The MISO Interference Channel: Competition versus Collaboration

Erik G. Larsson and Eduard A. Jorswieck

Abstract—We consider the problem of coordinating two competing multiple-antenna wireless systems (operators) that operate in the same spectral band. We formulate a rate region which is achievable by scalar coding followed by power allocation and beamforming. We show that all interesting points on the Pareto boundary correspond to transmit strategies where both systems use the maximum available power. We then argue that there is a fundamental need for base station cooperation when performing spectrum sharing with multiple transmit antennas. More precisely, we show that if the systems do not cooperate, there is a unique Nash equilibrium which is inefficient in the sense that the achievable rate is bounded by a constant, regardless of the available transmit power.

Next we model the problem of agreeing on beamforming vectors as a non-transferable utility (NTU) cooperative game-theoretic problem, with the two operators as players. Specifically we compute numerically the Nash bargaining solution, which is a likely resolution of the resource conflict assuming that the players are rational. Numerical experiments indicate that selfish but cooperating operators may achieve a performance which is close to the maximum-sum-rate bound.

I. INTRODUCTION

A. Background

We are concerned with the following scenario: Two independent wireless systems operate in the same spectral band. The first system consists of a base station BS1 that wants to convey information to a mobile MS1. The second system consists of another base station BS2 that wants to transmit information to a mobile MS2. The systems share the same spectrum, so the communication between BS1 → MS1 and BS2 → MS2 is going to take place simultaneously on the same channel. Thus MS1 will hear a superposition of the signals transmitted from BS1 and BS2, and conversely MS2 will also receive the sum of the signals transmitted by both base stations. This setup is recognized as an interference channel (IFC) [1]–[3]. In the setup we consider, BS1 and BS2 have \( n \) transmit antennas each, that can be used with full phase coherency. MS1 and MS2, however, have a single receive antenna each. Hence our problem setup constitutes a multiple-input single-output (MISO) IFC. See Figure 1.

We shall assume that transmission consists of scalar coding followed by beamforming, and that all propagation channels are frequency-flat. This leads to the following basic model for the matched-filtered, symbol-sampled complex baseband data received at MS1 and MS2:

\[
\begin{align*}
    y_1 &= h_{11}^T w_1 s_1 + h_{21}^T w_2 s_2 + e_1 \\
    y_2 &= h_{22}^T w_2 s_2 + h_{12}^T w_1 s_1 + e_2
\end{align*}
\]

where \( s_1 \) and \( s_2 \) are transmitted symbols, \( h_{ij} \) is the (complex-valued) \( n \times 1 \) channel-vector between BS\(_i\) and MS\(_j\), and \( w_i \) is the beamforming vector used by BS\(_i\). The variables \( e_1, e_2 \) are noise terms which we model as i.i.d. Gaussian with zero mean and variance \( \sigma^2 \). We assume that each base station can use the transmit power \( P \), but that power cannot be traded between the base stations. Without loss of generality, we shall take \( P = 1 \). This gives the power constraint

\[
\| w_i \| \leq 1, \quad i = 1, 2
\]

Throughout, we define the signal-to-noise ratio (SNR) as \( 1/\sigma^2 \). Various schemes that we will discuss require that the transmitters (BS\(_1\) and BS\(_2\)) have different forms of channel state information (CSI). However, at no point we will require phase coherency between the base stations.

The fundamental question we want to address is the following. If BS\(_1\) and BS\(_2\) operate in an uncoordinated manner, how should they choose their beamforming vectors...
$w_1, w_2$? There is an obvious conflict situation associated with this choice since a vector $w_1$ which is good for BS$_1 \rightarrow$ MS$_1$ may generate substantial interference for MS$_2$ and vice versa. The main contribution of this work is to discuss this conflict situation in a game-theoretic framework. In the course of doing so, we will also present an achievable rate region for the MISO IFC and a characterization of this region.

B. Related work

Information-theoretic studies of the IFC have a long history [1]–[3], [5]. These references have provided various achievable rate regions, which are generally larger in the more recent papers than in the earlier ones. However, the capacity region of the general IFC channel is still an open problem. For certain limiting cases, for example when the interference is weak or very strong, respectively, the sum capacity is known [6]. For weak interference the interference can simply be treated as additional noise. For very strong interference, successive interference cancellation can be applied at one or more of the receivers. Multiple-input multiple-output (MIMO) IFCs have also recently been studied in [7], from the perspective of spatial multiplexing gains.

Recently an increasing body of literature has looked at resource conflict problems in wireless communications using tools from game theory (see, for example [8]). Most of this work deals with networking aspects of communications. There is some available work, however, that studies the IFC from a game-theoretic perspective. In what follows, we summarize the relevant literature that we are aware of. Distributed algorithms for spectrum sharing in a competitive setup (using noncooperative game theory [9]) were developed by [10] and [11]. A more general analysis of the spectrum sharing problem was performed in [12]. All three [10]–[12] dealt with single-antenna transmitters and receivers, and looked at the problem from a noncooperative game-theoretic point of view. The MIMO IFC has also been studied from a noncooperative game-theoretic perspective in [13] and [14], who presented results on equilibrium rates and proposed distributed algorithms. These noncooperative approaches [10]–[13] generally lead to decentralized schemes for computing stable operating points, so-called Nash equilibria. Unfortunately, these equilibria are often rather inefficient outcomes, as measured by the achievable sum-rate, for example. Less work is available on cooperative game theory for IFCs, especially for multiple-antenna IFCs. Some results can be found in [15] who treated the spectrum sharing problem using cooperative (bargaining) game theory and [16] who proposed a decentralized algorithm for finding the bargaining solution. Both [15], [16] considered the case of single antennas at the transmitter and at the receiver. Apart from this, the area of cooperative strategies for the IFC appears largely open. (We shall note [17] that deals with the multiple-access channel (MAC) using coalitional game theory. However the MAC differs fundamentally from the IFC.)

Contributions: We study the MISO IFC both from a noncooperative (competitive) game theoretic perspective, and from a cooperative (bargaining) point of view. We show that the outcome of the noncooperative game is a unique Nash equilibrium but that this is rather bad from an overall system perspective (see Section III-A). We then consider the same problem using cooperative (Nash axiomatic bargaining) theory and show that this can significantly improve the outlook of the problem (see Section III-B).

II. Achievable Rates and Operating Points

A. An Achievable Rate Region

In what follows we will assume that all receivers treat co-channel interference as noise, i.e., they make no attempt to decode and subtract the interference. (See [22] for extensions.) The main justification for this assumption is that in most envisioned applications, MS$_1$ would use receivers with a simple structure. Additionally, one can argue that interference cancellation is difficult in an environment where the receivers do not know the coding and modulation schemes used by the interfering transmitters. For a given pair of beamforming vectors $\{w_1, w_2\}$, the following rates are then achievable, by using codebooks approaching Gaussian ones:

$$R_1 = \log_2 \left( 1 + \frac{|w_1^T h_{11}|^2}{|w_2^T h_{21}|^2 + \sigma^2} \right)$$

(2)

for the link BS$_1 \rightarrow$ MS$_1$, and

$$R_2 = \log_2 \left( 1 + \frac{|w_2^T h_{22}|^2}{|w_1^T h_{12}|^2 + \sigma^2} \right)$$

(3)

for BS$_2 \rightarrow$ MS$_2$. For fixed channels $\{h_{ij}\}$, we define the achievable rate region as

$$\mathcal{R} = \bigcup_{w_1, w_2 : \|w_i\|^2 \leq 1} (R_1, R_2).$$

We stress that this is not the capacity region, because it does not take into account the possibility of performing interference cancellation at the receivers, and it does not take into account the possibility of going beyond Gaussian signaling. However the rates in $\mathcal{R}$ are achievable with simple receiver signal processing, that treats interference as noise. The outer boundary of $\mathcal{R}$ is called the Pareto boundary, because it consists of Pareto optimal operating points. A Pareto optimal point is a point at which one cannot improve the rate of one link without simultaneously decreasing the rate of the other. We denote the Pareto

1Strictly speaking, only rates $R_1 - \epsilon$ are achievable, for some $\epsilon$. Since the main purpose of this paper is to explain fundamental limitations and possibilities associated with spectrum conflicts, rather than to develop coding theorems for IFCs, we shall say (with some sacrifice of rigor) that the rates $R_i$ are “achievable”.

WeA1.2
boundary by $R^*$. Note that for fixed $h_{ij}$, the region $R$ is compact, since the set $\{w_1, w_2\}$ subject to the power constraint (1) is compact and the mapping from $\{w_1, w_2\}$ to $\{R_1, R_2\}$ is continuous. However, the region $R$ is in general not convex.

We define the convex hull of $R$ as follows:

$$R = \bigcup_{0 \leq \tau \leq 1} (\tau R_1 + (1-\tau)R'_1, \tau R_2 + (1-\tau)R'_2).$$

Also we denote the Pareto boundary of $\bar{R}$ with $\bar{R}^*$. The region $\bar{R}$ can be interpreted as the set of achievable outcomes if the two systems BS$_1$ $\rightarrow$ MS$_1$ and BS$_2$ $\rightarrow$ MS$_2$ are allowed to split the available degrees of freedom (time or bandwidth in practice) offered by the channel in two parts, and use the beamforming vectors $\{w_1, w_2\}$ (corresponding to a rate point $(R_1, R_2)$) during a fraction $\tau$ of the time, and another set of beamforming vectors $\{w'_1, w'_2\}$ (corresponding to a different rate point $(R'_1, R'_2)$) during the rest of the time (i.e., during a fraction $1-\tau$ of the total time). Implicit in this interpretation is the assumption that the power constraint (1) is unchanged, i.e., the constraint is on the peak power rather than on the long-term average of power. Another interpretation of $\bar{R}$ is in terms of correlated mixed strategies: $\bar{R}$ is the set of average rates that can be achieved if the two systems decide on two arbitrary rate points in $R$ and then flip a synchronized coin to decide which one of these two points to operate at. The importance of working with $\bar{R}$ instead of $R$ will become clear when we formulate the beamforming problem as a bargaining problem (Section III-B).

Figures 2(a) and 2(b) show two examples of the rate region $\bar{R}$. In the first example the channels are chosen so that $\bar{R}$ is convex; in the second example $\bar{R}$ is nonconvex. The figures also show the convex hull $\bar{R}$. (The other rate points in the figures will be explained in what follows.) These figures were generated by computing $(R_1, R_2)$ over a grid of beamforming vectors, as explained in more detail in Section IV.

B. Characterization of the Pareto Boundary

A first question to ask is whether any point on the Pareto boundary of $\bar{R}$ (or $\bar{R}$) can be reached unless both BS$_1$ and BS$_2$ spend the maximum allowable power, i.e., whether $(R_1, R_2) \in \{\bar{R}^*, R^*\}$ always requires $\|w_1\|^2 = \|w_2\|^2 = 1$. We will show that the only points on the Pareto boundary which can be achieved without having $\|w_1\|^2 = \|w_2\|^2 = 1$ are points where the tangent to the Pareto boundary is either strictly vertical or strictly horizontal. The importance of this observation is that apart from pieces of the Pareto boundary that are strictly vertical or horizontal, it is enough to consider parameterizations of the boundary for which $\|w_1\|^2 = \|w_2\|^2 = 1$. More precisely we have the following proposition.

**Proposition 1:** a) Consider a point $(R_1, R_2)$ in the rate region $\bar{R}$ which corresponds to a set of beamforming vectors $(w_1, w_2)$ for which $\|w_1\|^2 < 1$ and $\|w_2\|^2 < 1$. Then there exists a beamforming vector $\hat{w}_1$, such that the rate operating point $(\hat{R}_1, \hat{R}_2)$ associated with $(\hat{w}_1, \hat{w}_2)$ satisfies $1 \geq \|\hat{w}_1\|^2 > \|w_1\|^2$, $\hat{R}_1 > R_1$ and $\hat{R}_2 = R_2$. In other words, it is possible to improve the rate of system 1 by changing $w_1$ in a way so that BS$_1$ uses more power, and simultaneously keep $w_2$ unchanged.

b) The result in (a) holds also if $\bar{R}$ is replaced by $\bar{R}$.

**Proof:** See [22].

Note that the converse is not true. Most points in the interior of $\bar{R}$ and $\bar{R}$ correspond to beamforming vectors for which both base stations use full power.

C. Some Special Operating Points

Some points in the rate region are especially interesting, and we discuss them as follows (in no particular order).
1) The single-user (SU) points \((R_{SU}^1, 0)\) and \((0, R_{SU}^2)\) are the rate points that result if only one user transmits, assuming the base station has full channel knowledge and performs maximum-ratio transmission beamforming (i.e., \(w_1 = h_{11}/\|h_{11}\|\) and \(w_2 = h_{22}/\|h_{22}\|\), respectively, as in single-user MISO transmission [4]). The associated rates are

\[
R_{SU}^1 = \log_2 \left( 1 + \frac{\|h_{11}\|^2}{\sigma^2} \right) \quad \text{and}
R_{SU}^2 = \log_2 \left( 1 + \frac{\|h_{22}\|^2}{\sigma^2} \right).
\]

Note that all convex combinations of the points \((R_{SU}^1, 0)\) and \((0, R_{SU}^2)\) lie on a straight line (see Figures 2(a)–2(b)). The points on this line correspond to orthogonal multiple access via time-sharing.

We can characterize the average rates associated with the single-user points as follows. Define

\[
G(\sigma^2, n) \triangleq \log_2(e) \exp(\sigma^2) \sum_{k=0}^{n-1} \sigma^{2k} \Gamma(-k, \sigma^2) \tag{4}
\]

where

\[
\Gamma(a, x) \triangleq \int_x^\infty t^{a-1} \exp(-t) dt
\]

is the incomplete Gamma function. Then, if \(\{h_{ii}\}\) have independent zero-mean Gaussian elements with unit variance (i.e., the fading is i.i.d. Rayleigh) we have

\[
E[R_{SU}^i] = G(\sigma^2, n) \tag{5}
\]

This follows by applying the results presented in [18, Section IV.B].

Equation (4) shows that the average single-user rate grows logarithmically with the SNR. Unfortunately, this is not of much interest since the single-user points are unstable outcomes of the resource conflict in the sense that if the systems operate at one of these points, then any of the systems can improve its rate by unilaterally changing its beamforming vector. This goes also for convex combinations of the single-user points: they are not stable operating points unless the systems have pre-agreed to use orthogonal time-sharing.

2) The best-user (BU) point \((R_{BU}^1, R_{BU}^2)\) is the rate point which is achieved if the system with the best channel (in the sense of largest channel norm) uses all resources and the other system stays quiet. More precisely we have

\[
R_{BU}^i = \begin{cases} R_{SU}^i, & R_{SU}^i \geq R_{BU}^i \\ 0, & \text{otherwise} \end{cases}
\]

\[
R_{BU}^2 = \begin{cases} R_{SU}^2, & R_{SU}^2 \geq R_{BU}^2 \\ 0, & \text{otherwise} \end{cases}
\]

It is clear that the average rate associated with the best-user point is at least as good as any of the single-user rates. However, like the single-user points, the best-user operating point is unstable as well.

3) The sum-rate (SR) point \((R_{SR}^1, R_{SR}^2)\) is the point at which \(R_1 + R_2\) is maximized. Geometrically, this the point where the Pareto boundary of \(R\) oscillates a straight line with slope \(-1\). (This point is not shown in Figures 2(a)–2(b).)

The expected sum-rates grow logarithmically with the SNR. This is clear by considering the following chain of inequalities:

\[
E[R_{SR}^1] \geq E[\max(R_{SU}^1, R_{SU}^2)] \geq \frac{1}{2} (E[R_{SU}^1] + E[R_{SU}^2]) \tag{6}
\]

and applying Lemma 1. In (6), the first inequality follows because the line with slope \(-1\) which touches \(R^*\) must lie to the upper right of both the points \((R_{SU}^1, 0)\) and \((0, R_{SU}^2)\). The second inequality in (6) is immediate. However, a more precise analytical characterization of the sum-rate point appears nontrivial. Fortunately, this is not of much interest anyway, because like the other rate points discussed above, the sum-rate operating point is also unstable.

4) The zero-forcing (ZF) point \((R_{ZF}^1, R_{ZF}^2)\) is the rate pair which is achieved if BS\(1\) chooses a transmit strategy that creates no interference at all for MS, and vice versa. If we assume that both base stations use maximum permitted power, then BS\(1\) should use a unit-norm beamforming vector \(w_1\) which is orthogonal to \(h_{21}\) and which at the same time maximizes \(|w_1^H h_{11}|\). This beamformer is given by [22]

\[
w_{ZF}^1 = \frac{\Pi_{h_{21}}^\bot h_{11}}{\|\Pi_{h_{21}}^\bot h_{11}\|} \tag{7}
\]

for BS\(1\), where \(\Pi_\bot^X = I - X (X^H X)^{-1} X^H\) denotes projection onto the orthogonal complement of the column space of \(X\). Similarly, BS\(2\) uses

\[
w_{ZF}^2 = \frac{\Pi_{h_{21}}^\bot h_{22}}{\|\Pi_{h_{21}}^\bot h_{22}\|}
\]

The corresponding rates are

\[
R_{ZF}^1 = \log_2 \left( 1 + \frac{|w_{ZF}^1 h_{11}|^2}{\sigma^2} \right) \quad \text{and}
R_{ZF}^2 = \log_2 \left( 1 + \frac{|w_{ZF}^2 h_{22}|^2}{\sigma^2} \right).
\]

We can characterize the performance with ZF as follows.

**Proposition 2:** Suppose the fading is i.i.d. Rayleigh and that all channels are independent. Then the average achievable rates if both users perform ZF are given by

\[
E[R_{ZF}^i] = E \left[ \log_2 \left( 1 + \frac{|w_{ZF}^i h_{ii}|^2}{\sigma^2} \right) \right] = G(\sigma^2, n - 1) \tag{8}
\]

where \(G(\cdot, \cdot)\) is defined in (4).

**Proof:** See [22]. □

Comparing Proposition 2 with the single-user rates (see (5)), we see that the transmitter interference cancellation
offered by ZF costs precisely one degree of freedom (since $n$ is reduced to $n−1$ in the argument of $G(·, ·)$). For a small number of antennas, e.g., $n = 2$, this will have a major impact on performance. For a large number of antennas, however, the reduction in the number of degrees of freedom associated with ZF is negligible, and the ZF rates will be close to the single-user rates on the average. The rate in (8) grows with increasing SNR without bound. However, ZF is not going to be a stable outcome of the spectrum resource conflict, for the same reasons as the sum-rate point was not stable.

III. BEAMFORMING AS A GAME THEORETIC PROBLEM

In this section we will treat the beamforming problem in a game-theoretic framework. We will separately discuss the two cases that the systems can cooperate, respectively not cooperate, in choosing their beamforming vectors. Whenever we refer to “cooperation” in this paper, we mean cooperation in the sense of the theory for non-zero-sum games [19].

A. Competitive (Non-cooperative) Solution

If BS$_1$, BS$_2$ do not cooperate then the only reasonable outcome of the spectrum conflict will be an operating point which constitutes a Nash equilibrium. This is a point where none of the base stations can improve its situation by unilaterally changing $w_i$, subject to the power constraint [19]. It is clear (and more generally shown in [12]) that at a Nash Equilibrium both users must use the entire available bandwidth and time, so we make that assumption for the rest of this subsection. A Nash equilibrium is then a pair of vectors $w^*_1$, $w^*_2$ such that

$$\log_2 \left( 1 + \frac{|w^T_1 h_{11}|^2}{|w^T_2 h_{21}|^2 + \sigma^2} \right) \geq \log_2 \left( 1 + \frac{|w^T_2 h_{12}|^2}{|w^T_2 h_{21}|^2 + \sigma^2} \right)$$

for all $w_1$ with $\|w_1\|^2 \leq 1$ and

$$\log_2 \left( 1 + \frac{|w^T_2 h_{21}|^2}{|w^T_2 h_{21}|^2 + \sigma^2} \right) \geq \log_2 \left( 1 + \frac{|w^T_2 h_{12}|^2}{|w^T_2 h_{21}|^2 + \sigma^2} \right)$$

for all $w_2$ with $\|w_2\|^2 \leq 1$. We have the following result.

**Proposition 3:** There is a unique, pure Nash equilibrium corresponding to the maximum-ratio transmission beamforming vectors

$$w^*_1 = \frac{h_{11}}{\|h_{11}\|} \quad \text{and} \quad w^*_2 = \frac{h_{22}}{\|h_{22}\|}.$$ 

**Proof:** See [22].

The corresponding rates at the equilibrium are

$$R^*_1 = \log_2 \left( 1 + \frac{|h_{11}|^2}{\|h_{22}\|^2 + \sigma^2} \right) \quad \text{and} \quad R^*_2 = \log_2 \left( 1 + \frac{|h_{12}|^2}{\|h_{21}\|^2 + \sigma^2} \right).$$

Note that by using $w^*_1$, BS$_1$ can guarantee the rate $R^*_1$ regardless of what beamforming vector BS$_2$ is using, and vice versa. (A discussion of Nash equilibria for the more general case of a MIMO IFC is given in [13, Proposition 3.1].)

The Nash equilibrium is contained in $R$, but in general it does not lie on the Pareto boundary. At low SNR, the Nash equilibrium is not a bad outcome since $\sigma^2$ will dominate over the interference terms in (9). Hence using $w^*_i$ (which amounts to maximum-ratio beamforming) will maximize the rate of each user. However, at high SNR, the equilibrium outcome of the game is generally poor for both systems. This observation is made precise in the following result.

**Proposition 4:** Suppose the systems operate in i.i.d. Rayleigh fading (the entries of $h_{ij}$ independent and complex Gaussian with zero mean and unit variance). Then the average Nash equilibrium rates are bounded by

$$E[R^*_i] = E \left[ \log_2 \left( 1 + \frac{|h_{11}|^2}{\|h_{22}\|^2 + \sigma^2} \right) \right] \leq \frac{\Psi(n) + \gamma + 1/n}{\log(2)}.$$  

(10)

for $i = 1, 2$, where $\Psi(x)$ is the Psi (DiGamma) function and $\gamma$ is Euler’s constant. The upper bound in (10) is tight for high SNR ($\sigma^2 \to 0$).

**Proof:** See [22].

The basic implication of Proposition 4 is that to achieve high rates in unlicensed bands, the systems need somehow to cooperate. For two transmit antennas ($n = 2$), the upper bound in (10) is given by

$$\frac{\Psi(n) + \gamma + 1/n}{\log(2)} \approx 2.16 \text{ bpcu}$$

This result corresponds well with the numerical result that we will present in Section IV (Figure 3). Another consequence of Proposition 4 is that since $\Psi(x) = O(\log(x))$, the equilibrium rate can grow at most logarithmically with the number of transmit antennas per base station, $n$. This means that adding more antennas help only marginally, if the systems compete with each other.

Note that generally a Nash equilibrium is not an optimal, or even desirable, solution in any sense (although misconceptions around this appear to exist). Rather, the equilibrium is a point where one is likely to end up operating if BS$_1$ and BS$_2$ compete with each other. All we can say
of this outcome is that the result can become no worse, if the base stations choose beamformers by unilateral action. In fact, in many games (including the one studied here) the Nash equilibrium is unique but it corresponds to an outcome which is bad for all players. See, the famous example of prisoner’s dilemma, for example [19].

B. Cooperative (Nash Bargaining) Solution

If BS$_1$ and BS$_2$ were able to cooperate, they could achieve rates higher than (\(R_1^{\text{NE}}, R_2^{\text{NE}}\)), say (\(R_1^{\text{NB}}, R_2^{\text{NB}}\)) (NB as in Nash Bargaining, to be defined). By reaching an appropriate agreement they could achieve any point in \(\bar{\mathcal{R}}\) or on the Pareto boundary. It is clear that if an agreement could be reached then \(R_i^{\text{NB}} \geq R_i^{\text{NE}}\), since otherwise at least one of the base stations would resort to the competitive (noncooperative) solution, which we know guarantees each link a rate of at least \(R_i^{\text{NE}}\). Thus we may restrict the search for collaborative solutions to the subregion \(\mathcal{R}^+\) that consists of all points of \(\mathcal{R}\) for which \(R_i \geq R_i^{\text{NE}}\), i.e. the set of points located to the upper right of the Nash equilibrium.

For example, the base stations could agree to operate at the sum-rate or ZF point. However, such an outcome is not likely to occur in practice, unless it is imposed by regulation. Additionally, if such a regulation is imposed it would be very hard to check whether the base stations comply with it. The reason is that generally one of the base stations would have to “give in” more than the other in order to agree on a specific operating point (such as the ZF or the sum-rate point). The basic issue is that if the players try to agree on a point on the Pareto boundary, then any incremental improvement for one leads to a reduction for the other.

We will examine this problem by using the axiomatic bargaining theory developed by economist John Nash in the 1950’s (and who was subsequently awarded the Nobel prize in economics) [19], [20]. Nash considered the general problem of establishing an agreement between players with conflicting objectives, under the assumption that there exists no utility (in our case “rate”, but in general, money for example) that one player could pay to the other in order to compensate the other for a non-favorable outcome. Thus, the players must agree on an outcome (\(R_1^{\text{NB}}, R_2^{\text{NB}}\)). If they fail, they will resort to playing non-cooperatively which generally results in an operating point no better than (\(R_1^{\text{NE}}, R_2^{\text{NE}}\)). This fallback point is generally called a “threat point” in bargaining theory, because it represents the outcome in the event the players would realize their threat not to cooperate.

Nash showed that under certain conditions there is a unique mapping between the convex hull of the achievable region (\(\bar{\mathcal{R}}\)), the treat point (which we take to be (\(R_1^{\text{NB}}, R_2^{\text{NB}}\)), and the cooperative (bargaining) outcome (\(R_1^{\text{NB}}, R_2^{\text{NB}}\)). The conditions stated by Nash are a set of axioms. Apart from technicalities, these essentially say that the cooperative outcome must lie on the Pareto boundary, and that the solution should be independent of irrelevant bargaining alternatives in the sense that if the solution is contained in a subset of \(\bar{\mathcal{R}}\), say \(\mathcal{R}'\), then the same bargaining solution would have been obtained if the feasible set had been \(\mathcal{R}'\) at the outset. Additionally, invariance to linear transformations is required (see [19] for details).

The Nash solution for the two player game at hand can be explicitly computed as follows:

\[
(R_1^{\text{NB}}, R_2^{\text{NB}}) = \max_{(R_1, R_2) \in \mathcal{R}^+} (R_1 - R_1^{\text{NE}})(R_2 - R_2^{\text{NE}}).
\]

In other words, the outcome of the bargaining is going to be the point where the Pareto boundary \(\mathcal{R}^+\) has exactly one intersection point with a curve of the form \((R_1 - R_1^{\text{NE}})(R_2 - R_2^{\text{NE}}) = c\) where \(c\) is a constant (chosen such that there is precisely one intersection point). Thus, given \(\mathcal{R}\) and \((R_1^{\text{NE}}, R_2^{\text{NE}})\) the Nash solution can in principle be found graphically. This is illustrated in Figures 2(a)–2(b). In these figures we can also see that the competitive solution \((R_1^{\text{NB}}, R_2^{\text{NB}})\) is generally much inferior to the Nash bargaining solution \((R_1^{\text{NB}}, R_2^{\text{NB}})\).

Note that the Nash bargaining solution is only defined for convex outcome regions. Thus, it requires that the systems are willing to perform time-sharing using a pair of two different beamforming vectors as explained in Section II-A (see the discussion where \(\bar{\mathcal{R}}\) was defined).

We finally remark on the notion of fairness versus Nash bargaining. If the Nash bargaining theory is applied with a threat point at the origin (corresponding to zero rates if no collaboration is reached, rather than using the equilibrium rates), then the NB solution will coincide with the outcome of the following maximization problem

\[
\max_{(R_1, R_2) \in \mathcal{R}^+} \log_2(R_1) + \log_2(R_2). \tag{11}
\]

The solution to (11) is sometimes called a “proportional fair” allocation. However, it is important to stress that the Nash bargaining solution has nothing to do with “fairness” in general. Rather it is an attempt to predict what will happen if the players act strictly rationally, i.e., they want to cooperate but nevertheless act with self-interest. Generally, a player who is already in a good position will gain more because he can be stronger in a negotiation (his threat is more effective). There are numerous examples in economics where the Nash bargaining solution would be considered unfair for most human observers [21].

IV. NUMERICAL RESULTS

We performed numerical experiments to gain insight into the phenomena analyzed in Sections II–III and the
corresponding scaling laws. For a fixed channel realization, the rate region $R$ was approximated by setting $w_i = [\alpha_i, \sqrt{1 - \alpha_i^2}]$ and then varying $\{\alpha_1, \alpha_2, \phi_1, \phi_2\}$ over a grid over $[0, 1] \times [0, 1] \times [-\pi, \pi] \times [-\pi, \pi]$ (in total $4^4$ points were searched for each channel realization). The Nash equilibrium and the rate points discussed in Section II-C are easy to compute. We found the Nash bargaining solution numerically by an interval-halving type search.

Illustrations of typical regions were given in Figures 2(a)–2(b). We also computed the average rates in i.i.d. Rayleigh fading. The computation was accomplished by numerical averaging over 2000 channel realizations. The result is shown in Figure 3. This figure confirms the conclusion of Proposition 4, regarding the high-SNR behavior of the Nash equilibrium. We can also see that all other rates grow with the SNR. Most interestingly, the Nash bargaining solution is about as good as zero-forcing, although it is quite far from the sum-rate point. (Neither of the two latter points would be achievable by voluntary bargaining, unless enforced by regulations.) This observation forms one of our major empirical conclusions.

V. CONCLUSIONS

In this paper we have considered the conflict situation that arises when two multiple-antenna systems must share the same (unlicensed) spectrum band. We have made two central points. First, we showed that if the systems do not cooperate, then the corresponding equilibrium rates are bounded regardless of how much transmit power the base stations have available. The important consequence of this is that there is a fundamental need for base station (system) collaboration in spectrum sharing with multiple antennas. Second, in numerical experiments, we found that the outcome of a Nash bargaining between the two systems is on the average close to the sum-rate bound. This indicates that in reality, selfish but cooperating systems may achieve close to max-sum-rate performance. It remains to develop protocols that the systems can use to communicate and actually reach the bargaining agreements whose existence we have predicted theoretically.

The work here can be extended in several directions. One may study the case where only partial (for example, long-term) channel state information is available. The theory may also be extended to multiple antennas at the receivers (i.e., MIMO). Another direction concerns the specific choice of cooperative game-theoretic axiomatic framework. We have chosen the Nash bargaining theory because it is well-established, and since it enabled us to compute the bargaining point numerically, with relative ease. Other approaches to cooperative games (e.g., $\lambda$-transfer theory) may also be possible.

REFERENCES


