Energy Efficiency in Random Opportunistic Beamforming

Zhijiat Chong and Eduard A. Jorswieck

Technische Universität Dresden, Institut für Nachrichtentechnik, Lehrstuhl Theoretische Nachrichtentechnik, 01062 Dresden, Germany
e-mail: {zhijiat.chong, eduard.jorswieck}@tu-dresden.de

Abstract—As a step towards decreasing power consumption in mobile communication networks, we investigate the energy efficiency (EE) of a scheduler that can be utilized at the transmitter in a broadcast channel: the random opportunistic beamforming algorithm. Using the ratio of rate to power as the EE metric and considering the constant circuit power, the EE at the transmitter decreases with the number of transmit antennas $M$. Two approaches of optimizing the power allocation when $M = 1$ are introduced. We show that the optimal EE scales double-logarithmically with the number of users, which has one antenna each. The appropriate expression for EE in communications is also discussed.

I. INTRODUCTION

With the fast emergence of the new generation of mobile communication, where providers offer higher rates, wider coverage and a larger variety of services, its power consumption is expected to rise substantially. Consequently, energy efficiency (EE) will become an inevitable concern in network design and architecture. This is essential not only from the economic aspect but also because of future energy availability and environmental reasons. Without considering users’ phones, the power consumption of a typical mobile communications network is estimated to be about 40 MW [1], where base stations have the major portion [2], [3]. Currently, 0.14% of the global CO$_2$ emissions are contributed by the mobile telecommunication industry [4]. In order to improve the situation, new algorithms are studied at physical and multiple access layers, which include resource and power allocation. Algorithms to achieve EE in OFDM systems and frequency-selective channels were presented in [5] and [6], respectively. Game theory was applied in [7] to analyze the EE of CDMA systems. In [8], the technique of switching between SIMO and MIMO to achieve EE in the uplink was investigated.

Our aim is to explore the possibilities of increasing the EE at base stations by implementing a suitable scheduler in cellular broadcast channels. As multi-antenna systems have been proved to provide higher rates, its popularity is anticipated to increase in the next generations of communication. The question of EE remains an issue. Particularly in this paper, we choose to investigate the EE of an established MISO scheduler at the transmitter known as random opportunistic beamforming (ROBF) [9].

One contribution of this paper is the proof of a clear trade-off between the EE and rate of the transmitter in this scheduling algorithm. Specifically, the EE decreases with the number of antennas used, and thus with the rate as well. As our second contribution, we show that the optimal EE of a maximum throughput scheduler (which is the ROBF with one antenna) scales with log log $n$ as the number of users $n \to \infty$. This multi-user diversity effect is identical to that of the optimal transmission rates. These results are based on the ratio of rate to total consumed power as the EE metric, which was also used in [5], [6], [7], [8]. Some reasons for the eligibility of this metric are found in [10], [11]. We use the power consumption model proposed in [12], where a constant power is spent for the employment of every antenna at the transmitter.

We will present the system and the channel model in Section II. Here, we will also describe the ROBF scheduling algorithm and discuss the EE measure. In Section III, we will show that transmission with one antenna yields the best EE when using beamforming. In Section IV, we present two methods of optimization when using the ROBF with one transmit antenna: dynamically optimizing the transmit power at every realization or time slot, and optimizing the transmit power which will be used constantly over time based on the average rate. We show analytically that the optimal EE achieved by the latter method scales asymptotically with log log $n$. In Section V, we compare the EE of both these methods using simulations and show that the optimal EE achieved by dynamic optimization scales in the same way as static optimization. Section VI concludes this paper.

II. PRELIMINARIES

A. System model

We consider a multiple-antenna Gaussian broadcast channel similar to [9]. Here, we focus on a multiple-input single-output (MISO) system, where there is one base station with $M$ transmit antennas and $n$ users with one antenna each. We assume a block-fading model where the channel matrix is constant throughout each coherence period $T$ and is independent from one period to the other. We will therefore omit the time index for the sake of convenience.

B. Channel model

The transmit signal is described as an $M \times 1$ vector $s$ of the transmit symbols. $y_k$ is the scalar of the received signal
at the kth receiver, \( k \in \{1, 2, \ldots, n\} \), which is related to the transmit signal as the following:
\[
y_k = \sqrt{p_k} \mathbf{h}_k \mathbf{s} + w_k, \quad k = 1, \ldots, n,
\]
(1)

where \( \mathbf{h}_k \) is the \( 1 \times M \) complex channel vector for the \( k \)th receiver, \( w_k \) the scalar additive noise. Each component of \( \mathbf{h}_k \) and \( w_k \) is independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and unit variance \( CN(0, 1) \). The SNR of each user is given by \( p_k \). In our model we assume that \( p_k \) is constant for all \( k \) (\( p_k = \rho \)).

C. Scheduling algorithm using random beamforming

In this algorithm, data streams \( s_m, m \in \{1, \ldots, M\} \), are to be transmitted to \( M \) out of \( n \) receivers via beamforming. These streams are weighted by \( M \) random orthonormal \( M \times 1 \) vectors \( \phi_m \) to form the signal vector such that
\[
\mathbf{s} = \sum_{m=1}^{M} \phi_m s_m. \tag{2}
\]
The beamforming vectors \( \{\phi_m\} \) are randomly generated using isotropic distribution [13], such that \( \Phi = (\phi_1, \ldots, \phi_M) \) forms a unitary matrix. The signal at the \( k \)th receiver is then
\[
y_k = \sum_{m=1}^{M} \mathbf{h}_k \phi_m s_m + w_k. \tag{3}
\]

During the training phase, signals \( s_m \) are transmitted through their corresponding beamforming vectors \( \phi_m \). Each receiver \( k \) calculates the \( M \) SINRs regarding \( s_m \) as the desired signal and treating the other \( s_i \)'s as interference, such that
\[
\text{SINR}_{k,m} = \frac{|\mathbf{h}_k \phi_m|^2}{1/\rho + \sum_{i \neq m} |\mathbf{h}_k \phi_i|^2} \quad m = 1, \ldots, M. \tag{4}
\]

As a function of the power, the SNR \( \rho \) can be written as \( p/\sigma^2 \), where \( p \) is the average signal power used for each data stream and \( \sigma^2 \) is the noise power. Since the absolute signal power has been factored out into the SNR term, we normalize every component of \( \mathbf{s} \) such that \( \mathbb{E}[|s_m|^2] = 1 \) for all \( m \), where \( \mathbb{E}[X] \) is the expected value of the random variable \( X \), which then yields \( \mathbb{E}[|s|^2] = M \).

Each receiver \( k \) selects the beamforming vector with the highest SINR, i.e. \( \arg \max_{m} \text{SINR}_{k,m} \), and returns both the index of this beamforming vector \( m \) and its corresponding SINR value back to the transmitter. For each beamforming vector \( m \), the transmitter then selects the user \( k \) with the highest SINR from the set of users who fed back \( m \) as their preference, i.e. \( \arg \max_{k} \text{SINR}_{k,m} \), with \( k \in \{k' : \arg \max_{m', \text{SINR}_{k',m'} = m\} \} \), and transmits data through \( m \) to them for a period of \( T \). After \( T \), another set of orthonormal vectors \( \{\phi_m\} \) is generated and the procedure above is repeated. Note that for \( M = 1 \), this scheduling is equivalent to the maximum throughput scheduler. We will show in Sec. III that the case with \( M = 1 \) is the most energy-efficient choice.

D. Energy efficiency

In the general sense, efficiency can be seen as the ratio of the goods produced to the resources consumed. In communications, the goods are the information units (nats) and the resources are the energy (Joule) consumed for transmitting data in a certain period of time. For nonvarying powers and rates, EE can be expressed as the ratio of the rate \( r \) to the power that achieves this rate:
\[
EE = \frac{r(p)}{pC + p} \quad \text{[nat Joule]}.
\]

The denominator consists of two terms, namely the constant power \( pC \) and the transmission power \( p \). The constant power can be understood as the requirement for enabling transmission and is always spent independently of the transmission power, which is justified considering the power consumption model of base stations [2]. It consists mainly of the power for the circuit electronics. According to the model in [12], \( pC \) is linearly proportional to the number of antennas used.

If the rate and the power are not constant in time, EE can be expressed as
\[
EE = \frac{\int_{T} r(p(t)) \, dt}{\int_{T} (p_C + p(t)) \, dt} = \frac{TE_t [r(p(t))]}{T(p_C + E_t [p(t)]},
\]
(6)
which is the ratio of the total amount of information units transmitted during a period \( T \), \( TE_t [r(p(t))] \), to the total energy expended \( Tp_C + TE_t [p(t)] \). Under the assumption that \( T \) is large enough for ergodicity and stationarity (of the channel distribution) to hold, \( E_t [x] = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{T} x(t) \, dt \) is the expected value of \( x \) averaged over time. Thus, \( R = E_t [r(p(t))] \) is the mean rate and \( E_t [p(t)] \) the mean transmit power.

III. ANALYSIS OF ENERGY EFFICIENCY WITH M ANTENNAS

Here, we will show that the EE of the random beamforming algorithm is maximum when \( M = 1 \). We will do this by analyzing the way the average rate changes with \( M \). According to [9],
\[
R \leq E \left[ \sum_{m=1}^{M} \log \left( 1 + \max_{1 \leq k \leq n} \text{SINR}_{k,m} \right) \right] = ME \left[ \log \left( 1 + \max_{1 \leq k \leq n} \text{SINR}_{k,m}^{(M)} \right) \right] = MR_u, \tag{7}
\]
where \( \text{SINR}_{k,m}^{(M)} \) is a representative of \( \text{SINR}_{k,m} \) for any \( m \) according to (4). More details about \( \text{SINR}_{k,m}^{(M)} \) are presented below. Equality for (7) holds, if the probability that user \( k \) has the maximum SINR for two or more beamforming vectors is zero. Otherwise, \( MR_u \) is regarded as the upper bound and is evaluated by obtaining the distribution of \( \text{SINR}_{k,m}^{(M)} \).

1For the sake of convenience in calculations later, we choose this information unit instead of the usual bit.
Lemma 1. \( R_u = \mathbb{E} \left[ \log \left( 1 + \max_{1 \leq k \leq n} \text{SINR}_k^{(M)} \right) \right] \) decreases as \( M \) increases.

Proof: We will show that \( \text{SINR}_k^{(M)} \) decreases as \( M \) increases for all \( k \). Recall from (4) that without loss of generality

\[
\text{SINR}_k^{(M)} = \frac{|h_k \Phi_i|^2}{1/\rho + \sum_{i=2}^{M} |h_k \Phi_i|^2},
\]

taking \( m = 1 \) as a representative for all \( ms \). Recall also that \( \Phi = (\Phi_1, ..., \Phi_M) \) is a unitary matrix and each component of \( h_k \) is distributed according to \( CN(0, 1) \). Thus, the random vector \( h_k \Phi^H \) has the same distribution as \( h_k \) and we can substitute \( |h_k \Phi_i|^2 \) with \( |h_k \Phi^H \Phi_i|^2 \) without any consequences on the distribution of \( \text{SINR}_k^{(M)} \). \( |h_k \Phi^H \Phi_i|^2 \) is now written as \( |h_{k,i}|^2 \), which is the absolute value squared of the \( i \)th component of \( h_k \), whose distribution is identical for all \( i \). Consider extending the dimension of \( h_k \) from \( M \) to \( M + 1 \) columns. With an analogous matrix \( \Phi = (\Phi_1, ..., \Phi_{M+1}) \), where \( \Phi_i \) is now an \( (M + 1) \times 1 \) vector, the corresponding SINR is expressed as

\[
\text{SINR}_k^{(M)} = \frac{|h_{k,1}|^2}{1/\rho + \sum_{i=2}^{M} |h_{k,i}|^2 + |h_{k,M+1}|^2}. \tag{9}
\]

Thus, increasing \( M \) clearly increases the interference and thus reduces \( \text{SINR}_k^{(M)} \) for any \( k \), including \( k^* = \arg \max_{1 \leq k \leq n} \text{SINR}_k^{(M)} \). Hence, the expected value \( R_u \) also decreases with \( M \).

We assume that if \( M \) transmit antennas are used for \( M \) beamforming vectors, the constant power used is \( M \) times what is used for one antenna, as suggested in [12], [8]. The average transmit power for each beamforming vector carrying one data stream is \( p = \rho \sigma^2 \). Thus, we formulate the EE as

\[
\text{EE} = \frac{MR_u}{M(pC + p)} = \frac{R_u}{pC + p}. \tag{10}
\]

Because of Lemma 1, we conclude that \( M = 1 \) is the most energy-efficient selection. This case will be our focus in the next section. Note that this conclusion is not necessarily valid if there is a sum power constraint where \( P = Mp \) is held constant in \( M \) instead of \( p \). The discussion of this matter is, however, beyond the scope of this paper.

IV. ENERGY EFFICIENCY OF A MAXIMUM THROUGHPUT SCHEDULER

With \( M = 1 \), random beamforming scheduling reduces to a maximum throughput scheduler. We will investigate the asymptotic properties of the EE of this scheduler in this section. The EE of this scheduler is measured using (6) with the average rate \( R \) being

\[
R = \mathbb{E} \left[ \log (1 + \rho \max_{1 \leq k \leq n} |h_k|^2) \right]. \tag{11}
\]

The random variable of the SINR is represented by \( X \), which is an i.i.d. random variable with \( \chi^2(2) \) distribution, and the random variable of the maximum SINR by \( Z \). The cdf of \( X \) is \( \text{cdf}_X(x) = 1 - \exp(-x) \). Using order statistics [14], it follows that the cdf of \( Z \) is given by \( \text{cdf}_Z(x) = (1 - \exp(-x))^n \). From this, we derive the cdf for \( r = \log (1 + \rho Z) \):

\[
\text{cdf}_r(x) = \text{cdf}_Z \left( \frac{\exp(x) - 1}{\rho} \right) = \left( 1 - \exp\left( -\frac{\exp(x) - 1}{\rho} \right) \right)^n. \tag{12}
\]

We examine two approaches for optimizing the average EE. One of them is finding the optimal power allocation with respect to EE at every time slot \( t \). This power allocation is given by

\[
p^* = \arg \max_p \log \left( \frac{1 + \frac{p}{\sigma^2} Z \cdot E} {pC + p} \right), \tag{13}
\]

which is itself a random variable that depends on \( Z \). The optimal EE is then

\[
\text{EE}_T = \mathbb{E} \left[ \log \left( 1 + \frac{Z \cdot \text{EE}} {pC + \text{EE}[p^*]} \right) \right]. \tag{14}
\]

We call this the dynamic optimization.

The other case is choosing a fixed optimal power allocation for all realizations based on the average rate. We call this the static optimization. The optimal EE obtained is expressed as

\[
\text{EE} = \max_p \mathbb{E} \left[ \log \left( 1 + \frac{p}{\sigma^2} Z \cdot \text{EE} \right) \right]. \tag{15}
\]

The analysis of the expression in (14) is difficult. Therefore, we will omit it and only investigate it via simulation in Sec. V. The EE achieved by (14) should outperform that of (15) since the former allows more freedom in choosing the optimal power allocation over time. In the simulations, however, we observe that this is not always the case. This will be discussed in Sec. V.

In the following, we will focus on the analysis of the EE given by (15). However, the rate part of EE as a function of power, which is given by

\[
\mathbb{E} \left[ \log \left( 1 + \frac{p}{\sigma^2} Z \cdot \text{EE} \right) \right] = \int_0^\infty x \cdot \text{pdf}_r(x) \, dx
\]

\[
= \int_0^\infty x \frac{n}{p/\sigma^2} \exp(x) \left( \frac{-\exp(x) - 1}{\rho} \right) \times \left( 1 - \exp\left( -\frac{\exp(x) - 1}{\rho} \right) \right)^{n-1} \, dx \tag{16}
\]

with \( \text{pdf}_r(x) = \frac{\rho}{p} \cdot \text{cdf}_r(x) \), is also difficult to handle. Therefore, we resort to investigating its lower and upper bounds, for which there are closed-form expressions. Since \( \exp(-x) \leq 1 \) for \( x > 0 \), we obtain the lower bound

\[
\mathbb{E} \left[ \log \left( 1 + \frac{p}{\sigma^2} Z \cdot \text{EE} \right) \right] \leq \int_0^\infty x \exp(-x) \cdot \text{pdf}_r(x) \, dx \tag{17}
\]

\[
\geq \arg \max \exp(-x) \cdot \text{pdf}_r(x) \tag{18}
\]

\[
= \log \left( 1 + \log(n) \frac{p}{\sigma^2} \right) = R. \tag{19}
\]
Based on numerical evidence, we observe that \( \exp(-x)pdf_f(x) \) is unimodal\(^2\) and right-skewed (positive skew). We thus conjecture that its mode (18) is less than (17). In the simulations in Sec. V, we see that this is justified. We compute (18) by setting the derivative of \( \exp(-x)pdf_f(x) \) to zero and obtain (19). The EE with respect to the rate lower bound is then

\[
EE = \frac{\log(1 + \frac{\log(n) \sigma_c}{\sigma_e})}{pC + p},
\]

(20)

EE is a strictly pseudo-concave function of \( p \) for \( p \in [0, \infty) \). Its pseudo-concavity results from the fact that the numerator is a strictly concave function and the denominator a convex function\(^3\). This implies that its modal value is unique and is found at \( p^* \), where \( \frac{dEE(p)}{dp} \bigg|_{p=p^*} = 0 \). Thus, the optimal power allocation is given by

\[
p^* = \frac{\log(n)pC - \sigma^2(T(n) + 1)}{\log(n)T(n)},
\]

(21)

where \( T(n) = W \left( \frac{\log(n)pC - \sigma^2}{\sigma_e} \right) \) and \( W(x) \) is the Lambert-W function\(^3\). The corresponding EE is then

\[
EE(p^*) = \frac{\log(\frac{\log(n)pC - \sigma^2}{T(n)\sigma_e})}{(T(n) + 1)(\log(n)\sigma_c - \sigma^2)}.
\]

(22)

**Theorem 2.** \( EE(p^*) \) scales with \( \log(n) \) as \( n \to \infty. \)

**Proof:** We can show that

\[
\lim_{n \to \infty} \frac{EE(p^*)}{\log(n)} = \frac{\log(\frac{\log(n)pC - \sigma^2}{T(n)\sigma_e})}{(T(n) + 1)(\log(n)\sigma_c - \sigma^2)\log(n)} = \frac{1}{pC}
\]

In the course of calculation, we used the following relation:

\[
\lim_{n \to \infty} \frac{T(n)}{\log(n)} = \frac{W(\frac{pC\log(n)}{\sigma_c} \log(n))}{\log(n)} = 0.
\]

The last line holds because the argument \( z \) in the function \( W(z) \) increases more rapidly than the function itself. \( \Box \)

Next, we derive the upper bound. Since \( \log(1 + \frac{p}{\sigma^2}Z) \) is a concave function, we obtain the upper bound of the average rate \( \bar{R} \) using Jensen's inequality:

\[
E \left[ \log(1 + \frac{p}{\sigma^2}Z) \right] \leq \log \left( 1 + \frac{p}{\sigma^2} E[Z] \right) = \bar{R}.
\]

(23)

Furthermore, \( \bar{R} \) can be expressed in closed form:

\[
\bar{R} = \log \left( 1 + \frac{\Psi(n + 1) + \gamma \cdot p}{\sigma^2} \right),
\]

(24)

where \( \Psi(x + 1) = -\gamma + \sum_{k=1}^{\infty} \left( \frac{1}{k} - \frac{1}{x+k} \right) \) is the digamma function and \( \gamma = 0.5772... \) is the Euler–Mascheroni constant. This is done by evaluating \( E[Z] = \int_0^\infty x \cdot pdf_g(x) \, dx \), where \( pdf_g(x) = \frac{\sigma_e}{\sigma_c} \cdot pdf_f(x) \), through exploiting the binomial identity such that \( (1 - \exp(-x))^{n-1} = \sum_{k=0}^{n-1} \binom{n-1}{k} (-\exp(-x))^k \). The corresponding EE is

\[
EE(p) = \frac{\bar{R}}{pC + p} = \log \left( 1 + \frac{(\Psi(n+1) + \gamma) \cdot p}{\sigma_c} \right).
\]

(25)

As done previously, we find the optimal power allocation by equating its first derivative to zero and obtain:

\[
p^* = \frac{pCq(n) - \sigma^2(1 + W(\cdot))}{q(n)W(\cdot)} \]

(26)

where \( W(\cdot) = W \left( \frac{pCq(n) - \sigma^2}{\sigma_e} \right) \) and \( q(n) = \Psi(n + 1) + \gamma \).

Correspondingly, the upper bound of EE is

\[
EE(p^*) = \frac{\log \left( \frac{q(n) - \sigma^2}{\sigma_e^2} \cdot W(\cdot) \right)}{pCq(n) - \sigma^2(1 + W(\cdot))}.
\]

(27)

**Theorem 3.** \( EE(p^*) \) scales with \( \log(n) \) as \( n \to \infty. \)

**Proof:** From the expansion \( \Psi(x) = \log(x) - \frac{1}{2x} - \frac{1}{12x^2} + \ldots \) according to [18], we obtain

\[
\lim_{n \to \infty} \frac{\Psi(n + 1) + \gamma}{\log(n)} = \infty; \lim_{n \to \infty} \frac{\Psi(n)}{\log(n)} = 1.
\]

(28)

By applying these and some common properties in the limit of a function, we can show that

\[
\lim_{n \to \infty} \frac{EE(p^*)}{\log(n)} = 0.
\]

Here, we also applied the fact that \( \log(f(x)) \), where \( f(x) \) is an increasing function, grows faster than \( \log(W(f(x))) \).

\( \Box \)

Since both the lower and upper bounds of the optimal EE asymptotically scale with \( \log(n) \), we conclude that EE also scales in the same way with \( n \).

**Corollary 4.** The optimal EE of maximum throughput scheduler scales with \( \log(n) \) as \( n \to \infty. \)

**V. SIMULATIONS**

In Fig. 1, we compare the values of the optimal values of EE (static optimization), \( EE \) (lower bound), \( EE \) (upper bound) and \( EE_T \) (dynamic optimization). These correspond to (15), (22), (27) and (14), respectively, for \( n = 2, 4, \ldots, 30 \). EE is calculated by numerical integration, whereas \( EE_T \) is computed by generating 10000 random realizations of \( Z \). Fig. 2 shows the average rates achieved corresponding to the optimal EE, \( EE \), \( EE \) and \( EE_T \) in Fig. 1. We observe that \( EE, EE \) and \( EE \) scale with \( \log\log(n) \) just as the rate, agreeing with the results stated in Section IV. We can also observe that \( EE_T \) scales with \( \log\log(n) \).

The optimal EE achieved by dynamic optimization, \( EE_T \), begins to outperform that of static optimization EE only at \( n = 8 \). This may seem surprising as one would expect it to always perform better because the system is optimized more frequently. This can be understood by considering the following. When there are few users, the chances of obtaining
a small maximum SINR is greater than when there are more users. This is evident since more users implies a higher degree of user diversity and thus the average maximum SINR also increases. At low SINR, the optimum power allocation according to (13) is higher than that at high SINR. If this occurs often, it is not efficient since the scheduler would often choose high power allocations that do not yield high rates. This rather deteriorates the overall EE than increases it. A lower degree of user diversity and thus the average maximum SINR decreases with the number of users. This is evident since more users implies a higher degree of user diversity and thus the average maximum SINR also increases.

VI. CONCLUSION

We have shown that under certain assumptions the EE of the transmitter in a broadcast channel using the ROBF decreases with the number of antennas employed. We also showed that the optimal EE of a maximum throughput scheduler scales with \( \log \log n \) as \( n \to \infty \). In simulations, two energy-efficient power allocation approaches were compared: the dynamic and static optimization. It can be observed that the dynamic optimization is basically superior to the latter except when the number of users is small. Future work includes analysis of the EE of the ROBF with multiple antennas on the receivers’ side and with more realistic channel statistics.

ACKNOWLEDGEMENT

The authors would like to thank our partners in the Cool Cellular project and Christian Isheden for helpful discussions. This work is supported by the German Federal State of Saxony in the excellence cluster Cool Silicon in the framework of the project Cool Cellular under grant number 14056/2367.

REFERENCES