In Casavola, Giannelli, and Mosca (2000), an open-loop min–max MPC algorithm was proposed for robustly regulating input-saturated discrete-time systems described by polytopic model uncertainties. The term open-loop is now commonly referred to MPC strategies based on the following control policy:

\[ u(t + k|t) = \begin{cases} u_k, & k = 0, 1, \ldots, N - 1, \\ Fx(t + k|t), & k \geq N \end{cases} \]

(1)

with \( N \) the control horizon, \( x(t + k|t) \) the set-valued \( k \)-step ahead state prediction and \( F \) the terminal feasible and robustly stabilizing state-feedback.

In Lemma 2 and Main Result of Casavola et al. (2000) it was erroneously claimed that the regions of attraction are nested, viz. the region for \( N + 1 \) includes the one corresponding to \( N \). This can be shown to be untrue in general by counterexamples.

As discussed in Mayne, Rawlings, Rao, and Scokaert (2000), the flaw hinges upon the poor feasibility properties of open-loop MPC solutions in the presence of polytopic model uncertainties.

There are several ways for recovering the feasibility and all other properties stated in the Lemma 2 and Main Result of Casavola et al. (2000). The one that will be described in this note appears as the simplest one in that it only consists of slightly modifying the earlier formulation with no increase of computational load. The amended formulation uses, instead of (1), a so-called feedback control policy. The latter consists, in its simplest form, of adopting a control strategy of the form

\[ u(t + k|t) = \begin{cases} Fx(t + k|t) + c_k, & k = 0, 1, \ldots, N - 1, \\ Fx(t + k|t), & k \geq N \end{cases} \]

(2)

with \( F \) a feasible and robustly stabilizing state-feedback and \( c_k \) free control moves selected on-line by the MPC solver.

Notice that under no model uncertainty, MPC schemes based on either (1) or (2) are equivalent in terms of optimality and feasibility. On the contrary, under polytopic uncertainties, this does not hold true.

In fact, under (1), the fact that the problem has solution for a certain horizon \( N \), say \( \{u_0, \ldots, u_{N-1}\} \), does not imply in general that a solution exists for \( N + 1 \), viz. the feasible set of commands \( \{u_0, \ldots, u_{N-1}, u_N\} \) could be empty for \( N + 1 \) even if non-empty for \( N \) and the same initial condition \( x(0) \).

On the contrary, under (2), if \( \{c_0, \ldots, c_{N-1}\} \) is a feasible solution for the horizon \( N \), it can be proved that \( \{c_0, \ldots, c_{N-1}, 0\} \) is a feasible solution, though not necessarily optimal, for \( N + 1 \). The reason is that, if it is a solution, the sequence \( \{c_0, \ldots, c_{N-1}\} \) steers in \( N \)-steps the initial state \( x(0) \) onto a terminal ellipsoidal region, say \( \mathcal{E} \), which is positively invariant for the closed-loop uncertain system \( A(p) + B(p)F \) denoting a polytope element. This is to say that \( \mathcal{X}_N^N(x(0)) \subset \mathcal{E} \), \( \mathcal{X}_N^N(x(0)) \) being the set of all \( N \)-steps ahead state predictions emanating from \( x(0) \) under (2). Then, \( \{c_0, \ldots, c_{N-1}, 0\} \) is feasible for the horizon \( N + 1 \), because

\( (A_i + B_1F)z + B_i c_N \in \mathcal{E} \quad \forall z \in \text{vert}(\mathcal{X}_N^N(x(0))) \quad \forall i \in \{1, \ldots, q\} \)

(3)

is surely satisfied by \( c_N = 0 \) on all \( q \) system vertices by positive invariance of \( \mathcal{E} \).

On the contrary, under the formulation (1), if the sequence \( \{u_0, \ldots, u_{N-1}\} \) steers \( x(0) \) into \( \mathcal{E} \) in \( N \)-steps, viz. \( \mathcal{X}_{u_N}^N(x(0)) \subset \mathcal{E} \), it is not true in general that there exists a single vector \( u_N \) capable of satisfying

\( A_i z + B_i u_N \in \mathcal{E} \quad \forall z \in \text{vert}(\mathcal{X}_N^N(x(0))) \quad \forall i \in \{1, \ldots, q\} \)

(4)
for all system vertices $i$. Notice that (4) amounts to many quadratic conditions to be jointly satisfied by a single vector $u_N$. The same number of conditions need to be satisfied also under (2) with the difference that $c_N = 0$ is always a feasible choice by construction.

The above arguments prove that the regions of attraction for all schemes based on (2) are nested and non-decreasing with $N$. The interested readers can refer to Casavola, Famularo, and Franzé (2002), where a feedback version of the MPC scheme of Casavola et al. (2000) is detailed for the more general context of LPV systems.

Going to a different issue, some colleagues pointed out to us that the terminal condition $\mathcal{X}_N^T (x(0)) \subset \mathcal{E}$ was not explicitly imposed into the LMI’s of the MPC schemes of Casavola et al. (2000) and Casavola et al. (2002). For this condition to hold, it is required to add the constraint $J_N \leq \rho(t)$, $J_N$ being an upper-bound to the terminal cost and $\rho(t)$ the radius of the terminal ellipsoidal set $\mathcal{E}$ at time $t$. Though one can explicitly add the above constraint, in all the many simulated examples carried out so far this constraint turned out to be redundant and, for this reason, it was overlooked in the original formulation.

References

