

SOME THEOREMS OF FITCH ON OMNIPOTENCE

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Frederic Fitch, in a fascinating article¹, most regrettably ignored by philosophers of religion, proves the following theorem on omnipotence: If for each situation that is the case it is logically possible that that situation was brought about by some agent, then whatever is the case was personally brought about by that agent. This is a mightily perplexing result. It seems to say that an omnipotent agent, in this sense, must personally have brought about every actual state of affairs that obtains. Yet many theologians have held that God is omnipotent while not being a universal agent. The free will defense,² for example, seems to require that there should be some actual states of affairs not (personally, at any rate) brought about by God. Whether and how God acts is puzzling in its own right.³ But in any case it has often been assumed that God is omnipotent, at least minimally in Fitch's sense, without being a universal personal agent.

Let us therefore carefully examine Fitch's proof. We need to establish two preliminary theorems before we can arrive at Theorem 3, the omnipotence theorem. And in order to establish these two lemmas, we must introduce some of Fitch's terminology. A class of propositions is said to be **closed with respect to conjunction elimination** if (necessarily) whenever the conjunction of two propositions is in the class so are the two propositions themselves.⁴ For example, the relation of believing (that obtains between an agent and a possible state of affairs) is closed with respect to conjunction elimination because if I believe 'p & q'

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¹ Frederic Fitch, 'A Logical Analysis of Some Value Concepts', *Journal of Symbolic Logic*, Vol. 28, No. 2, June 1963, pp. 135-142.

² Vide J. L. Mackie, 'Evil and Omnipotence', *Mind*, Vol. LXIV, 1955. Reprinted in Nelson Pike (ed.), *God and Evil*, Englewood Cliffs, Prentice-Hall, 1964. See also Douglas Walton, 'Modalities in the Free Will Defense', *Religious Studies*, forthcoming.

³ Gordon D. Kaufman, 'On the Meaning of "Act of God"', *Harvard Theological Review*, Vol. 61, 1968, pp. 175-201.

⁴ Fitch, 136f.

then it follows that I believe p and that I believe q .⁵ In symbols,

$$B(p \ \& \ q) \rightarrow (B \ p \ \& \ B \ q)$$

Thus for any class of operators L that obtain between an agent and a state of affairs, L is closed with respect to conjunction elimination where

$$L(p \ \& \ q) \rightarrow (L \ p \ \& \ L \ q)$$

for any p and q . Fitch also postulates that the relation of doing (personally bringing it about that p) is closed with respect to conjunction elimination. If I do both p and q then I do p and also I do q .

$$S(p \ \& \ q) \rightarrow (S \ p \ \& \ S \ q)$$

Here we read 'S p ' as 'the agent **a** personally brings it about that p obtains'.⁶ Fitch understands S as being a truth-entailing, that is,

$$S \ p \rightarrow p$$

Defining the general notion, Fitch postulates that a class of propositions is said to be a **truth class** if (necessarily) every member of it is true. Thus if L is a truth class, we have it that

$$L \ p \rightarrow p$$

Hence S is such a truth class. Truth and logical necessity are also obviously truth classes.

We can now approach Theorem 1 which I shall quote below in entirety with its proof.

Theorem 1. If L is a truth class which is closed with respect to conjunction elimination, then the proposition [$p \ \& \ \sim(L \ p)$], which asserts that p is true but not a member of (where p is any proposition), is itself necessarily not a member of L .

Proof. Suppose, on the contrary, that [$p \ \& \ \sim(L \ p)$] is a member of L ; that is, suppose ($L \ [p \ \& \ \sim(L \ p)]$). Since L is closed with respect to conjunction elimination, the propositions p and $\sim(L \ p)$ must accordingly both be members of L , so that the propositions ($L \ p$) and ($L \ (\sim(L \ p))$) must both be true. But from the fact that L is a truth class and has $\sim(L \ p)$ as a member, we conclude that $\sim(L \ p)$ is true, and this contradicts the result that ($L \ p$) is true. Thus from the assumption that

⁵ Vide Jaakko Hintikka, 'Knowledge, Belief, and Logical Consequence', *Ajatus*, Vol. 32, 1970, pp. 32-47.

⁶ In Fitch's notation, the usual subscript denoting an individual is dropped. Instead of 'S a p ' we simply have 'Sp' meaning '**a** brings it about that p '.

$[p \ \& \ \sim (L p)]$ is a member of L we have derived contradictory results. Hence that assumption is necessarily false.⁷

Formally, the proof of Theorem 1 can be exhibited in five steps as follows:

- (1) $L [p \ \& \ \sim (L p)]$ Assumption
- (2) $L p$ (1), Conj. Elim.
- (3) $L \sim (L p)$ (1), Conj. Elim.
- (4) $\sim (L p)$ (3), $L p \rightarrow p$
- (5) $L p \ \& \ \sim (L p)$ (2), (4) Conj. Intro.

Thus by **reductio** we can conclude that:

- (6) $\sim M L [p \ \& \ \sim (L p)]$

where 'M' is read 'it is logically possible that'.

The second theorem reads as follows:

Theorem 2. If L is a truth class which is closed with respect to conjunction elimination, and if p is any true proposition which is not a member of L , then the proposition $[p \ \& \ \sim (L p)]$, is a true proposition which is necessarily not a member of L .

Proof. The proposition $[p \ \& \ \sim (L p)]$ is clearly true, and by Theorem 1 it is necessarily not a member of L .⁸

We can perhaps see a little clearer how this proof works if we write a 'T' in the right upper corner of a proposition (state of affairs) that is true (obtains).⁹ The theorem then reads:

$$T2: \sim L p^T \rightarrow \sim M L [p \ \& \ \sim (L p)]^T$$

That the consequent obtains (disregarding the 'T') simply follows from the fact that it is an instance of (6), that is, Theorem 1.

That the part in brackets of the consequent is true is apparent in that both conjuncts of it follow trivially from the antecedent. By the antecedent, we have it that $\sim L p$, and since p , according to the antecedent, is true, we have it that p .

Now let us proceed to Theorem 3.

Theorem 3. If an agent is all-powerful in the sense that for each situation that is the case, it is logically possible that that situation was brought about by that agent,

⁷ Fitch, p. 138.

⁸ Fitch, p. 138.

⁹ This is not used by Fitch.

then whatever is the case was brought about (done) by that agent.

Proof. Suppose that p is the case but was not brought about by the agent in question. Then, since doing is a truth class closed with respect to conjunction elimination, we conclude from Theorem 2 that there is some actual situation which could not have been brought about by that agent, and hence that the agent is not all-powerful in the sense described.¹⁰

Theorem 3 is proven simply by substituting the operator S (bringing it about) for the general operator L . If we assume:

$$(1) \sim S p^T$$

namely that p obtains but was not brought about by the agent in question, then it follows simply by substitution in T 2 that we have

$$(2) \sim M S [p \& \sim (S p)]^T$$

This asserts that there is an actual state of affairs, represented by the proposition in the square brackets that obtains (note the T in the right corner) but that it is logically impossible for the agent to bring about. Fitch concludes from (2) that

$$(3) \sim M S p^T$$

For the agent in question, it is not logically possible that he should fail to bring about any actual state of affairs. Necessarily, everything that obtains has been brought about by him. Thus any omnipotent agent (in this sense) is also thereby necessarily a universal agent of actual states of affairs. Formally, this result comes about as follows:

$$(4) \sim S p^T \rightarrow \sim M S p^T \quad (1), (3), \text{ Conditionalization}$$

$$(5) M S p^T \rightarrow S p^T \quad (4), \text{ Contraposition}$$

(5) states that for a given agent, if it is possible that he brings about any actual state of affairs then it follows that he personally brings about that actual state of affairs. Any omnipotent being is also a universal agent.

Fitch's theorem may not seem too significant because it is generally felt that God's omnipotence consists not merely in the logical possibility of his doing anything but in the physical possibility of his doing anything. It is not enough that it should be logically possible for God to do anything, but it should be required

¹⁰ Fitch, p. 138.

that there can be no physical obstruction to his action either.¹¹ For any given state of affairs it could never be the case that a set of antecedent conditions of God's bringing about that state of affairs, taken together with the set of nomic universals, entails the proposition that God does not bring about the state of affairs. There are no physical obstacles to God's action. This sense of 'physical possibility' has never been very clearly explained,¹² but what is clear is that something like it, rather than merely logical possibility, is the true sense in which it is possible for God to do anything. Whatever 'physical possibility' is, minimally, we can say that it includes logical possibility. That is for this sense of possibility, M^* ,

$$(1) \quad M^* p \rightarrow M p$$

And by substitution in (1) we have

$$(2) \quad M^* S p^T \rightarrow M S p^T \text{ Sub. } [S p^T/p]$$

Then from Fitch's theorem

$$(3) \quad M S p^T \rightarrow S p^T$$

and (2), the transitivity of \rightarrow yields

$$(4) \quad M^* S p^T \rightarrow S p^T \quad (2), (3), \text{ H.S.}$$

Thus if it is physically possible that an agent brings about every actual state of affairs, that agent has brought about every actual state of affairs. The import of this theorem for philosophical theology is even more evident than Fitch's. If God is omnipotent, in perhaps the most usual or standard sense, then Aquinas' traditional solution to the problem of evil, that God permits rather than brings about evil, cannot be defended. Indeed, if (4) obtains, it is hard to see how any free will defense could possibly succeed.

I cannot pretend that I know definitively what all the implications for philosophical theology are of Fitch's theorem and my corollary of it, although it seems to me that these results must be importantly relevant to the problem of evil and related problems of the notion of omnipotence in a number of ways. I will content myself with adumbrating two primary implications here.

First, Fitch's Theorem 1 provides interesting new grounds for rejecting a claim of Aquinas that God can bring about any state

¹¹ For an excellent recent discussion and review of various traditional conceptions of omnipotence, see Peter Geach, 'Omnipotence', *Philosophy*, January 1973, Vol. 48, pp. 7-20, and Peter Geach, 'An Irrelevance of Omnipotence', *Philosophy*, October 1973, Vol. 48, pp. 327-333. References in the sequel are to the former article.

¹² For a helpful discussion, see Myles Brand, 'On Having the Opportunity', *Theory and Decision*, Vol. 2, 1972, pp. 307-313.

of affairs that is logically possible.¹³ Fitch has shown that

$$\sim M S [p \ \& \ \sim (S p)]$$

even though the state of affairs represented within the brackets is logically possible. It is logically possible that the door is open where God has not brought it about that the door is open (unless it is assumed at the outset that God does everything). Yet it is logically impossible that God personally brings it about both that the door is open and that God does not bring it about that the door is open. Of course this Thomist doctrine has often been rejected,¹⁴ perhaps even by Aquinas himself elsewhere,¹⁵ but so far as I know it has never been rejected on the grounds of anything like Theorem 1.

Second, Theorem 1 and its corollary expose the absurdity of a common but spurious notion of omnipotence. In this sense, to say that God is omnipotent is to say that for any actual state of affairs, God could have been the agent of that state of affairs. To put it another way, let us say that every actual state of affairs is such that either it was brought about by some agent or it just happened. In the latter case, we might say that it was brought about by the null agent. Now to say that God is omnipotent, in this sense, is to say that God is an alternate for any agent. Given any agent of an actual event, even the null agent, God can be put in for this agent. He is the universal agent-substitute. For anything that happened, God can be conceived as being the agent of it.

This notion is demonstrably absurd because if God is omnipotent in this sense, there are no other agents, nor can anything just happen. It follows from the conception of omnipotence that God is the author of literally every state of affairs. Thus the notion that God's power can consist in such agent-substitutability is absurd.

This result, however, by no means implies that all conceptions of omnipotence are inchoate or contradictory. Indeed, I would like to suggest that it indicates that we should concentrate our attention on explicating another quite distinct model of omnipotence while rejecting the above spurious conception. I suggest that we need to think of God's omnipotence not as directed to actual (past) states of affairs, but as directed to unactualized (future) states of affairs. Given any future state of affairs p , God can bring it about that p or bring it about that $\sim p$. There are no obstacles to his actualization of any possible (that is, unactualized)

¹³ *Summa Theologica*, Ia q. xxv art. 3. **Vide** Geach, p. 12f. What I take to be the same view has been held by Alvin Plantinga (see note 16).

¹⁴ **Vide** Wade Savage, 'The Paradox of the Stone', *Philosophical Review*, Vol. LXXVI, No. 1, January 1967, 74-79.

¹⁵ See the discussion in Geach, p. 12f.

state of affairs. It should be a **desideratum** of this concept of omnipotence (which we might call the **nihil obstat** conception) that it not be true of every state of affairs that God personally brings it about (or that God will bring it about), except perhaps in some suitably vicarious sense. Importantly, God's power is viewed as being directed to an unactualized future (though he himself is timeless) rather than to the actualized past. Equally importantly, God's power is not the power to substitute for (even future) agents, but the power to bring it about either that p or that $\sim p$. Even where p has actually obtained at t , God's power is such that only at a previous time, $t-A$, could he have brought it about that $\sim p$.

It is important to emphasise that the power of an omnipotent agent must not be thought of as extending over actual states of affairs that have occurred, since in this instance otherwise viable accounts of omnipotence are driven to absurdity. Consider Plantinga's view that God is omnipotent just in case God can create any state of affairs p such that God brings it about that p is consistent.¹⁶ On this view, an agent is omnipotent where for that agent

$$(P) \quad M \ S \ p$$

But we remember that Fitch's theorem reads

$$M \ S \ p^T \rightarrow S \ p^T$$

Obviously if we allow p^T as a legitimate substitution instance of p in (P), we get the undesirable result

$$S \ p^T$$

namely that the agent in question has brought about everything that has happened. The same result is forthcoming on the **nihil obstat** account of omnipotence which asserts that an agent is omnipotent where

$$M^* \ S \ p$$

By the corollary to Fitch's theorem we have it that

$$M^* \ S \ p^T \rightarrow S \ p^T$$

Thus substituting p^T for p yields the same result. The same result is a consequence of a third conception of omnipotence entertained by Duns Scotus, who was concerned to determine whether we can prove that God can produce directly whatever can be caused.¹⁷ We might reconstruct this conception by ruling that an agent is omnipotent in this sense where

$$M^* \ p \rightarrow M^* \ S \ p$$

An agent is omnipotent where if a state of affairs is physically possible then it is physically possible that it be brought about by

¹⁶ Alvin Plantinga, 'The Free Will Defense', **Philosophy in America**, ed. Max Black, London, Allen and Unwin, 1965, p. 209.

the agent. Here again, if we allow p^T as an instance of p , we have it that

$$M^* p^T \rightarrow M^* S p^T$$

But assuming our corollary to Fitch's theorem

$$M^* S p^T \rightarrow S p^T$$

it follows by hypothetical syllogism that

$$M^* p^T \rightarrow S p^T$$

Since we also have it that any actual state of affairs must be physically possible

$$p^T \rightarrow M^* p^T$$

it also follows that every actual state of affairs is an action of the agent in question

$$p^T \rightarrow S p^T$$

Or to put it more simply,

$$S p^T$$

Thus even these otherwise quite plausible accounts of omnipotence are vitiated if we allow the substitution in question, that is, if we allow the agent's power to extend over actual states of affairs.

Our conclusion from Fitch's theorem that omnipotence is best thought of as directed only to unactualized states of affairs is thus doubly reinforced. Even otherwise promising explications of omnipotence, such as the *nihil obstat* conception, admit of Fitch-like consequences if actualized states of affairs are included in the domain of the variables. An omnipotent being is better thought of as having unlimited power over which unactualized states of affairs he will bring about.¹⁸ At least, I would add, that is the conclusion that Fitch's theorems suggest to me. It is not clear to me that this conclusion is the only one that can be unequivocally and finally drawn from Fitch's results. What I would principally like to leave in the reader's mind is the importance of Fitch's theorems in any serious attempt to understand the concept of omnipotence and the problem of evil. Here I leave off and commend them to the attention of philosophical theologians and atheologists.

¹⁷ Felix Alluntis and Allan B. Wolter, 'Duns Scotus on the Omnipotence of God', *Studies in Philosophy and the History of Philosophy*, Vol. 5, 1970, pp. 178-222.

¹⁸ For an illuminating discussion of divine control of past events, see Geach, p. 16f. I take it that our conclusions here strongly support Geach's view that what is past ceases to be alterable even by God (Geach, p. 17).