Uncertainty Analysis using Condor Citvap calculation codes

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I. INTRODUCTION

The development of nuclear reactors for energy production and for research applications requires a great capability of modelling for a safety analysis, performance, reliability and cost optimization of the systems involved; and essential components for modelling these systems are nuclear data and engineering data (or sometimes called design data or design parameters). Through the analysis of this kind of data and their respective uncertainties, nuclear reactors can be designed to achieve high efficiencies satisfying the appropriate safety standards and performance requirements.

There are different types of uncertainty on these parameters; which can be fabrication tolerance for every component on a nuclear reactor or a fuel element, uncertainties in nuclear data and uncertainty at the reactor's operation state (for example, uncertainty in the system's pressure or temperature, etc.). It's worth mentioning that there are guides [1] that specify how to proceed with uncertainty analysis of every reactor parameter.

A. Nuclear data uncertainty

Nuclear data, especially cross sections, are parameters that can vary from few percentages to tens of percentages for isotopes involved in nuclear reactor calculations. An example of this is shown in Figure 1 that presents 50 samples in black of the ^{208}Pb scattering cross section from TENDL-2013 and the reference value in red. For further details about how the uncertainty information from nuclear data is stored see [1].

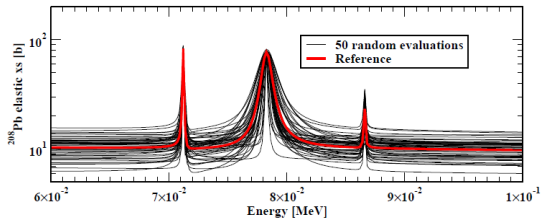


Figure 1. 50 samples from TENDL-2013 of ^{208}Pb scattering cross section

The uncertainty on nuclear data has direct effects on reactor calculations because it is a very important parameter in the calculation of the neutron distribution and it also represents the direct interaction between neutrons and matter.

For this paper, the only isotopes analyzed where Hydrogen, Aluminum, Uranium 234, Uranium 235, Uranium 236 and Uranium 238.

B. Design data uncertainty: Fabrication tolerance and probability distributions

Through the entire fabrication process, design parameters; such as aluminum mass for compact conformation, enrichment, cladding and plate width, and several other parameters associated with fuel element fabrications; take different values that are defined between fabrication tolerances. Each of these parameters presents a probability distribution between these tolerances and is important to know how this random values and probability distributions affect every observable in the reactor. An example of these probability distributions is the Uranium mass for a fuel plate inside type 2 fuel elements, presented in Figure 2.

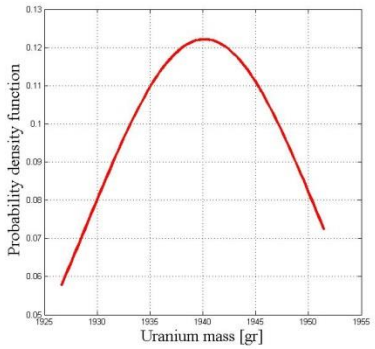


Figure 2. Probability distribution of Uranium mass in a fuel plate.

In this summary, the uncertainties in design parameters associated with the reactor OPAL (see reference [3]) were analyzed.

II. DIFFERENT METHODOLOGIES FOR UNCERTAINTY ANALYSIS

A. General Perturbation Theory

In a nuclear reactor, the behavior of the neutron population can be known by solving equation (1), where **A** is known as the *absorption and loss operator*, **P** is known as the *production operator*, $\mathbf{k_{eff}}$ represents the *effective multiplication*

factor of the nuclear reactor, and **Ø** represents the neutron flux at a given position $\vec{r}$, with a certain direction $\widehat{\Omega}$ and energy E.

$$\left(\boldsymbol{A}-\frac{\boldsymbol{P}}{\boldsymbol{k_{eff}}}\right)\emptyset=\mathbf{0} \qquad (1)$$

For the neutron flux in equation (1), it is defined its *adjoint operator* $Ø^+$, so that this operator verifies equations (2) and (3), where $\mathbf{A}^+$ and $\mathbf{P}^+$ represent the adjoint operators of **A** and **P** respectively; $\mathbf{H}^+$ represents the adjoint operator of a linear operator **H,** and $\langle\,,\rangle$ represent the inner product defined in between 2 operators, which correspond to an integration over all the reactor and over all possible neutron energies and directions**.**

$$\left(\boldsymbol{A}^+-\frac{\boldsymbol{P}^+}{\boldsymbol{k_{eff}}}\right)\emptyset^+=\mathbf{0} \qquad (2)$$

$$\langle\emptyset^+,\boldsymbol{H}\emptyset\rangle=\langle\boldsymbol{H}^+\emptyset^+,\emptyset\rangle \qquad (3)$$

These operators can be used to perform perturbation calculus related to *observables*. Every observable represents any relevant parameter related to the reactor's performance and security measures; such as core reactivity, control rod worth, peak factor, etc.

Using the properties of the adjoint flux and considering small perturbations on the operators in equation (1), it is possible to calculate perturbations on any observable **R** if it takes the form of $\boldsymbol{R}=\langle\boldsymbol{S}^+,\emptyset\rangle$, $\boldsymbol{R}=\langle\boldsymbol{S_1}^+,\emptyset\rangle/\langle\boldsymbol{S_2}^+,\emptyset\rangle$ or $\boldsymbol{R}=\langle\emptyset^+,\boldsymbol{H_1}\emptyset\rangle/\langle\emptyset^+,\boldsymbol{H_2}\emptyset\rangle$, where $\mathbf{H_1}$ and $\mathbf{H_2}$ represent linear operators similar to **H**, **A** and **P**, and $\mathbf{S}^+$, $\mathbf{S_1}^+$ and $\mathbf{S_2}^+$ represent vector operator, defined as those operators whose inner product with the neutron flux represents a scalar value.

A complication rises from the fact that every observable that we analyze has to be represented as a result of an inner product or as a ratio between inner products, and this increases the complexity on perturbation calculus and limits the amount of observables available for analysis with perturbation theory.

To overcome these problems associated with Perturbation Theory, a second methodology of uncertainty calculation is presented below.

B. Total MonteCarlo

This methodology is based on sampling N times every reactor parameter using their probability distribution, and recalculate N times every observable of the reactor to analyze with new values of reactor parameters and make a statistical analysis of every observable.

The main problem that comes from using this methodology is the computational cost of every reactor simulation, and some observables that are not sensible to design parameter's variation are recalculated in every simulation; this makes Total Montecarlo an inefficient method of analyzing uncertainties from a computational cost point of view.

However, despites this disadvantages, this method is much simpler to apply than perturbation theory, every observable can be analyzed (such as the peak factor, which is an observable whose perturbation can't be easily analyzed using General Perturbation Theory) and this methodology takes into account every nonlinear effect associated with the dynamic of the nuclear reactor, unlike General perturbation theory whose main assumption is that every perturbation is linear and small. At last, this methodology is easy to add to the current calculation codes.

Because of these advantages that Total Montecarlo presents over General perturbation theory, Total Montecarlo was used to analyze uncertainties associated with design parameters in the OPAL reactor, specifically in its fuel elements.

III. CALCULATIONS USING CONDOR-CITVAP

In order to optimize the design of nuclear reactor's cores, valuable information can be obtained from sensitivity analysis and uncertainty analysis, where we can observe the behavior of every observable when we sample engineering parameters or nuclear data.

A. Description of the calculation codes

CONDOR is a cell calculation code used by INVAP that solves the neutron transport equation using the Collision probability method and Heterogeneous Response in 2D. This code uses microscopic cross section libraries processed by NJOY to generate condensed and homogenized macroscopic cross sections that preserve reaction rates; after that, the code HXS generates a library from these macroscopic cross sections and takes them to the core calculation code named CITVAP, which is a code developed from CITATION II and solves the Diffusion equation in 1D, 2D and 3D geometries. For post-process of every observable, there is a program named ARCANE that takes outputs from CITVAP for post-processing the data. A graphic explanation of this calculation line is presented in Figure 3.

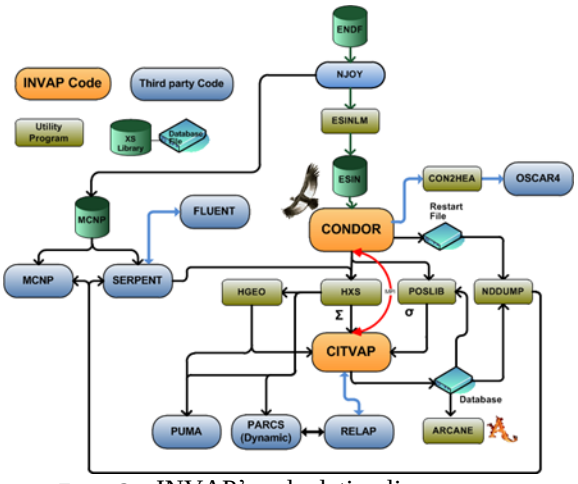


Figure 3. INVAP's calculation line

Both CONDOR and CITVAP had to acquire the capability of generating random numbers. For further knowledge about this nuclear calculation codes see [9] and the adaptations that were necessary in order to use Total Montecarlo uncertainty analysis see [3].

B. Modeling the OPAL reactor: cell and core calculations

All fuel elements parameters were classified in 3 types:

1) Fixed parameters: those parameters whose values do not change in every simulation.
- Pitch between fuel plates
- Active length
- Pitch between fuel elements

2) Variable parameters: those parameters whose values are sampled in every simulation.

i) Geometry
- Width and thickness of the frame of every fuel element
- Width and thickness of every plate
- Cadmium wire's radius
- Distance between plate and meat
- Width and thickness of the slot of every plate
- Width of the cadmium wire slot

ii) Composition
- Uranium mass
- Uranium enrichment
- Silicon mass in fuel plate
- Aluminum mass in fuel plate
- Impurities
- Weight percentage of Uranium 234 and 236.

3) Balance calculated parameters: those parameters whose values are fixed once other parameters values are fixed or sampled.

i) Geometry
- Meat width
- Dimension of every water channel between plates
- Meat thickness

ii) Composition
- Fuel total mass
- Mass of Uranium 238

For uncertainties analysis at cell level calculations, the model that was used is presented in Figure 4 where a single fuel plate is reflected in every direction.

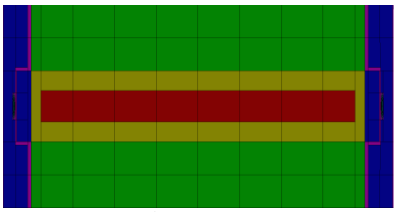


Figure 4. Single fuel plate cell model

For uncertainties analysis at core level calculation, an entire fuel element was used, presented in Figure 5.

Figure 5. Fuel element cell model for core calculation

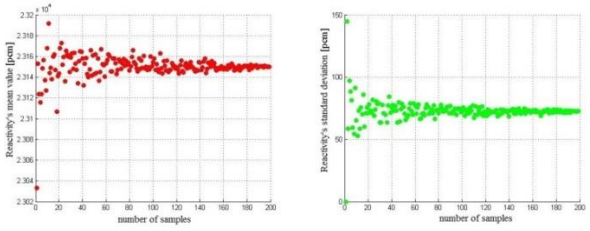
C. Analysis and Results

In this section, all cell calculations were made using only CONDOR, while for core calculations, fuel elements were calculated using CONDOR and later CITVAP.

1) Observables on Cell calculation

a) Convergence

An analysis was made about how many samples were necessary to reach a convergence criterion of variations in the mean value and standard deviation lower than 1%, for that, the observable was considered to be the reactivity of the single plate model, while the only variable parameter was the Uranium enrichment. The convergence behavior is shown in Figure 6.



(a) (b)

Figure 6. Convergence behavior of the single plate model's reactivity. (a) Mean value of reactivity. (b) Standard deviation of reactivity.

The number of simulations necessary to accomplish the convergence criterion was stablished at 200 samples.

b) Design parameters sensitivity

A sensitivity analysis was made by fixing the values of every design parameter except a single parameter and viewing which parameter creates more variance to the observable. For the reactivity of the single plate model, the most sensitive parameter is the Uranium enrichment, while for the peak factor; the most sensitive parameter is the meat width.

c) Nuclear data sensitivity

Fixing all design parameters, a sensitivity analysis was made sampling the cross sections of one of the isotopes previously mentioned in I.A and fixing the others. An example of this sampling is shown in Figure 7 which has different absorption cross sections corresponding to Uranium 234.

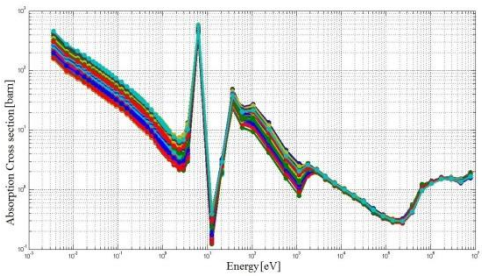


Figure 7. Absorption cross section of Uranium 234 vs neutron's energy

For reactivity in the single plate model, the most sensitive isotope is Uranium 235, while for the peak factor; the most sensitive isotope is Hydrogen.

2) Observables on Core calculations

At cell calculations, 200 fuel elements were sampled with different design parameters, from 3 different types of fuel elements, corresponding with the specifications in [3]. At core calculation, CITVAP generates pseudo random integer numbers, each number representing a different fuel element. Different kinetic parameters mentioned below and observables were calculated taking 100 core simulations.

a) Feedback Coefficients

Such coefficients were the isothermal temperature coefficient, and the void coefficient.

b) Control rod worth

100 control rod worth's were calculated making core simulations with control rod configurations from [7].

At every percentage of insertion, the control rod worth has a probability distribution due to uncertainties associated with different parameters.

c) Observables on critical configurations

From [7] they were taken 42 critical control rod configurations, and core reactivities and peak factors were calculated.

IV. CONCLUTIONS

Different methodologies for uncertainty analysis were examined, emphasizing advantages and disadvantages in each of them.

In the case of General perturbation theory, sensitivity analysis and uncertainty analysis can be done faster and more efficiently than using Total Montecarlo. However, Perturbation theory has limitations with respect to the magnitude of the perturbations involved and the observables.

In the case of Total Montecarlo, this methodology does not have limitations related to observables, but it takes more computational time than General perturbation theory. For this paper, we chose this methodology because it was easier to add to the calculation codes than General perturbation theory.

With Total Montecarlo, we were able to analyze the behavior of different observables as a response of variation in design parameters and nuclear data, presenting their probability distributions and mean values.

At last, this paper presents possible improvements for the calculation codes and uncertainty analysis. Specifically the capability of Latin hypercube sampling of reducing the number of samples needed to achieve convergence criterions comparing with random sampling, and the possibility representing an observable's probability distribution, which will be presented in the final version of this paper.

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