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Propagation of a fast electron cloud in a solar-like plasma of decreasing density

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Abstract

The influence of plasma inhomogeneity on the quasilinear dynamics of a spatially limited electron beam is investigated. It is shown that the electron beam propagates in a plasma of decreasing density as a beam–plasma structure with a decreasing velocity. Plasma inhomogeneity leads to the beam–plasma structure having energy losses.

1. Introduction

The electron beam in a plasma is one of the most colourful phenomena in plasma physics. The problem of electron beam propagation in a plasma is an essential issue for astrophysical, as well as laboratory, plasmas. One of the brightest appearances of low-density electron beams in relatively cold and dense plasma are solar type-III bursts [1–3]. The electrons accelerated during the solar flares travel in the solar corona and are a source of radio emission. The current understanding of the problem assumes that propagating electron beams generate a high level of Langmuir waves, which partly transform into observable radio emission at plasma and double plasma frequencies via nonlinear plasma processes [1, 4]. The signatures of the beams present radio emission with fast frequency drift. The frequency drift is associated with the decrease of plasma density and consequently the local plasma frequency declines as the beam propagates away from the Sun.

The characteristic parameters of the beams and the surrounding plasma have such values that the treatment can be conducted in the framework of weak turbulence theory [5, 6]. The parameter that determines the rate of beam–plasma interaction is the quasilinear time $\tau = n/\omega_{\rm pe}n'$, where n' and n are the beam and plasma densities. The quasilinear time is much smaller than the electron propagation time for typical parameters of the solar plasma and beams generating type-III bursts, and therefore quasilinear relaxation is normally considered to be the main process governing the beam dynamics [5–10]. The influence of plasma inhomogeneity on beam dynamics has normally not been taken into account due to the smallness of the spatial size of electron beams in comparison to the scale of plasma inhomogeneity [6, 10, 13, 14]. However, near the starting frequencies of the bursts, where the plasma gradient has a maximum, the influence of plasma inhomogeneity is significant.

Generally, the presence of a plasma gradient leads to two main physical effects. First, the increment of beam–plasma instability (or quasilinear time) is dependent on distance, which changes the rate of the interaction between the electron and plasma waves. Second, a given Langmuir wave with wavenumber $k(x)$ experiences a shift $\Delta k(x)$ due to the course of its propagation. The former effect plays the main role [15]. For a theoretical description of laboratory experiments on beam transport [16–18] and some astrophysical applications [19] it is necessary to solve a boundary problem on the stationary injection of the electron beam into the plasma half-space. It has been shown [16, 17] that the decreasing plasma density leads to the drift of Langmuir waves towards smaller phase velocities, while the growing plasma density increases the phase velocity that leads to the appearance of accelerated particles. The time of beam–plasma interaction generally increases due to plasma inhomogeneity due to waves shifting out of resonance with the beam and therefore the influence of the plasma inhomogeneity is considered to be a process suppressing quasilinear relaxation [18, 20]. In the solar corona the plasma inhomogeneity plays a secondary role while quasilinear relaxation is a dominant process. In order to understand observational properties (the starting frequencies of the bursts, the frequency drift rate, the beam propagation over large distances), the dynamics of a spatially limited electron beam should be considered. Moreover, we need to investigate the long time (on the time scale $t \gg d/v_0$, where d is the spatial size of the beam and v_0 is the beam velocity) dynamics of an electron cloud in the solar corona plasma.

Additional interest in this problem has been stimulated by the fact that an electron beam with initial electron distribution function $g_0(v)$ can propagate in a plasma as a beam–plasma structure without energy or particle losses [7, 11]. In a uniform plasma the structure propagates with a constant velocity while the electrons form a plateau and generate a high level of Langmuir waves at every spatial point [7, 9].

In this paper we demonstrate that the presence of a plasma inhomogeneity leads to the beam–plasma structure having energy losses and to the decrease of the structure velocity. The results obtained show that the plasma inhomogeneity is a physical process which plays an important role in beam dynamics in the solar corona.

2. Quasilinear equations for inhomogeneous plasma

We consider an initial value problem of electron beam propagation in a plasma with longitudinal inhomogeneity. The electron distribution function of the electron cloud at the initial time moment $t = 0$,

$$f(v, x, t = 0) = g_0(v) \exp(-x^2/d^2) \tag{1}$$

has a spatial length scale d. The initial distribution function that leads to the formation of a beam-plasma structure [12] is

$$g_0(v) = \begin{cases} \dfrac{2n'}{v_0^2} v & v < v_0 \\ 0 & v > v_0 \end{cases} \tag{2}$$

where v_0 is the maximum velocity of the beam. The initial spectral energy density is homogeneous and of the thermal level

$$W(v, x, t = 0) \approx \frac{T}{2\pi^2 \lambda_{\mathrm{De}}^2} \tag{3}$$

where T is the electron temperature of the surrounding plasma and λ_{De} is the electron Debye length. The one-dimensional character of beam propagation is supported by the strong magnetic field along which electrons propagate [6]. However, the magnetic field is not strong enough to change the dispersion properties of the Langmuir waves. The one-dimensional character of beam propagation is also supported by the results of three-dimensional numerical simulations [21].

We assume that the Langmuir wavelength variation at a given wavelength is a small fraction of itself

$$\left|\frac{d\lambda}{dx}\right| \ll 1 \tag{4}$$

or, in other words, we describe wave propagation in the geometrical optics (WKB) approximation [22, 23]. Using the fact that the frequency of a Langmuir wave does not change over its propagation in a plasma, we readily derive from (4)

$$\frac{v}{|L|} \ll 3\omega_{pe}(x)\left(\frac{v_{Te}}{v}\right)^2 \tag{5}$$

where

$$L \equiv \omega_{pe}(x)\left(\frac{\partial \omega_{pe}(x)}{\partial x}\right)^{-1} \tag{6}$$

is the scale of the local inhomogeneity, ω_{pe} is the local plasma frequency and v_{Te}, v are the electron thermal and wave phase velocities, respectively. Under these conditions the kinetics of Langmuir wavepackets (quasiparticles) and electrons can be described by the following system of equations [16]:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \frac{4\pi^2 e^2}{m^2}\frac{\partial}{\partial v}\frac{W}{v}\frac{\partial f}{\partial v} \tag{7}$$

$$\frac{\partial W}{\partial t} + \frac{3v_{Te}^2}{v}\frac{\partial W}{\partial x} + \frac{v^2}{L}\frac{\partial W}{\partial v} = \frac{\pi\omega_{pe}}{n}v^2 W\frac{\partial f}{\partial v} \qquad \omega_{pe} = kv \tag{8}$$

where $f(v, x, t)$ is the electron distribution function and $W(v, x, t)$ is the spectral energy density of Langmuir waves. $W(v, x, t)$ plays the same role for waves as the electron distribution function does for particles. The system (7) and (8) describes the resonant interaction $\omega_{pe} = kv$ of electrons and Langmuir waves. The last two terms on the left-hand side of (8) describe the propagation of Langmuir waves and the corresponding shift in wavenumber. On the right-hand side of equations (7) and (8) we have omitted spontaneous terms due to their smallness in comparison with the induced terms [5].

The system of kinetic equations (7) and (8) is nonlinear with three characteristic time scales. The first is the quasilinear time $\tau \approx n/n'\omega_{pe}$ that is determined by the interaction of particles and waves, the second scale is the characteristic time scale of the plasma inhomogeneity $L/v_0 \gg \tau$ and we are interested in the dynamics of the electron cloud at the time scale $t \gg d/v_0$.

2.1. Plasma density model

In order to obtain the density dependency of the solar corona we follow the heliospheric density model [24]. On the one hand, this is quite a simple isothermal model. On the other hand, it gives results which agree well with observations (see [24] for details). These two features justify the choice of the model.

Following [24] the density profile $n(r)$ is found by numerically integrating the equations for a stationary spherical symmetric solution,

$$r^2 n(r) v(r) = C = \text{constant} \tag{9}$$

$$\frac{v(r)^2}{v_c^2} - \ln\left(\frac{v(r)^2}{v_c^2}\right) = 4\ln\left(\frac{r}{r_c}\right) + 4\frac{r_c}{r} - 3 \tag{10}$$

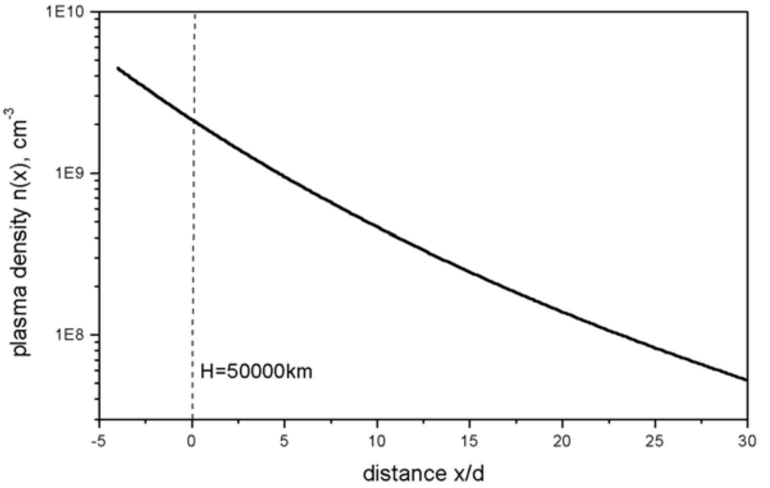

Figure 1. Solar plasma density profile; numerical solution of equations (9) and (10). The initial position of the electron cloud ($x = 0$) is $H = 50\,000$ km above the Sun.

where $v_c \equiv v(r_c) = (k_B T/\tilde{\mu} m_p)^{1/2}$, $r_c = GM_s/2v_c^2$, T is the electron temperature and M_s is the mass of the Sun. The constant C appearing in (9) is fixed by satellite measurements near the Earth's orbit (at $r = 1$ AU $n = 6.59$ cm^{-3}). Thus, following the authors of [24], the constant C is determined to be $C = 6.3 \times 10^{34}$ s^{-1}. For our calculations we take the temperature to be 10^6 K. To solve equations (9) and (10) numerically the *rugula falsi* method is used [25].

The local density profile $n(x)$ is presented in figure 1. The initial location of the electron cloud ($x = 0$) is taken at the altitude $H = 50\,000$ km above the Sun.

3. Homogeneously distributed beam and numerical solution of the kinetic equations

To construct a conservative finite-difference scheme with correct asymptotic behaviour, we initially consider a simplified problem excluding propagation of particles and waves. The main interaction in the system is the particle–wave interaction governed by the quasilinear terms on the right-hand side of kinetic equations (7) and (8).

To exclude transport terms we consider the problem when the electron beam is homogeneously distributed in space. The problem with a homogeneous surrounding plasma has been investigated a few times [6, 32]. It is well known that the initially unstable electron distribution function $g_0(v)$ (2) leads to a plateau formation in the electron distribution function

$$f(v, t \approx \tau) = \begin{cases} \dfrac{n'}{v_0} & v < v_0 \\ 0 & v > v_0 \end{cases} \tag{11}$$

and the generation of Langmuir waves

$$W(v, t \approx \tau) = \begin{cases} \dfrac{mn'}{v_0 \omega_{pe}} \displaystyle\int_0^v \left(1 - \frac{v_0}{n'} g_0(v)\right) \mathrm{d}v & v < v_0 \\ 0 & v > v_0 \end{cases} \tag{12}$$

for the quasilinear time τ. In the case of inhomogeneous plasma we can also consider relaxation of the homogeneously distributed beam. Thus, kinetic equations will take the form

$$\frac{\partial f}{\partial t} = \frac{4\pi^2 e^2}{m^2} \frac{\partial}{\partial v} \frac{W}{v} \frac{\partial f}{\partial v} \tag{13}$$

$$\frac{\partial W}{\partial t} - \frac{v^2}{L_0} \frac{\partial W}{\partial v} = \frac{\pi \omega_{pe}}{n} v^2 W \frac{\partial f}{\partial v} \qquad \omega_{pe} = kv \tag{14}$$

where $L_0 = |L|$ is taken to be a positive constant. In equations (13) and (14) the terms responsible for wave and particle transportation are omitted. It should be noted that our assumption is physically incorrect. The change in spectrum of Langmuir waves is only due to the spatial movement of the waves with the group velocity. However, from the mathematical point of view it is well justified. The group velocity of Langmuir waves $3v_{Te}^2/v$ is a small value and the effects connected with spatial movement can be neglected.

Equations (13) and (14) describe two physical effects: quasilinear relaxation (with characteristic time τ) and the drift of Langmuir waves in velocity space (with characteristic time $\tau_2 = L_0/v$). For our parameters of the beam and plasma $\tau_2 \gg \tau$. Therefore, the influence of plasma inhomogeneity can be considered to be the evolution of the final stage of quasilinear relaxation.

Thus, after the time of quasilinear relaxation a plateau is established for the electron distribution function and high level of Langmuir waves are generated. Since the quasilinear processes are fast, we have a plateau at every time moment

$$f(v, t > \tau) = \begin{cases} \dfrac{n'}{v_0} & v < v_0 \\ 0 & v > v_0. \end{cases} \tag{15}$$

The wave spectrum changes with time and, using the fact that we have a plateau at every time moment, equation (14) can be reduced to

$$\frac{\partial W}{\partial t} - \frac{v^2}{L_0} \frac{\partial W}{\partial v} = 0. \tag{16}$$

The spectral energy density generated during the relaxation stage (12) plays the role of the initial condition for (16). Integrating equation (16), we obtain the solution for $t \gg \tau$

$$W(v, t) = \begin{cases} \dfrac{m}{\omega_{pe}} (1/v - t/L_0)^{-3} \displaystyle\int_0^{1/(1/v - t/L_0)} \left[\frac{n'}{v_0} - g_0(v) \right] \mathrm{d}v & v < u(t) \\ 0 & v > u(t) \end{cases} \tag{17}$$

where

$$u(t) = \frac{v_0}{1 + v_0 t / L_0} \tag{18}$$

is the maximum velocity of Langmuir waves. Note that the electron distribution function is time independent and presents a plateau (15).

For the solution of kinetic equations (13) and (14) we use the finite-difference method [27, 28]. We introduce the grid points v_i, for $i = 1, \ldots, N$, with a uniform mesh width of $\Delta v = v_{i+1} - v_i$. The discrete time t_k is also uniformly spaced with a time step Δt. To approximate the quasilinear terms we used the explicit scheme used in [14]. This scheme demonstrates high accuracy of the upper border for plateau-like distributions. The term connected with the shift of the wave spectrum plays the role of the transport term in the velocity space. It is approximated by the 'monotonic' scheme [29–31]. This scheme is considered to be the best in these kinds of problems [26].

The numerical solution of the equations (13) and (14) with the initial electron distribution function (2) is presented in figure 2. Indeed, comparing the numerical results and the simplified

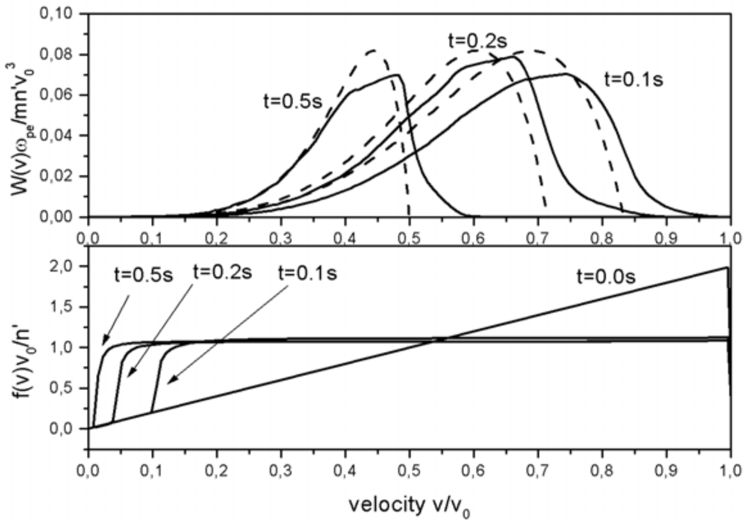

Figure 2. Numerical solution of equations (13) and (14) at four time moments: $t = 0$, 0.1 s, 0.2 s, 0.5 s. The initial electron beam density $n' = 100$ cm^{-3}, the plasma inhomogeneity length $L_0 = 5 \times 10^9$ cm and the plasma density $n = 4.96 \times 10^8$ cm^{-3}. Dashed curves correspond to the simplified analytical solution (17).

solution (17) we have a good agreement. The electron distribution function and the spectral energy density demonstrate the correct asymptotic behaviour at $t \gg \tau$. The plateau for the wide range of velocities is formed after the short time $t = 0.1$ s and remains almost unchanged up to the end of the calculations. For the time $t > 0.1$ s the drift of the Langmuir wave spectrum towards smaller phase velocities becomes observable. At $t = 0.5$ s the maximum phase velocity is half of the initial beam velocity.

4. Propagation of the electron cloud in the inhomogeneous plasma

The results obtained in the previous section justify the choice of the finite-difference scheme for quasilinear terms. The transport terms in kinetic equations (7) and (8) are approximated using a monotonic scheme [29–31]. The additional resolution requirement emerges when combining quasilinear relaxation with particle propagation. For a grid spacing in velocity space Δv, a spatial separation of size $\Delta v t$ occurs in the numerical solution after a time t. Therefore, to ensure that the continuous distribution does not become spiky, we require $\Delta v t < \Delta x$, where we have introduced a uniform mesh in coordinate space $\Delta x = x_{j+1} - x_j$ for $j = 1, \dots, M$.

Generally, as in the case of homogeneous plasma, the propagation of the electron cloud in the inhomogeneous solar-like plasma (figure 1) leads to the formation of a beam–plasma structure. The electron distribution function and the spectral energy density of Langmuir waves are presented in figures 3 and 4. In these figures the electron distribution function and the spectral energy density of Langmuir waves are presented as contour plots in the (x, v) plane at various time moments. At every spatial point the electron distribution function is a plateau from almost zero velocity up to some maximum velocity. This maximum velocity is not a constant value as it takes place in a uniform plasma (figure 5), but a decreasing function of distance. This is a direct result of the plasma inhomogeneity.

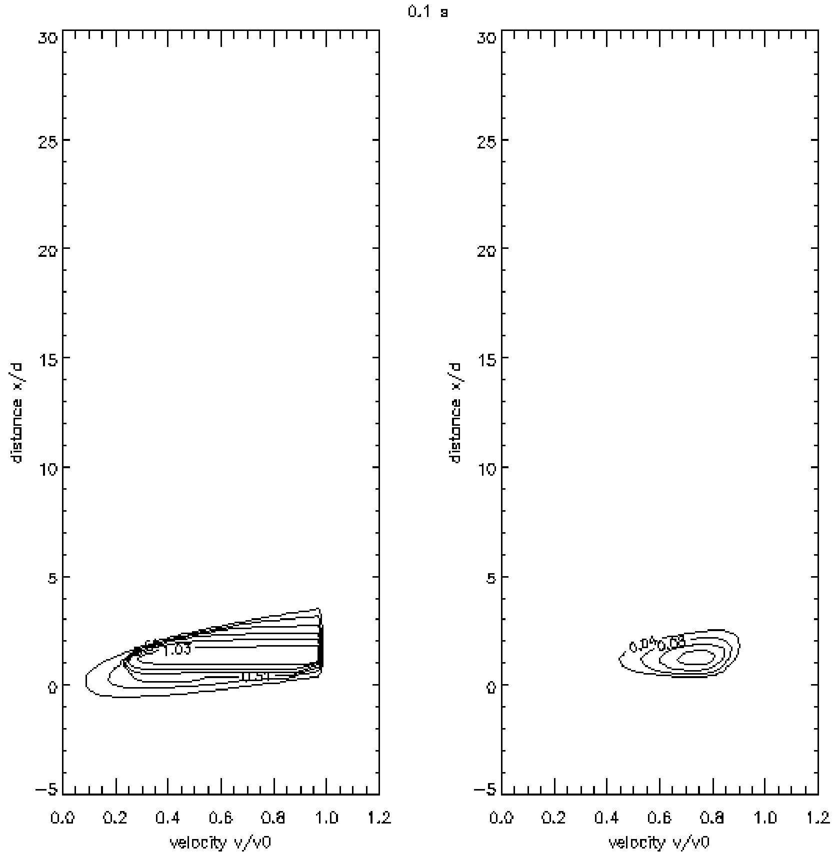

Figure 3. The electron distribution function $f(v, x)v_0/n'$ (left) and the spectral energy density of Langmuir waves $W(v, x)\omega_{pe}/mn'v_0^3$ (right) at $t = 0.1$ s. Numerical solution of kinetic equations $d = 10^9$ cm, $n' = 100$ cm^{-3}, $v_0 = 2 \times 10^{10}$ cm s^{-1}.

At the initial time moment $t = 0$ we have no Langmuir waves generated. The plateau of the electron distribution function and the high level of plasma waves appear as a result of quasilinear relaxation at the time moment $t = 0.1$ s (figure 3). For the time of quasilinear relaxation, the electrons have not moved far from the region of their initial location. The plasma inhomogeneity has also demonstrated no influence on the beam or Langmuir wave evolution for this time. At this stage the numerical solution agrees well with the analytical solution for homogeneous plasma (see [10]). The electron distribution function

$$f(v, x, t) = \begin{cases} \dfrac{n'}{v_0} \exp\left(-\dfrac{(x - v_0 t/2)^2}{d^2}\right) & v < v_0 \\ 0 & v > v_0 \end{cases} \tag{19}$$

and the spectral energy density of Langmuir waves

$$W(v, x, t) = \begin{cases} \dfrac{mn'}{v_0 \omega_{pe}} v^4 \left[1 - \dfrac{v}{v_0}\right] \exp\left(-\dfrac{(x - v_0 t/2)^2}{d^2}\right) & v < v_0 \\ 0 & v > v_0 \end{cases} \tag{20}$$

for a uniform plasma can be applied for $t < \tau_2$. Indeed, electrons form a plateau in the electron distribution function and the spectrum of plasma waves has a maximum close to

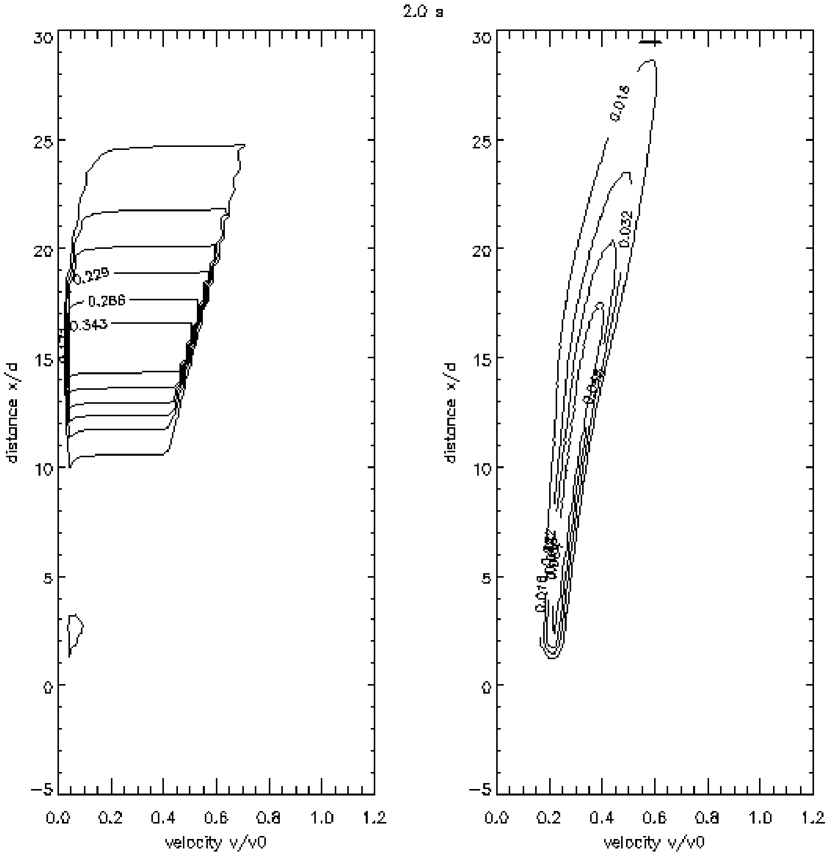

Figure 4. The same as in figure 3 but at the time moment $t = 2.0$ s.

$0.75v_0$ (figure 6). Differentiating the solution for the homogeneous plasma (20) with respect to v, we readily obtain the maximum at $0.8v_0$.

The characteristic time of Langmuir wave drift in velocity space towards small phase velocities τ_2 is about 0.1 s. Therefore, the impact of plasma inhomogeneity is observable at $t > \tau_2$. At distances far from the initial location of the electrons the influence of plasma inhomogeneity becomes significant. In figure 4 we see that there are no plasma waves with phase velocity of more than $\approx 0.7v_0$.

4.1. Electron distribution function and the spectrum of Langmuir waves

The evolution of the electron distribution function and the spectral energy density qualitatively follows the simplified solution of the previous section. At every spatial point we observe two physical processes. The first process is the spatial movement of the beam–plasma structure as a whole and the second is the shift of the Langmuir wave spectrum due to plasma inhomogeneity.

Let us consider the time evolution of plasma waves and the electron distribution function at a given point $x = 11d$ (figure 6). In contrast to the previous section we have a finite size electron cloud. Therefore, the first electrons arrive at this point at $t = 0.5$ s. The electrons which have arrived form a plateau in the electron distribution function and excite Langmuir waves (see figure 6). At the front of the stream the electrons which have arrived have a positive

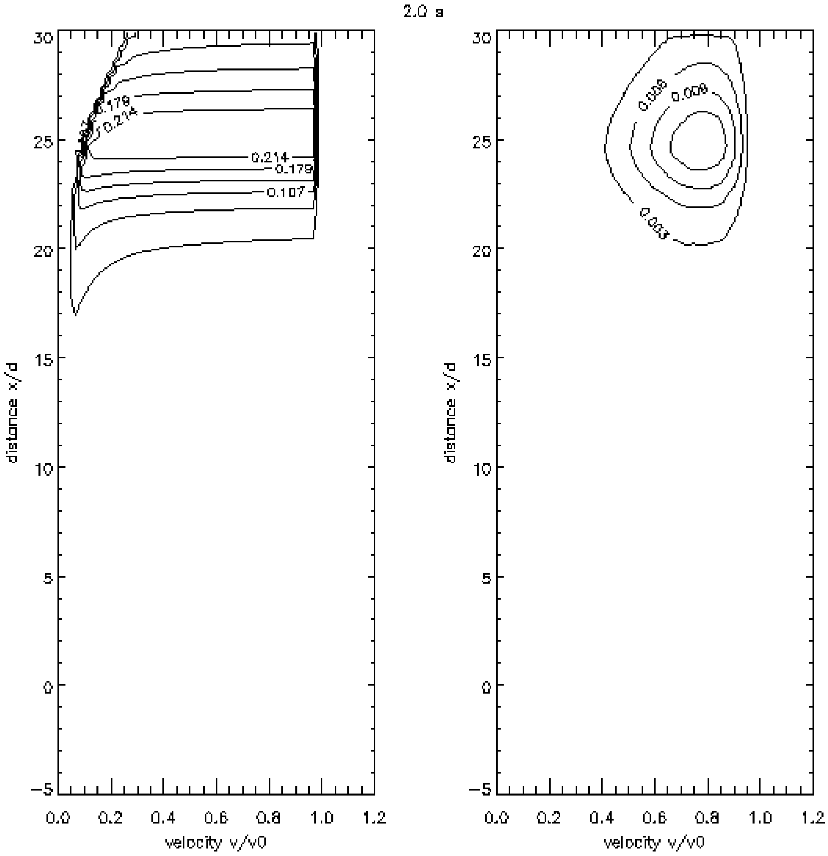

Figure 5. The electron distribution function (left) and the spectral energy density of Langmuir waves (right) at $t = 2.0$ s in the case of homogeneous plasma $n = 4.96 \times 10^8$ cm^{-3}. Numerical solution of kinetic equations $d = 10^9$ cm, $n' = 100$ cm^{-3}, $v_0 = 2 \times 10^{10}$ cm s^{-1}.

derivative $\partial f/\partial v$ (see [9] for details) that supplies the growth of Langmuir waves. At the time moments $t = 1.0$ s and $t = 1.2$ s the plateau height and the level of plasma waves continue to grow. Approximately at the moment $t = 1.2$ s the maximum electron density as well as the maximum wave energy density are achieved. From this moment onwards, electrons mainly with $\partial f/\partial v < 0$ arrive at the point. The plateau height declines and, due to the negative $\partial f/\partial v$ of the arriving electrons, the level of plasma waves also decreases ($t = 1.6$ s in figure 6). However, since the time size of the beam–plasma structure is more than the characteristic time of the Langmuir spectrum change, Langmuir waves drift towards smaller phase velocities. Some electrons have corresponding Langmuir waves with $v < u$, but the electrons with velocities $v > u$ have no resonance waves. The electrons accompanied by waves propagate as a beam–plasma structure with velocity $u/2$, while the latter propagate freely with velocity v. Indeed, we see in figure 6 that electrons with $v > u$ leave the given point, which leads to the absence of electrons with $v > u$. Therefore, the maximum plateau velocity following the Langmuir wave maximum phase velocity decreases with time. Moreover, the fast electrons at the edge of the plateau not accompanied by Langmuir waves with the same phase velocities freely overlap the electrons involved in quasilinear relaxation and relax further from the structure maximum. This effect increases the spatial size of the structure and makes

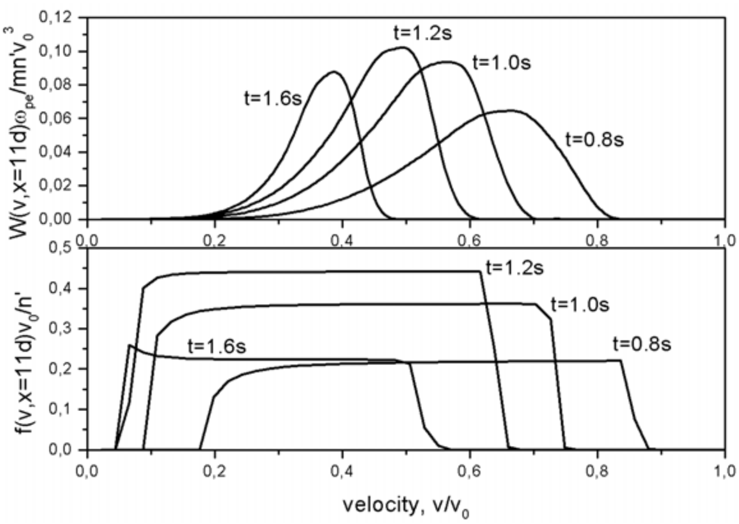

Figure 6. The electron distribution function and the spectral energy density of Langmuir waves at $x = 11d$ for various time moments. Numerical solution of the kinetic equations $d = 10^9$ cm, $n' = 100$ cm^{-3}, $v_0 = 2 \times 10^{10}$ cm s^{-1}.

the front longer than the back (figure 4). Note that in the case of homogeneous plasma the structure is symmetrical (figure 5).

4.2. The velocity of a beam–plasma structure

We see that the plasma inhomogeneity changes the spectrum of plasma waves and the electron distribution function. The maximum velocity of the plateau and Langmuir waves decreases with time. The electrons with $v < u$ propagate as a single structure accompanied by Langmuir waves. The speed of the structure can be approximated as $\approx u/2$, where u is the maximum velocity of the plateau near the maximum of the electron density. The drift of Langmuir waves occurs at every spatial point and leads to a decrease of the velocity of the beam–plasma structure. In the case of uniform plasma the maximum velocity of Langmuir waves and the plateau remains unchanged (figure 8). For a uniform plasma the velocity of the structure is a constant $\approx v_0/2$, while it is a decreasing function for an inhomogeneous plasma (figure 7).

4.3. Energy density of the particles and waves

As we see, negligible spatial movement of the waves causes a significant shift of their spectrum towards smaller phase velocities. As a result, a portion of the Langmuir waves appear to be out of resonance with the particles. The back of the structure has more low-velocity plasmons that can be absorbed by the electrons, and the fastest electrons have no Langmuir waves with the corresponding phase velocity to absorb. Therefore, part of the beam–plasma structure energy is lost by the structure in the form of low-velocity Langmuir waves (compare with the homogeneous plasma in figure 5). In the case of a homogeneous plasma the spectrum of the Langmuir waves generated at the front is the same spectrum which will be absorbed at the back [7, 9].

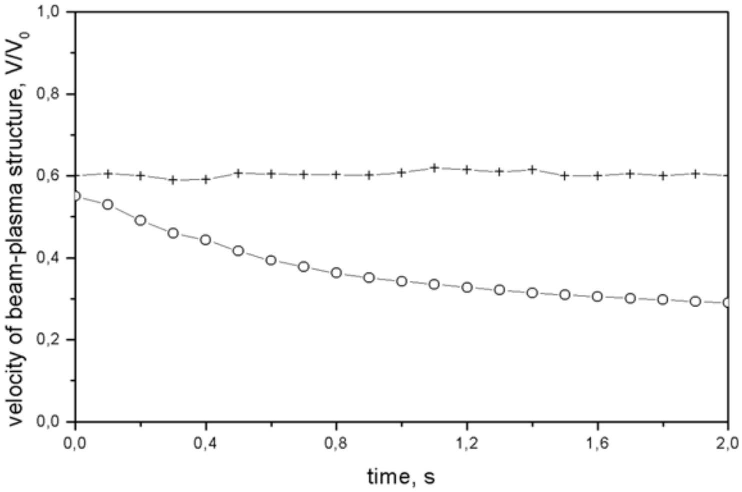

Figure 7. The velocity of the beam–plasma structure for inhomogeneous (circles) and homogeneous $n = 4.96 \times 10^8$ cm^{-3} (crosses) plasmas. All beam parameters are as in figures 3–6.

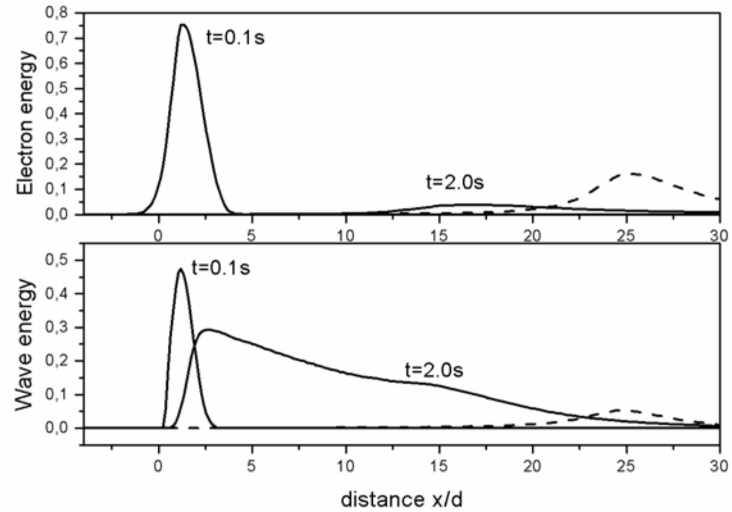

Figure 8. The energy density distribution of electrons and Langmuir waves at $t = 0.1$ s and $t = 2$ s for inhomogeneous plasma (full curves). Numerical solution of kinetic equations $d = 10^9$ cm, $n' = 100$ cm^{-3}, $v_0 = 2 \times 10^{10}$ cm s^{-1}. Numerical solution for homogeneous plasma $n = 4.96 \times 10^8$ cm^{-3} (dashed curves).

The distributions of the energy density of the electrons

$$E_e(x, t) = \frac{m}{2} \int_0^{v_0} v^2 f(v, x, t)\,dv \tag{21}$$

and the Langmuir waves

$$E_w(x, t) = \int_0^{\infty} W\,dk = \omega_{pe} \int_0^{v_0} \frac{W(v, x, t)}{v^2} dv \tag{22}$$

are presented in figure 8 at various time moments. Fast quasilinear relaxation results in the fact that approximately one third of the initial beam energy is transformed into wave energy by $t = 0.1$ s (figure 8). This result is in a good agreement with the solution in a homogeneous plasma. Integrating solutions (19) and (20), we obtain the same result.

The decrease of the number of waves in resonance with the beam electrons leads to energy losses. In figure 8 we see a non-zero level of Langmuir waves in the region where the structure has passed. Despite the fact of considerable energy loss, the beam remains the source of the growing plasma level (see figure 6). The growth of the spectral energy density is supplied by the declining local plasma frequency.

5. Summary

Finally, we have to mention one question concerning the applicability of the kinetic equations (7) and (8). The right-hand side part of equation (8) is a Liouville equation demonstrating the conservation of the number of Langmuir waves

$$N(v, x, t) = W(v, x, t)/\hbar\omega$$

where $\hbar\omega$ is the energy of a quasiparticle. This equation is derived assuming that $\hbar\omega$ is a constant value

$$\omega(k, x) = \omega_{pe}(1 + \Delta n/2n + 3k^2 v_{Te}^2/2\omega_{pe})$$

or, in other words, the change of plasma density should be a small fraction of itself [15, 22]. In our investigation we apply equation (8) to consider a wide range of plasma frequencies. To answer this question we have to mention the fact that the Langmuir waves are slow ($v_{gr} \ll v$) and therefore a given plasmon can pass the distance $v_{gr}t$ at which the plasma density only changes slightly. Indeed, the numerical results show that the Langmuir waves do not propagate far from the point where they were excited. The Langmuir waves generated near $x \approx 0$ remain close to this point at $t = 2$ s.

The results obtained demonstrate that plasma inhomogeneity has a rather strong influence on beam dynamics. The main factor determining the influence of the plasma inhomogeneity is the shift of the Langmuir wave phase velocity due to its spatial movement. As a result a portion of the Langmuir waves generated by the beam becomes out of resonance with the electrons. The dependency of the quasilinear time on distance has a weaker influence.

The plasma inhomogeneity plays the role of a dissipative medium for electron beam propagation. Numerical solution shows that the propagation of the electron beam leads to the formation of a beam–plasma structure with the velocity decreasing with distance. The beam–plasma structure propagates whilst losing its energy in the form of Langmuir waves. The initial kinetic energy of the beam is transformed into the energy of Langmuir oscillations.

The results obtained may allow us to quantitatively connect the observable properties of bursts (frequency drift rate and starting frequency) with the basic plasma and beam parameters (plasma density, density gradient, beam density and beam velocity). Observations [4] show us that the source velocity of type III bursts or the velocity of the beam–plasma structure that we are considering is a slowly decreasing function. Our results show that the plasma inhomogeneity can be the physical process which leads to the velocity decrease.

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