A NEW ADAPTIVE STEP SIZE MCMA BLIND EQUALIZER ALGORITHM BASED ON ABSOLUTE ERROR AND ITERATION NUMBER

Thamer M. Jamel 1, Mohammed Abed Shabeeb 2
1Department of Electrical Engineering, University of Technology, Baghdad, Iraq.
2Foundation of Technical Education, Technical Institute-Nejif
Al-Nejif Al-Ashraf, Iraq

ABSTRACT

Blind equalization is a technique for adaptive equalization of a communication channel without the aid of the usual training sequence. The Modified Constant Modulus Algorithm (MCMA) is one of adaptive blind equalization algorithms. The drawbacks of the fixed step size of (MCMA) are slow convergence speed and high misadjustment. In order to overcome the tradeoff between fast convergence rate and low level of misadjustment of MCMA algorithm, we propose an enhanced technique based on an absolute difference error and iteration number to adjust a step size. The new proposed algorithm is called Combined Iteration and Absolute Error MCMA (CIAE-MCMA). Then we applied it for 16 QAM and 64 QAM adaptive blind equalizer systems. Simulation of adaptive blind equalizer with a typical telephone channel is evaluated to compare the performance of the proposed algorithm with MCMA and other two variable step size MCMA (VSS- MCMA) algorithms. It is observed from the simulation results, that the proposed algorithm has better performance compared with other algorithms in terms of fast convergence rate, low level of misadjustment and small BER.

Keywords: Adaptive blind equalizer, MCMA, variable step size.

1. INTRODUCTION

For bandwidth-limited channels, it is usually found that inter-symbol interference is the main determining factor in the design of high inter-symbol interference appears in all QAM systems. QAM signals are sensitive to inter-symbol interference (ISI) caused by multi-
path propagation and the fading of the channel, so it is necessary to use equalization technique to mitigate the effect of ISI [1]. For bandwidth-efficient communication systems, operating in high inter-symbol interference (ISI) environments adaptive equalizers have become a necessary component of the receiver architecture. The basic data communications process can be explained with the simplified baseband equalizer block diagram of Fig. 1[2].

A typical QAM data transmission system consists of a transmitter, a channel, and a receiver, where the unknown channel represents all the interconnections between the transmitter and the receiver (Fig.1). The transmitter generates a zero mean, independent input data sequence \( s \), each element of which comes from a finite alphabet \( A \) of the QAM symbols (or constellation). The data sequence is sent through the channel that its output \( x \) is the receiver input. The received symbol \( x(n) \) is corrupted by intersymbol interference (ISI) and Gaussian white noise.

![Baseband model of the adaptive digital communication system][1]

**Fig. 1** Baseband model of the adaptive digital communication system [2]

In order to counter inter-symbol interference effect, the observed signal may first be passed through a filter called the equalizer that its characteristics are the inverse of the channel characteristics. Filters, with adjustable parameters, are usually called adaptive filters, especially when they include algorithms that allow the filter coefficients to adapt to the changes in the signal statistics, the equalizers; thereby using adaptive filters are called adaptive equalizers. For adaptive filtering, the FIR filter is the most practical and widely used. The reason is the FIR filter has only adjustable zeros (the stability of FIR) [2].

The equalizer removes the distortion caused by the channel by estimating the channel inverse. The equalizer output \( y(n) \) is sent to a decision device which results in the received symbol estimate \( \hat{s}(n) \). Blind equalizer as opposed to data trained equalizer, is able to compensate amplitude and delay distortion of a communication channel using only channel output sample and knowledge of basic statistical properties of the data symbol [2]. The main advantage of blind equalization is that a training sequence is not needed. Hence no bandwidth is wasted by its transmission. Although various blind equalization techniques exist, the best known algorithms are the constant modulus algorithm (CMA), the generalized Sato algorithm (GSA), modified constant modulus algorithm (MCMA), Stop and Go (SGA) and etc.
However, since the CMA and MCMA methods yield slow convergence speed and high MSE, several blind algorithms are available to improve performance of the CMA and MCMA during the different stages of adaptation. One of these is a variation of step size. There are many methods available to adjust step size of different algorithms during adaptation [3-12].

One of these methods is proposed by Jones.D. L. [3] who controls the step size by using the channel output signal vector energy. Chahed et al. [4] adjusts the step size by using a time varying step size parameter depending upon squared Euclidian norms of the channel output vector and on the equalizer output. Xiong et al. [5] employed the lag error autocorrelation function between the current and previously output error of the blind system. Zhao, B. [6], proposed that the variable step size of CMA algorithm is controlled by difference between current and previous MSE. Kevin Banović. [7] proposed adjustment process to step size based on the length of the equalizer output radius. An alternative scheme that considers a nonlinear function of instantaneous error for adjusting the step-size parameter is proposed by Liyi et al. [8]. Meng Zhang [9], proposed a fine projection blind equalization CMA algorithm based on quantization estimation errors and variable step-size. Variable step size MCMA is proposed by Wei Xue, [10] in which the step size is adjusted according to the region where the received signal lies in the constellation plane.

In this paper, a new variable step size method is used for MCMA algorithm in order to enhance the performance of the traditional MCMA algorithm and to overcome its drawbacks which are the slow convergence rate, large steady state mean square error (MSE) and phase-blind nature [13]. The proposed algorithm is called Combined Iteration number with Absolute Error MCMA (CIAE-MCMA). In this proposed algorithm, the step size is controlled by two parameters; the first parameter is the absolute difference between current and previous error, while the second one is the iteration number. As it will be shown later in the simulations, the proposed algorithm has better performance as compared with MCMA and other variable step size (VSSMCMA) algorithms which were proposed in [6 and 8].

This paper is organized as follows: basic concept of the Modified constant modulus algorithm (MCMA) is given in section 2. The proposed algorithm and its analysis are described in section 3. In section 4, a simulation system which includes system model, channel model, other VSS-MCMA algorithms and simulation results are presented. Finally, section 5 provides main conclusions.

2. MODIFIED CONSTANT MODULUS ALGORITHM (MCMA)

Modified constant modulus algorithm (MCMA) shows the improved performance of the convergence behavior and can correct the phase error and frequency offset at the same time [14]. MCMA algorithm modifies the cost function of CMA in the form of real and imaginary parts, the modified cost function can be written as [14]:

\[ J(n) = J_R(n) + J_I(n) \]  \hspace{1cm} (1)

Where \( J_R(n) \) and \( J_I(n) \) are the cost function of real and imaginary parts of the equalizer output \( y(n) = y_R(n) + y_I(n) \) respectively and they are defined as:

\[ J_{\|}(n) = E \left( \left( |y_R(n)|^2 - R_{mR} \right)^2 \right) \] \hspace{1cm} (2)
Where \( R_{m_R} \) and \( R_{m_I} \) are the real constants determined by the real and imaginary parts of the transmitted data sequence respectively and is determined by the mean distance between the symbols and the real axis as follows [14]:

\[
R_{m_R} = \frac{E[|a(n)|^4]}{E[|a(n)|^2]}
\]

(4)

Where \( a(n) \) is the signal to be transmitted. The mean distance between the symbols and the imaginary axis is identical:

\[
R_{m_I} = \frac{E[|a(n)|^4]}{E[|a(n)|^2]}
\]

(5)

And the error signal is given by:

\[
e_R(n) = y_R(n)(|y_R(n)|^2 - R_{m_R})
\]

(6)

\[
e_I(n) = y_I(n)(|y_I(n)|^2 - R_{m_I})
\]

(7)

\[
e_{MCMA}(n) = e_R(n) + j * e_I(n)
\]

(8)

In contrast, the cost function of MCMA separates the output of equalizer to the real and the imaginary parts and estimates the error signal for real and imaginary parts respectively.

3. A NEW PROPOSED ADAPTIVE STEP SIZE MCMA ALGORITHM

As explained previously, this paper proposes a new algorithm in order to enhance the performance of the traditional MCMA algorithm and to overcome its drawbacks.

3.1 Algorithm formulation

Most of variable step size algorithms used the error signal \( e(n) \) to directly control the step size. The error signal is calculated as:

\[
e(n) = \hat{a}(n) - y(n) = \hat{a}(n) - W^T(n)X(n)
\]

(9)

Where \( W(n) = [W_0(n) \ W_1(n) \ W_2(n) \ldots \ W_{L-1}(n)]^T \) is filter coefficients, \( L \) being the order of the filter, \( y(n) = X^T(n)W(n) \) is adaptive filter output. \( e(n) \) is an error signal , \( X^T(n) = [X(n) \ X(n-1) \ X(n-2) \ldots \ X(n-L+1)] \) is input data and \( \hat{a}(n) \) is the desired signal. The proposed algorithm calculates the difference value between current and previous remaining errors, and then it will use to adjust the step size as:

\[
\mu(n + 1) = \mu(n) * \rho
\]

(10)
Where \( \mu(n) \) is time varying step size and the term (\( \rho \)) is:

\[
\rho = \beta \ast \exp^{-(f+g)} \\
\mu(n + 1) = \mu(n) \ast \exp^{-(|\epsilon(n) - \epsilon(n-1)|/\gamma)} 
\]

(11)

Where \( \beta \) is the proportionality factor. It is used to control the value scope of \( \mu(n + 1) \). When \( 0 \leq \exp^{-(f+g)} \leq 1 \), the value scope of \( \mu_l \) (Initial step size) satisfies \( 0 \leq \mu(n + 1) \leq \beta \ast \mu_l \). In order to guarantee the algorithm restrain, the step size must satisfy \( 0 \leq \mu(n) \leq 2/3\text{tr}(R) \) [15]. Where \( R \) is the input signal autocorrelation matrix, \( \text{tr}(R) \) is the trace of \( R \). The rule of using the term \( [e(n) - e(n-1)] \) is that the difference is large in the iteration initial period, then it is gradually reducing along with algorithm restraining. When the algorithm enters the stable state, \( [e(n) - e(n-1)] \) achieves the minimum. Therefore; \( \mu(n) \) has a corresponding change rule with difference, and the adaptive step-size \( \mu(n) \) control was realized. The step-size is large in the algorithm iteration initial period, and the convergence rate is faster. After algorithm restraining, the step-size was reduced to enhance the restraining precision. An exponential function is carefully designed in this proposed algorithm in order to adjust the value of the step size. An additional parameter was used together with absolute difference error for adjusting the step size; that is the iteration number. The parameter \( \gamma \) in (11) is a constant used to control the converges rate. Then the calculated value of \( \mu(n+1) \) is bounded between two values \((\mu_{max}) \) and \((\mu_{min}) \) as:

\[
\begin{align*}
\mu(n + 1) &= \mu_{max} \text{ if } \mu(n) > \mu_{max} \\
\mu(n + 1) &= \mu_{min} \text{ if } \mu(n) < \mu_{min} \\
\text{otherwise } \mu(n + 1) &= \mu(n + 1)
\end{align*}
\]

(12)

\( \mu(n + 1) \) is set to \( \mu_{min} \) or \( \mu_{max} \) when it falls below or above these lower and upper bounds, respectively. Table I below illustrates the steps required for the proposed algorithm:

### 3.2 Analysis of Variable Step-size rule

The new algorithm utilized a nonlinear function of remainder error besides the iteration number to control the step-size. To analyze the equation (11), we start with the first part: \( f = (-|e(n) - e(n-1)|) \), at the beginning of the adaptation process, the difference between current and previous error is high so the value of this part \( \rho = \exp^{-(f)} \) starts with a large value and as iteration number increases, it becomes a small value, as shown in Fig. 2. The second part \( g = (n/\gamma) \) depends on the number of iterations. At the beginning, the iteration number is small, so the value of \( (\rho = \exp^{-(n/\gamma)}) \) is equal to the initial value of the step size (\( \mu \)) and as the number of iteration increases, the value of \( (\rho) \) decreases. The relation between \( (\rho) \) and the iteration number is illustrated in Fig. 3.

By combining both parts as in (11) i.e. \( (\rho = \exp^{-(f+g)}) \), we observe that they cause large step size at a low number of iterations (dedicate a faster convergence rate) and a small value at the steady state reduces low MSE and low misadjustment at the steady state as shown in Fig. 4.
Table 1. A New Proposed CIAE-MCMA Algorithm

CIAE-MCMA ALGORITHM

1. Initialization:
- Given the input vector:
  \[
  X(k) = [x(k), x(k-1), \ldots, x(k-L+1)]^T
  \]
- \(N\) is iteration number, \(L\) is the order of the taps FIR filter;
- for accuracy: \(P = N - L\),
- Set the parameters \(\gamma, \beta\) constant \(\mu, \mu_i, \mu_{max}, \mu_{min}\) for adjusting the step size.
- \(a(k)\) is the output of QAM modulation with dimension \([1xN]\)
- Set \(W(0) = \text{zeros}[1xN]^T\)
  \(e(k) = \text{zeros}[1xP]\)

\[
\begin{align*}
  R_r &= \text{mean}(\text{abs}(\text{real}(a)).^4)/\text{mean}(\text{abs}(\text{real}(a)).^2); \\
  R_i &= \text{mean}(\text{abs}(\text{imag}(a)).^4)/\text{mean}(\text{abs}(\text{imag}(a)).^2);
\end{align*}
\]

2. For \(k= 1, 2, \ldots\) Iterations

Compute the following:
- \(y = W^*X\) : Equalizer Output
- \(e_r(k) = \text{real}(y)* ( R_r - \text{real}(y)^2)\)
- \(e_i(k) = \text{imag}(y)* ( R_i - \text{imag}(y)^2)\)
- \(e(k) = e_r(k) + j*e_i(k)\)
- \(\mu(1) = \mu_i\)
- \(\mu(k+1) = \mu(k)*e^{-(\exp(-|e(k) - e(k-1)|))+(k/\gamma)}\)
- \(\mu(k+1) = \mu_{max} \text{ if } \mu(k) > \mu_{max}\)
- \(\mu(k+1) = \mu_{min} \text{ if } \mu(k) < \mu_{min}\)
- otherwise \(\mu(k+1) = \mu(k+1)\)
- \(w = w + \mu(k+1)*e(k)^*X;\)
- \(\mu(k) = \mu(k+1)\)
Fig. 2 Updating the step size with iteration number using absolute difference error part only.

Fig. 3 Updating the step size with iteration number using iteration number part only.
4. SIMULATION SYSTEM

4.1 System Model

Figure 1 provides a block diagram of adaptive blind equalizer for the digital communications system that will be simulated in this section. For the work presented in this paper, we consider the channel and equalizer are both constrained to be linear as well as time invariant FIR filters. Noise in the channel is modeled as zero-mean additive white Gaussian noise (AWGN).

4.2 Telephone Channel Model

The channel used for testing comes from Proakis [16]. This channel is given by the 11-tap impulse response $h = [0.04 -0.05 0.07 -0.21 -0.5 0.72 0.36 0.0 0.21 0.03 0.07]$ and it is real-valued. It is a typical response of a good quality telephone channel and exhibits a non-linear phase distortion [16].

4.3 Another VSS MCMA Algorithms

Two variable step size algorithms are explored here to make a comparison with the proposed algorithm in this section:-

4.3.1 VSS-MCMA1 Algorithm

This variable step size algorithm is proposed by Wei Xue and Xiaoniu Yang in 2010 [8]. The step size is adjusted according to the region where the received signal lies in the constellation plane.

During the transient stages the output of equalizer will be scattered around a large area of the transmitted data symbols. However, in the steady state the output of equalizer will lie in a close
neighborhood of the transmitted data symbols. In the variable step size scheme, \( M \) regions \( D_1, D_2, \ldots, D_M \) are chosen in the constellation for M-QAM signals. The region \( D_i (i = 1; 2; \ldots; M) \) represents a small circular area around the point of the transmitted data symbol constellation with the radius \( d \).

\[
\begin{align*}
\mu(n + 1) &= \mu_0 & \text{if } y(k) \notin UD_i \\
\mu(n + 1) &= \mu_1 & \text{if } y(k) \in D_i
\end{align*}
\]

4.3.2 VSS-MCMA2 Algorithm

This variable step size algorithm is proposed by Zhang Liyi and Chen Lei1 2009 [6]. The new algorithm utilized a nonlinear function of remainder error to control the step-size as follows:

\[
\begin{align*}
\mu(n) &= \beta \left[ 1 - e^{-\sigma|e(n)|} \right] \\
\mu(n) &= \mu_{max} & \text{if } \mu(n) > \mu_{max} \\
\mu(n) &= \mu_{min} & \text{if } \mu(n) < \mu_{min}
\end{align*}
\]

(14)

Where, \( \beta \) is the proportionality factor that is used to control the value scope of \( \mu(n) \).

4.4 Simulation mythology

The parameters used for MCMA, VSSMCMA1, VSSMCMA2 and CIAE-MCMA algorithms were chosen to achieve a better performance in terms of a fast convergence time and a low level misadjustment in order to make fairly comparable between these algorithms. They are chosen as follows:

- Number of symbols equal to 3000.
- The signal to noise ratio for all simulations is 30 dB. The noise source used for all simulations was white Gaussian noise with zero mean and with unity variance.
- The length of the equalizer was taken as (11) taps.
- To carry out the BER performance of different algorithms for 16-QAM and 64-QAM, the signal to noise ratio at the primary input for all simulations is alternated between 0 and 30 dB with an increment step equals to 2 dB.
- The remaining factors used for these algorithms for telephone channel are shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3 Parameters used in telephone channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>( \mu )</td>
</tr>
<tr>
<td>( \mu_{max} )</td>
</tr>
<tr>
<td>( \mu_{min} )</td>
</tr>
<tr>
<td>( \mu_0 )</td>
</tr>
<tr>
<td>( \mu_1 )</td>
</tr>
<tr>
<td>( \delta )</td>
</tr>
<tr>
<td>( \gamma )</td>
</tr>
<tr>
<td>( \sigma )</td>
</tr>
<tr>
<td>( \beta )</td>
</tr>
</tbody>
</table>

4.5 Simulation results

4.5.1 Case 1

In order to illustrate the performance and activity measurement for our proposed algorithm, it must be compared with other algorithms which are traditionally MCMA, the variable step size modified constant modulus algorithms (VSSMCMA1) and (VSSMCMA2) respectively. The Equalizer output for four algorithms is shown in Fig. 5 (16-QAM) and Fig. 6 (64-QAM) respectively using telephone channel.
It can be observed that a better constellation is gotten for CIAE-MCMA algorithm compared with other algorithms. Figure 7 shows the learning curves (MSE) curves for all algorithms. It is clear that the proposed algorithm has fast convergence rate and low level of misadjustment compared with the other algorithms. The relation between the signal noise ratio (SNR) and the bit error ratio (BER) for all algorithms is shown in Fig. 8 (16-QAM) and Fig. 9 (64-QAM). From these figures, we observed that BER performance was improved using the proposed algorithm.

![Equalizer output using all algorithms for 16-QAM scheme](image)

**Fig. 5** Equalizer output using all algorithms for 16-QAM scheme (a) MCMA (b) VSSMCMA1 (c) VSSMCMA2 (d) proposed algorithm.

![Equalizer output using all algorithms for 64-QAM scheme](image)

**Fig. 6** Equalizer output using all algorithms for 64-QAM scheme (a) MCMA (b) VSSMCMA1 (c) VSSMCMA2 (d) proposed algorithm.
Fig. 7 Learning curves for all algorithms for (a) 16-QAM (b) 64-QAM scheme
Fig. 8 BER performance using all algorithms for 16-QAM system.

Fig. 9 BER performance for all algorithms using the 64-QAM system.

4.6 Effect of change the parameter ($\gamma$)

Figure 10 illustrates the effect of changing a constant ($\gamma$) that is used in the proposed algorithm. From the Fig. 10, it is found that the best value of ($\gamma$) to achieve fast convergence and low level of misadjustment is equal to 500 when using a fixed value for the parameter $\beta$ which is equal to 0.99.
CONCLUSIONS

In this paper, a variable step size modified constant modulus algorithm (CIAE-MCMA) is proposed. The step size of the algorithm is adjusted according to the combined absolute difference error with iteration number. The (CIAE-MCMA) can obtain both fast convergence rate, and a small steady state MSE compared with traditional MCMA and other variable step size MCMA algorithms. The simulation results for 16-QAM and 64-QAM signals demonstrate the effectiveness of the (CIAE-MCMA) in the equalization performance. Moreover, the optimum parameters for the proposed algorithms are $\beta$ equals to 0.99 and $\gamma$ equals to 500.

REFERENCES


