Stanujkic, D., Zavadskas, K.E., Brauers, W.K.M., Karabasevic, D. (2015), "An Extension of the Multimoora Method for Solving Complex Decision-Making Problems based on the Use of Interval-valued Triangular Fuzzy Numbers", *Transformations in Business & Economics*, Vol. 14, No 2B (35B), pp.355-375.

---------TRANSFORMATIONS IN --------
BUSINESS & ECONOMICS

AN EXTENSION OF THE MULTIMOORA METHOD FOR SOLVING COMPLEX DECISION-MAKING PROBLEMS BASED ON THE USE OF INTERVAL-VALUED TRIANGULAR FUZZY NUMBERS

[1]Dragisa Stanujkic
Megatrend University
Faculty of Management in Zajecar
Park suma Kraljevica bb
19000 Zajecar
Serbia
Tel.: +381 19 430 805
Fax: +381 19 430 803
E-mail: dragisa.stanujkic@fmz.edu.rs

[2]Edmundas Kazimieras Zavadskas
Vilnius Gediminas Technical University
Research Institute of Smart Building Technologies
Sauletekio al. 11
LT-10223 Vilnius
Lithuania
E-mail: edmundas.zavadskas@vgtu.lt

[3]Willem K.M. Brauers
University of Antwerp
Faculty of Applied Economic
University of Antwerpen
F. Birontlaan 97
B2600 Antwerpen
Belgium.
E-mail: willem.brauers@uantwerpen.be

[4]Darjan Karabasevic
Megatrend University
Faculty of Management in Zajecar
Park suma Kraljevica bb
19000 Zajecar
Serbia
E-mail: darjankarabasevic@gmail.com

[1]**Dragisa Stanujkic**, PhD, is an Associate Professor at Graduate School of Computer Sciences and Faculty of Management in Zajecar, Megatrend University, Serbia. His current research has been focused on the decision-making theory, expert systems and intelligent decision support systems.

[2]**Edmundas Kazimieras Zavadskas**, Prof., the Head of the Department of Construction Technology and Management at Vilnius Gediminas Technical University, Lithuania. PhD in Building Structures (1973). Dr. Sc. (1987) in Building Technology and Management. Research interests: building technology and management, decision-making theory, automation in design and decision support systems.

[3]**Willem K.M. Brauers**, PhD in economics, master of arts (in economics) of Columbia University (New York), master in economics, in management and financial sciences, in political and diplomatic sciences and bachelor in philosophy (all of the University of Leuven). He is professor ordinarius at the Faculty of Applied Economics of the University of Antwerp.

[4]**Darjan Karabasevic**, PhD candidate at the Faculty of Management in Zajecar, Megatrend University, Serbia.

Received: April, 2015
1st Revision: May, 2015
2nd Revision: August, 2015
Accepted: September, 2015

ABSTRACT. ***Many real-world decision-making problems are complex and placed in a fuzzy environment, especially when it is related to predictions and assessments. Therefore, an extension of the MULTIMOORA method, primarily adapted for the use of interval-valued triangular fuzzy numbers as well as the use of the group decision-making approach and linguistic variables, is proposed in this paper. In order to demonstrate the applicability and effectiveness of the proposed approach, an example of comminution circuits design selection is considered.***

KEYWORDS: MCDM, MULTIMOORA, ratio system, reference point, full multiplicative form, interval-valued triangular fuzzy numbers.

JEL classification: D81, C61, C44.

Introduction

Decision-making is often associated with selecting of the best alternative out of a set of available alternatives based on multiple and often conflicting criteria (Stanujkic, 2015b). Since the 1970s, the Multiple Criteria Decision Making (MCDM) approach has rapidly been developed (Lertprapai, 2013), and as a result of that, many significant MCDM methods have been proposed. The following methods can be mentioned as some of the most prominent ones: the SAW (MacCrimmon, 1968), Compromise Programming (Zeleny, 1973), the AHP (Saaty, 1980), TOPSIS (Hwang, Yoon, 1981), PROMETHEE (Brans, Vincke, 1985), ELECTRE (Roy, 1991), COPRAS (Zavadskas *et al.*, 1994), VIKOR (Opricovic, 1998), ARAS (Zavadskas, Turskis, 2010) and WASPAS (Zavadskas *et al.*, 2012). A comprehensive overview of some prominent MCDM methods is given in Zavadskas, Turskis (2011) and Stanujkic *et al.* (2013).

The Multi-Objective Optimization by a Ratio Analysis (MOORA) was introduced by Brauers and Zavadskas (2006) on the basis of a previous research by Brauers (2004). As one of the significant characteristics of this method, it can be mentioned that it consists of two parts named the Ratio System Approach (RSA) and the Reference Point Approach (RPA) used simultaneously in order to rank alternatives or select the most suitable one can be mentioned.

Brauers, Zavadskas (2010a) further developed this method under the name of MULTIMOORA (MOORA plus the full multiplicative form) by adding the third part named the Full Multiplicative Form (FMF).

The MULTIMOORA method has been applied in numerous studies for the purpose of solving a wide range of different problems in the field of economics (Brauers *et al.*, 2014a; 2014b; Brauers, Ginevicius, 2013; Brauers *et al.*, 2012; 2007; Brauers, Zavadskas, 2011), construction (Zavadskas *et al.*, 2013a; 2013b; Brauers *et al.*, 2013; Kracka, Zavadskas, 2013), regional development (Balezentis *et al.*, 2013), sustainability management (Stankeviciene *et al.*, 2014; Stankeviciene, Cepulyte, 2014) and establishing sustainable energy policy (Streimikiene *et al*., 2012; Balezentiene *et al*., 2013), health-care waste treatment (Liu *et al.*, 2015; Liu *et al.*, 2014b), supply chain management (Sahu *et al*., 2014), personnel selection (Balezentis *et al.*, 2012a; 2012b), material selection (Karande, Chakraborty, 2012) and comparative assessment of technologies etc.

In order to enable its use in solving a larger number of complex real-world decision-making problems, a number of extensions have been proposed. Brauers *et al*. (2011) proposed the first fuzzy extension of the MULTIMOORA method. In that extension, the MULTIMOORA method was modified to enable the use of triangular fuzzy numbers. Balezentis *et al.* (2012a; 2012b) further modified the fuzzy MULTIMOORA and proposed the extension that enables the use of linguistic variables and the group decision-making approach. Balezentis, Zeng (2013) proposed an extension of the MULTIMOORA method based on interval-valued fuzzy numbers. Finally, Balezentis *et al*. (2014) proposed an extension based on intuitionistic fuzzy numbers.

In addition to these extensions, some other extensions, such as Liu *et al.* (2014a; 2014b) as well as some extensions related to the MOORA method or certain parts of the MULTIMOORA method (Stanujkic *et al*., 2012b; Stanujkic, 2015a) can be identified as well.

A comprehensive overview of the development of the MULTIMOORA method including its extensions is provided by Balezentis, Balezentis (2014).

In order to propose an extension of the MULTIMOORA method for solving complex decision-making problems based on the use of interval-valued triangular fuzzy numbers, the rest of this paper is organized as follows: in Section 1, some basic definitions and notations are given. In Section 2, the ordinary MULTIMOORA method is presented, whereas in Section 3, the interval-valued fuzzy extension of the MULTIMOORA method is proposed. In Section 4, an example is considered with the aim to explain in detail the proposed methodology. The conclusions are presented in the end.

1. Preliminaries

In this section, some basic definitions and notations relevant for forming the new extension of the MULTIMOORA method proposed in Section 3 are given.

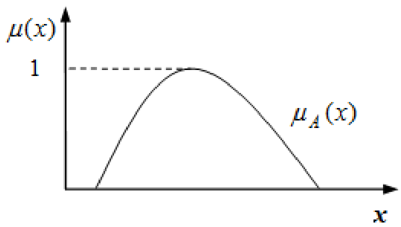
1.1 Interval-valued fuzzy sets

In the classical set theory, an element can either belong or not belong to a set. Unfortunately, many real-world decision-making problems are often related to the influence of uncertainty and, therefore, such problems cannot be easily expressed using classical sets.

Zadeh (1965) introduced the Fuzzy Sets (FS) theory, which allows a partial membership in a set. The fuzzy set *A*, illustrated in *Figure 1*, is completely defined by a set of pairs, such as:

$$A = \left\{ \left\langle x, \mu_A(x) \right\rangle \middle| \, x \in X \right\}, \tag{1}$$

where $\mu_A(x) \in [0,1]$.



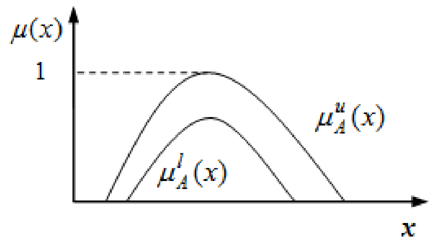
Source: created by the authors.

Figure 1. **The Fuzzy Set**

The FS theory was extended later. The concept of the interval-valued fuzzy set was proposed by Turksen (1986; 1996) and Gorzalczany (1987). The interval-valued fuzzy set A, illustrated in *Figure 2*, can be displayed as

$$A = \left\{ \left\langle x, \left[\mu_A^l(x), \mu_A^u(x) \right] \right\rangle \middle| x \in X \right\}, \qquad (2)$$

where: $\mu_A^l(x)$ denotes the lower limit of the degree of the membership, $\mu_A^l(x) \in [0,1]$; $\mu_A^u(x)$ denotes the upper limit of the degree of the membership, $\mu_A^u(x) \in [0,1]$, and $\mu_A^l(x) \le \mu_A^u(x)$.



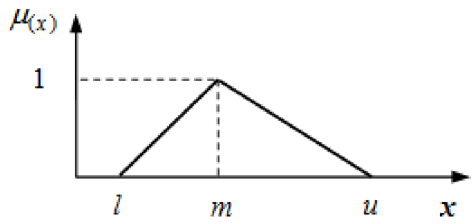
Source: created by the authors.

Figure 2. **The Interval-Valued Fuzzy Set**

1.2 Interval-valued triangular fuzzy numbers

As previously stated, the Classical Sets theory is based on the use of crisp numbers, and numbers are not suitable for solving complex real-world decision-making problems.

A triangular fuzzy number. Triangular fuzzy numbers were introduced in the FS theory. The triangular fuzzy number, shown in *Figure 3*, is fully characterized by a triplet of real numbers (l, m, u), where the parameters *l*, *m* and *u* denote the smallest possible value, the most promising value and the largest possible value that describes a fuzzy event (Dubois, Prade, 1980; Berry,1992; Kahraman *et al*., 2004; Ertugrul, Karakasoglu, 2009).



Source: created by the authors.

Figure 3. **The Triangular Fuzzy Number**

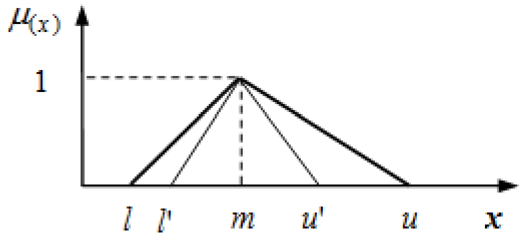
An interval-valued triangular fuzzy number. According to Yao and Lin (2002), the Interval-Valued Triangular Fuzzy Number (IVTFN) can be presented as follows:

$$\tilde{A} = \left[\tilde{A}^l, \tilde{A}^u \right] = \left[(a_l', a_m', a_u'; \omega_A'), (a_l, a_m, a_u; \omega_A) \right], \qquad (3)$$

where: $\tilde{A}^L$ and $\tilde{A}^U$ denote the lower and the upper triangular fuzzy numbers, $\tilde{A}^L \subset \tilde{A}^U$, respectively; l', m' and u' denote the smallest possible value, the most promising value and the largest possible value that describes a fuzzy event $\tilde{A}^L$; l, m and u denote the smallest possible value, the most promising value and the largest possible value that describes a fuzzy event $\tilde{A}^U$; ω_A' and ω_A denote the maximum values of the lower $\mu_{\tilde{A}^L}(x)$ and the upper $\mu_{\tilde{A}^U}(x)$ membership functions.

A normalized interval-valued triangular fuzzy number. The particular case of the IVTFNs are the normalized IVTFNs, $\omega'_A = \omega_A = 1$, with the same mode. The normalized IVTFN shown in Figure 4 can be presented as follows:

$$\tilde{A} = [\tilde{A}^l, \tilde{A}^u] = [(l, l'), m, (u', u)]. \quad (4)$$



Source: created by the authors.

Figure 4. **The Normalized Interval-Valued Triangular Fuzzy Number with the Same Mode**

1.3 Interval-valued triangular fuzzy number the basic operations of normalized interval-valued triangular fuzzy numbers

It is supposed that $\tilde{A} = [(a_l, a'_l), a_m, (a'_u, a_u)]$ and $\tilde{B} = [(b_l, b'_l), b_m, (b'_u, b_u)]$ are two normalized IVTFNs. Then, the basic operations on these fuzzy numbers (Chen 1997; Chen, Chen, 2008) are defined as follows:

$$\tilde{A} + \tilde{B} = [(a_l + b_l, a'_l + b'_l), a_m + b_m, (a'_u + b'_u, a_u + bu)], \quad (5)$$

$$\tilde{A} - \tilde{B} = [(a_l - b_u, a'_l - b'_u), a_m - b_m, (a'_u - b'_l, a_u - b_l)], \quad (6)$$

$$\tilde{A} \times \tilde{B} = [(a_l b_l, a'_l b'), a_m b_m, (a'_u b'_u, a_u b_u)], \quad (7)$$

$$\tilde{A} \div \tilde{B} = \left[\left(\frac{a_l}{b_u}, \frac{a'_l}{b'_u}\right), \frac{a_m}{b_m}, \left(\frac{a'_u}{b'_l}, \frac{a_u}{b_l}\right)\right], \quad (8)$$

$$k \times \tilde{A} = [(ka_l, ka'_l), ka_m, (ka'_u, ka_u)] \; . \quad (9)$$

1.4 The distance between Interval-Valued Triangular Fuzzy Numbers

In order to rank alternatives, it is necessary in some cases that the distance between two fuzzy numbers should be determined.

The maximum distance between normalized IVTFNs. Let $\tilde{A} = [(a_l, a'_l), a_m, (a'_u, a_u)]$ and $\tilde{B} = [(b_l, b'_l), b_m, (b'_u, b_u)]$ be two IVTFNs; then, the maximum distance between them is as follows (Stanujkic *et al*., 2014):

$$d_{\max}(\tilde{A}, \tilde{B}) = \max\left(|a_l - b_l|, |a'_l - b'_l|, |a_m - b_m|, |a'_u - b'_u|, |a_u - b_u| \right). \quad (10)$$

1.5 The defuzzification of interval-valued triangular fuzzy numbers

In the past few decades, a number of defuzzification methods were proposed. However, these methods were mainly intended for the defuzzification of trapezoidal and triangular fuzzy numbers, such as the methods proposed by Liou, Wang (1992) and Opricovic, Tzeng (2003).

A simple procedure for the defuzzification of IVTFNs was proposed by Stanujkic (2015b) as follows:

$$gm(\tilde{B}) = \frac{l + l' + m + u' + u}{5}. \tag{11}$$

The procedure for the defuzzification proposed by Stanujkic (2015a) is somewhat more complex, but it as well provides some greater opportunities:

$$fm(\tilde{A}) = \frac{1}{5}\left[(1-\alpha)l + (1-\beta)l' + \gamma m + \beta u' + \alpha u\right], \tag{12}$$

where: α, β, and γ are the coefficients, $\alpha, \beta \in [0,1]$, and $\gamma \geq 0$.

1.6 Aggregation operators

A significant number of aggregation operators were proposed in the literature (Yager, 1988; 1993; 2003; Herrera *et al*., 2003; Xu, 2004a; 2007). The Weighted Averaging (WA) operator (Harsanyi, 1995), the Ordered Weighted Averaging (OWA) operator (Yager, 1988), the Weighted Geometric (WG) operator and the Ordered Weighted Geometric Averaging (OWGA) operator are some of the most prominent aggregation operators.

Xu (2003; 2004b) adapted the WA and the WG operators for their application in a case when input arguments are triangular fuzzy information (Li, Li, 2012).

The Triangular Fuzzy Weighted Averaging operator. Let $\tilde{A} = (l, m, u)$ be a collection of triangular fuzzy numbers. The Triangular Fuzzy Weighted Averaging (TFWA) operator (Li *et al*., 2011) of the dimensions *n* is a mapping $TFWA: R^n \to R$ that has an associated weighting vector $\omega_j = (\omega_1, \omega_2, ..., \omega_n)^T$ with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$, such as

$$TFWA(\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n) = \sum_{j=1}^{n} \omega_j \tilde{A}_j = \left(\sum_{j=1}^{n} \omega_j l_j, \sum_{j=1}^{n} \omega_j m_j, \sum_{j=1}^{n} \omega_j u_j \right). \tag{13}$$

The Triangular Fuzzy Weighted Geometric operator. Let $\tilde{A} = (l, m, u)$ be a collection of triangular fuzzy numbers. The Triangular Fuzzy Weighted Geometric (TFWG) operator (WeiLi, 2012) of the dimensions *n* is a mapping $TFWG: R^n \to R$ that has an associated weighting vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$, such as

$$TFWG(\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n) = \prod_{j=1}^{n} (\tilde{A}_j)^{\omega_j} = \left(\prod_{j=1}^{n} (l_j)^{\omega_j}, \prod_{j=1}^{n} (m_j)^{\omega_j}, \prod_{j=1}^{n} (u_j)^{\omega_j} \right). \tag{14}$$

Based on Eq. (13) and Eq. (14), the following aggregation operators can be defined for IVTFNs:

The Interval-Valued Triangular Fuzzy Weighted Averaging operator. Let $\tilde{A} = [(l, l'), m, (u', u)]$ be a collection of IVTFNs. The Interval-Valued Triangular Fuzzy Weighted Averaging (IVTFWA) operator of the dimensions *n* is a mapping $IVTFWA: R^n \to R$ that has an associated weighting vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$, such as

$$IVTFWA(\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n) = \sum_{j=1}^{n} \omega_j \tilde{A}_j = \left[\left(\sum_{j=1}^{n} \omega_j l_j, \sum_{j=1}^{n} \omega_j l'_j \right), \sum_{j=1}^{n} \omega_j m_j, \left(\sum_{j=1}^{n} \omega_j u'_j, \sum_{j=1}^{n} \omega_j u_j \right) \right]. \quad (15)$$

The Interval-Valued Triangular Fuzzy Weighted Geometric operator. Let $\tilde{A} = [(l, l'), m, (u', u)]$ be a collection of IVTFNs. The Interval-Valued Triangular Fuzzy Geometric Averaging (IVTFWG) operator of the dimensions n is a mapping $IVTFGA: R^n \rightarrow R$ that has an associated weighting vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$, such as

$$IVTFGA(\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n) = \prod_{j=1}^{n} (\tilde{A}_j)^{\omega_j} = $$
$$= \left[\left(\prod_{j=1}^{n} (l_j)^{\omega_j}, \prod_{j=1}^{n} (l'_j)^{\omega_j} \right), \prod_{j=1}^{n} (m_j)^{\omega_j}, \left(\prod_{j=1}^{n} (u'_j)^{\omega_j}, \prod_{j=1}^{n} (u_j)^{\omega_j} \right) \right]. \quad (16)$$

1.7 Linguistic variables

In a series of papers, Zadeh (1975a; 1975b; 1975c) introduced the concept of linguistic variables. According to Zadeh, linguistic variables are defined as variables whose values are words or sentences in either a natural or an artificial language.

The concept of a linguistic variable is very suitable for dealing with many real-world decision-making problems that are usually complex, slightly defined and related to uncertainties (Olcer, Odabasi, 2005; Yong, 2006). Therefore, in the published papers written by many authors, such as Wang, Chang (1995), Chen (2000), Wang, Elhag (2006), Mahdavi *et al*. (2008), different linguistic variables (linguistic terms scales), usually based on the use of triangular or trapezoidal fuzzy numbers, are proposed.

In the literature, linguistic variables based on the use of interval-valued fuzzy numbers are proposed as well. Wei and Chen (2009) proposed a nine-level linguistic terms scale based on the use of IVTFNs. Ashtiani *et al*. (2009), Kuo (2011), Kuo, Liang (2012) proposed a seven-level linguistic terms scale based on the use of IVTFNs.

In this approach, the linguistic scale with five linguistic gradations shown in *Table 1* was proposed.

Table 1. Linguistic variables for the ratings

Linguistic variables	Interval-valued triangular fuzzy numbers
Very high (VH)	[(0.75, 0.8), 0.9, (0.9, 0.90)]
High (H)	[(0.55, 0.6), 0.7, (0.8, 0.85)]
Medium (M)	[(0.35, 0.4), 0.5, (0.6, 0.65)]
Low (L)	[(0.15, 0.2), 0.3, (0.4, 0.45)]
Very low (VL)	[(0.10, 0.1), 0.1, (0.2, 0.25)]

Source: own calculations.

2. The MULTIMOORA Method

As previously stated, the MULTIMOORA method integrates three parts, which, according to Brauers, Zavadskas (2006), can be presented as follows:

The Ratio System Approach. In the RSA, the overall importance of the *i*-th alternative can be determined as follows:

$$y_i = y_i^+ - y_i^-, \quad (17)$$

with:

$$y_i^+ = \sum_{j \in \Omega_{\max}} r_{ij} \text{ , and} \tag{18}$$

$$y_i^- = \sum_{j \in \Omega_{\min}} r_{ij} \text{ ,} \tag{19}$$

where: y_i denotes the overall importance of the i-th alternative with respect to all criteria according to the RSA; y_i^+ and y_i^- denote the sum of the normalized performance ratings of the importances obtained on the basis of the benefit and cost criteria; r_{ij} denotes the normalized performance ratings of the i-th alternative with respect to the j-th criterion; $\Omega_{\max}$ and $\Omega_{\min}$ denote a set of the benefit and a set of the cost criteria; i = 1,2, … m; m is the number of alternatives, j = 1,2, ..., n; n is the number of the criteria.

The normalized performance ratings are calculated as follows

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}} \text{ ,} \tag{20}$$

where x_{ij} is the performance rating of the i-th alternative to the j-th criterion.

The compared alternatives are ranked on the basis of their y_i in descending order, and the alternative with the highest value of y_i is the best ranked.

The Reference Point Approach. The optimization based on the RPA can be shown as follows:

$$d_i^{\max} = \max_j (| r_j^* - r_{ij} |) \text{ ,} \tag{21}$$

where: $d_i^{\max}$ denotes the maximum distance to the reference point of the i-th alternative; r_j^* denotes the j-th coordinate of the reference point, as follows:

$$r_j^* = \begin{cases} \max_i r_{ij} \text{ ; } & j \in \Omega_{\max} \\ \min_i r_{ij} \text{;} & j \in \Omega_{\min} \end{cases} \text{ .} \tag{22}$$

The compared alternatives are ranked on the basis of their $d_i^{\max}$ in ascending order, and the alternative with the lowest value of $d_i^{\max}$ is the best ranked.

The Full Multiplicative Form. In the FMF, the overall utility of the i-th alternative can be determined as follows:

$$u_i = \frac{A_i}{B_i} \text{ ,} \tag{23}$$

with:

$$A_i = \prod_{j \in \Omega_{\max}} x_{ij} \text{ , and} \tag{24}$$

$$B_i = \prod_{j \in \Omega_{\min}} x_{ij} \text{ ,} \tag{25}$$

where: u_i denotes the overall utility of the i-th alternative with respect to all criteria according to the FMF; A_i denotes the product of the weighted performance ratings of the benefit criteria, and B_i denotes the product of the weighted performance ratings of the cost criteria of i-th alternative.

In the particular case of decision-making problems that does not include the cost criteria, B_i is equal to 1.

As in the RSA, the compared alternatives are ranked on the basis of their u_i in descending order, and the alternative with the highest value of u_i is the best ranked.

The final ranking of alternatives based on the MULTIMOORA method. Different parts of the MULTIMOORA method usually provide different ranking orders. Brauers and Zavadskas (2011) proposed the theory of dominance in order to summarize the ranks provided by different parts of the MULTIMOORA method and determine the final ranking order of alternatives.

2.1 Importance given to objectives

In order to express the different importance (significance) of objectives, i.e., criteria in MCDM models, Brauers, Zavadskas (2009) introduced the Significance Coefficient. As a result, some previously mentioned equations suffered certain adjustments. Based on Zavadskas *et al*. (2013a) and Stanujkic *et al*. (2012a, 2012b), these adaptations can be presented as follows:

In the RSA, Eq. (18) and Eq. (19) are as follows:

$$y_i^+ = \sum_{j \in \Omega_{\max}} w_j r_{ij} \text{ , and} \qquad (26)$$

$$y_i^- = \sum_{j \in \Omega_{\min}} w_j r_{ij} \text{ ,} \qquad (27)$$

where w_j denotes the weight of the *j*-th criterion.

In the RPA Eq. (21) is as follows:

$$d_i^{\max} = \max_j (w_j \mid r_j^* - r_{ij} \mid) \text{ .} \qquad (28)$$

In the FMF, Eq. (24) and Eq. (25) are as follows:

$$A_i = \prod_{j \in \Omega_{\max}} w_j r_{ij} \text{ , and} \qquad (29)$$

$$B_i = \prod_{j \in \Omega_{\min}} w_j r_{ij} \text{ .} \qquad (30)$$

3. An Extension of the MULTIMOORA Method Based on Interval-Valued Triangular Fuzzy Numbers

For solving a number of complex real-word decision-making problems, it is necessary to take into account opinions of a larger number of decision-makers (experts). The real-world decision-making problems are as well often associated with predictions, uncertainty and imprecision, which is why linguistic variables are often used for an evaluation of alternatives in relation to the criteria.

As already indicated, a large number of linguistic scales have been proposed in the literature with different numbers of linguistic variables that have been transformed into different types of usually fuzzy numbers. Depending on the method of the ranking of such fuzzy numbers or the procedure used for defuzzification, the significant advantages that the use of fuzzy or interval-valued fuzzy numbers provides can sometimes be minimized. It is as well evident that significant benefits can be achieved using the non-symmetric IVTFNs.

In this approach, the five-level linguistic scale, i.e., five linguistic variables, has been proposed. A relatively small number of levels have been chosen with the aim to stimulate decision makers to modify the default parameters of IVTFNs and express their attitudes more accurately.

The detailed step-by-step computational procedure of the MULTIMOORA method based on the group decision-making approach, linguistic variables and IVTFNs can be precisely expressed through the following steps:

Step 1. Identify available alternatives and select evaluation criteria. In this step, a team of decision makers identify available alternatives and choose criteria for their evaluation.

Step 2. Determine the relative importance of evaluation criteria. The procedures for determining the weights of criteria are usually not integrated into many significant MCDM methods as their integral part. However, the weights of criteria can have a significant impact on the obtained results; for that reason, different authors suggested different techniques (Ma *et al*. 1999) and methods, such as the SMART technique (Edwards 1977), pairwise comparisons (Saaty, 1977), the Entropy method (Hwang, Yoon 1981), the SWING method (Von Winterfeldt, Edwards 1986), the Delphi method (Hwang, Lin 1987), the SMARTER technique (Edwards, Barron 1994) and the SWARA technique (Kersuliene *et al*. 2010) for determining relative weights of criteria.

In this approach, any one of the aforementioned methods or techniques can be used for determining the weights of criteria.

Step 3. Construct an interval-valued fuzzy decision matrix for each decision maker**.** In this step, each one of the decision makers involved in the evaluation makes his or her own evaluation matrix, where alternatives are evaluated against criteria by applying the linguistic variables from Table 1. After that, the linguistic variables are transformed into the corresponding IVTFNs and, in accordance with their preferences, if necessary, some parameters of IVTFNs are modified taking into account the condition $0 \le l \le l' \le m \le u \le u' \le 1$.

As the result of this, each decision maker/expert has formed his or her own evaluation matrix, whose elements are IVTFNs.

Step 4. Construct a group decision-making matrix. The integration of an individual evaluation matrix into a group decision-making matrix can be carried out by using an aggregation operator. In this approach, the IVTFWA aggregation operator is proposed.

Step 5. Determine the ranking order of alternatives based on the RSA. The ranking of alternatives and the selection of the best one based on the RSA of the proposed extension of the MULTIMOORA method can be precisely expressed using the following sub-steps:

Step 5.1. Calculate $\tilde{y}_i^+$ and $\tilde{y}_i^-$. This optimization begins from the group decision-making matrix. Then, using the IVTFWA operator, $\tilde{y}_i^+$ and $\tilde{y}_i^-$ can be determined as follows:

$$\begin{aligned} \tilde{y}_i^+ &= IVTFWA(\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_n) = \sum_{j \in \Omega_{\max}} w_j \tilde{r}_{ij} = \\ &= \left[\left(\sum_{j \in \Omega_{\max}} w_j l_{ij}, \sum_{j \in \Omega_{\max}} w_j l'_{ij} \right), \sum_{j \in \Omega_{\max}} w_j m_{ij}, \left(\sum_{j \in \Omega_{\max}} w_j u'_{ij}, \sum_{j \in \Omega_{\max}} w_j u_{ij} \right) \right], \end{aligned} \tag{31}$$

$$\tilde{y}_i^- = IVTFWA(\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_n) = \sum_{j \in \Omega_{\min}} w_j \tilde{r}_{ij} =$$
$$= \left[\left(\sum_{j \in \Omega_{\min}} w_j l_{ij}, \sum_{j \in \Omega_{\min}} w_j l'_{ij} \right), \sum_{j \in \Omega_{\min}} w_j m_{ij}, \left(\sum_{j \in \Omega_{\min}} w_j u'_{ij}, \sum_{j \in \Omega_{\min}} w_j u_{ij} \right) \right], \quad (32)$$

where: $\tilde{y}_i$ denotes the overall interval-valued fuzzy importance of the *i*-th alternative; $\tilde{y}_i^+$ and $\tilde{y}_i^-$ denote the interval-valued fuzzy importance of the *i*-th alternative obtained on the basis of the benefit and cost criteria, and $\tilde{r}_{ij}$ denotes the interval-valued fuzzy normalized performance rating of the *i*-th alternative with respect to the *j*-th criterion, i.e., the *j*-th element of the *i*-th column of the group decision-making matrix constructed in the Step 4 of the proposed computational procedure.

Step 5.2. Calculate the overall interval-valued fuzzy importance for each alternative. The overall interval-valued fuzzy importance based on the RSA can be determined as follows

$$\tilde{y}_i = \tilde{y}_i^+ - \tilde{y}_i^- . \quad (33)$$

Step 5.3. Defuzzify the overall interval-valued fuzzy importance based on the RSA. The results obtained using Eq. (33) are IVTFNs. In order to enable the ranking of the considered alternatives, these IVTFNs should be transformed into a form suitable for ranking, i.e., crisp numbers, which can be done using Eq. (11) or Eq. (12) considered in Sub-Section 1.5.

Additionally, by assigning different values to the coefficients α, β and γ in Eq. (12), decision makers have an opportunity to express their opinions more accurately.

Step 5.4. Rank the alternatives and select the best one. After defuzzification, the ranking of the alternatives can be performed in the same way as in the RSA of the ordinary MULTIMOORA method.

Step 6. Determine the ranking order of the alternatives based on the RPA. The ranking of the alternatives and the selection of the best one based on the RPA of the proposed extension of the MULTIMOORA method can be precisely expressed through the following sub-steps:

Step 6.1. Determine the interval-valued fuzzy reference point. In this approach, each coordinate of the interval-valued fuzzy reference point $r^* = \{\tilde{r}_1^*, \tilde{r}_2^*, \dots, \tilde{r}_n^*\}$ is in the form of IVTFNs, $\tilde{r}_j^* = [(r_{j1}, r_{j2}), r_{j3}, (r_{j4}, r_{j5})]$, whose values are determined as follows

$$r_{jk} = \left\{ \max_i r_{ij}^k \mid j \in \Omega_{\max}), (\min_i r_{ij}^k \mid j \in \Omega_{\min}) \right\}, \quad (34)$$

where: $\tilde{r}_j^*$ denotes the *j*-th coordinate of the interval-valued triangular fuzzy reference point; r_{jk} denotes the *k*-th parameter of the *j*-th coordinate of the interval-valued fuzzy reference point; r_{ij}^k denotes the *k*-th parameter of the interval-valued fuzzy performance rating of the *i*-th alternative with respect to the *j*-th criterion, *k*=1,2, ...,5.

Step 6.2. Determine the maximum distance from each alternative to all the coordinates of the reference point, as follows:

$$d_{ij}^{\max} = d_{\max}(\tilde{r}_{ij}, \tilde{r}_j^*)\, w_j , \quad (35)$$

where $d_{ij}^{\max}$ denotes the maximum distance of the *i*-th alternative obtained on the basis of the *j*-th criterion determined by Eq. (10).

Step 6.3. Determine the maximum distance of each alternative, as follows:

$$d_i^{\max} = \max_j d_{ij}^{\max} . \tag{36}$$

Step 6.4. Rank the alternatives and select the best one. In this step, the ranking of the alternatives can be done in the same way as in the RPA of the ordinary MULTIMOORA method.

Step 7. Determine the ranking order of the alternatives based on the FMF. The ranking of the alternatives and the selection of the best one based on FMF of the proposed extension of the MULTIMOORA method can be precisely expressed through the following sub-steps:

Step 7.1. Calculate $\tilde{A}_i$ and $\tilde{B}_i$. As in the case of the optimization based on the RSA, the optimization begins from the group decision-making matrix. Then, using the IVTFGA operator, $\tilde{A}_i$ and $\tilde{B}_i$ can be determined as follows:

$$\begin{aligned}\tilde{A}_i &= IVTFGA(\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_n) = \prod_{j \in \Omega_{\max}} (\tilde{r}_{ij})^{w_j} \\ &= \left[\left(\prod_{j \in \Omega_{\max}} (l_{ij})^{w_j}, \prod_{j \in \Omega_{\max}} (l'_{ij})^{w_j}\right), \prod_{j \in \Omega_{\max}} (m_{ij})^{w_j}, \left(\prod_{j \in \Omega_{\max}} (u'_{ij})^{w_j}, \prod_{j \in \Omega_{\max}} (u_{ij})^{w_j}\right)\right],\end{aligned} \tag{37}$$

$$\begin{aligned}\tilde{B}_i &= IVTFGA(\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_n) = \prod_{j \in \Omega_{\min}} (\tilde{r}_{ij})^{w_j} \\ &= \left[\left(\prod_{j \in \Omega_{\min}} (l_{ij})^{w_j}, \prod_{j \in \Omega_{\min}} (l'_{ij})^{w_j}\right), \prod_{j \in \Omega_{\min}} (m_{ij})^{w_j}, \left(\prod_{j \in \Omega_{\min}} (u'_{ij})^{w_j}, \prod_{j \in \Omega_{\min}} (u_{ij})^{w_j}\right)\right].\end{aligned} \tag{38}$$

Step 7.2. Determine the overall interval-valued fuzzy utility for each alternative, as follows:

$$\tilde{u}_i = \frac{\tilde{A}_i}{\tilde{B}_i}, \tag{39}$$

where $\tilde{u}_i$ denotes the overall interval-valued fuzzy utility of the *i*-th alternative.

Step 7.3. Defuzzify the overall interval-valued fuzzy utility of each alternative. The results obtained using Eq. (39) are IVTFNs. In order to rank the alternatives, these IVTFNs should be defuzzified in the same way as in the RSA approach.

Sep 7.4. Rank the alternatives and select the best one. After defuzzification, the ranking of the alternatives can be performed in the same way as in the FMF of the ordinary MULTIMOORA method.

Step 8. Determine the final ranking order of the alternatives. The final ranking order of the alternatives can be determined as in the case of the ordinary MULTIMOORA method.

4. A Numerical Example

In order to demonstrate the applicability and efficiency of the proposed approach, its applicability in solving a particular problem is shown in this section on the example adopted from Stanujkic *et al*. (2014) and Stanujkic (2013). In order to briefly demonstrate the advantages of the proposed methodology, this example is slightly modified.

Suppose that a mining and smelting company is planning to build a new flotation plant. Therefore, a team of three experts is formed with the aim to perform a preliminary evaluation of three generic Comminution Circuit Designs (CCDs),

- A_1, CCDs based on the combined use of rod mills and ball mills;
- A_2, CCDs based on the use of ball mills, and
- A_3, CCDs based on the use of semi-autogenous mills,

and to propose the most appropriate one, on the basis on the following evaluation criteria:

- C_1, Grinding efficiency;
- C_2, Economic efficiency;
- C_3, Technological reliability;
- C_4, Capital investment costs; and
- C_5, Environmental impact.

After that, the team of experts evaluates the three generic CCDs in relation to the selected evaluation criteria. The evaluation results obtained from the three experts are shown in *Tables 2* to *Table 4*.

Table 2. The ratings of the three generic CCDs obtained from the first expert

Criteria / Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	VH	M	VH	VH	VH
A_2	H	H	H	VH	H
A_3	VH	VH	VH	M	M

Source: own calculations.

Table 3. The ratings of the three generic CCDs obtained from the second expert

Criteria / Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	H	M	H	H	M
A_2	M	M	H	M	L
A_3	M	M	H	H	L

Source: own calculations.

Table 4. The ratings of the three generic CCDs obtained from the third expert

Criteria / Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	H	M	VH	H	VH
A_2	M	M	H	M	VH
A_3	VH	H	VH	H	H

Source: own calculations.

Linguistic variables are then transformed into appropriate IVTFNs and some parameters of IVTFNs are modified according to the preferences of the respondents.

Table 5 shows the ratings in the form of the IVTFNs obtained as the result of the transformation of the linguistic variables from *Table 2*.

Table 5. The ratings of the three generic CCDs obtained from the first expert in the form of IVTFNs

Criteria	A_1	A_2	A_3
C_1	[(0.75,0.8),0.9,(0.9,0.90)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.75,0.8),0.9,(0.9,0.90)]
C_2	[(0.35,0.4),0.5,(0.6,0.65)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.55,0.6),0.7,(0.8,0.85)]
C_3	[(0.75,0.8),0.9,(0.9,0.90)]	[(0.75,0.8),0.9,(0.9,0.90)]	[(0.35,0.4),0.5,(0.6,0.65)]
C_4	[(0.75,0.8),0.9,(0.9,0.90)]	[(0.75,0.8),0.9,(0.9,0.90)]	[(0.35,0.4),0.5,(0.6,0.65)]
C_5	[(0.75,0.8),0.9,(0.9,0.90)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.35,0.4),0.5,(0.6,0.65)]

Source: own calculations.

Table 6 shows the ratings obtained from the first of the three experts after tuning some parameters of the IVTFNs.

Table 6. The ratings of the alternatives obtained from the first expert after tuning some parameters of the IVTFNs

Criteria	A_1	A_2	A_3
C_1	[(0.85,0.85),0.9,(0.9,0.9)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.55,0.8),0.85,(0.9,0.9)]
C_2	[(0.5,0.5),0.55,(0.6,0.65)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.4,0.4),0.7,(0.8,0.85)]
C_3	[(0.75,0.8),0.9,(0.9,0.9)]	[(0.75,0.8),0.8,(0.85,0.85)]	[(0.3,0.4),0.5,(0.6,0.75)]
C_4	[(0.75,0.8),0.9,(0.9,0.9)]	[(0.75,0.8),0.9,(0.9,0.9)]	[(0.5,0.5),0.5,(0.6,0.75)]
C_5	[(0.75,0.8),0.8,(0.85,0.85)]	[(0.6,0.65),0.7,(0.7,0.75)]	[(0.45,0.45),0.5,(0.6,0.65)]

Source: own calculations.

The performance ratings obtained from the three experts are displayed in *Table 7*.

Table 7. The ratings of the three generic CCDs obtained from the three experts in the form of IVTFNs

Cr.	Alt.	Expert 1	Expert 2	Expert 3
C_1	A_1	[(0.85,0.85),0.9,(0.9,0.9)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.55,0.6),0.7,(0.8,0.85)]
	A_2	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.35,0.4),0.5,(0.6,0.65)]	[(0.35,0.4),0.5,(0.6,0.65)]
	A_3	[(0.55,0.8),0.85,(0.9,0.9)]	[(0.35,0.4),0.5,(0.6,0.65)]	[(0.75,0.8),0.9,(0.9,0.9)]
C_2	A_1	[(0.5,0.5),0.55,(0.6,0.65)]	[(0.35,0.4),0.5,(0.6,0.65)]	[(0.35,0.4),0.5,(0.6,0.65)]
	A_2	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.35,0.4),0.5,(0.6,0.65)]	[(0.35,0.4),0.5,(0.6,0.65)]
	A_3	[(0.4,0.4),0.7,(0.8,0.85)]	[(0.35,0.4),0.5,(0.6,0.65)]	[(0.55,0.6),0.7,(0.8,0.85)]
C_3	A_1	[(0.75,0.8),0.9,(0.9,0.9)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.75,0.8),0.9,(0.9,0.9)]
	A_2	[(0.75,0.8),0.8,(0.85,0.85)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.55,0.6),0.7,(0.8,0.85)]
	A_3	[(0.3,0.4),0.5,(0.6,0.75)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.75,0.8),0.9,(0.9,0.9)]
C_4	A_1	[(0.75,0.8),0.9,(0.9,0.9)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.55,0.6),0.7,(0.8,0.85)]
	A_2	[(0.75,0.8),0.9,(0.9,0.9)]	[(0.35,0.4),0.5,(0.6,0.65)]	[(0.35,0.4),0.5,(0.6,0.65)]
	A_3	[(0.5,0.5),0.5,(0.6,0.75)]	[(0.55,0.6),0.7,(0.8,0.85)]	[(0.55,0.6),0.7,(0.8,0.85)]
C_5	A_1	[(0.75,0.8),0.8,(0.85,0.85)]	[(0.35,0.4),0.5,(0.6,0.65)]	[(0.75,0.8),0.9,(0.9,0.9)]
	A_2	[(0.6,0.65),0.7,(0.7,0.75)]	[(0.1,0.1),0.1,(0.2,0.25)]	[(0.75,0.8),0.9,(0.9,0.9)]
	A_3	[(0.45,0.45),0.5,(0.6,0.65)]	[(0.1,0.1),0.1,(0.2,0.25)]	[(0.55,0.6),0.7,(0.8,0.85)]

Source: own calculations.

The group performance ratings shown in Table 8 are obtained by applying the IVTFWA operator, i.e., by applying Eq. (15), in which case the same weight $\omega_j = 0.33$ is assigned to all the experts.

Table 8. The group decision-making matrix

Crit.	w_j		A_1	A_2	A_3
C_1	0.24	max	[(0.64,0.67),0.75,(0.82,0.85)]	[(0.41,0.46),0.56,(0.66,0.70)]	[(0.54,0.66),0.74,(0.79,0.80)]
C_2	0.17	max	[(0.39,0.42),0.51,(0.59,0.64)]	[(0.41,0.46),0.56,(0.66,0.70)]	[(0.42,0.46),0.62,(0.72,0.77)]
C_3	0.24	max	[(0.67,0.72),0.82,(0.85,0.87)]	[(0.61,0.66),0.72,(0.8,0.840)]	[(0.52,0.59),0.69,(0.75,0.82)]
C_4	0.21	min	[(0.61,0.66),0.75,(0.82,0.85)]	[(0.47,0.52),0.62,(0.69,0.72)]	[(0.52,0.56),0.62,(0.72,0.80)]
C_5	0.14	min	[(0.61,0.66),0.72,(0.77,0.79)]	[(0.47,0.51),0.56,(0.59,0.62)]	[(0.36,0.37),0.42,(0.52,0.57)]

Source: own calculations.

The data shown in *Table 8* represent the starting point for the use of the RSA, the RPA and the FMF parts of the MULTIMOORA method.

The ranking based on the RSA

The ranking results and the ranking order of the alternatives obtained on the basis of the RSA, i.e., by applying Eqs. (31) to (33), are presented in *Table 9*.

The overall crisp importances of y_i are obtained using Eq. (11).

Table 9. The ranking orders of the alternatives obtained on the basis of the RS approach

	$\tilde{y}_i^+$	$\tilde{y}_i^-$	$\tilde{y}_i$	y_i	Rank
A_1	[(0.38,0.40),0.46,(0.50,0.52)]	[(0.21,0.23),0.26,(0.28,0.29)]	[(0.09,0.12),0.20,(0.27,0.31)]	0.202	2
A_2	[(0.31,0.34),0.40,(0.46,0.49)]	[(0.16,0.18),0.21,(0.22,0.24)]	[(0.07,0.11),0.19,(0.28,0.32)]	0.199	3
A_3	[(0.33,0.37),0.45,(0.49,0.52)]	[(0.16,0.17),0.19,(0.22,0.25)]	[(0.07,0.15),0.25,(0.32,0.36)]	0.236	1

Source: own calculations.

The ranking based on the RPA

As previously stated, the ranking of alternatives based on the RPA approach as well starts from *Table 8*. After that, the interval-valued fuzzy reference point shown in *Table 10* is determined by applying Eq. (34).

Table 10. The interval-valued fuzzy reference point

Criteria	r_i^*
C_1	[(0.15,0.16),0.18,(0.19,0.20)]
C_2	[(0.07,0.07),0.10,(0.12,0.13)]
C_3	[(0.16,0.17),0.19,(0.20,0.20)]
C_4	[(0.10,0.11),0.13,(0.14,0.15)]
C_5	[(0.05,0.05),0.06,(0.07,0.08)]

The maximum distances from each alternative to the *j*-th coordinate of the reference point determined by applying Eq. (35) are shown in the columns II-VI of *Table 11*.

The maximum distances of each alternative to the reference point obtained by applying Eq. (36) are shown in the column VII of *Table 11*. The ranking order of the alternatives is shown in the column VIII of *Table 11*.

Table 11. The ranking orders of the alternatives obtained on the basis of the RPA approach

I	II	III	IV	V	VI	VII	VIII
	r_1^*	r_2^*	r_3^*	r_4^*	r_5^*	d_i^{max}	Rank
A_1	0.00	0.02	0.00	0.03	0.04	0.04	2
A_2	0.06	0.01	0.02	0.00	0.02	0.06	3
A_3	0.02	0.00	0.04	0.02	0.00	0.04	1

Source: own calculations.

The ranking based on the FMF

The ranking results and the ranking order of the alternatives obtained on the basis of the FMF approach, i.e., by applying Eqs. (37) to (39), are shown in *Table 12*.

Table 12. The ranking orders of the alternatives obtained on the basis of the FMF approach

	$\tilde{A}_i$	$\tilde{B}_i$	$\tilde{u}_i$	u_i	Rank
A_1	[(0.69,0.73),0.79,(0.84,0.86)]	[(0.84,0.86),0.9,(0.92,0.93)]	[(0.74,0.78),0.88,(0.97,1.02)]	0.88	3
A_2	[(0.61,0.65),0.73,(0.8,0.83)]	[(0.77,0.79),0.83,(0.86,0.87)]	[(0.7,0.76),0.87,(1,1.07)]	0.89	2
A_3	[(0.64,0.7),0.78,(0.83,0.86)]	[(0.75,0.77),0.8,(0.85,0.88)]	[(0.72,0.81),0.97,(1.08,1.14)]	0.95	1

Source: own calculations.

Table 13. The final ranking order of the alternatives according to the MULTIMOORA method

	RS	RP	FMF	Rank
A_1	2	2	3	2
A_2	3	3	2	3
A_3	1	1	1	1

Source: own calculations.

The final ranking order of the alternatives which summarizes the three different ranks provided by the respective parts of the MULTIMOORA method is shown in *Table 13*.

Conclusions

The MULTIMOORA method, as well as its predecessor, the MOORA method, is proven in the cases of solving many different decision-making problems. In order to enable its application in the solving of a larger number of complex decision-making problems, numerous extensions have been proposed for the MULTIMOORA method.

Compared to crisp and ordinary fuzzy numbers, the interval-valued fuzzy numbers provide much greater opportunities for solving complex decision-world problems, especially the problems related to predictions or forecasting. Therefore, an extension of the MULTIMOORA method that enables the application of interval-valued triangular fuzzy numbers is proposed in this paper.

The usability and efficiency of the proposed extension is presented on the example of the comminution circuit design selection. Using the proposed extension of the

MULTIMOORA method, decision-makers are as well enabled to analyse different scenarios and make the most appropriate selection.

Finally, it should be noted that the proposed extension of the MULTIMOORA method can be used for solving a much larger number of complex decision-making problems.

References

Ashtiani, B., Haghighirad, F., Makui, A. (2009), "Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets", *Applied Soft Computing*, Vol. 9, No 2, pp.457-461.

Balezentiene, L., Streimikiene, D., Balezentis, T. (2013), "Fuzzy decision support methodology for sustainable energy crop selection", *Renewable and Sustainable Energy Reviews*, Vol. 17, pp.83-93.

Balezentis, A., Balezentis, T., Brauers, W.K.M. (2012a), "MULTIMOORA-FG: A multi-objective decision making method for linguistic reasoning with an application to personnel selection", *Informatica*, Vol. 23, No 2, pp.173-190.

Balezentis, A., Balezentis, T., Brauers, W.K.M. (2012b), "Personnel selection based on computing with words and fuzzy MULTIMOORA", *Expert Systems with Applications*, Vol. 39, No 9, pp.7961-7967.

Balezentis, T., Balezentis, A. (2014), "A Survey on Development and Applications of the Multi-criteria Decision Making Method MULTIMOORA", *Journal of Multi-Criteria Decision Analysis*, Vol. 21, No 3-4, pp.209-222.

Balezentis, T., Misiunas, A., Balezentis, A. (2013), "Efficiency and productivity change across the economic sectors in Lithuania (2000–2010): the DEA–MULTIMOORA approach", *Technological and Economic Development of Economy*, Vol. 19, No 1, pp.S191-S213.

Balezentis, T., Zeng, S. (2013), "Group multi-criteria decision making based upon interval-valued fuzzy numbers: an extension of the MULTIMOORA method", *Expert Systems with Applications*, Vol. 40, No 2, pp.543-550.

Balezentis, T., Zeng, S., Balezentis, A. (2014), "MULTIMOORA-IFN: A MCDM method based on intuitionistic fuzzy number for performance management", *Economic Computation & Economic Cybernetics Studies & Research*, Vol. 48, No 4, pp.85-102.

Berry, M.W. (1992), "Large-scale sparse singular value computations", *International Journal of Supercomputer Applications*, Vol. 6, No 1, pp.13-49.

Brans, J.P., Vincke, P. (1985), "A preference ranking organization method: The PROMETHEE method for MCDM", *Management Science*, Vol. 31, No 6, pp.647-656.

Bauers, W.K.M., Zavadskas, E.K. (2010), "Project management by MULTIMOORA as an instrument for transition economies". *Technological and Economic Development of Economy*, Vol. 16, No 1, pp.5-24.

Brauers, W.K.M. (2004), *Optimization Methods for a Stakeholder Society, a Revolution in Economic Thinking by Multi-Objective Optimization*, Kluwer Academic, Boston.

Brauers, W.K.M., Balezentis, A., Balezentis, T. (2011), "MULTIMOORA for the EU Member States updated with fuzzy number theory", *Technological and Economic Development of Economy*, Vol.17, No 2, pp.273-304.

Brauers, W.K.M., Balezentis, A., Balezentis, T. (2012), "European Union Member States preparing for Europe 2020. An application of the MULTIMOORA method", *Technological and Economic Development of Economy*, Vol. 18, No 4, pp.567-587.

Brauers, W.K.M., Ginevicius, R. (2013), "How to invest in Belgian shares by MULTIMOORA optimization", *Journal of Business Economics and Management*, Vol. 14, No 5, pp.940-956.

Brauers, W.K.M., Ginevicius, R., Podviezko, A. (2014a), "Development of a methodology of evaluation of financial stability of commercial banks" *Panoeconomicus*, Vol. 61, No 3, pp.349-367.

Brauers, W.K.M., Kildiene, S., Zavadskas, E.K., Kaklauskas, A. (2013), "The construction sector in twenty European countries during the recession 2008–2009– country ranking by MULTIMOORA", *International Journal of Strategic Property Management*, Vol. 17, No 1, pp.58-78.

Brauers, W.K.M., Zavadskas, E.K. (2010a), "Project management by MULTIMOORA as an instrument for transition economies", *Technological and Economic Development of Economy*, Vol. 16, No 1, pp.5-24.

Brauers, W.K.M., Zavadskas, E. K. (2006), "The MOORA method and its application to privatization in a transition economy", *Control and Cybernetics*, Vol. 35, No 2, pp.445-469.

Brauers, W.K.M., Zavadskas, E.K. (2009), "Robustness of the multi-objective MOORA method with a test for the facilities sector" *Technological and Economic Development of Economy*, Vol. 15, No 2, pp.352-375.

Brauers, W.K.M., Zavadskas, E.K. (2011), "Robustness in Economic Development Studies: The Case of Tanzania", in: *New State of MCDM in the 21st Century*, Springer Berlin Heidelberg. pp.199-213.

Brauers, W.K.M., Zavadskas, E.K., Kildiene, S. (2014b), "Was the Construction Sector in 20 European Countries Anti-cyclical during the Recession Years 2008-2009 as Measured by Multicriteria Analysis (MULTIMOORA)? ". *Procedia Computer Science*, Vol. 31, pp.949-956.

Chen, C.T. (2000), "Extensions of the TOPSIS for group decision-making under fuzzy environment", *Fuzzy Sets and Systems*, Vol. 114, No 1, pp.1-9.

Chen, S.J., Chen, S.M. (2008), "Fuzzy risk analysis based on measures of similarity between interval-valued fuzzy numbers", *Computers and Mathematics with Applications*, Vol. 55, No 8, pp.1670-1685.

Chen, S.M. (1997), "Fuzzy system reliability analysis based on vague set theory", in: proceedings of the *IEEE International Conference on Systems, Man, and Cybernetics*, 12–15 October, 1997, Orlando, USA, pp.1650-1655.

Dubois, D., Prade, H. (1980), *Fuzzy Sets and Systems: Theory and Application*, Academic Press.

Edwards, W. (1977), "How to use multiattribute utility measurement for social decisionmaking", *Systems, Man and Cybernetics, IEEE Transactions on*, Vol. 7, No 5, pp.326-340.

Edwards, W., Barron, F.H. (1994), "SMARTS and SMARTER: Improved simple methods for multiattribute utility measurement", *Organizational Behavior and Human Decision Processes*, Vol. 60, No 3, pp.306-325.

Ertugrul, I., Karakasoglu, N. (2009), "Performance evaluation of Turkish cement firms with fuzzy analytic hierarchy process and TOPSIS methods", *Expert Systems with Applications*, Vol. 36, No 1, pp.702-715.

Gorzalczany, M.B. (1987), "A method of inference in approximate reasoning based on interval-valued fuzzy sets", ***Fuzzy sets and systems*****, Vol. 21, No 1, pp.1-17.**

Harsanyi, J.C. (1995), "Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility", ***Journal of Political Economy*****, Vol. 63, No 4, pp.309-321.**

Herrera, F., Herrera-Viedma, E., Chiclana, F. (2003), "A study of the origin and uses of the ordered weighted geometric operator in multicriteria decision making", ***International Journal of Intelligent Systems*****, Vol. 18, No 6, pp.689-707.**

Hwang, C.L., Lin, M.J. (1987), ***Group decision making under multiple criteria: Methods and applications*****, Springer-Verlag, Berlin.**

Hwang, C.L., Yoon, K. (1981), ***Multiple Attribute Decision Making - Methods and Applications*****, Springer-Verlag, Berlin.**

Kahraman, C., Cebeci, U., Ruan, D. (2004), "Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey", ***International Journal of Production Economics*****, Vol. 87, No 2, pp.171-184.**

Karande, P., Chakraborty, S. (2012), "Application of multi-objective optimization on the basis of ratio analysis (MOORA) method for materials selection", ***Materials & Design*****, Vol. 37, pp.317-324.**

Kersuliene, V., Zavadskas, E.K., Turskis, Z. (2010), "Selection of rational dispute resolution method by applying new step - wise weight assessment ratio analysis (SWARA)", ***Journal of Business Economics and Management*****, Vol. 11, No 2, pp.243-258.**

Kracka, M., Zavadskas, E.K. (2013), "Panel building refurbishment elements effective selection by applying multiple-criteria methods", ***International Journal of Strategic Property Management*****, Vol. 17, No 2, pp.210-219.**

Kuo, M.S. (2011), "A novel interval-valued fuzzy MCDM method for improving airlines' service quality in Chinese cross-strait airlines", ***Transportation Research Part E: Logistics and Transportation Review*****, Vol. 47, No 6, pp.1177-1193.**

Kuo, M.S., Liang, G.S. (2012), "A soft computing method of performance evaluation with MCDM based on interval-valued fuzzy numbers", ***Applied Soft Computing*****, Vol. 12, No 1, pp.476-485.**

Lertprapai, S. (2013), "Review: Multiple Criteria Decision Making Method with Applications", ***International Mathematical Forum*****, Vol. 8, No 7, pp.347- 355.**

Li, S., Chang, L., Song, Z. (2011), "Comprehensive evaluation system for physical education teaching effectiveness with triangular fuzzy information", ***IJACT: International Journal of Advancements in Computing Technology*****, Vol. 3, No 7, pp.101-107.**

Li, W., Li, Q. (2012), "The Research of Software Project Risk Assessment with Triangular Fuzzy Information", ***IJACT: International Journal of Advancements in Computing Technology*****, Vol. 4, No 6, pp.93-99.**

Liou, T.S., Wang, M.J. (1992), "Ranking fuzzy numbers with integral value", ***Fuzzy Sets and Systems*****, Vol. 50, No 3, pp.247-255.**

Liu, H. C., Fan, X.J., Li, P., Chen, Y.Z. (2014a), "Evaluating the risk of failure modes with extended MULTIMOORA method under fuzzy environment", ***Engineering Applications of Artificial Intelligence*****, Vol. 34, pp.168-177.**

Liu, H.C., You, J.X., Lu, C., Chen, Y.Z. (2015), "Evaluating health-care waste treatment technologies using a hybrid multi-criteria decision making model", ***Renewable and Sustainable Energy Reviews*****, Vol. 41, pp.932-942.**

Liu, H.C., You, J.X., Lu, C., Shan, M.M. (2014b), "Application of interval 2-tuple linguistic MULTIMOORA method for health-care waste treatment technology evaluation and selection", ***Waste Management,*** **Vol. 34, No 11, pp.2355-2364.**

Ma, J., Fan, Z. P., Huang, L.H. (1999), "A subjective and objective integrated approach to determine attribute weights", ***European Journal of Operational Research,*** **Vol. 112, No 2, pp.397-404.**

MacCrimmon, K.R. (1968), ***Decision-making among multiple-attribute alternatives: a Survey and consolidated approach. RAND memorandum,*** **RM-4823-ARPA.**

Mahdavi, I., Mahdavi-Amiri, N., Heidarzade, A., Nourifar, R. (2008), "Designing a model of fuzzy TOPSIS in multiple criteria decision making", ***Applied Mathematics and Computation,*** **Vol. 206, No 2, pp.607-617.**

Olcer, A.I., Odabasi, A.Y. (2005), "A new fuzzy multiple attributive group decision making methodology and its application to propulsion/manoeuvring system selection problem", ***European Journal of Operational Research,*** **Vol. 166, No 1, pp.93-114.**

Opricovic, S. (1998), ***Multicriteria optimization of civil engineering systems,*** **Faculty of Civil Engineering, Belgrade, (In Serbian).**

Opricovic, S., Tzeng, G.H. (2003), "Defuzzification within a multicriteria decision model", ***International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems,*** **Vol. 11, No 5, pp.635-652.**

Roy, B. (1991), "The Outranking Approach and the Foundation of ELECTRE Methods", ***Theory and Decision,*** **Vol. 31, No 1, pp.49-73.**

Saaty, T.L. (1977), "A scaling method for priorities in hierarchical structures", ***Journal of mathematical psychology,*** **Vol. 15, No 3, pp.234-281.**

Saaty, T.L. (1980), ***The analytic hierarchy process: planning, priority setting, resource allocation,*** **McGraw-Hill.**

Sahu, A.K., Datta, S., Mahapatra, S.S. (2014), "Use of IVFNs and MULTIMOORA method for supply chain performance measurement, benchmarking and decision-making: an empirical study", ***International Journal of Business Excellence,*** **Vol. 7, No 2, pp.237-280.**

Stankeviciene, J., Cepulyte, J. (2014), "Sustainable value creation: coherence of corporate social responsibility and performance of socially responsible investment funds", ***Economic Research-Ekonomska Istrazivanja,*** **Vol. 27, No 1, pp.882-898.**

Stankeviciene, J., Sviderske, T., Miecinskiene, A. (2014), "Dependence of sustainability on country risk indicators in EU Baltic Sea region countries", ***Journal of Business Economics and Management,*** **Vol. 15, No 4, pp.646-663.**

Stanujkic, D. (2013), "An extension of the MOORA method for solving fuzzy decision making problems", ***Technological and Economic Development of Economy,*** **Vol. 19, Supplement 1, pp.S228-S255.**

Stanujkic, D. (2015a), "An extension of the ratio system approach of MOORA method for group decision-making based on interval-valued triangular fuzzy numbers", ***Technological and Economic Development of Economy,*** **Vol. 32, No 1 (Accepted, June 2014; Should be published in Volume 32, Issue 1, 2016)**

Stanujkic, D. (2015b), "Extension of the ARAS Method for Decision-Making Problems with Interval-Valued Triangular Fuzzy Numbers", ***Informatica,*** **Vol. 26, No 2, pp.335-355.**

Stanujkic, D., Djordjevic, B., Djordjevic, M. (2013), "Comparative analysis of some prominent MCDM methods: A case of ranking Serbian banks" *Serbian Journal of Management*, Vol. 8, No 2, pp.213-241.

Stanujkic, D., Magdalinovic, N., Jovanovic, R., Stojanovic, S. (2012a), "An objective multi-criteria approach to optimization using MOORA method and interval grey numbers", *Technological and Economic Development of Economy*, Vol. 18, No 2, pp.331-363.

Stanujkic, D., Magdalinovic, N., Milanovic, D., Magdalinovic, S., Popovic, G. (2014), "An efficient and simple multiple criteria model for a grinding circuit selection based on MOORA method", *Informatica*, Vol. 5, No 1, pp.73-93.

Stanujkic, D., Magdalinovic, N., Stojanovic, S., Jovanovic, R. (2012b), "Extension of ratio system part of MOORA method for solving decision-making problems with interval data", *Informatica*, Vol. 23, No 1, pp.141-154.

Streimikiene, D., Balezentis, T., Krisciukaitiene, I., Balezentis, A. (2012), "Prioritizing sustainable electricity production technologies: MCDM approach", *Renewable and Sustainable Energy Reviews*, Vol. 16, No 5, pp.3302-3311.

Turksen, I.B. (1986), "Interval valued fuzzy sets based on normal forms", *Fuzzy Sets and Systems*, Vol. 20, No 2, pp.191-210.

Turksen, I.B. (1996), "Interval-valued strict preference with Zadeh triples", *Fuzzy Sets and Systems*, Vol. 78, No 2, pp.183-195.

Von Winterfeldt, D., Edwards, W. (1986), *Decision analysis and behavioral research*, Cambridge University Press, Cambridge, 604 p.

Wang, M.J.J., Chang, T.C. (1995), "Tool steel materials selection under fuzzy environment", *Fuzzy Sets and Systems*, Vol. 72, No 3, pp.263-270.

Wang, Y.M., Elhag, T. (2006), "Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment", *Expert Systems with Applications*, Vol. 31, No 2, pp.309-319.

Wei, S.H., Chen, S.M. (2009), "Fuzzy risk analysis based on interval-valued fuzzy numbers", *Expert Systems with Applications*, Vol. 36, No 2, pp.2285-2299.

WeiLi, Q.L. (2012), "The Research of Software Project Risk Assessment with Triangular Fuzzy Information", *IJACT: International Journal of Advancements in Computing Technology*, Vol. 4, No 6, pp.93-99.

Xu, Z. (2003), "A priority method for triangular fuzzy number complementary judgment matrix", *System Engineer-Theory & Practice*, Vol. 231, No 10, pp.86-89.

Xu, Z. (2004a), "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment", *Information Sciences*, Vol. 168, No 1, pp.171-184.

Xu, Z. (2004b), *Uncertain multiple attribute decision making: methods and applications*, Tsinghua university press, Beijing.

Xu, Z. (2007), "Intuitionistic fuzzy aggregation operators", *Fuzzy Systems. IEEE Transactions on*, Vol. 15, No 6, pp.1179-1187.

Yager, R.R. (1988), "On ordered weighted averaging aggregation operators in multi-criteria decision making", *IEEE Transactions in Systems, Man and Cybernetics*, Vol. 18, No 1, pp.183-190.

Yager, R.R. (1993), "Families of OWA operators", *Fuzzy Sets and Systems*, Vol. 59, No 2, pp.125-148.

Yager, R.R. (2003), "Induced aggregation operators", *Fuzzy Sets and Systems,* Vol. 137, No 1, pp.59-69.

Yao, J.S., Lin, F.T. (2002), "Constructing a fuzzy flow-shop sequencing model based on statistical data", *International Journal of Approximate Reasoning,* Vol. 29, No 3, pp.215-234.

Yong, D. (2006), "Plant location selection based on fuzzy TOPSIS" *The International Journal of Advanced Manufacturing Technology,* Vol. 28, No. 7-8, pp.839-844.

Zadeh, L.A. (1965), "Fuzzy sets", *Information and Control,* Vol. 8, No 3, pp.338-353.

Zadeh, L.A. (1975a), "The concept of linguistic variable and its application to approximate reasoning – I", *Information Sciences,* Vol. 8, No 3, pp.199-249.

Zadeh, L.A. (1975b), "The concept of linguistic variable and its application to approximate reasoning – II", *Information Sciences,* Vol. 8, No 4, pp.301-357.

Zadeh, L.A. (1975c), "The concept of linguistic variable and its application to approximate reasoning – III", *Information Sciences,* Vol. 9, No 1, pp.43-80.

Zavadskas, E.K., Turskis, Z. (2010), "*A new additive ratio assessment (ARAS) method in multicriteria decision-making", Technological and Economic Development of Economy,* Vol.16, No 2, pp.159-172.

Zavadskas, E.K., Antucheviciene, J., Saparauskas, J., Turskis, Z. (2013a), "MCDM methods WASPAS and MULTIMOORA: verification of robustness of methods when assessing alternative solutions", *Economic Computation and Economic Cybernetics Studies and Research,* Vol. 47, No 2, pp.5-20.

Zavadskas, E.K., Kaklauskas, A., Sarka, V. (1994), "The new method of multicriteria complex proportional assessment of projects", *Technological and Economic Development of Economy,* Vol. 1, No 3, pp.131-139.

Zavadskas, E.K., Turskis, Z., Kildiene, S. (2014). "State of art surveys of overviews on MCDM/MADM methods", *Technological and Economic Development of Economy,* Vol. 20, No 1, pp.165-179.

Zavadskas, E.K., Turskis, Z. (2011), "Multiple criteria decision making (MCDM) methods in economics: an overview", *Technological and Economic Development of Economy,* Vol. 17, No 2, pp.397-427.

Zavadskas, E.K., Turskis, Z., Antucheviciene, J., Zakarevicius, A. (2012), "Optimization of Weighted Aggregated Sum Product Assessment", *Elektronika ir elektrotechnika,* No 6, pp.3-6.

Zavadskas, E.K., Turskis, Z., Volvaciovas, R., Kildiene, S. (2013b), "Multi-criteria assessment model of technologies", *Studies in Informatics and Control,* Vol. 22, No 4, pp.249-258.

Zeleny, M. (1973), "Compromise programming, in Multiple Criteria Decision Making", in: J.L. Cochrane, M. Zeleny (eds.), University of South Carolina Press, Columbia, pp.262-301.

INTERVALAIS IŠREIŠKIAMAIS TRIKAMPIAIS NERAIŠKIAISIAIS SKAIČIAIS PAGRĮSTAS MULTIMOORA METODO PRAPLĖTIMAS, SKIRTAS SPRĘSTI SUDĖTINGUS SPRENDIMŲ PRIĖMIMO UŽDAVINIUS

Dragisa Stanujkic, **Edmundas Kazimieras Zavadskas**, **Willem K.M. Brauers**, **Darjan Karabasevic**

SANTRAUKA

Daug realaus pasaulio sprendimų priėmimo uždavinių yra sudėtingi ir sprendžiami neaiškiai, ypač tada, kai tai susiję su prognozėmis ir vertinimais. Todėl šiame straipsnyje siūlomas MULTIMOORA metodo išplėtimas, kuris,visų pirma, pritaikytas naudoti intervalais vertinamų trikampių neraiškiuosius skaičius. Praplėstas metodas taip pat yra skirtas priimti grupinius sprendimus, juose taikant žodžiais išreiškiamus kintamuosius. Siūlomo metodo tinkamumą ir veiksmingumą įrodostraipsnyje pateiktas smulkinimo grandinės projekto atrankos pavyzdys.

REIKŠMINIAI ŽODŽIAI: MCDM, MULTIMOORA, santykinė sistema, atskaitos taškas, pilnoji sandaugos forma, intervalas vertinami trikampiai neraiškieji skaičiai.