Simplified computational routine to correct the modal decoupling in transmission lines and power systems modelling

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Abstract: Modal analysis is widely approached in the classic theory of power systems modelling. This technique is also applied to model multiconductor transmission lines and their self and mutual electrical parameters. However, this methodology has some particularities and inaccuracies for specific applications, which are not clearly described in the technical literature. This study provides a brief review on modal decoupling applied in transmission line digital models and thereafter a novel and simplified computational routine is proposed to overcome the possible errors embedded by the modal decoupling in the simulation/modelling computational algorithm.

1 Introduction

The Electromagnetic Transient Program (EMTP) is one of the most important tools for transmission line design, study of insulation coordination and development of electric power components and systems. In 1960s, Dommel published several papers which many years after originated the EMTP [1, 2], since then many authors have contributed with improvements and new computational models, resulting in the current versions of the EMTP. Nowadays, many versions are available, such as the commercial software Microtran, developed in the University of British Columbia, Canada. Another well-known commercial version of the EMTP is the PSCAD, developed jointly by the Manitoba Hydro (Manitoba HVDC Research Centre) and the University of Manitoba. An open-source version of the EMTP is available as well: the Alternative Transient Program (ATP). The constant evolution of the computational resources results in the Real-Time Digital Simulators (RTDS), which represent the actual state of the art in computational tools to study electromagnetic transients in power systems.

This great diversity of tools to simulate and to project electric power systems has been possible because of the numerous contributions from several institutions, universities and researches around the world. In fact, these new contributions have been continuously proposed which means that new proposals for further improvements on power systems modelling are always in the state of the art. Based on this statement, this paper proposes an efficacious procedure to improve the accuracy of multiconductor transmission line models to simulate electromagnetic transients.

The explicit or intrinsic use of modal analysis techniques is widely considered to model three-phase power systems and the electromagnetic coupling among the line phases. Thus, successive transformations between phase and mode domains are carried out through several computational routines in the EMTP; that is the line parameters, voltages and currents are successively converted between the phase and mode domains [3]. The alternation between phase and mode values is performed using modal transformation matrices, which are usually frequency-dependent since the transmission line parameters are also variable with the frequency because of the skin effect in the wires and the soil effect [4–6]. However, depending of the line geometry and system characteristics, a real and constant modal transformation matrix can be used [7].

The line models available in the EMTP are basically implemented based on distributed or lumped parameters [1, 2, 8, 9]. The first type is directly developed from the frequency-dependent line parameters, the time-domain simulations are carried out using inverse transforms and convolutions [10]. The multiconductor line models by distributed parameters are usually based on a frequency-dependent transformation matrix and provide accurate results. However, these models have several restrictions to model time-variable and nonlinear components, for example most of the power devices and electromagnetic phenomena. These elements modelling is well-known in the time domain, but their representations in the frequency domain are complex and most often impracticable [8, 9]. On the other hand, models based on lumped parameters are developed directly in the time domain which represents a great advantage on the distributed-parameters line models. Time-domain line models
usually require the use of real and constant transformation matrices for the successive phase-mode transformations in the simulation process [8, 11, 12]. However, the approach using real and constant matrices is not an accurate method when applied to simulate electromagnetic transients composed of a wide range of frequencies [3, 4]. In fact, this characteristic represents one of the main restrictions of line models based on lumped parameters.

First, from the qualitative descriptions given above, this paper presents a brief review of modal analysis theory applied for multiconductor transmission line modelling. Thereafter, a simple variation is proposed for the use of the transformation matrices in the modal transformation algorithm to correct the errors intrinsic to the modal decoupling of the line, without the handling of matrix elements and complex correction processes [13].

2 Three-phase transmission line modelling using modal decoupling

In the modal domain, a multiconductor transmission line can be represented by its propagation mode, that is the n phases of the line are decoupled into n-independent propagation modes. This procedure is the conventional method usually applied to model multiconductor lines for electromagnetic simulations. Thus, a transmission line with n phases can be decoupled into n exact propagation modes and each mode can be modelled as a single-phase line. This simplification is the most usual method to model symmetrical and untransposed transmission systems, where the complex representation of the mutual parameters and electromagnetic coupling between phases can be easily solved [4].

Initially, the basic equations of a transmission line are introduced [4]

$$\frac{d^2[V_{ph}]}{dx^2} = [Z][Y][V_{ph}]; \quad \frac{d^2[Y]}{dx^2} = [Y][Z][I_{ph}] \quad (1)$$

In (1), [Z] and [Y] are per unit of length (p.u.l.) longitudinal impedance and shunt admittance matrices of the line, respectively. The vectors [V_{ph}] and [I_{ph}] are the transversal voltages and the longitudinal currents expressed in the modal domain, respectively. These equations are valid for complex representation of sinusoidal alternating electrical magnitudes and considering a quasi-stationary behaviour of the electromagnetic field in the orthogonal direction to line axis [4].

To solve (1) for an n-phase transmission line, the coupled equations in (1) are decoupled in n decoupled equations. The decoupling can be achieved using a modal transformation matrix [T_{j}^T] to convert the matrix product [Y][Z] to a diagonal form [3]

$$[T_{j}^T][Y][Z][T_{j}] = [\lambda_j] \quad (2)$$

where [\lambda_j] is the diagonal matrix composed of the eigenvalues of [Y][Z]. This means that the columns of [T_{j}] are autovectors related to the matrix operation [Y][Z] as well.

After some matrix manipulations and substituting (2) into (1), widely covered in the reference literature, the basic equations can be written in the mode domain as follows [3, 4]

$$\frac{d^2[V_{m}]}{dx^2} = [Z_m][Y_m][V_{m}]; \quad \frac{d^2[Y_m]}{dx^2} = [Y_m][Z_m][I_m] \quad (3)$$

In (3), the vectors [Y_{m}] and [I_{m}] are the transversal voltages and the longitudinal currents expressed in the modal domain, respectively. Terms [Z_{m}] and [Y_{m}] are the p.u.l. longitudinal impedance and the p.u.l. shunt admittance diagonal matrices, respectively, where the main diagonal is composed of inductances and admittances of the modal components a, b and zero. The relationship between phase and modal values are [3, 4]

$$[Z_{m}] = [T_{j}^T][Z][T_{j}] \quad (4)$$

$$[Y_{m}] = [T_{j}^{-1}][Y][T_{j}] \quad (5)$$

$$[V_{m}] = [T_{j}^T][V_{ph}] \quad (6)$$

$$[I_{m}] = [T_{j}^{-1}][I_{ph}] \quad (7)$$

In (4)-(7), [T_{j}^T] is the transposed matrix of [T_{j}], whereas [T_{j}]^{-1} is the inverse matrix of [T_{j}]. The step by step calculation of the frequency-dependent matrix [T_{j}] using the Newton–Raphson method is widely described in [4]. After the phases decoupling, the n-phase line can be represented by its n-exact propagation modes and each modal component can be represented as a single-phase transmission line, modelled by lumped or distributed parameters.

Concerning a transposed transmission line or then an untransposed system, but with vertical symmetry plane, a real and constant matrix can be used as a transformation matrix. This approach is very useful for models developed directly in the time domain [11–13]. A real and constant matrix conventionally used to model transmission systems is the well-known Clarke’s matrix

$$[T_{ck}] = \begin{bmatrix} 2/\sqrt{6} & 0 & 1/\sqrt{3} \\ -1/\sqrt{6} & 1 & 1/\sqrt{3} \\ -1/\sqrt{6} & -\sqrt{2} & 1/\sqrt{3} \end{bmatrix} \quad (8)$$

For an untransposed three-phase transmission line with vertical symmetry plane, the p.u.l. line impedance [Z] and admittance [Y] has the following structures

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12} & Z_{22} & Z_{23} \\ Z_{13} & Z_{23} & Z_{33} \end{bmatrix}; \quad [Y] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12} & Y_{22} & Y_{23} \\ Y_{13} & Y_{23} & Y_{33} \end{bmatrix} \quad (9)$$

As described in (9), there is symmetry in the elements below and above the main diagonal because of the symmetrical vertical plane of the transmission system. Therefore using the Clarke’s matrix as the modal transformation matrix in (4) and (5)

$$[Z_{m}] = [T_{ck}^T][Z][T_{ck}] \quad (10)$$

$$[Y_{m}] = [T_{ck}^{-1}][Y][T_{ck}]^{-1} \quad (11)$$

From (10) and (11), the modal impedance and admittance
matrices are described as

$$[Z_m] = \begin{bmatrix}
Z_\alpha & 0 & Z_{\alpha 0} \\
0 & Z_\beta & 0 \\
Z_{\alpha 0} & 0 & Z_0
\end{bmatrix}$$  \hfill (12)

$$[Y_m] = \begin{bmatrix}
Y_\alpha & 0 & Y_{\alpha 0} \\
0 & Y_\beta & 0 \\
Y_{\alpha 0} & 0 & Y_0
\end{bmatrix}$$  \hfill (13)

In (12) and (13), the $\beta$-component is an exact mode because it is completely decoupled from other modes. The same statement is not true for $\alpha$ and zero components, as show the mutual remaining terms $Z_{\alpha 0}$ and $Y_{\alpha 0}$. The modes $\alpha$ and zero are known as quasi-modes [3].

In Ref. [3], a thorough study of modal analysis applied for transmission line modelling is presented. This referred study showed that the mutual remaining terms, resulted from the decoupling procedure using Clarke’s matrix, can be neglected without significant inaccuracies; that is when the mutual terms $Z_{\alpha 0}$ and $Y_{\alpha 0}$ are null, the quasi-modes $\alpha$ and zero can be considered as exact propagation modes, such as mode $\beta$. Therefore $[Z_m]$ and $[Y_m]$ are diagonal matrices and modelling/simulation procedure can be illustrated by the following scheme.

Fig. 1 shows that the transmission line is decoupled into its exact propagation modes, which are modelled as a function of their frequency-dependent parameters, as indicated by the blocks indicated by the modes $\alpha$, $\beta$ and zero. The electrical values at phases 1, 2 and 3 at sending end of the line are converted to the modal domain, all simulation process is performed in the modal domain, considering each propagation mode as a single-phase line and thereafter the results are converted back to the phase domain, where the output voltages and currents can be properly analysed. Thus, it is possible to observe that there are at least three successive modal transformations through the modelling/simulation process.

The modes $\alpha$, $\beta$ and zero, described in Fig. 1, are conventionally represented as an equivalent two-port circuit ($ABCD$ matrix) as a function of their respective propagation functions $\gamma_m$ and characteristic impedances $Zc_m$. The subscript $m$ is referring the modes $\alpha$, $\beta$ and zero (Fig. 2).

The terms $V_{Am}$ and $V_{Bm}$ are voltages at sending and receiving ends of a given propagation mode $m$, respectively. Terms $I_{Am}$ and $I_{Bm}$ are the modal currents at sending and receiving ends, respectively. The propagation function and the characteristic impedance are expressed as follows

$$\gamma_m = \sqrt{Z_m Y_m}; \quad Zc_m = \sqrt{Z_m Y_m}$$  \hfill (14)

where $Z_m$ and $Y_m$ are the modal impedance and admittance, respectively.

The two-port expressions for the voltages and the currents at both terminals are given as follows

$$V_{Am} = V_{Bm} \cos h(\gamma_m \ell) - Zc_m I_{Bm} \sin h(\gamma_m \ell)$$  \hfill (15)

$$I_{Am} = -I_{Bm} \cos h(\gamma_m \ell) + \frac{V_{Bm}}{Zc_m} \sin h(\gamma_m \ell)$$  \hfill (16)

The constant $\ell$ is the line length.

A general matrix expression for (15) and (16) can be given in the $ABCD$ matrix representation for the modes $\alpha$, $\beta$ and zero as follows (see (17))

The time-domain expressions of (15) and (16) are obtained by inverse Laplace transforms, resulting in the following convolution integrals [14]

$$i_{Am}(t) = \int_{-\infty}^{\infty} y_{AA}(t-\tau) v_{Am}(\tau) \, d\tau + \int_{-\infty}^{\infty} y_{AB}(t-\tau) v_{Bm}(\tau) \, d\tau$$  \hfill (18)

$$i_{Bm}(t) = \int_{-\infty}^{\infty} y_{BA}(t-\tau) v_{Am}(\tau) \, d\tau + \int_{-\infty}^{\infty} y_{BB}(t-\tau) v_{Bm}(\tau) \, d\tau$$  \hfill (19)

$$\begin{bmatrix}
V_{\alpha A} \\
I_{\alpha A} \\
V_{\alpha B} \\
I_{\alpha B}
\end{bmatrix} = \begin{bmatrix}
\cosh(\gamma_\alpha \ell) & -Zc_\alpha \sinh(\gamma_\alpha \ell) & 0 & 0 & 0 \\
\frac{1}{Zc_\alpha} \sinh(\gamma_\alpha \ell) & -\cosh(\gamma_\alpha \ell) & 0 & 0 & 0 \\
0 & 0 & \cosh(\gamma_\beta \ell) & -Zc_\beta \sinh(\gamma_\beta \ell) & 0 & 0 \\
0 & 0 & \frac{1}{Zc_\beta} \sinh(\gamma_\beta \ell) & -\cosh(\gamma_\beta \ell) & 0 & 0 \\
0 & 0 & 0 & 0 & \cosh(\gamma_0 \ell) & -Zc_0 \sinh(\gamma_0 \ell) \\
0 & 0 & 0 & 0 & \frac{1}{Zc_0} \sinh(\gamma_0 \ell) & -\cosh(\gamma_0 \ell)
\end{bmatrix} \begin{bmatrix}
V_{\beta A} \\
I_{\beta A} \\
V_{\beta B} \\
I_{\beta B}
\end{bmatrix}$$  \hfill (17)
where \( i_m \) and \( v_m \) express the currents and voltages in the time domain at the sending end (terminal \( A \)) and at the receiving end (terminal \( B \)). The input and transfer admittances are expressed as follows

\[
y_{AA} = y_{BB} = Z c_m \cot h(y_m \ell) \\
y_{AB} = y_{BA} = -Z c_m \csc h(y_m \ell)
\]  

(20) 

The term \( y_{AA} \) is the input admittance seen at the sending end whereas the receiving end is shorted; the same procedure is applied to obtain the admittance \( y_{BB} \). The term \( y_{AB} \) is the transfer admittance between terminals \( A \) and \( B \) considering the terminal \( A \) shorted, analogously for \( y_{BA} \).

3 Conventional algorithm to simulate electromagnetic transients on multiconductor transmission lines using modal decoupling

In Fig. 3, the conventional electromagnetic transient simulation routine is introduced.

Usually, most of the EMTP models use the Clarke’s matrix as a transformation matrix through the entire computational process. This procedure is not totally correct and the possible errors from this conventional routine are further evaluated.

The first step 1a represents the calculation of the line mutual and self-parameters in the frequency domain as a function the earth return impedance and the skin effect on the wires [5, 6]. As a second procedure indicated by step 1b, the phase values of voltage and current are usually converted to the mode domain from (6) and (7) using the Clarke’s matrix, respectively. Thus, the input voltages and currents at the sending ends of the modes \( \alpha, \beta \) and zero, in the step 1b, are calculated as follows

\[
[V_{\alpha m}] = [T_{ck}]^{-1}[V_{\alpha ph}] \\
[I_{\alpha m}] = [T_{ck}]^{-1}[I_{\alpha ph}]
\]  

(22) 

(23) 

Vectors \([V_{\alpha m}]\) and \([I_{\alpha m}]\) are composed of the voltages and currents at the sending end of the propagation modes \( \alpha, \beta \) and zero. Vectors \([V_{\alpha ph}]\) and \([I_{\alpha ph}]\) are voltages and currents at the sending end of the phases 1, 2 and 3 (as described in Fig. 1).

The second stage is the line parameters decoupling into the modes \( \alpha, \beta \) and zero, based on (10) and (11), using the Clarke’s matrix. The resulting matrices \([Z_{\alpha}]\) and \([Y_{\alpha}]\), expressed in (12) and (13), are not completely decoupled because of the remaining quasi-modes \( Z_{ck0} \) and \( Y_{ck0} \). Although, both terms are conventionally neglected and matrices \([Z_{\alpha}]\) and \([Y_{\alpha}]\) can be considered as exact matrices. From this way, the third step consists in the modelling of propagation modes as three independent single-phase lines using conventional models, available or not in the EMTP [8–12]. Following the propagation modes modelling, the input signals obtained in step 1b are considered in step 4b to simulate the electromagnetic transients in the modal domain. The output signals in the modal domain (step 5) are then converted back to the phase domain at the receiving end (terminal \( B \)) of the line, also based on expressions (22) and (23)

\[
[V_{\beta ph}] = [T_{ck}]^{-1}[V_{\beta m}] \\
[I_{\beta ph}] = [T_{ck}]^{-1}[I_{\beta m}]
\]  

(24) 

(25) 

The sum of all these steps represents the full routine to simulate electromagnetic transients for most of the well-established multiconductor transmission line models. Although, it is important to emphasize that the computational routine in Fig. 3 is susceptible to two types of inaccuracies: errors from the line representation and errors from the intrinsic modal transformations.

The errors from the line representation are related to the modelling of the electrical parameters of the line. The approaches by lumped or distributed parameters are subject to truncation errors and inaccuracies because of the variables/functions discretisation. Concerning the line represented by lumped elements, the continuous functions related to the line electrical parameters are discretised as a function of the line length. On the other hand, when a distributed-parameter model is considered, the continuous function \( \omega \) is discretised by a constant or variable frequency interval. These oscillations are totally or partially solved using window functions [14]. The second type of error is because of the approach using transformation matrices. The use of a real and constant transformation matrix is especially required for models implemented direct in the time domain, using fitting techniques to represent the frequency-dependent parameters of the line and propagation modes, for example models referred in [11, 12], where the line parameters are directly fit in the time domain using lumped electric-circuit elements [8]. The proposed correction method is particularly developed to solve this second type of inaccuracy.
4 Evaluation of the intrinsic errors using Clarke’s matrix

The use of the Clarke’s matrix as a modal transformation matrix is widely applied to model multiconductor transmission systems directly in the time domain. The successive modal transformations in the simulation procedure are highlighted in the diagram of Fig. 3 by the grey blocks. Conventionally, the approach using Clarke’s matrix is extended to steps 1b, 2 and 5. This paper proposes an alternation between the use of the frequency-dependent modal transformation matrix (exact matrix) and the Clarke’s matrix in order to eliminate the errors occurred from the approach using real and constant matrices.

As discussed before, the line decoupling using the Clarke’s matrix results in the line quasi-modes $\alpha$ and zero and in the exact mode $\beta$. The remaining mutual coupling/parameters between modes $\alpha$ and zero are then neglected. This approach leads to errors in the line decoupling, especially at high frequencies, which are measured and further discussed. To evaluate the errors related to the routine using the approach by the Clarke’s matrix, two distinct algorithms are considered based on the diagram in Fig. 3. The first algorithm takes into account the use of the Clarke’s matrix in steps 2, 1b and 5, exactly as described in Fig. 3. The second routine is implemented considering the exact frequency-dependent matrix at the same steps. This last routine is taken as reference to measure the errors resulted from the first routine, using the Clarke’s approach. Firstly, a frequency-domain analysis is performed based on the two-port representation of each propagation mode of the three-phase line and thereafter a time-domain transient analysis is presented using the Universal Line Model (ULM) [10, 14], considering a step impulse and a unitary impulse as input signals. This procedure represents a wide-frequency scan, showing the main time-domain variations from the modelling using the Clarke’s matrix compared with the modelling using the frequency-dependent exact matrix. The reference routine, characterised by the use of the exact frequency-dependent matrix at all stages of the computational process, is described in details in Fig. 4.

The geometrical and physical characteristics of a real transmission system, given by an untransposed 440-kV line, are taking into account to perform the proposed frequency/time analyses [11].

4.1 Frequency-domain analysis

As shown in both algorithms (Figs. 3 and 4), the first step is the calculation of the frequency-dependent parameters of the line. Thereafter, the line decoupling is carried out by distinct procedures using the Clarke’s matrix and using the frequency-dependent transformation matrix. Based on the matrices $[Z_m]$ and $[Y_m]$ obtained from both decoupling procedures, the propagation characteristics represented by the characteristic impedance of each mode $Z_m$ and the propagation function $\gamma_m$ can be calculated using the expressions given in (14). Thus, the modes $\alpha$, $\beta$ and zero are calculated and compared based on the line decoupling using both modal transformation matrices. In the following analyses, the conventional procedure using the Clarke’s matrix (Fig. 3) is called the ‘Clarke’s approach’ and the algorithm using the exact transformation matrix is called as ‘reference routine’.

The propagation functions $\gamma_\alpha$ and $\gamma_0$ are calculated from the modes $\alpha$ and zero, respectively, applying the modal decoupling in the second step of the algorithm using the Clarke’s matrix (10) and (11) and the exact matrix (4) and (5). With the variations obtained in the results, a relative error is calculated measuring the differences in the attenuation (real component of the complex $\gamma_m$) and phase delay (imaginary component of the complex $\gamma_m$) associated with the propagation functions of each propagation mode. The errors intrinsic to the Clarke’s approach related to the reference routine, for the attenuation and for the phase delay, are then calculated as follows

$$E_{\text{att}} = \frac{\text{Re}(\gamma_{m})_{\text{ck}}}{\text{Re}(\gamma_{m})_{\text{ref}}}$$

$$E_{\text{delay}} = \frac{\text{Im}(\gamma_{m})_{\text{ck}}}{\text{Im}(\gamma_{m})_{\text{ref}}}$$

Terms $E_{\text{att}}$ and $E_{\text{delay}}$ are the relative errors of the attenuation and phase delay, respectively. In (25), $\text{Re}(\gamma_{m})_{\text{ck}}$ and $\text{Re}(\gamma_{m})_{\text{ref}}$ are the real part of the complex propagation function of the mode $m$ calculated from the Clarke’s approach and the reference routine, respectively. In (26), $\text{Im}(\gamma_{m})_{\text{ck}}$ and $\text{Im}(\gamma_{m})_{\text{ref}}$ are the imaginary part of the complex propagation function of the mode $m$ obtained from the Clarke’s approach and the reference routine, respectively.

As described before, the mode $\beta$ is an exact mode independently of the modal transformation matrix used in the line decoupling. Thus, the parameters calculated for the mode $\beta$, using both decoupling procedures, are similar, that is there are not variations in $\gamma_\beta$ from modal decoupling using Clarke’s matrix or the exact matrix [3, 7].
Relative errors $E_{att}$ and $E_{delay}$, associated with the propagation function $\gamma_\alpha$, are described in Fig. 5 up to 100 MHz.

The errors intrinsic to the line decoupling in the mode zero are also evaluated in terms of attenuation and phase delay as follows (Fig. 6).

An interesting behaviour is observed in Figs. 5 and 6: the more expressive variations are observed in the attenuation factor of the propagation function associated with the mode $\alpha$, especially for frequencies above 10 kHz, as shown in Fig. 5. On the other hand, the phase delay of the propagation functions $\gamma_\alpha$ and $\gamma_\beta$ have variations up to 1%. The attenuation of $\gamma_\alpha$, in Fig. 6, shows also a discrete variation up to 1%. This means that the main error in the use the Clarke’s matrix as a modal transformation matrix is in the attenuation of the propagation mode $\alpha$. In fact, the attenuation is more accentuated to the modal decoupling using the Clarke’s matrix, which means that probably major errors using this approach will be highlighted in simulations considering fast and impulsive signals, composed of a wide range of frequencies, far above 10 kHz. These statements can be properly verified in the next time-domain evaluations.

### 4.2 Time-domain analysis

According with results presented in the last section, major errors were verified in the attenuation of the mode $\alpha$ at frequencies above 10 kHz. To verify what these frequency-domain variations mean in the time domain, simulations were carried out using the ULM based on two distinct input voltage signals: a unitary step and an impulse [10]. The first input signal is characterised by low frequencies, mainly up to 100 Hz, whereas the unitary impulse represents a wide range of frequencies. An illustration describing the line modelling and simulation process with the intrinsic phase/mode/phase transformations as in Fig. 1, is presented as follows:

The line configuration and simulation process in Fig. 7 represent an open-circuit test, where the receiving end of the line is open or characterised by a high-impedance load. The sending end of phases 2 and 3 are shorted and the sending end of phase 1 is connected to a voltage source $U(\omega)$. The two blocks indicated by $[T]$ represent the phase/mode/phase transformations. Emphasizing that the input signal, represented by $U(\omega)$ and the switch $S$, is in the frequency domain, whereas the output voltages at the phases $V_1(t)$, $V_2(t)$ and $V_3(t)$ are in the time domain and can be also obtained from (24), in the vector $[V_{ph}]$, associated with the time-domain voltages at the receiving end of the line.

First, the voltage source $U(\omega) = 1/\omega$ and the switch $S$ represent a step voltage signal at the sending end of phase 1. The transient voltage at the receiving end of phase 1 is shown in Fig. 8, considering the Clarke’s approach and the reference routine.

Curves 1 and 2 in Fig. 8 are approximated; this behaviour was expected accounting that major variations in the frequency domain were previously observed at frequencies above 10 kHz and emphasizing that a step signal is basically composed of frequencies up to 100 Hz.

As a next analysis, an unitary impulse ($U(\omega) = 1$) is applied at the sending end of the phase 1, also considering the modal transformations using the Clarke’s approach and the reference routine. The output signals at the receiving end of the phase 1, considering both representations, are described in Fig. 9.

An abrupt voltage peak is observed in the result obtained from the reference routine, more than 30% above the voltage peak obtained using the Clarke’s approach. These variations reflect the characteristics previously verified in...
at high frequencies, as analysed in Fig. 

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Fig. 9 Transient response at the receiving end of phase 1 from an impulse voltage signal: reference routine (curve 1) and Clarke’s approach (curve 2)

the frequency-domain attenuation associated with the propagation mode \( \alpha \) at high frequencies, as analysed in Fig. 5.

5 Simplified routine to correct the errors associated with the modal decoupling

As discussed in the introduction, there are several well-established transmission line models based on modal analysis techniques using the approach by the Clarke’s matrix. Although this subject-matter has been studied over a few decades, alternative modelling techniques have been even more discussed over the last years to improve the current computational models or to propose new equivalent representations \([7, 13, 15]\). There are efficient methods, however, they are sometimes very complex, unstable and have a hard computational processing, for example, the procedure given in \([13]\). The mentioned method is based on the correction of the eigenvectors for each discrete frequency, which are equivalently approximated by the constant and real columns of the Clarke’s matrix. It presents a reasonable performance to various cases, although, for practical situations, the successive eigenvectors corrections become difficult, computationally costly and unstable.

The features mentioned above motivate a proposal of a more simplified routine. Firstly, the routine stage where the errors are embedded in the algorithm was detected. In sequence, an exact correction is applied only in this stage, without unnecessary modifications in other stages of the algorithm.

The basic algorithm, described by the structure of the routines presented in Figs. 3 and 4 (Clarke’s approach and reference routine), can be separated into two main parts. The first stage refers to the modal decoupling of the line parameters (step 2 in Figs. 3 and 4). The second stage is the conversion of the voltage and current values from the phase domain to the modal domain (step 1b in Figs. 3 and 4) followed by the inverse transformation given at the last step of the main algorithm (step 5 in Figs. 3 and 4), when the output signals are obtained in the phase domain. From this simple division, the errors can be isolated only into the first stage, related to the line parameters decoupling, and then easily solved by a simple alternation in the use of the modal transformation matrix.

The simplified correction consists basically in use of the frequency-dependent transformation matrix only in the line modal decoupling (step 2 in Figs. 3 and 4), which represents the first stage of the algorithm, as previously discussed. The following steps of the algorithm, representing the second stage prior determined, are carried out using the Clarke’s matrix, that is, the remaining steps indicated by 1b and 5 in the diagrams in Figs. 3 and 4. This idea seems at a first sight to be intuitive and trivial; however, the most classical references on transmission line theory and power system modelling do not provide an exact statement about this point or even where the errors from the successive modal transformations are inserted in the simulation algorithm \([1, 2, 6, 8, 9]\).

The proposed routine, alternating the use of Clarke’s and exact matrices, is described by a new block diagram (Fig. 10): The diagram presented in Fig. 10 describes the proposed algorithm in details. The steps are divided into two stages, as delimited in the referred figure. Emphasizing that only the first stage uses the exact transformation matrix while the second stage, describing the line modelling and simulation, was given by using the Clarke’s matrix.

The same simulations performed in the last section are performed from the reference routine, in which the exact matrix is applied for all modal transformations, and considering the proposed algorithm, detailed in Fig. 10. Firstly, a step voltage signal applied at the sending end of phase 1, as carried out for the prior simulations, and the output transients at the receiving end of phase 1 are obtained as followed in Fig. 11.

The dotted curve indicated by 1 is the transient simulated using the proposed algorithm. Curve 2 is the output obtained from the reference routine (the same result prior presented in Fig. 8). Thus, it is possible to verify that curves 1 and 2 are graphically overlapped, which validates the proposed correction procedure firstly for low frequencies.

![Fig. 10 New computational algorithm for three-phase representation, line modelling and simulation using modal transformations](image-url)
From the same way as in Section 4.2, a time-domain analysis is carried out comparing results obtained from the proposed algorithm and from the reference routine with an input signal represented by an unitary impulse. Conceptually, this type of signal is composed of a very wide range of frequencies, which enables an evaluation of the proposed correction procedure/algorithm from low up to very high frequencies, as a full scan of frequencies in the time domain. The same procedure is carried out applying an unitary voltage impulse at the sending end of phase 1 and the voltage transient is obtained at the receiving end of the same phase as followed in Fig.12:

The curves 1 and 2 for an unitary impulse are also graphically overlapped. Thus, it is possible to validate the method using both matrices as a correction procedure. The great advantage of this hybrid algorithm is that only the line decoupling (step 2 of the diagrams in Figs. 3 and 4) is implemented based on the frequency-dependent transformation matrix and the remaining simulation algorithm is developed using a real and constant matrix. This alternative shows to be a good improvement, especially for frequency-dependent line models direct in the time domain, which require strictly the use of real and constant matrices for the successive modal transformations through the basic algorithm [8, 11, 12].

6 Conclusions

A practical and simple procedure was presented to overcome the errors associated with the modal decoupling applied to multiconductor transmission lines modelling for electromagnetic transient simulations. The complete simulation process was detailed step by step by a block diagram and, from the conventional computational algorithm, the correction procedure was proposed based on a simple alternation in the use of the transformation matrices along the modelling and simulation algorithm. Basically, the correction procedure consists in locate the exact stage where the errors are inserted in the algorithm, isolate and correct them by a simplified correction procedure. This proposal seems to be intuitive and trivial, although, the main references on transmission line theory and power system modelling do not even mention this point and where the errors associated with the successive modal transformations are embedded in the simulation process. This statement has been strongly emphasized in the paper.

The analysis developed in this research shows that the errors are inserted in the modelling/simulation routine by the line parameters decoupling using the Clarke’s approach, indicated by the second step in the main block diagram (Fig. 3). This means that the errors are not a sum of the successive modal transformations along the process, but a punctual error occurred in the line parameters decoupling which is propagated forward along the modelling/simulation process.

Many conventional computational routines associated with well-established line models were developed based on the Clarke’s approach, which results in the inaccuracies evaluated in this paper. Based on the frequency- and time-domain analyses, the errors due to the Clarke’s approach were firstly detected and after corrected by using the exact transformation matrix only in the modal decoupling step. In sequence, all remaining modal transformations can be carried out using the Clarke’s approach without major errors. This correction procedure represents a good tool to eliminate the errors usually observed in transmission line models based on lumped elements, which are mostly developed based on the explicit used of the Clarke’s matrix as a modal transformation matrix in the line decoupling and voltage/current values.

7 Acknowledgments

This research received support of CAPES foundation, Ministry of Education of Brazil (Processes 4570/11-1 and 6851/12-6). Acknowledgements to the Editor and two anonymous Reviewers that provide us important comments and suggestions.

8 References