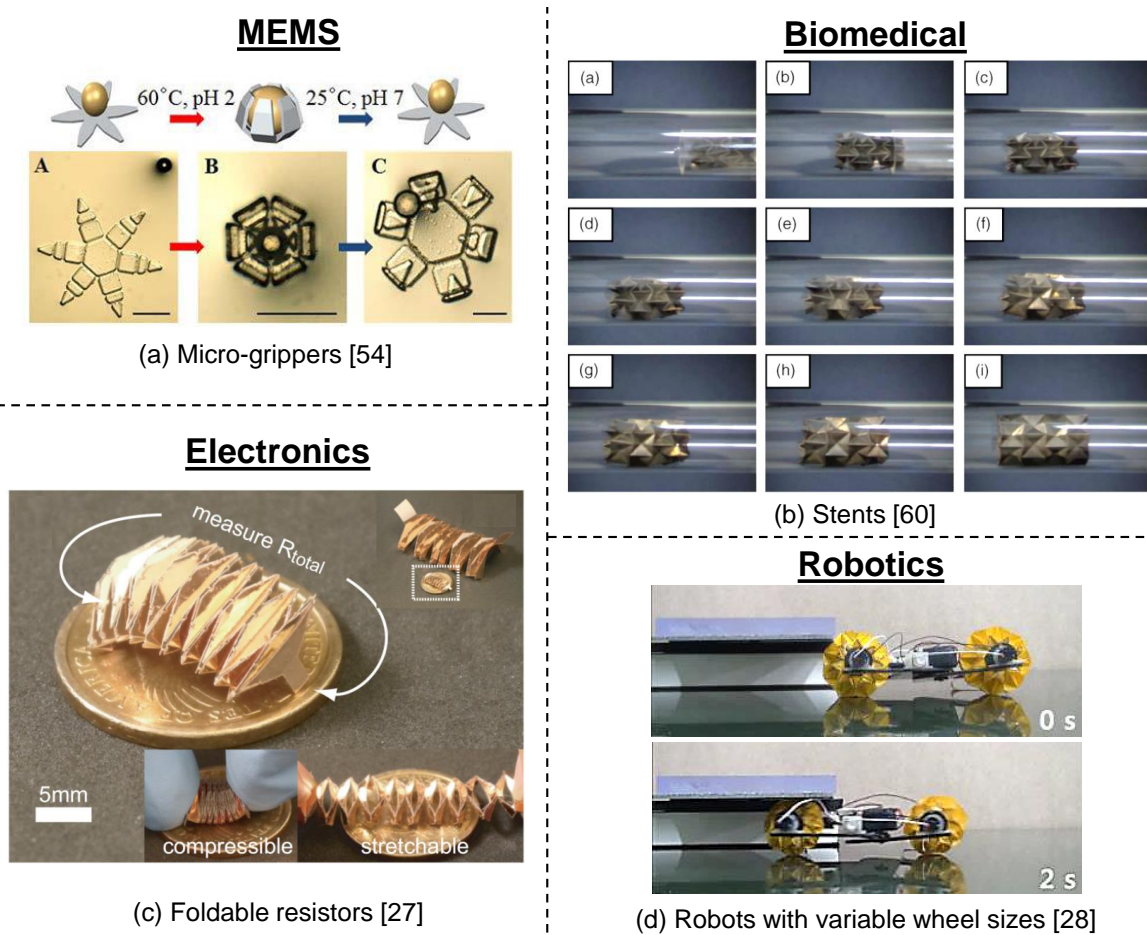


Figure 1. Examples of current and potential applications of origami-inspired structures with and without self-folding capability. (b) Reprinted from [60], Copyright (2006), with permission from Elsevier. (d) ([28] Reproduced by permission of the American Society of Mechanical Engineers ASME).

folding capabilities are essential. A *self-folding* structure is one that has the capability of folding and/or unfolding to and/or from a desired shape without external manipulations. One approach for the development of self-folding systems is to leverage the use of active materials agents of fold generation. *Active materials* are materials that convert various forms of energy into mechanical work [65]. This coupling can be categorized as *direct* (mechanical response due to field-induced eigenstrains in the active material) or *indirect* (mechanical response due to field-induced change in stiffness or other properties). In either case, active materials allow the mechanical work associated with the folding operations to be obtained via the application of a non-mechanical field (e.g., thermal, electrical, optical, chemical, etc.).

Certain basic origami concepts should be defined prior to the discussion of active self-folding structures in the subsequent sections. In origami, the locations of localized folds on the sheet are formally called *creases* [66, 67]. The creases, folding directions, folding magnitude, and folding sequence determine the ultimate shape of the structure.

Typically, creases are defined by their endpoints, formally called *vertices*. Sheet regions bounded by the creases are known as *faces*. To determine the fold direction of a crease, a “mountain-valley” assignment is typically used. For *mountain* folds, faces on either side of the crease can be thought of as rotating into the page, while for *valley* folds, they can be thought of as rotating out of it. A *crease pattern* is a schematic that shows all the creases on a sheet required to fold a structure, typically with mountain-valley assignments. These concepts are depicted figure 2.

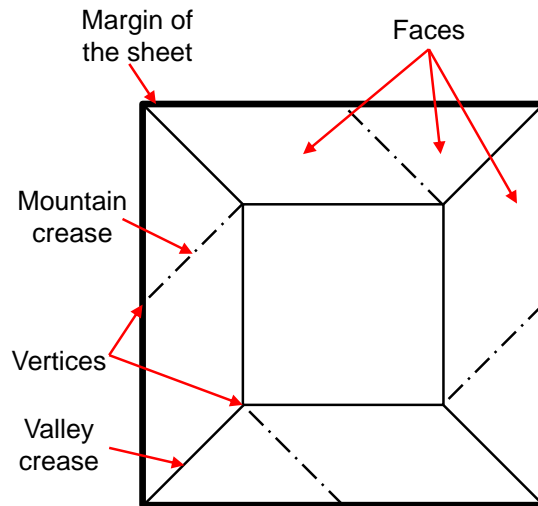


Figure 2. Schematic of a pinwheel crease pattern illustrating various origami concepts.

Two parameters that describe the magnitude of a fold are the folding angle and the radius of curvature at the fold line. These parameters are shown schematically in figure 3. In the view that a finite thickness sheet cannot provide sharp folds and for the purposes of discussion, it is assumed that a finite region centered at the fold is bent and has a radius of curvature R . The internal fold angle θ_i is the angle at the intersection of two line segments stretching collinearly with respect to the folding faces. The external fold angle θ_e is defined as $180^\circ - \theta_i$.

The objectives of this paper are to provide a survey and classification of current active self-folding structural approaches and design tools that might be applied to them, as well as to provide open questions and challenges in the field of origami-inspired engineering with active materials. Figure 4 is a schematic of the process for developing origami-inspired active structures and how it relates to the organization of this paper. Specifically, the remainder of this paper is organized as follows: section 2 presents a description of active materials and describes how they might be used for self-folding structures, section 3 presents examples of active self-folding structures, section 4 presents a review of design tools for folding structures, and section 5 presents the conclusions and open questions for the field of origami-inspired engineering.

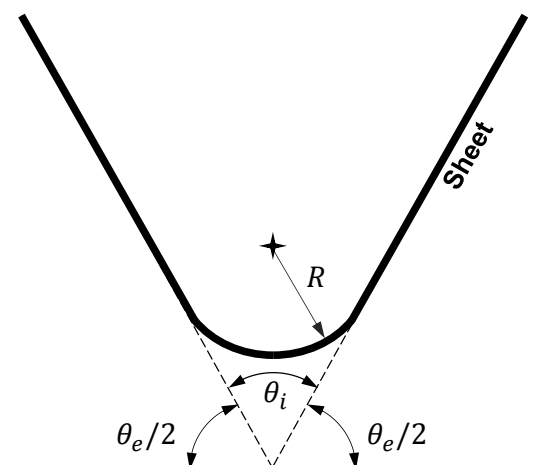


Figure 3. Parameters that define the magnitude of a fold.

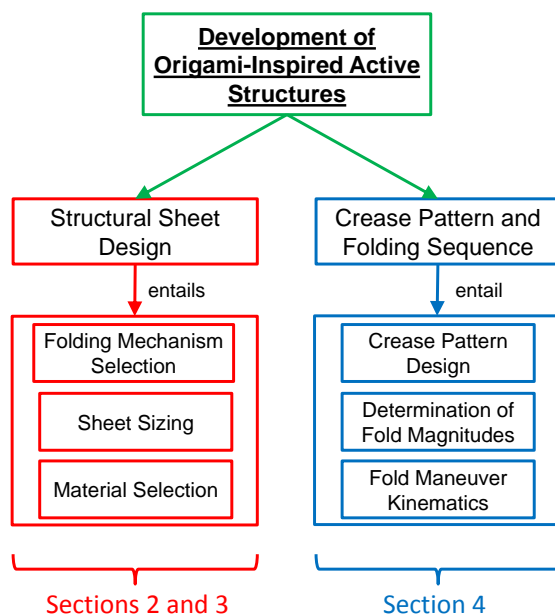


Figure 4. Process for developing origami-inspired active structures as it relates to the organization of this work.

2. Self-Folding

Self-folding is the capability of a structure to fold and/or unfold without the application of external manipulations. As mentioned in the preceding section, active materials enable for self-folding systems since they inherently convert other forms of energy into mechanical work allowing folding and, in some cases, unfolding operations. Active materials are also typically energy dense and geometrically simple when used as actuators [68, 69, 70]. Compactness is an important design consideration for self-folding systems since flat reference configurations are often essential.

2.1. Single Fold Concepts

Concepts for generating individual folds using active materials are provided here while particular examples follow in section 3. The concepts are divided into two categories: hinge type and bending type. Most hinge-type active folds are associated with one of three local actuator concepts: i) variable length active rod or spring connected to the two faces joined by the hinge (figure 5a), ii) active torsional element at the hinge (figure 5b), and iii) active element with preset folded shape (figure 5c). Throughout this paper these fold concepts will be referred to as “extensional”, “torsional”, and “flexural”, respectively.

The extensional concept (figure 5a) uses the active material in a rod or spring form with its two ends attached to the faces connected by the hinge and the length of the active element controls the rotation of the hinge. The torsional concept (figure 5b) uses the active material as a torsional spring or a rod that provides twist at the hinge. The twist angle of the active material thus directly controls the rotation of the hinge. In the flexural concept (figure 5c), the active material has been manufactured or trained to have a preset folded configuration but is then deformed to an initially flat configuration. Upon application of the activation field, the active material itself returns to its preset folded configuration and being bonded to the faces of the passive material, induces the local hinge to do the same.

All concepts can be further improved to allow for folding in both directions relative to the sheet normal by adding corresponding antagonistic active components. In the torsional concept, this can be achieved by installing two active elements that generate twist in opposite directions (e.g., [71]). For the extensional and flexural concepts, it can be achieved by pairs of active elements in opposition to each other (e.g., on opposite sides of the sheet for the extensional concept). However, mechanical restrictions may arise since activation of one active element may subject its associated antagonistic active element to excessive stress or deformation, hindering the folding operation and possibly leading to material failure (e.g., plastic deformation, damage). Another challenge is that of maintaining both sufficient geometric offset and localized/targeted driving field imposition such that opposing hinge moments do not result.

Other applications and approaches do not assume the existence of discrete hinge mechanisms but are rather based on direct local sheet bending caused by the actuation of the active material. Such concepts are shown in figures 5d and 5e. Unlike the *hinge type* fold concepts, the direct bending approach (i.e., without hinges) may offer the advantage of massive foldability. In other words, folds can occur at any location or orientation to which the driving field is applied (unless mechanically restricted). In hinge type concepts, however, folds are clearly restricted to structurally pre-determined hinge locations. We will refer to the concept of figure 5d as “multi-layer” and that of figure 5e as “single layer”.

The multi-layer concept considers self-folding using a two-layer laminate with one passive layer and one active layer (figure 5d). A *passive* layer generates negligible

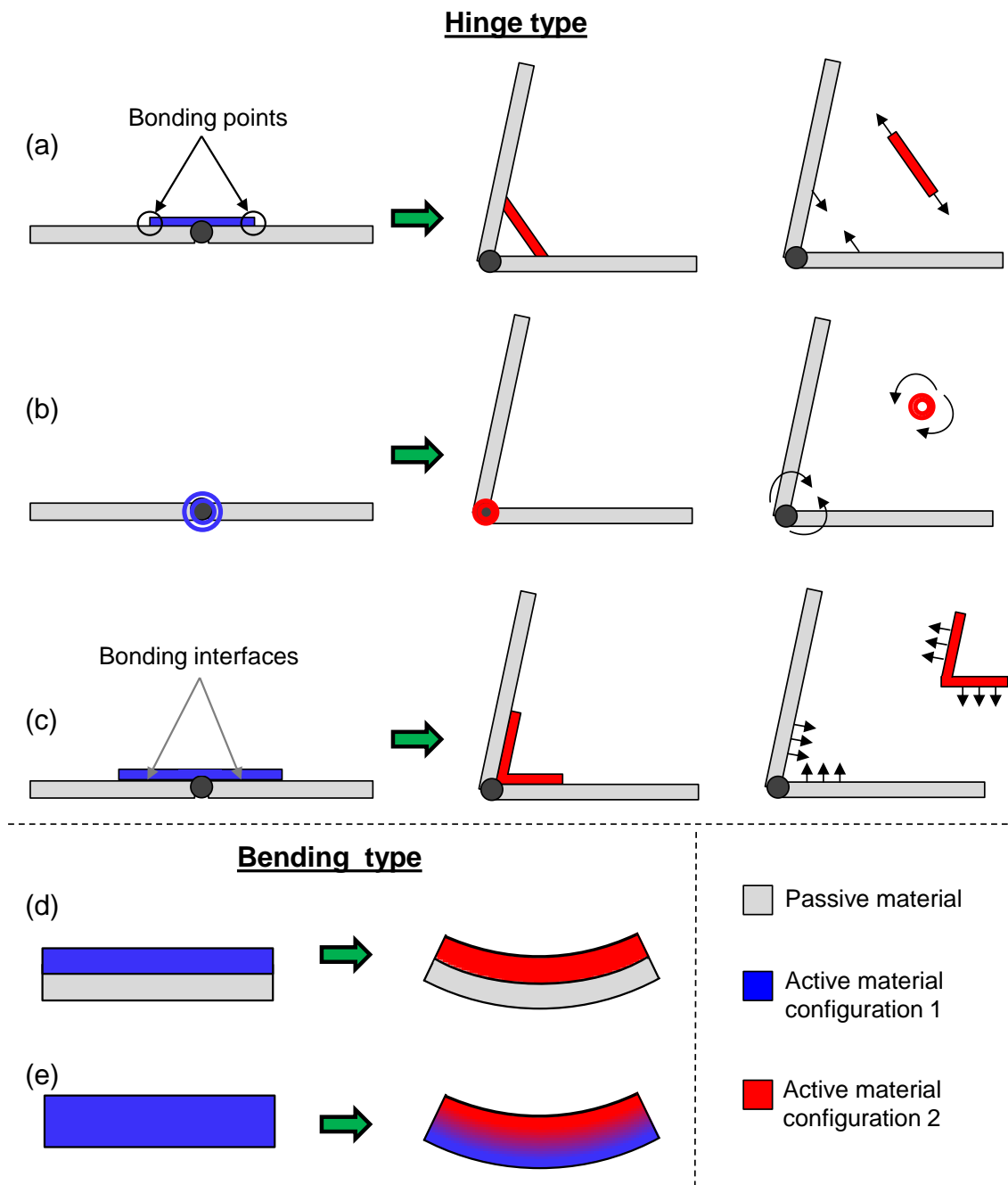


Figure 5. Basic active fold concepts. Hinge type: (a) Extensional (variable length active rod or spring connected to the two faces), (b) Torsional (active torsional element at the hinge), and (c) Flexural (active element with preset folded shape). Individual simplified free body diagrams of the hinge-face structure and the active element are also shown. Bending type: (d) Bilayer consisting of an active and a passive layer, and (e) Single layer subjected to graded driving field.

mechanical work compared to the active layer under the application of the actuation inducing field. When such a field (thermal, magnetic, etc.) is applied, the active layer is driven to deform, generally axially, while the passive layer is not. This difference in

expansion or contraction between the two layers generates localized bending of the sheet. This concept can be further expanded to allow for folds in both directions relative to the sheet normal in a manner similar to that employed for hinge-type folds. Specifically, three-layer designs with two opposing outer layers of active material separated by a passive material can be used. A key challenge for such three-layer designs is to isolate the driving field within only one of the two active outer layers. For this purpose, the middle layer may serve in another role as an insulator with respect to the driving field, preventing the field applied to one active layer from reaching the opposing active layer.

The single layer concept considers self-folding via bending without hinges using a sheet with a single active layer subjected to a graded driving field (figure 5e). Such a gradient generates a distribution of actuation strain through the sheet thickness, causing the sheet to bend. This design allows for folds in both directions relative to the sheet normal based on the direction of the driving field gradient. However, folding via this approach is generally more difficult as compared to the multi-layer concept since it is not practical to maintain gradients in some physical fields (e.g., temperature) at specific locations for a considerable period of time [72].

It should be noted that folding using active materials is not restricted to the five concepts presented in this section. Examples of self-folding systems that use other types of folding concepts (e.g., hinged faces with magnetic patches that are kinematically manipulated by the direction of the magnetic field without requiring deformation of the magnetic patches [73, 74]) are presented in section 3‡.

2.2. Active Materials for Self-folding Systems

When considering self-folding from a flat to a deformed configuration using active materials, several critical design drivers should be considered: actuation strain, actuation stress, and the capability of generating and/or manipulating the desired field at the chosen scale. This section provides a description of field/actuation strain/actuation stress relations.

In order to understand how active self-folding systems are developed, constitutive relations that relate the externally applied fields to the obtained actuation strain are needed. The following expression relates the total strain $\boldsymbol{\varepsilon}$ to different fields [65, 78, 79, 80, 81]§:

$$\begin{aligned} \boldsymbol{\varepsilon} = & \mathbf{S}\boldsymbol{\sigma} + \boldsymbol{\alpha}(T - T_0) + \mathbf{d}^p \mathbf{E} \\ & + \mathbf{d}^m \mathbf{H} + \sum_{i=1}^n \mathbf{e}^i (c^i - c_0^i) + \boldsymbol{\varepsilon}^{ms}, \end{aligned} \quad (1)$$

where \mathbf{S} is the fourth order compliance tensor, $\boldsymbol{\sigma}$ is the second order stress tensor,

‡ Although it is not strictly folding, it should be noted that out-of-plane displacements and changes in Gaussian curvature can be obtained by the application of uniform through thickness but inhomogeneous in-plane strain fields in a sheet [75, 76, 77].

§ This additive decomposition of strain is valid only in the case of small strains. We will limit the conversations of this work to such an assumption.

$\boldsymbol{\alpha}$ is the second order thermal expansion tensor, T is the absolute temperature, T_0 is the reference temperature, \mathbf{d}^p is the third order piezoelectric coefficients tensor, \mathbf{E} is the electric field vector, \mathbf{d}^m is the third order piezomagnetic coefficients tensor, \mathbf{H} is the magnetic field vector, \mathbf{e}^i is the second order tensor of expansion due to concentration of i chemical species, c^i is the concentration of the i chemical species, c_0^i is the reference concentration of the i chemical species, and $\boldsymbol{\varepsilon}^{ms}$ is the second order tensor of strains caused by changes in the material micro- and/or nano-structure. This last contribution ($\boldsymbol{\varepsilon}^{ms}$) may be generated or recovered due to phase transformation [82], variant reorientation [83, 84], change in the crosslinked structure in certain polymers [85, 86], etc. These strains are coupled in certain extent to external variables such as $\boldsymbol{\sigma}$, T , \mathbf{H} , \mathbf{E} , and are often associated with other internal variables (e.g., phase or variant fraction).

The total strain $\boldsymbol{\varepsilon}$ can clearly be separated into two distinct parts: the strain due to elastic deformation $\boldsymbol{\varepsilon}^{el}$ and the strain due to actuation $\boldsymbol{\varepsilon}^{act}$. This separation is given by $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{el} + \boldsymbol{\varepsilon}^{act}$, where the elastic strain is given by $\boldsymbol{\varepsilon}^{el} = \mathbf{S}\boldsymbol{\sigma}$. Taking this into account, one obtains the following expression for the actuation strain:

$$\begin{aligned} \boldsymbol{\varepsilon}^{act} = & \boldsymbol{\alpha}(T - T_0) + \mathbf{d}^p \mathbf{E} + \mathbf{d}^m \mathbf{H} \\ & + \sum_{i=1}^n \mathbf{e}^i (c^i - c_0^i) + \boldsymbol{\varepsilon}^{ms} \end{aligned} \quad (2)$$

It is observed that there are various possible ways to control an active self-folding structure depending on the active material chosen:

- Use direct coupling by applying fields that induce actuation strain without modifying the material micro- and nano-structure (e.g., modify the electric field \mathbf{E} to increase actuation strain due to piezoelectric effect ($\mathbf{d}^p \mathbf{E}$) or modify the temperature to generate thermal expansion or contraction $\boldsymbol{\alpha}(T - T_0)$).
- Use direct coupling by applying fields to modify the micro- and nano-structure of the material (e.g., apply temperature to induce phase transformation in a shape memory alloy (SMA) and generate/recover transformation strains).
- Use indirect coupling and apply fields that modify certain material properties (e.g., use temperature to modify the material compliance \mathbf{S} and alter the elastic strains. Examples of indirect coupling: [87, 88, 89, 90]).

2.2.1. Comparative Single Fold Analysis

To make an assessment of the actuation strain and actuation stress for different active materials, those properties should be mapped into fold-specific quantitative characteristics (e.g., bending or torsional moment, folding angle or radius of curvature at the fold). Analytical expressions for the mechanical analysis of a single fold can be obtained for the torsional and multi-layer concepts (see figure 5) after making certain simplifications. The influence of secondary material regions or structural components (e.g., sensors, connectors) that might be essential in the full physically realized self-folding systems is neglected.

Equations relating actuation strain and actuation stress to various self-folding metrics are presented in table 1 where φ , R , M_t , and M_b are the torsional angle (equal to folding angle θ_e for the torsional concept), radius of curvature, torsional moment, and bending moment, respectively. Those values are directly or inversely proportional to the active material actuation strain ε^{act} and actuation stress σ^{act} . The derivations for the relations associated with a single fold via the torsional concept and multi-layer concept are presented in Appendix A and Appendix B, respectively. The other parameters in the equations shown in table 1 are constants that depend on geometry and/or material properties. The constants r and L correspond to the cross-section radius and axial length of the torsional active element at the hinge, respectively. The constants h , a_1 , and w correspond to the total sheet thickness, the thickness of the active layer, and the width of the sheet in the direction parallel to the fold line, respectively. The dimensionless constants C_1 and C_2 are defined as follows

$$C_1 = \frac{3(1+m)^2 + (1+mn)(m^2 + 1/(mn))}{6(1+m)^2}, \quad (3)$$

$$C_2 = 2 + \frac{6ha_1^2E_1}{E_1a_1^3 + E_2a_2^3}, \quad (4)$$

where m is the ratio of thickness of the active layer to the thickness of the passive layer, n is the ratio of the elastic modulus of the active layer to the passive layer, a_2 is the thickness of the passive layer, and E_1 and E_2 are the elastic modulus of the active and the passive layer, respectively.

Table 1. Relations between actuation stress and actuation strain with different fold metrics.

Fold concept	Actuation strain assessment	Actuation stress assessment
Torsional concept	$\frac{r}{L}\varphi = 2\varepsilon^{act}$	$\frac{M_t}{r^3} = \frac{\pi\sigma^{act}}{2}$
Multi-layer concept	$\frac{R}{C_1h} = \frac{1}{\varepsilon^{act}}$	$\frac{M_bC_2}{wha_1} = \sigma^{act}$

2.2.2. Active Materials Assessment

Figure 6 and figure 7 compare the actuation strain and actuation stress of common active materials [65] for the torsional and multi-layer fold concepts, respectively. The bottom axes of both figure 6 and figure 7 represent the actuation strain (related to the stroke of an actuator) and the left axes represent the actuation stress σ^{act} , assuming here that the active material is operating under constant stress. The top and right axes of figure 6 show metrics that assess the material performance in the torsional concept (according to the results of table 1). The top and right axes of figure 7 show

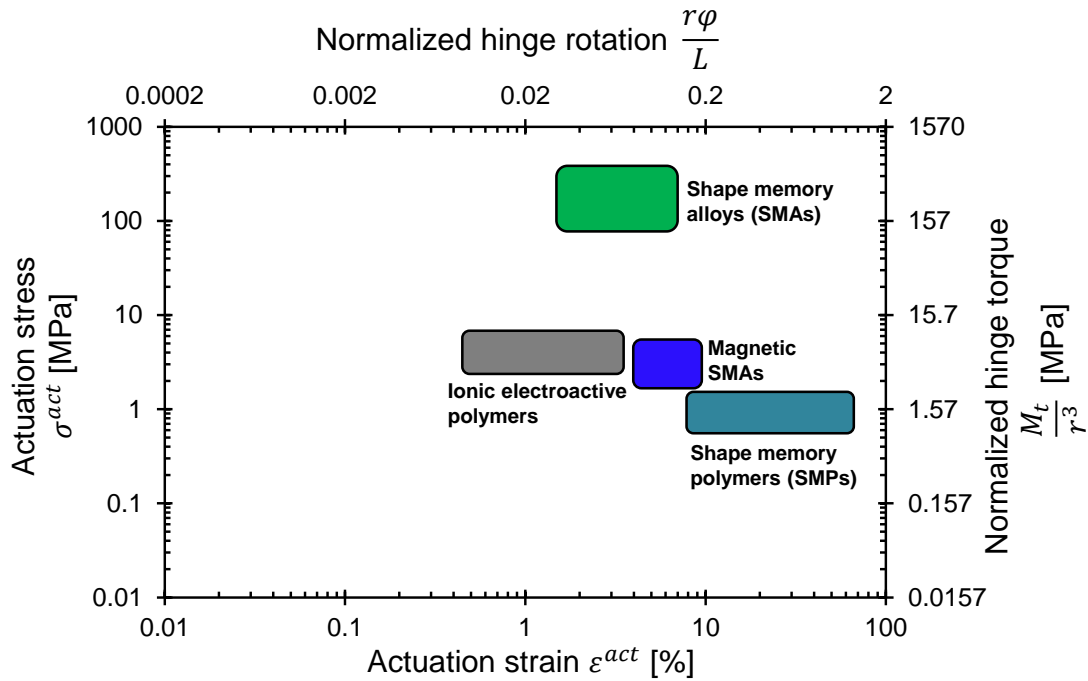


Figure 6. Active materials performance for the torsional fold concept. The diagram shows typical ranges of actuation strain and actuation stress for different common active materials [65]. The top and right axes represent the normalized hinge rotation and the normalized hinge torque, respectively.

quantities that assess the material performance as an active layer in the multi-layer concept (table 1). In this plot, it is observable that ceramic-type active materials (electrostrictive ceramics, magnetostrictive ceramics, and piezoelectric ceramics) may not be suitable for self-folding systems because they would provide bending radii that are orders of magnitude below other common active materials, though they may do so at much higher frequencies [91, 92]. Often, actuation stress limitations may be reduced by the usage of compliant mechanisms [93, 94, 95, 96, 97, 98]. For instance, shape memory polymers (SMPs) [85, 99, 100], which are shown in figure 6 and figure 7 as having the lowest actuation stress compared to the other common active materials, have been demonstrated to be a suitable active material for self-folding (e.g., [101]).

The assessments in table 1 were made using analytical models and they serve as a quantitative first order assessment of how different materials will perform in self-folding systems under certain idealizations (e.g., the influence of structural components such as connectors and sensors on the folding performance is neglected). To provide with further information regarding the full system integration and capabilities of active materials within self-folding structures, examples of previously demonstrated systems are presented in the subsequent section.

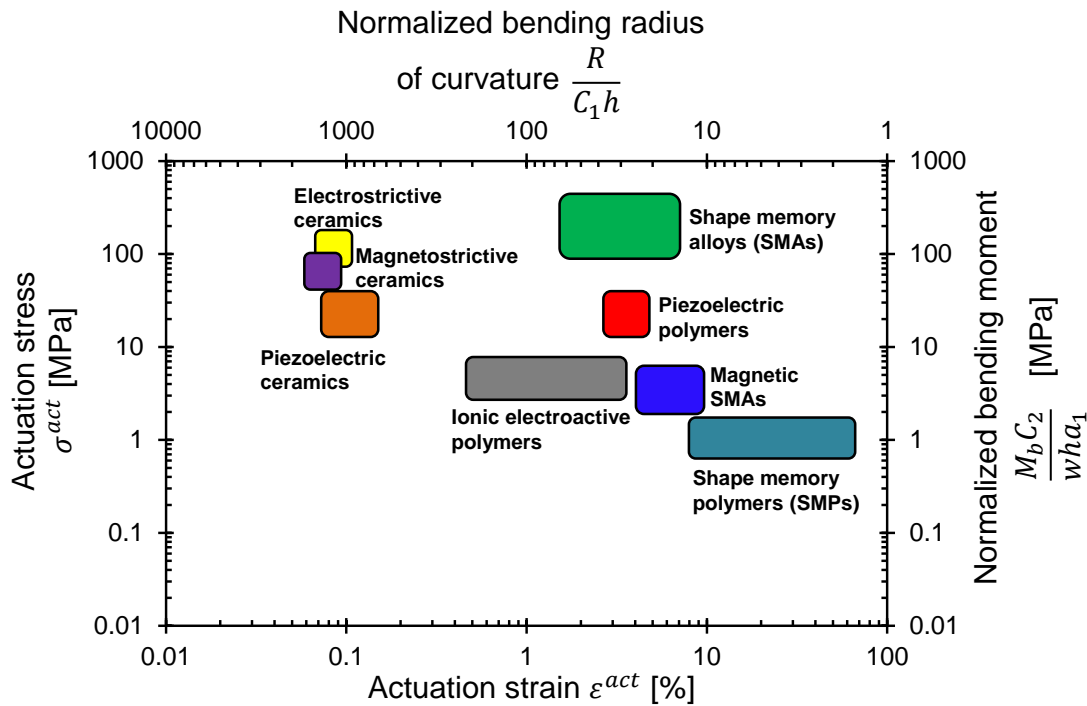


Figure 7. Active materials performance for the multi-layer fold concept. The diagram shows typical ranges of actuation strain and actuation stress for different common active materials [65]. The top and right axes represent the normalized bending radius of curvature and the normalized bending moment, respectively.

3. Active Self-folding Structures

Here we describe existing examples of self-folding structures classified according to the physical field that induces the folds. The goal is to provide researchers and practitioners with a systematic view of the state-of-the-art in this important and evolving area. Thermal, chemical, optical, electrical, and magnetic field-induced self-folding structures are presented. Tables 2 through 6 summarize the characteristics of the largest classes of self-folding systems considered. Examples of each type are characterized in terms of their inducing field, fold concept, reversibility of the folds, material system, characteristic sheet thickness, and current or potential application.

3.1. Thermally-activated Self-folding

The design space for thermally induced self-folding systems is large in terms of the methods available for localized supply of heat which include conduction, Joule heating, convection, radiation, etc. Although there are several ways to alter the temperature field in a structure, the diffusive nature of heat represents a design challenge requiring the consideration of methods for controlling the spatial distribution of temperature over time (e.g., by adding thermal insulators to maintain a large temperature zone concentrated in certain regions of the self-folding system).

Table 2. Examples of thermally-activated self-folding structures.

Fold concept	Reversible	Material system	Characteristic thickness [μm]	Application	Reference
Flexural	Yes	SMA	500.0	Potential multi-purpose morphing structures	[102, 103]
Multi-layer and single layer	Yes	SMA	1000.0	Potential multi-purpose morphing structures	[104, 105, 106, 107]
Other	Yes	SMA	50.0	Stent	[60, 108, 109]
Other	Yes	SMA	100.0	Robotics fabrication	[110, 111]
Extensional	Yes	SMA	1000.0	Robotics	[28, 112]
Flexural	Yes	SMA	1000.0	Robotics	[113]
Extensional, flexural, and multi-layer	Yes	SMA	2000.0	Flexible mobile devices	[114]
Multi-layer	Yes	SMA	100.0	Flexible mobile devices	[115]
Multi-layer	Yes	SMA	500.0	Decorative paper structures	[116, 117, 118]
Multi-layer	Yes	SMA and SMP	1000.0	Potential multi-purpose morphing structures	[119]
Flexural	Yes	SMP	1000.0	Potential multi-purpose morphing structures	[101, 120, 121, 122]
Flexural and multi-layer	Yes	SMP	500.0	Novel fabrication methods	[123]
Other	Yes	SMP	500.0	Potential multi-purpose morphing structure	[124]
Single layer	Yes (once)	SMP	250.0	Potential multi-purpose morphing structure	[125]
Multi-layer	Yes	SMA	25.0	Microgripper	[126, 127]
Multi-layer	Yes	Thermoresponsive polymer	Not available (in the order of 1000.0)	Potential multi-purpose morphing structures	[128]
Multi-layer	Yes	Polymer-metal bilayer	5.0	Capture/release small scale devices	[129, 130]
Multi-layer	Yes	Thermoresponsive polymer	0.5	Capture/release small scale devices	[131, 132, 133]

At the macro-scale, thermally-induced self-folding has been achieved mostly by the usage of SMAs and SMPs. On the SMA side, Rus and coworkers [102, 103] developed a

concept of self-folding origami with universal crease patterns that consisted of a single sheet with repeated triangular tiles connected by pre-engineered actuated hinges. The repeated triangular tiles configuration provides flexibility in the number of shapes into which the sheet can fold. Thin ($100\ \mu\text{m}$) Nitinol foil was used as the actuating hinge. This system was shown to be successful through demonstrations of the single planar sheet folding towards conventional origami shapes such as a boat or a plane (figure 8a). An and Rus provided a design and programming guide for the development of self-folding sheets of this kind [134]. In their work, they describe and analyze algorithms that generate designs and programs for the sheet.

Other example of SMA-based self-folding origami is the concept of massively foldable self-folding sheet developed by Hartl, Malak, Lagoudas and others [104, 106, 135, 136]. The concept consists of a composite laminate with two outer layers of SMA separated by a compliant and insulating layer (e.g., an elastomer). The outer layers of the SMA may consist of thin pre-strained SMA films [106, 137] or meshes of pre-strained SMA wires [104, 138]. With this three layer design, the side of the laminate being heated determines the direction of the fold relative to the laminate normal. It was shown via finite element analysis that structures composed of this laminate are able to morph, raise against gravity, and form three-dimensional structures as demonstrated in [105]. Simulations and experimental demonstrations of this concept are shown in figure 8b. More recently, SMA-SMP composite laminates have been investigated with this concept to create a shape memory composite capable of locking in its folded shape upon a heating/cooling cycle [119].

Kuribayashi and coworkers [60, 108] performed the design, manufacturing, and characterization process of a self-deployable origami stent graft. This stent is made from a single foldable SMA foil with mountain and valley folds. The deployment of this stent design can be achieved by shape memory effect activated at the body temperature or by making use of pseudoelasticity. The authors demonstrated that the proposed design successfully deploys as expected. This concept adds to the extensive use of SMAs in the biomedical field [140, 141, 142].

Robots with SMA-based self-folding components have also been designed. Rus and coworkers [110] presented an origami-inspired technique that allows for the application of two-dimensional fabrication methods to build three-dimensional robotic systems. The laser-machined origami robots use only a flat sheet as the base structure for building complicated bodies. They showed the fabrication and assembly process of a robot that can undergo worm-like peristaltic locomotion [111]. In such design, NiTi spring actuators placed on the body moved parts of the robot on demand. Lee and coworkers [28] designed a deformable wheel robot with the ball-shaped waterbomb origami pattern where the size was altered by activating an SMA spring actuator. By changing the size of its wheels the robot was able to navigate paths with various space limitations. Firouzeh and coworkers [113] developed a four-fold sheet-like robot that folds using SMA actuators. The device also showed promising locomotion characteristics.

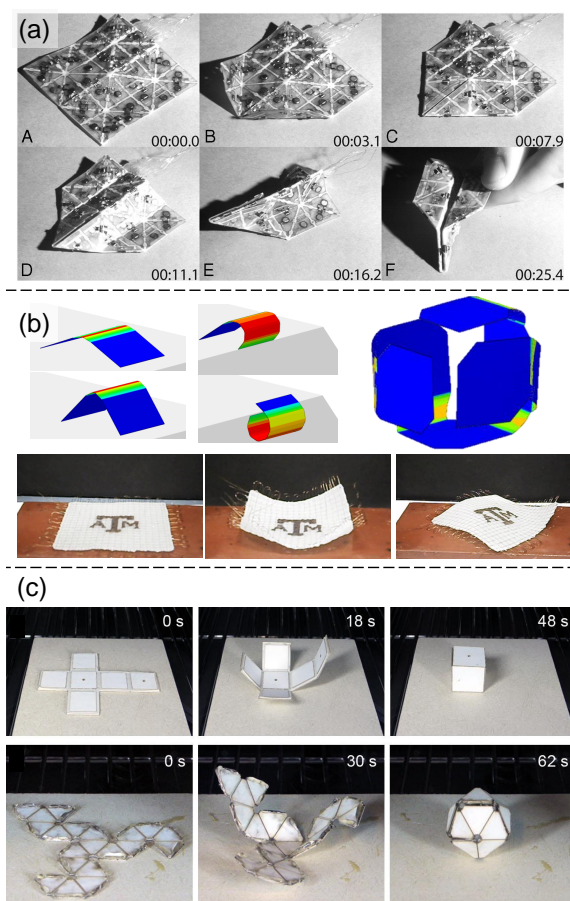


Figure 8. Thermally-induced self-folding structures: (a) Sheet with SMA hinges. A flat sheet maneuvers to fold towards an airplane shape [102] (Reproduced by permission of the Proceedings of the National Academy of Science of the United States of America PNAS). (b) Massively foldable SMA-based laminate including worm and rolling locomotion examples and self-folded cube simulations [139] and experimental demonstrations of the sheet folding towards a “bowl shape” and an “S-shape” [135]. (c) SMP-based self-folding structures morphed under uniform heating [121].

Shape memory alloy-based self-folding has also been used for paper animation. Qi and coworkers [116, 117] utilized SMA wires to self-fold different shapes such as cranes. Their works provide a set of design guidelines and information regarding circuitry details. The interesting shapes of folding structures can also be potentially applied to other areas such as flexible display devices [143, 144, 145, 146]. For example Roudaut and coworkers [114] and Gomes and coworkers [115] proposed new flexible display designs for mobile devices that can adapt their shape via SMA actuation driven fold-like operations to create what they term *shape resolution* in mobile devices.

Of course, SMAs are not the only option for thermally induced folding. Shape memory polymers provide actuation at a higher strain but lower force (see figure 6 and figure 7). One example of thermally-activated self-folding with SMPs is the work of Demaine, Rus, Wood, and others [101]. They developed a method of self-folding hinges consisting of SMP, paper, and resistive circuits. In addition, they created a model for

the torque exerted by such composite hinges, which was experimentally validated. The SMP-based self-folding composites show the capability of creating complex geometries and locking shapes via proper sequential folds design. They have studied different ways to heat the SMP including radiation heating (visible light), electrical methods, and uniform heating [120, 121]. Examples of uniformly heated SMP-based self-folding composites are shown in figure 8c.

Behl and coworkers have also demonstrated self-folding using the shape memory effect in SMPs [124]. In their demonstration, the permanent (set) shape of the SMP is the desired folded configuration while the temporary shape is associated with the unfolded configuration. Initially at a temperature below the glass transition temperature, the SMP can be heated above this temperature, morphing from its flat temporary configuration to its folded permanent configuration.

Light sources have also been used to heat thermally-activated self-folding devices based on the conversion of light to heat [72], where the thermal energy is then converted to mechanical energy. Liu and coworkers fabricated self-folding structures that employ localized absorption of light in the infrared spectrum cast over a compositionally homogenous sheet of SMP. The uniform externally applied stimulus (i.e., unfocused light) generates a focused folding response via localized designed light absorption [125]. Their approach uses mass-produced materials without the need for multiple fabrication steps, where the folded regions were defined by the presence of black ink patterned by a printing process. The polymer regions located beneath inked areas heat faster than the areas elsewhere and eventually heat beyond the glass transition temperature of the SMP. After such temperature is exceeded, these local SMP regions relax and the film bends. The original flat configuration can be recovered by heating the entire sheet above the SMP transition temperature.

There are a number of self-folding examples using thermally-induced actuation at the small scale. Lee and coworkers designed a microgripper that uses Ni-Ti-Cu SMA self-folding to open and close [126, 127]. The microgripper is fabricated by alignment and selective eutectic bonding of two preprocessed silicon wafers. Deposited SMA films serve as the outer layers of the microgrippers, acting as actuators. Applications for the microgripper include assembling small parts for manufacturing, minimally invasive biopsy tissue sampling, remote handling of small particles in extreme environments, among others [126]. Clearly the single outer layer of SMA provides bending actuation in a manner analogous to the bilayer design shown in figure 5d.

The multi-layer concept has also been exploited by other researchers for self-folding at the small scales. For instance, Gracias and coworkers fabricated folding structures at the micro-scale able to perform sequential folding via heating of pre-stressed hinges using lasers [128]. Their hinges were composed of Cr/Au-polymer bilayers while their rigid regions were composed of single Au layers. Upon laser irradiation, the polymer layer softens and the bilayer bends due to existing pre-stress generated during the bilayer fabrication process. Kalaitzidou and coworkers [129, 130] also developed self-folding polymer-metal bilayer structures. Their bilayer sheets consisted of polydimethylsiloxane

(PDMS [147, 148]) and gold (Au) layers that had self-folding capabilities. The PDMS layer had a thickness of several micrometers while the thickness of the Au layer was in the order of nanometers. Upon changes in temperature, the bilayer folds or unfolds due to dissimilar thermal expansion of the two layers. They also created a PDMS-silicon carbide (SiC) bilayer with similar behavior to demonstrate that their concept can be applied using any two materials with dissimilar thermal expansions. The ability of the bilayers to capture, transport and release different solids was demonstrated indicating their potential application as delivery tubes [130]. A similar bilayer approach was adopted by Ionov and coworkers when fabricating polycaprolactone (PCL) - poly-(N-isopropylacrylamide) (PNIPAM) self-folding polymer bilayers [131, 132]. PCL is hydrophobic (i.e., tending to repel/reject water [149]) while the solubility of PNIPAM can be changed reversibly with temperature by going above/below its low critical solution temperature (LCST) [131]. This temperature dependent behavior allows PNIPAM to swell or collapse in the presence of water with changes in temperature. Combining PNIPAM with the PCL hydrophobic layer allows for temperature controlled folding and unfolding of structures created with this bilayer.

3.2. Chemically-activated Self-folding

Self-folding using chemical stimulus has also been explored by multiple researchers. Most of these systems are based on the multi-layer fold concept and utilize the degradation or swelling behavior of certain polymers under the presence of specific substances or pH level [150, 151]. Examples of these systems are presented in table 3.

Self-folding films sensitive to pH are commonly designed using weak polyelectrolytes as active polymers [151]. One example are the microtubes fabricated by Kumar and coworkers [152]. They considered a three-layer laminate sheet of poly-(dimethylsiloxane) (PDMS) / polystyrene (PS) / poly(4-vinylpyridine) (P4 VP) for self-rolling of microtubes. The folding-based mechanism for rolling-up of the microtubes was based on the different degree of swelling in the constituting polymer layers. PS demonstrates minimal water uptake while P4 VP is less hydrophobic and swells in acidic aqueous solutions because of protonation of polymer chains. When a P4 VP layer swells, its volume increases and, if fixed to a PS layer, will cause the resulting polymer laminate to fold. In their work, Kumar and coworkers converted the PDMS layer into silica after rolling via oxidative pyrolysis. This process allowed them to produce a stiff and thermally and chemically stable microtube. This approach was also demonstrated to be useful for the fabrication of all metallic microtubes by using a PDMS-PS polymer bilayer as template by depositing a thin layer of metal onto the bilayer, rolling by the polymer swelling, and then removing the bilayer [153] where the effects of layer thickness and other parameters on the microtubes fabrication procedure have been investigated [154]. Micro- and nano-tubes fabricated by this method are promising for applications including nano-syringes for intra-cellular surgery and nano-jet printing [164]. This folding-based fabrication approach shows flexibility in the axial

Table 3. Examples of chemically-activated self-folding structures.

Fold concept	Reversible	Material system	Characteristic thickness [μm]	Application	Reference
Multi-layer	No	Hydrogel	0.5	Microtubes fabrication for small scale devices	[152, 153, 154, 155]
Multi-layer	No	Hydrogel	20.0	Drug release device	[156, 157]
Multi-layer	Yes	Hydrogel	50.0	Micro-containers	[158]
Multi-layer	Yes	Hydrogel	5.0	Potential multi-purpose morphing structures	[159]
Torsional and multi-layer	Yes	Hydrogel	200.0	Potential multi-purpose morphing structures	[160]
Multi-layer	No	Polymer degradation	Not available (in the order of 100)	Small scale grippers	[161, 162]
Multi-layer	No	pH responsive polymer	0.5	Capture/release small scale devices	[163]

shape of the obtained microtube (i.e., the tubes do not necessarily have to be straight). An example of this is the fabrication of toroidal hollow-core microcavities obtained using the polymer bilayer approach [155]. In another work, Shim and coworkers [158] created robust microcarriers using hydrogel bilayers that exhibited reversible folding behavior. The bilayer hydrogel system consisted of a layer of poly(2-hydroxyethyl methacrylate-co-acrylic acid), p(HEMA-co-AA), and a layer of poly(2-hydroxyethyl methacrylate), p(HEMA). Planar films composed of this bilayer were able to fold towards microcontainers by swelling of the p(HEMA-co-AA) layer at a pH of 9.

A similar self-folding bilayer approach was adopted by He and coworkers in the fabrication of an oral delivery device [156]. The main part of the device consisted of a finger like bilayer composed of pH-sensitive hydrogel based on crosslinked poly(methacrylic acid) (PMAA), which swells significantly when exposed to body fluids, and a second non-swelling layer based on poly(hydroxyethyl methacrylate) (PHEMA). Studies regarding the degree of folding as a function of the bilayer composition have been performed [159]. In such studies, the swelling layer was prepared with a mixture of poly(ethylene glycol methacrylate) (PEGMA) and poly-(ethylene glycol dimethacrylate) (PEGDMA). By controlling the ratio between the two components of this “active” layer, different degrees of folding were achieved. Two different micro-scale structures folding using this bilayer are shown in figure 9a. The bilayer approach was also utilized by Ionov and coworkers to

synthesize polycaprolactone/polysuccinimide polymer bilayer self-rolled tubes. Both polycaprolactone and polysuccinimide are generally hydrophobic. In physiological buffer surrounding environment, polysuccinimide hydrolyzes forming water-swelling polyaspartic acid that yields to the rolling of the polymer bilayer and the formation of tubes [163].

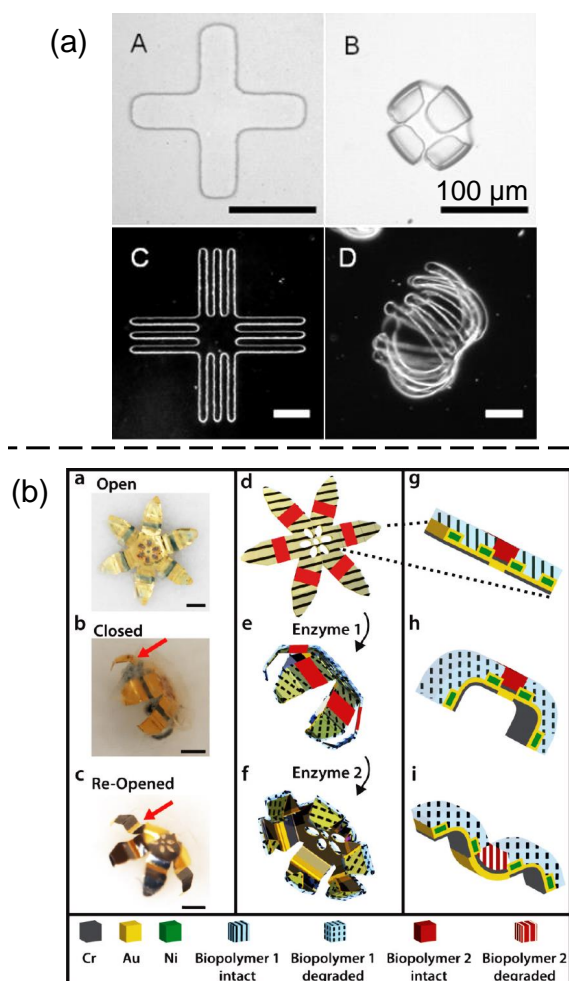


Figure 9. Chemically-activated self-folding examples: (a) Optical micrographs of a cross-shaped microwell and a multiple extensions microwell in their folded and unfolded configurations. All scale bars represent $100\ \mu\text{m}$. Reprinted with permission from [159]. Copyright (2005) American Chemical Society. (b) Optical images and schematic representation of micro-grippers folded by selective polymer degradation. Scale bars represent $200\ \mu\text{m}$. The folding process is also illustrated for a single hinge. Reprinted with permission from [161]. Copyright (2010) American Chemical Society.

Jeong and coworkers developed a PDMS - hydrophilic polyurethane (PU)/2-hydroxyethyl methacrylate (HEMA) bilayer composite that folds when submerged in hexane solvent due to the large swelling of the PDMS layer of such substance [160]. This idea allowed them to create programmed two-dimensional sheets that can self-fold into a cube, pyramid, and helix forms.

A self-folding gripper that opens and closes by the actuation of polymer hinges was fabricated by Gracias and coworkers [161, 162]. The actuation of the polymer hinges was triggered by their sensitivity to the presence of enzymes, where they utilize two different polymer types with two mutually exclusive enzyme sensitivities. The two polymers were placed at hinges in such a way that bending in opposite directions is activated given the appropriate stimulus. When one polymer is selectively degraded by its associated enzyme, its modulus decreases and the gripper closes. When the other polymer is degraded via the action of its own distinct enzyme, its respective hinge bends and the gripper opens. This process is shown in figure 9b.

3.3. Optically-activated Self-folding

Previously in section 3.1, we discussed light driven actuation that worked on the principle of radiative heating. Here, however, we consider actuation driven by microstructural changes directly caused by light irradiation.

Table 4. Example of optically-activated self-folding structures.

Fold concept	Reversible	Material system	Characteristic thickness [μm]	Application	Reference
Single layer	No	Photo-responsive polymer	Not available (in the order of 1000.0)	Potential multi-purpose morphing structures	[165]
Multi-layer	Yes	GaAs-based alloys	1.0	Micro-mirrors	[22, 56]

Folding of polymer films with light was investigated by Ryu and coworkers [165]. The mechanism used for self-folding in their work was localized photo-induced stress relaxation. The considered polymer was composed of Pentaerythritol tetra(3-mercaptopropionate) (PETMP), 2-methylene-propane-1, 3-di(thioethylvinylether) (MDTVE), and ethylene glycol di(3-mercaptopropionate) (EGDMP) mixed with photoinitiators. Straining the sheet and subsequently irradiating it with light dissociates specific photoinitiators into free radicals that react with and cleave the MDTVE functionalities along the polymer backbone [165]. Such events irreversibly rearrange the network connectivity and macroscopically result in stress relaxation that generates the folds. The authors were able to create folded arcs and a closed cube-shaped box using this self-folding approach. Further information about optically induced deformation in polymer films can be found elsewhere [166, 167]. Polymers are not the only option when considering optical actuation. For instance, Zanardi Ocampo and coworkers fabricated optically-actuated micro-mirrors based on laminates with strained GaAs-based alloys. Actuation of such laminates was achieved by laser irradiation [56, 22].

3.4. Electrically-activated Self-folding

With the recent advanced in electronics and MEMS technologies, limitations for the generation and manipulation of electric fields across scales have diminished. These advances along with the extensive research in dielectric elastomers [168, 169] and other electro-active polymers [170] have made possible for the development of electrically-activated self-folding structures. Examples of these structures are provided in table 5.

Table 5. Examples of electrically-induced self-folding structures.

Fold concept	Reversible	Material system	Characteristic thickness [μm]	Application	Reference
Other	Yes	Dielectric elastomer	1000.0	Robotics	[29]
Multi-layer	Yes	Dielectric elastomer	350.0	Potential multi-purpose morphing structures	[73, 171, 172]
Other	Yes	Electroactive polymer	20.0	Robotics/ Locomotion	[173]
Extensional	Yes	Dielectric elastomer	2000.0	Flexible mobile devices	[114]
Multi-layer	Yes	Electroactive polymer	0.5	Small scale tools	[174, 175]

On the large scale, White and coworkers [29] demonstrated the feasibility of implementing dielectric elastomer actuators for a bending component that can be applied to modular robotics. They present the design and experimental analysis of a dielectric elastomer actuator that consists of two layers of dielectric elastomer separated from each other. It was noted that improvements could be made with respect to the bending performance by optimizing the geometrical parameters of the device and/or adding more dielectric elastomer layers. A different design for a dielectric elastomer employed for self-folding structural actuation was investigated by Frecker and coworkers [73, 171, 172]. They developed a bending actuator that consisted of three primary layers: the active dielectric elastomer, a passive substrate (scotch tape), and compliant electrodes (carbon grease). The bending samples configured in this manner were able to generate a sample length normalized tip displacement of slightly above 1, demonstrating the promising qualities of this as a self-folding concept. Figure 10a shows the folding behavior of the dielectric elastomer-based bending sample. Self-folding with dielectric elastomers based on the extensional concept (figure 5a) were developed by Roudaut and coworkers for flexible mobile device displays (previously discussed in section 3.1). Their concept considered bending of a flexible mobile device via contraction and expansion of a linear dielectric elastomer actuator connected to two sides of the device [114].

More complex deformations were demonstrated by Okuzaki and coworkers, who

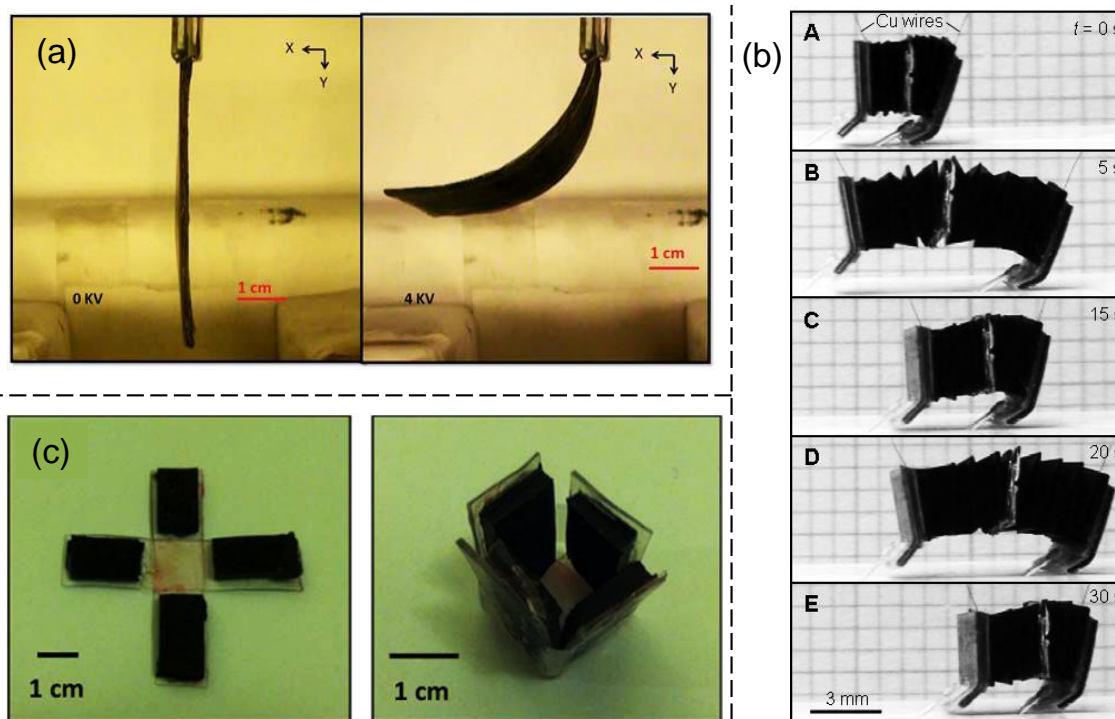


Figure 10. Electrically and magnetically-activated self-folding structures: (a) A self-folding sheet with dielectric elastomer actuator under no applied electrical field (left) and with applied electrical field (right) [73] (Reproduced by permission of the American Society of Mechanical Engineers ASME). (b) Photo series of a biomorphic origami robot showing the robot translating using accordion-like movement activated via electrically-induced folds [173] (Reproduced by permission of IOP Publishing. All rights reserved). (c) Magnetically induced folding of a PDMS-MAE box under zero magnetic field (left) and under non-zero magnetic field (right) [73] (Reproduced by permission of the American Society of Mechanical Engineers ASME).

created a biomorphic robot fabricated by a folding a conducting polymer film [173]. The folding actuation was generated by electrically-induced changes in the elastic modulus of a conducting humidosensitive polymer film. Those changes of elastic modulus were due to absorption and desorption of water vapor molecules depending on the imposed electrical field. They demonstrated the feasibility of the concept by fabricating different prototypes such as an origami robot that moves rectilinearly with a caterpillar-like motion. Such motion was achieved by repeated expansion and contraction of its accordion-like body via folding/unfolding of electrically sensitive Polypyrrole (PPy [176, 177]) (figure 10b).

At the small scale, Inganäs and Lundström utilized conducting polymers as actuators within electrically-driven polymer-gold folding bilayers [174, 175]. They used conducting PPy as the active layer. The bilayer is submerged in Na^+DBS^- electrolyte solution (where DBS represents dodecylbenzenesulfonate). When a negative voltage potential is applied, the PPy film fills with Na^+ and is expanded to its original size, thus the flattens. When a positive voltage potential is applied, the volume of the PPy

layer reduces and the bilayer bends.

3.5. Magnetically-activated Self-folding

Table 6. Example of magnetic-induced self-folding structures.

Fold concept	Reversible	Material system	Characteristic thickness [μm]	Application	Reference
Other	Yes	Magneto-active elastomer	3000.0	Potential multi-purpose morphing structures	[73, 74]

Magneto-active elastomers (MAE [178]) have been investigated as actuators for self-folding materials by von Lockette and coworkers [73, 74]. The MAE materials were fabricated by mixing 30% (by weight) barium ferrite (BaM) particles into a silicone rubber matrix. The passive substrate consisted simply of a silicone RTV compound. The MAE-based self-folding composite was able to bend with a sample length normalized tip displacement of slightly above 0.5. Composites consisting of PDMS sheets with MAE patches were able to achieve locomotion under application a oscillating magnetic field. In addition, a cross-shaped PDMS sheet with four MAE patches on its sides that raised under the application of magnetic field was shown to fold into a box (figure 10c). However, it should be noted that in this particular example, the MAE patches were used only for their ability to be magnetized rather than for any magnetic to mechanical energy conversion.

The subsequent section provides a survey of another level in the design process of a self-folding structure, which entails crease pattern design and fold planning.

4. Crease Pattern/Fold Kinematics Design

Previously in section 2, we addressed the design of individual folds as well as the material selection for active self-folding systems. This section is concerned with the design of crease patterns and the analysis of folding maneuvers to achieve certain shapes or functionalities, irrespective of the active material or other mechanism being used to effect the folding behavior.

To realize a useful active self-folding structure, designers must consider several issues beyond the choice of active material and folding concept. The motivation for a design problem is to fulfill one or more desired functions. For example, designers might achieve a storage/deployment functionality for satellite solar panels using a folding structure. Important considerations are to determine what is the final folded shape (or shapes, in the case of a reconfigurable structure), to identify a crease pattern that can achieve the desired shape(s), and a sequence of folding actions that results in the desired shape(s). Designers must achieve all of this subject to local and system-level

failure criteria and requirements for interfacing the structure with other parts of the system (e.g., mating points). Table 7 is a summary of this design problem.

Table 7. High-level definition of the design problem for active self-folding structures.

<i>Given</i>	Desired design functionality
<i>Find</i>	<ol style="list-style-type: none"> 1. Folded shape(s) that achieve desired functionality 2. Crease pattern that can produce said shape(s) 3. Sequence in which to execute folds (fold kinematics) 4. Appropriate active fold concept(s) (section 2.1) and detailed geometry 5. Appropriate active material(s) (sections 2.2 and 3) and passive structure materials
<i>Subject to</i>	Failure criteria and interface requirements

Although many origami design approaches have their limits with respect to actual material folding behavior (e.g., they commonly neglect thickness or assume an infinitely small fold radius), they provide useful options for addressing the design of folded structures in general. While this survey is far from complete, we do attempt to address the options most suitable for the design of self-folding systems with a focus on the most recent works.

4.1. Software Developments

Designers need tools and methods tailored to each part of the design problem of self-folding structures (table 7). The previous sections of this article have focused primarily on domain knowledge about fold concepts and the various active materials available for achieving self-folding behavior. Existing computer aided design (CAD) and finite element analysis (FEA) tools can be used directly or extended for use in the analysis and design of active folds. However, the challenges of identifying crease patterns and fold sequencing require unique tools and methods. This section is a review of the prior work on origami engineering tools in support of these design activities.

One of the most well known software packages for fold pattern design is Robert Lang’s TreeMaker [179]. Lang described the theoretical foundation on which TreeMaker is based: the tree method [179]. This method allows for the design of an origami *base*, which is defined as a non-stretching transformation of the sheet into 3-space such that all facets remain flat. The *base* can be fully defined by the location of creases, their angles, and the orientation and location of each facet [179]. The base is then folded and shaped into an origami model. TreeMaker determines a crease pattern that results in the desired base via optimization methods. An example of the application of this algorithm is shown in figure 11a, where the tree graph of a bug is shown together with the computed crease pattern generated by TreeMaker. Further information and advances of this algorithm can be found in the literature [180, 181].

Another approach for crease pattern design is to directly impose the exact final shape that the sheet should take after folding. Demaine and coworkers [182] developed

a mathematical framework for folding a sheet into a thin strip that is essentially “wrapped” around the desired shape. In their work, they showed that one can obtain a crease pattern for a particular flat silhouette or for a three-dimensional polyhedral surface. They proposed different algorithms for wrapping the desired shape with the sheet, these include one that uses any sheet area arbitrarily close to the surface area of the desired shape, another that maximizes the width of the sheet strip subject to certain constraints, etc. [183].

Tachi presented a practical method for “origamizing” objects [184, 185] (i.e., obtaining the crease pattern that folds a single sheet of material into a given polyhedral surfaces without any cut). It is based on the idea of using folds in an initially planar sheet to create flaps that are “tucked” (hidden) to form the desired three-dimensional shape. The Origamizer software solves the inverse problem of providing a crease pattern from the input, which may be an arbitrary polyhedral surface. In his work Tachi investigated the conditions required for constructing a valid crease pattern.

Tachi has also contributed methods and software for creating freeform foldable structures that can approximate curved surfaces [186, 187, 188]. In such processes, curved bodies such as disks and cylinders can be obtained from collapsible variations of rigid-foldable surfaces. One of the proposed techniques consists of obtaining a freeform variation of rigid-foldable and bidirectionally flat-foldable disk surfaces, which is a combination of generalized Miura-Ori [189] and eggbox patterns [190]. A software that provides these capabilities has been created. In addition to folding pattern generation, it also allows for the visualization of the folding/unfolding process for a given model. In a more recent work [191], Tachi proposed a method to produce origami tessellations from given polyhedral surfaces. The method consists of first separating faces of the desired shape and then inserting folded parts between them. The obtained configuration is then modified by satisfying geometric constraints of developability^{||}, folding angle limitations, and self-intersections. Such method is not only applicable to paper but also sheets that do not allow 180° degree folding. In another work, Tachi developed a software for interactive simulation of origami based on rigid origami [194]. The simulation program can generate the continuous process of folding a flat sheet into a 3D shape by calculating the configuration from the crease pattern. The configuration of the sheet is then determined by the crease angles and the trajectory is calculated by projecting the crease angles into the constrained configuration space.

Ishida and coworkers developed a method using conformal transformations to design crease patterns of circular membranes that can be wrapped up compactly [195]. The method follows an analogous procedure for the generation of fluid flow streamline profiles using velocity potentials and streamline functions. In their work, similar potential and streamline functions are used to generate the fold lines profile in circular shells. The graphical analogy between fluid flows and origami pattern is shown in figure 11b where it is observed that unidirectional flow is analogous to a crease pattern for a cylindrical

^{||} A surface is developable if it has zero Gaussian curvature everywhere [192, 193].

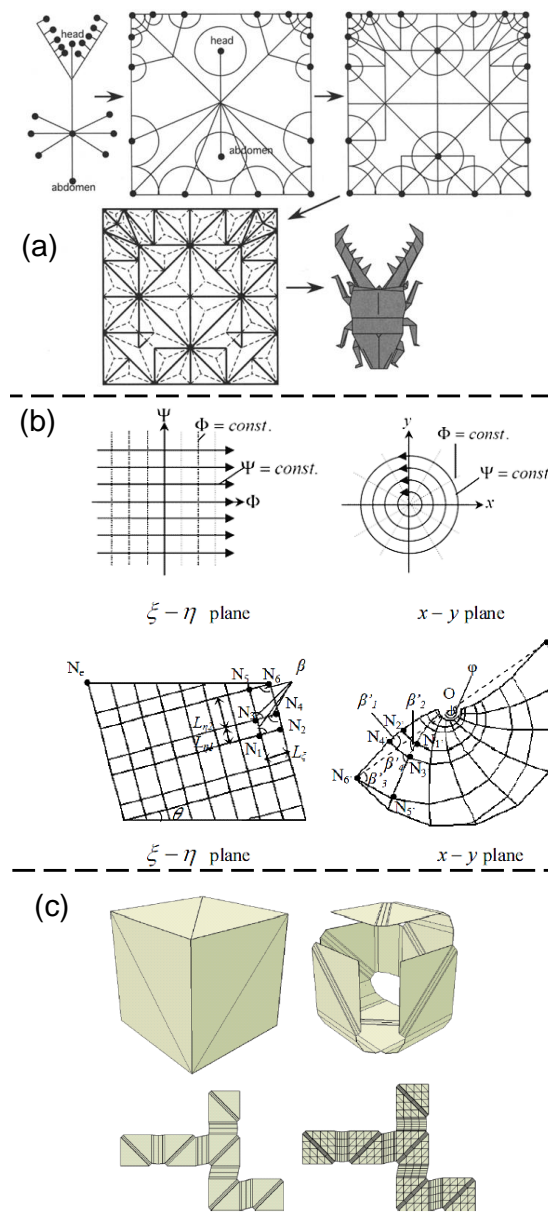


Figure 11. Examples of folding pattern/sheet design tools. (a) Illustration of the tree algorithm using one of Meguro’s bugs. (Not all creases are shown.) [181] (Springer and The Mathematical Intelligencer, 27, 2005, page 93, Origami design secrets: mathematical methods for an ancient art, Robert J. Lang, fig. 2, © 2005 Springer Science+Business Media, Inc. is given to the publication in which the material was originally published, by adding this figure. With kind permission from Springer Science and Business Media). (b) Conformal transformation on fluid flows and analogy to origami patterns: (left) uniform flow with analogous crease pattern for a cylindrical shell, and (right) flow with a vortex filament with its analogous crease pattern for conical shell [195] (Reproduced by permission of the American Society of Mechanical Engineers ASME). (c) Procedure to obtain a single-panel unfolding from a cube: initial mesh, spanning tree, unfolding, and refinement for finite element analysis (Reprinted from [105], Copyright (2013), with permission from Elsevier).

shell while flow with a vortex filament is analogous to a crease pattern for a conical shell. The method enables the design of complex deployable structures systematically and efficiently from simple structures, controlling angles among fold lines. In their work they demonstrate the feasibility and applicability of their algorithm by creating plastic film and stainless steel plate folded models.

An origami computational interface called *Eos* [196, 197, 198, 199] that permits a user to fold sheets virtually as if they were folding paper by hand has been developed by Ida and coworkers. The software allows for visualization and interaction of origami constructions. *Eos* provides two methods for folding: the mathematical fold and the artistic fold. The former folding method is based on axiomatic definitions of origami folds [196, 200, 201]. Such method requires fair knowledge of nomenclature and coding definitions for different folding options. The artistic fold method is more user-friendly and allows one to specify relatively straightforward folding features such as whether fold is a valley or a mountain, which lines are to be folded, and to what angle they should be folded. This method is preferred for the creation of artistic origami.

Akleman and coworkers developed a method to *unfold* a given convex polygonal shape into a one-piece planar (i.e., developable) surface [105]. The algorithm first triangulates (i.e., divides into triangles) the initial shape to guarantee that all faces of the shape are planar. A “dual mesh” is then constructed from the triangulated shape such that every face becomes a vertex and every original vertex becomes a face [202]. A dual graph consisting of the edges and vertices of the dual mesh is then created along with an associated spanning tree that includes all vertices from the dual graph. Every vertex of the dual graph is then two-dimensionally thickened into a triangle and every edge of the spanning tree is two-dimensionally thickened into a developable quadrilateral. The spanning tree is then unfolded into two-dimensional planar shapes [203]. The algorithm has been extended such that each two-dimensional planar shape can be further subdivided into smaller quadrilaterals and triangles for input into finite element software, which allows full mechanical analysis of the folding (or self-folding) process. An example of this procedure for the creation of a single-panel unfolding for a cube is shown in figure 11c. The feasibility of the algorithm was tested by considering the complete unfolding/re-folding process associated with a dome and a cube shapes.

Fuchi and coworkers [204] proposed a folding pattern design method based on topology optimization [205]. In their approach, the existence and type of fold lines are assumed; this given configuration is known as the “ground structure”. The folding angles are the design variables. For a given ground structure and set of folding angles, the three-dimensional geometry of the folded sheet can be computed using origami approaches similar to those described previously in this section. A topology optimization method is then used in an iterative effort to find the optimal set of folding angles that, combined with the ground structure, provide the desired folded geometry.

4.2. Studies of Folding and the Behavior of Folded Structures

One main consideration when designing a crease pattern for conventional origami designs is *flat-foldability*. A *flat-foldable* origami structure must be able to fold into a shape that has dihedral angles of zero between pairs of adjacent faces. The imposition of flat foldability constraints onto the design of origami structures ensures that such structures can be created from an initially flat sheet or, conversely, be deployed toward a fully flat configuration. It has been shown [206] that crease patterns created under this constraint can be applied for the design of physically realized sheets that have non-zero thickness and may not provide perfectly flat folds. According to the previous works of Demaine and O’Rourke [66], Schneider [207], and Poma [208], global flat-foldability must satisfy the following conditions:

- (i) *Local flat-foldability*: considering a single vertex, it has been shown that is flat-foldable if the alternating angles surrounding the vertex sum to 180° . This is known as the Kawasaki’s theorem [209, 210, 211]. In general, a crease pattern is everywhere locally foldable if there is a mountain-valley assignment so that each vertex locally folds flat [183].
- (ii) *2-colorable*: This condition refers to the binary designation assigned to the faces (finite areas) bounded by the creases (see figure 2) referred to as “colors”. To ensure global flat foldability no neighboring faces should share the same color.
- (iii) Able to stack all faces flat without penetration or collision between faces during the folding process.

The kinematics of both folding and the mechanical response of folded structures is also an extensive area of study and is relevant to active self-folding systems, especially in terms of their potential applicability. For example, Schenk and coworkers studied the kinematics of Miura-ori folding patterns [52] and showed that a folded shell structure folded in this pattern provides a negative Poisson’s ratio for in-plane deformations and a positive Poisson’s ratio for out-of-plane bending. An example of these results are provided in figure 12a where the negative in-plane Poisson’s ratios in a Miura-ori pattern for different folding stages and pattern geometries are shown. In addition, they also considered a folded structure based on a stacking of individual folded layers taking into account kinematic compatibility between layers. They showed that such multilayer folded structure is able to fold and unfold uniformly and can be designed to lock its motion in a specific configuration.

Using a similar approach while further addressing more physical considerations, Qiu and coworkers performed a kinematic analysis of origami carton-type packages [212, 213]. They addressed design issues of origami macro-scale packaging and presented mathematical models to predict their folding characteristics. They idealized creases as torsional springs with arbitrary stiffness during folding. Experiments were performed to validate their models. Such work has potential value in origami packaging design and elastic type origami elements in robotics.

The kinematics of origami-inspired robotic linkages were addressed by several authors. Qin and Dai studied an eight bar robotic mechanism where folded panels acted as links and the fold creases themselves acted as revolute joints [214]. They explored the configuration space of this mechanism under different geometric constraints. Moses and coworkers investigated the kinematics of origami-inspired rotors consisting of a snake-like strip connected to two fixed platforms at its ends and having a rotating body in the middle [215]. The entire mechanism can be folded from a single initially planar sheet, and the body at the middle of the strip rotates only due to folding maneuvers of the strip. Potential applications of this idea include rotating propellers for micro underwater or fluid immersed robots and high mobility wheel legs for crawling vehicles [215].

Researchers have also explored origami models that are folded so that in their final folded/deployed state they exhibit motion [216, 217]. The term *action origami* [218], has been used to describe such focus. Bowen and coworkers [216, 219, 220] studied a large number of action origami models and proposed a classification scheme for them. The classification is based in the idealization of vertices of intersecting folding lines as spherical mechanisms. In this formalism, each vertex is considered to lie at the center of an imagined sphere and the motion of the folds connected to each vertex are tracked by considering their projections onto that the sphere. The classification takes into account characteristics of the folding pattern such as whether the fold lines form a loop, the linearity of the fold line chains, whether they are periodic, and other aspects.

With several examples in nature of origami-like processes [224, 225, 226, 227, 228, 229], bioinspiration has also provided ideas for fold designs [230]. For instance, De Focatiis and Guest [231] presented a folding pattern design of deployable structures inspired by a model of deploying tree leaves. They investigated the effects of combining several corrugated leaf patterns to produce deployable surfaces, such as solar sails, solar panels, and antennas. Nature inspired approaches for crease pattern analysis are also being developed by McAdams and coworkers [232, 233] where they consider pixelated multicellular representations to define origami structures and a crease generation method based on the analogy between a crease pattern and ice-cracks on a frozen lake surface. The first method consists of defining a crease pattern via a set of distributed cells. The cells define the crease pattern by their color (each face contains a collection of cells with the same color). A crease restoration algorithm to extract the equivalent crease pattern from the multicellular representation was proposed. The second method is based on the analogy between ice-cracks on a frozen lake surface where each crack is equivalent to a crease and each forking point to a vertex [233]. To form the creases and vertices in an “ice-cracking”-like origami crease pattern, a vertex is picked as the starting location and the rest of the creases and vertices are grown in an analogous manner to crack growth and fork formation in ice. A genetic algorithm [234, 235] encodes the geometric information of forming the creases and vertices according to the development sequence based on the “ice-cracking” process which can be adapted to accelerate the emergence of optimal design outcomes through the evolutionary design process.

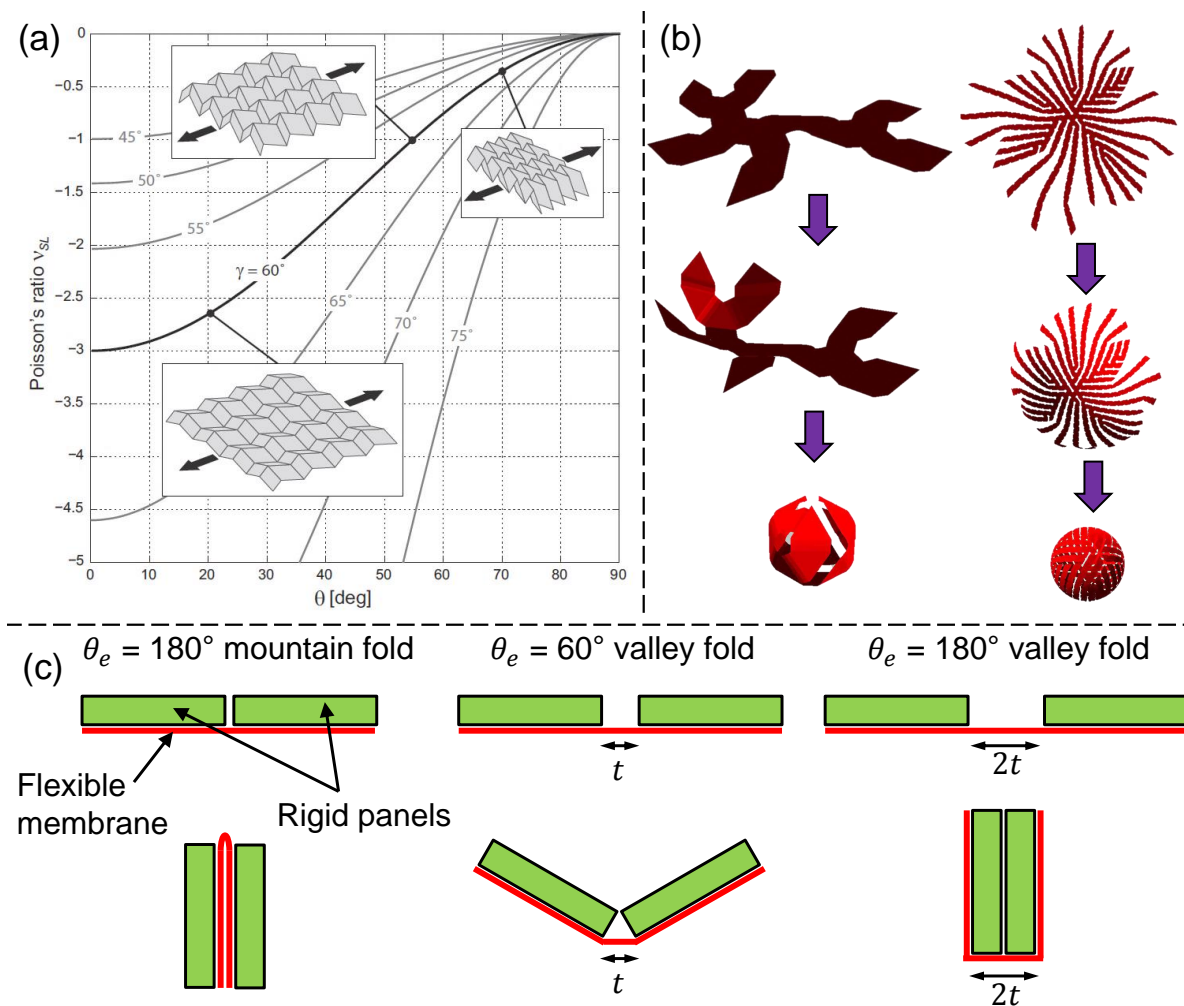


Figure 12. Examples of folding kinematics studies. (a) In-plane Poisson's ratio of Miura-ori folded sheet for different folding states. The results show that Miura-ori folded sheets have metamaterial characteristics independent of the sheet material [52] (Reproduced by permission of the Proceedings of the National Academy of Science of the United States of America PNAS). (b) Fold planning [221, 222] examples: motion paths to fold an icosahedron and a sphere from single panels (courtesy of Nancy Amato and her research group at TAMU). (c) Fold designs of membrane-panel model for thickness accommodation. The rigid panels are depicted in green while the flexible membranes are represented as red lines; 180° mountain fold, 180° valley fold, and 60° valley fold are shown. Based on the works of [223].

One issue not addressed in the studies and developments mentioned up to this point is that of *fold planning*, which assesses the physical feasibility of folding operations in complex forms. Amato and coworkers investigated motion planning algorithms [221, 222, 236, 237] that are applicable to the fold planning problem. Their motion planning algorithm can obtain, as input, the geometry of the sheet in both its folded and unfolded shapes and can generate a continuous path through the state space that will allow the sheet to move from one configuration to the other without self-intersecting itself. The motion planner is based on *Probabilistic RoadMaps*

(PRM) [238] and has been improved through the years to increase computational efficiency. Two examples of successfully determined folding paths are shown in figure 12b and includes examples are folding paths for an icosahedron and a sphere.

The majority of what has been described so far in this section is relevant for folding of only very thin sheets. However, the examples described in section 3 largely address self-folding of “thick” sheets. Fortunately, there are a number of works addressing the kinematics and fold design pertaining to thick sheets. For example, Guest and Pellegrino [239] developed a methodology for wrapping thin membranes around a central hub. They begin with the assumption of zero thickness for the generation of a folding pattern and then show a way to correct the proposed folding pattern to accommodate for a sheet of finite thickness. Tachi proposed a method for creating a 3D structure composed of finite thickness panels. The kinematic behavior of an associated zero-thickness model of the same structure is preserved by embedding said zero-thickness mechanism within a set of finite thickness panels [240]. Thus, the proposed mechanism folds in the same way of the base zero-thickness model though with some limitations such as that the final folded state of the structure cannot have coplanar panels (e.g., the panels must remain within a non-zero internal folding angle and/or a finite distance between them). Zirbel and coworkers [241, 223, 242] addressed the accommodation of thickness in origami-based deployable arrays, motivated by the need to fold thick rigid panels that cannot bend during folding or deployment (e.g., solar panels made of brittle materials). In their work they present a mathematical model for the modification of folding patterns to accommodate material thickness in the context of the design, modeling, and testing of a deployable system. Examples of panel-membrane hinge designs for common folds are shown in figure 12c. Hinge designs for 180° mountain folds, 180° valley folds, and 60° valley folds are shown. They demonstrate the applicability of their model using a 1/20th scale prototype of a deployable solar array for space applications. Their physical construction included gap widths at the folding lines (not present in zero thickness origami models) to accommodate for thickness.

5. Conclusions

Although the art of origami is ancient, the science and technology associated with origami-inspired engineering is new and developing rapidly. As identified in this review, smart materials can play a significant role in the realization of self-folding origami-inspired structures. Researchers have demonstrated self-folding behavior in many active material systems with inducing fields that include thermal, chemical, optical, electrical, and magnetic. Several combinations of materials, geometry, and inducing field are feasible, yielding an array of design options. Considerations such as whether folds must be reversible, restrictions on sheet thickness, boundary conditions, availability of a particular inducing field, and the overall folded structure concept are among the drivers for design decisions. Evaluation of common active materials and self-folding structures for various fold concepts confirms that there is no dominant active

material or mechanism for self-folding applications and that the material and mechanism selection are application-dependent (e.g., dependent on the desired folding radii or angle, surrounding environment).

Mechanically, one can use smart materials to produce folding along a hinge or to approximate folds through a small-radius bending action in a composite laminate structure or single-layer active material using a graded field. Concepts in the latter category are less similar to creases in paper, but can be similar enough to benefit from origami design principles. The lack of pre-engineered hinge locations in bending fold concepts also makes them more adaptable in principle. However, concepts with pre-engineered hinge locations can produce folds that are more idealized and their more restricted structure can simplify aspects of the design process such as fold planning.

A number of design tools exist for origami, many of which are useful for designing self-folding structures. Most assume a desired folded form is known and thus address one of the two origami sub-problems: determining where folds should go and determining the order in which folds should be made. Early investigations into the mathematics of origami addressed basic questions such as whether a particular shape is foldable and relied on strong idealizations such as having a folding sheet of zero thickness. Newer research into topics such as folding with thickness, the kinematics of origami structures, and bio-inspired folded structures is yielding results that will help bridge the gap between idealized origami and the reality of smart self-folding structures.

Although the research on origami principles and folding structures has been extensive, multiple challenges and open questions remain. Three questions are of particular importance in the context of smart self-folding systems:

- Increased understanding of the mechanics of folding in thick structures toward improved flat foldability: Even though multiple theories and software exist for the design and analysis of zero thickness folding structures (see section 4), very few exist for the development of folding structures with non-zero thickness. Thick origami folding is a challenging problem and has to be addressed for the development of applications that require compact storage in non-planar configurations (e.g., stacking of faces) and conventional origami structures where the thickness of the sheet is not several orders of magnitude below the characteristic length of the sheet.
- Function-driven design of self-folding structures: In both its artistic goals and mathematical theory, the theme of origami has traditionally captured form and geometric considerations. For example multiple software products exist for the determination of crease patterns and/or folding sequences that will allow a sheet to be mapped into a certain goal three-dimensional shape. However, engineering applications of origami principles are more often motivated and constrained by *function* rather than *form*. Only a few examples have been addressed such as the determination of folded shapes for crash box applications [44, 45, 46], and folded shapes towards simple objectives such as maximizing self-folded structural height under gravity [206]. However, these limited examples apply to specific sheet

materials and constraints and lack the generality desired of an engineering tool to determine ideal crease patterns/folding sequences for the optimal performance of specific functions. Perhaps increased efforts by engineering design researchers, such as those who draw inspiration from the functions of folding in nature [233, 243, 244, 245], will help to expand these capabilities.

- Development of remote field-induced reconfigurable foldability: Every one of the active material-based folding or fold-like structures described herein relies on the imposition, local or global, of a single given physical field (e.g., thermal, magnetic, etc.) Sheets are pre-engineered to respond in a single particular manner to such field application. Therefore, *reconfigurability* (i.e., from one morphed/folded shape to another) under global fields becomes difficult, this may prevent the level of truly reconfigurable self-folding structures at the small scale or other situations where local field imposition becomes unfeasible. Thus, reconfigurability under global (remote) fields would be an important advancement. This could be accomplished by designing for multi-field actuation (e.g., thermal *and* electric [246], thermal *and* magnetic [247]) or by allowing discrete folding regions to be tuned so as to respond to changes in a single global field type (e.g., as in changes in magnetic field frequency). Multi-field self-folding is being addressed by some researchers in both large [73] and small scales [54] but there is still much to be done in this area.

Active material technologies have already enabled a wide range of engineering applications, many of them accurately described as being based on “smart structures”. As the community that champions such solutions progresses, it is likely that researchers may find great advantage in looking beyond traditional design approaches and permitting inspiration from alternative sources, such as the art and theory of origami. As has been discussed, creating recoverable folds in complex material systems is more difficult than creasing paper; inducing localized self-folding is more challenging still. Application of past discoveries and current tools toward future morphing structures challenges, however, will allow engineers to address the open issues posed and offer new technological solutions to the world at large.

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Appendix A. Analysis of Folding Using an Active Torsional Element

For the analysis of the torsional self-folding concept (figure 5b), the active torsional element is isolated from the other bodies (hinge and connected faces are not modeled). A boundary value problem of the individual torsional element is considered by modeling

the interactions of the element with other bodies in terms of applied moments. The active torsional element is represented as a rod of circular cross-section with radius r and length L . The rod is composed of an active material and is initially unstressed. Under the uniform application of the inducing field, the rod twists and, if constrained to some degree at its ends, is subjected to equal and opposite end moments M_t . Such constraints are associated with the resistance of the sheet to the fold-like operation (e.g., caused by friction at the hinge, inertial forces, interactions of the sheet with other bodies, the effects of gravity, etc.). The initial and final configurations of the torsional element are shown in figure A1.

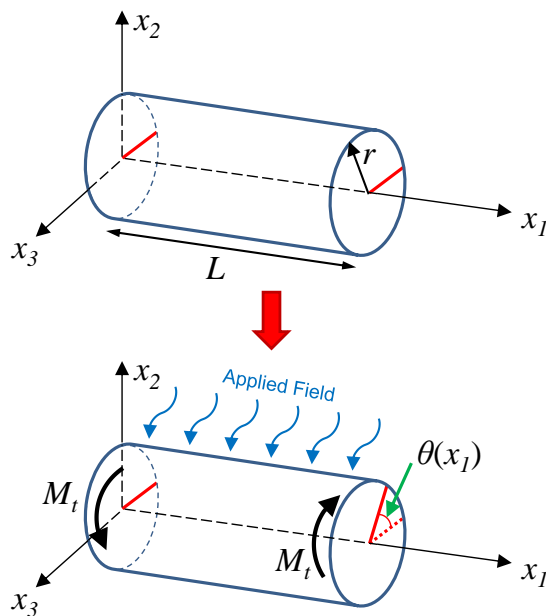


Figure A1. Local twisting of an active circular bar under the application of a locally uniform field.

The centroidal axis of the bar coincides with the x_1 -axis. By the rotational symmetry of the problem, it is reasonable to assume that the motion of each cross-sectional region with thickness dx_1 is restricted to a rigid body rotation about the x_1 -axis [248]. From equilibrium, in the absence of body forces, it can be found that the local twist angle θ varies linearly with x_1 ; therefore, $d\theta/dx_1 = \text{constant} = \theta'$. Under such displacement assumption, the only nonzero strains, which are due to elastic strains (denoted by superscript “*el*”) and mechanism-agnostic *actuation strains* (denoted by superscript “*act*”) are given by:

$$\varepsilon_{12}^{el} + \varepsilon_{12}^{act} = -\frac{1}{2}x_3\theta', \quad \varepsilon_{13}^{el} + \varepsilon_{13}^{act} = \frac{1}{2}x_2\theta'. \quad (\text{A.1})$$

The corresponding nonzero stress components are:

$$\sigma_{12} = G \left(-\frac{1}{2}x_3\theta' - \varepsilon_{12}^{act} \right), \quad \sigma_{13} = G \left(\frac{1}{2}x_2\theta' - \varepsilon_{13}^{act} \right) \quad (\text{A.2})$$

where G is the shear modulus of the active material bar, assumed to be constant. To obtain bounds for an actuation stress and strain assessment, two different cases are explored. To relate actuation strain to the folding performance of the torsional element, it is assumed that the magnitude of the actuation shear strain far exceeds that of the elastic shear strain due to the choice of active material and/or the assumption of relatively small resistive moments at the fold ($\sigma_{12} = \sigma_{13} = 0$). Considering either equation A.1 or A.2 under this assumption:

$$\varepsilon_{12}^{act} = -\frac{1}{2}x_3\theta', \quad \varepsilon_{13}^{act} = \frac{1}{2}x_2\theta'. \quad (\text{A.3})$$

The magnitude of the maximum principal actuation strain ε^{act} at any point in the bar under the strain state given in equation A.3 is:

$$\varepsilon^{act} = \frac{1}{2}\theta'(x_2^2 + x_3^2)^{1/2}. \quad (\text{A.4})$$

The maximum value of ε^{act} will occur when $\sqrt{(x_2^2 + x_3^2)} = r$. Therefore, the limit of the change in angle per unit length the bar axis θ' multiplied by r is given by

$$r\theta' = 2\varepsilon^{act}. \quad (\text{A.5})$$

The total twist angle φ (that is expected to translate into a folding angle) is obtained by integrating θ' along the entire length of the bar

$$\varphi = \int_0^L \theta' dl = \theta' \int_0^L dl = \theta' L. \quad (\text{A.6})$$

By substituting equation A.5 into A.6, the following dimension-normalized relation between ε^{act} and the total twist angle φ is obtained

$$\frac{r}{L}\varphi = 2\varepsilon^{act}. \quad (\text{A.7})$$

To obtain a bound for the assessment of actuation stress, the case in which the sheet is fully constrained from folding is considered. In this case $\theta(x_1) = \theta' = 0$. Using equations A.1 and A.2 the following is obtained:

$$\sigma_{12} = G(-\varepsilon_{12}^{act}) = G\varepsilon_{12}^{el}, \quad \sigma_{13} = G(-\varepsilon_{13}^{act}) = G\varepsilon_{13}^{el}. \quad (\text{A.8})$$

The preceding equation corresponds to the elastic solution of the problem. The relations between the stresses and the applied moments are given as follows [248]:

$$\sigma_{12} = -\frac{M_t x_3}{I_p}, \quad \sigma_{13} = \frac{M_t x_2}{I_p}, \quad (\text{A.9})$$

where I_p is the polar second moment of the circular cross-section ($I_p = \pi r^4/2$). The maximum normal or shear stress due to actuation σ^{act} occurs at the boundary of the bar and are related to the reaction moment M_t by [248]:

$$\frac{M_t}{r^3} = \frac{\pi \sigma^{act}}{2}. \quad (\text{A.10})$$

Equations A.7 and A.10 provide an assessment of the folding characteristics of concept (b) (twist angle, obtained moment) as function of actuation stress and strain of the active material.

Appendix B. Analysis of Folding Using an Active/Passive Bilayer

For the analysis of the multi-layer self-folding concept (figure 5d), bending of an active-passive bilayer sheet is considered. The base analytical solution for this case is obtained from Timoshenko [249]. In 1925, Timoshenko analyzed bending a bilayer sheet due to unequal thermal expansion. The bilayer sheet is depicted in figure B1. The sheet of total thickness h has a layer of material I with thickness a_1 and a layer of material II with thickness a_2 so that $a_1 + a_2 = h$. The width of sheet is denoted as w .

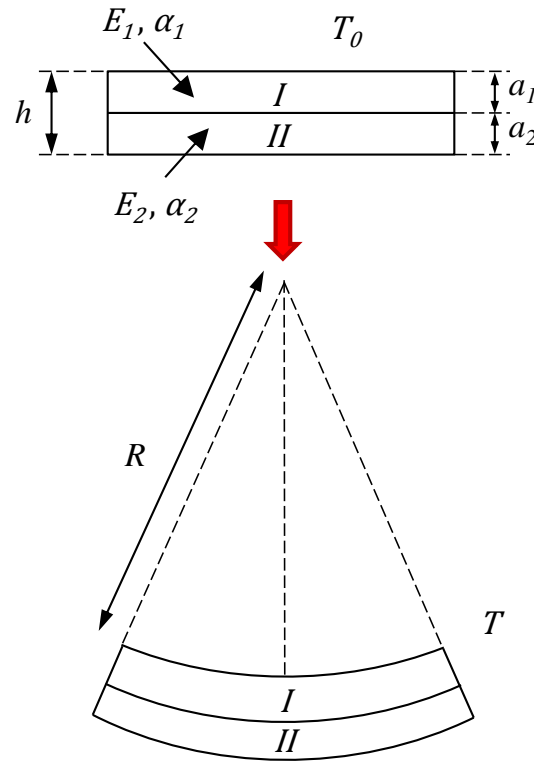


Figure B1. Bending of a bilayer sheet under unequal field-induced expansion.

Material I has an elastic modulus and thermal expansion coefficient of E_1 and α_1 , respectively. Similarly, material II has an elastic modulus and thermal expansion coefficient of E_2 and α_2 , respectively. When the bilayer film is uniformly heated from temperature T_0 to temperature T , it bends to a configuration with radius of curvature R due to unequal thermal expansion of the constituent layers (i.e., when $\alpha_1 \neq \alpha_2$). The

relationship between the obtained radius of curvature R and the geometric parameters and material properties of the film was found by Timoshenko [249] to be

$$\frac{1}{R} = \frac{6(\alpha_2 - \alpha_1)(T - T_0)(1 + m)^2}{h \left(3(1 + m)^2 + (1 + mn)(m^2 + 1/(mn)) \right)}. \quad (\text{B.1})$$

where $m = a_1/a_2$ and $n = E_1/E_2$. Here we consider material I as the active material while material II is the passive material. In equation B.1 the eigenstrain is assumed to be due to thermal expansion of the constituent layers; however, the end state of such a bilayer beam would be the same regardless of the origin of the eigenstrain (neglecting out-of-plane effects). Thus we use the following simple substitution

$$\alpha_2 \approx 0, \quad \varepsilon^{act} \equiv -\alpha_1(T - T_0), \quad (\text{B.2})$$

where ε^{act} is the actuation strain. The first expression in equation B.2 emerges from the assumption that the passive material provides negligible strain under the applied field. The second expression implies that the active layer is contracting with a strain of ε^{act} . When substituting the expressions in equation B.2 into equation B.1, the following is obtained[¶]

$$\frac{1}{R} = \frac{6\varepsilon^{act}(1 + m)^2}{h \left(3(1 + m)^2 + (1 + mn)(m^2 + 1/(mn)) \right)}. \quad (\text{B.3})$$

The radius of curvature normalized with the film thickness is then given as follows

$$\frac{R}{h} = \frac{3(1 + m)^2 + (1 + mn)(m^2 + 1/(mn))}{6\varepsilon^{act}(1 + m)^2}. \quad (\text{B.4})$$

To simplify the preceding equation, the constant C_1 , defined in equation 3, is substituted into equation B.4 and the following is obtained

$$\frac{R}{C_1 h} = \frac{1}{\varepsilon^{act}}. \quad (\text{B.5})$$

Equation B.5 provides a non-dimensional analytical expression for the assessment of the folding behavior of a bilayer film with a layer of active material and a layer of passive material. Clearly, as actuation strain increases, the radius of curvature of the bent sheet decreases.

For an assessment of actuation stress σ^{act} , the maximum stress experienced (σ^{max}) by the active layer is found. An expression for the maximum stress was also found by Timoshenko [249] and it consists of two parts: the stress due to axial force and the stress due to bending. It is given by

$$\sigma^{act} = \sigma^{max} = \frac{P}{wa_1} + \frac{a_1 E_1}{2R}, \quad (\text{B.6})$$

[¶] Assuming expansion of the active layer will change the sign of R (meaning that the final configuration is convex down) but the magnitude of R will be unaffected.

where P is the total axial force in the active layer. It can be shown [249] that the total bending moment M_b relates to P and R as follows:

$$P = \frac{2M_b}{h}, \quad R = \frac{E_1 I_1 + E_2 I_2}{M_b} = \frac{w(E_1 a_1^3 + E_2 a_2^3)}{12M_b}, \quad (\text{B.7})$$

where I_1 and I_2 are the moments of inertia of the cross-sections of layer I and II, respectively. The term $E_1 I_1 + E_2 I_2$ represents the effective bending stiffness of the bilayer. The moments of inertia were substituted in equation B.7 using the result $I_i = wa_i^3/12$ where $i = 1, 2$. Substituting the results from equation B.7 into equation B.6 the following is obtained

$$\sigma^{act} = \frac{M_b}{wha_1} \left(2 + \frac{6ha_1^2 E_1}{E_1 a_1^3 + E_2 a_2^3} \right). \quad (\text{B.8})$$

To simplify the form of equation B.8, the constant C_2 , defined in equation 4, is substituted into equation B.8 and the following relation between actuation stress and bending moment is obtained:

$$\sigma^{act} = \frac{M_b C_2}{wha_1}. \quad (\text{B.9})$$

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