Outage Probability of NOMA with Partial HARQ over Time-Correlated Fading Channels

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***Abstract*—This paper investigates the outage performance of downlink non-orthogonal multiple access (NOMA) system over time-correlated Rayleigh fading channels, where the users have heterogeneous quality of service requirements, e.g., a latency-critical user with a low target rate and a delay-tolerant user with a large target rate. In order to meet the different requirements of the users, a partial hybrid automatic repeat request (HARQ) scheme is proposed. The closed-form expressions of outage probabilities for NOMA without and with re-transmission are derived. With the developed outage probabilities, a condition on the superiority of NOMA to orthogonal multiple access (OMA) is obtained. Simulation results demonstrate the accuracy of developed analytical results. It is shown that the performance of NOMA is superior to orthogonal multiple access if the derived condition is satisfied and HARQ-CC can enhance the performance of NOMA over time-correlated fading channels.**

I. Introduction

Different from the conventional orthogonal multiple access (OMA) technologies, the available resources (both frequency and time) can be shared among multiple users in NOMA systems. Taking the advantage of superposition coding and successive interference cancellation (SIC) techniques [1], NOMA achieves significant improvements in terms of spectral efficiency as well as connections [2].

Note that the power allocation and the feedback of channel state information (CSI) are crucial to guarantee successful decoding with SIC in downlink NOMA systems. Recently, the power allocation and the performance analysis of downlink NOMA systems with perfect instantaneous CSI have drawn significant attention [2]–[4], In [2], the outage probability and the ergodic sum-rate have been derived. The developed analytical results show that the diversity order of each user is proportional to its decoding order of SIC. However, the feedback of the perfect instantaneous CSI is impractical due to the rapidly changing wireless channel and large feedback delay. Inspired by these, the authors in [3] focused on the performance of NOMA involving two types of partial CSI, i.e., estimation error and second-order statistics (SOS). It is shown that the users achieve no diversity gain in the first type and one in the second type. Therefore, such imperfect CSI might lead to a considerable performance degradation.

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To improve the reliability of NOMA systems with imperfect CSI, HARQ techniques have been combined with NOMA, e.g., [5], [6]. Specifically, in [5], the outage performance of NOMA with SIC and HARQ with incremental redundancy (HARQ-IR) have been evaluated. Furthermore, the upper bound of outage probability and the power allocation strategy of HARQ-NOMA with the statistical CSI have been studied in [6]. However, most of the above works assume that each user has the same maximum number of re-transmissions and the wireless channels are independent among different transmission rounds. In fact, the users typically have heterogeneous quality of service (QoS) requirements (e.g., delay, reliability) and the channels are not independent among different re-transmissions.

In this paper, we investigate the performance of a downlink NOMA system over time-correlated fading channels, where a partial HARQ with chase combining (HARQ-CC) is adopted. Unlike [5]–[7], where all of the users in NOMA have to re-transmit if anyone fails to decode its signal, more flexible partial HARQ scheme is proposed to serve the users in NOMA according to their QoS requirements. In particular, the latency-critical user with a low target rate transmits different signals among different transmissions; while the delay-tolerant user performs re-transmission due to its large target rate. We first derive the closed-form expressions of outage probabilities for NOMA without and with re-transmission over time-correlated fading channel. With the developed results, the impact of the time-correlated coefficient of the channels on the outage probability is identified and a condition on the superiority of NOMA to OMA is achieved. It is shown that the performance of NOMA is superior to OMA over time-correlated fading channels when the condition is satisfied.

II. System Model

We consider a downlink NOMA system over time-correlated Rayleigh fading channels, where the base station (BS) and the users are equipped with a single antenna. Since the complexity of SIC is $\mathcal{O}(8M^3)$(M is the number of users) [8]. It is not realistic to include all users into a single NOMA group. A low complexity NOMA scheme [9] is employed here, where every two users are paired to perform NOMA [9] and orthogonal frequency resources are allocated to different user pairs. Without loss of generality, one pair of users, e.g., a latency-critical user, u, with a low target rate and a delay-tolerant user, v, with a large target rate, is focused. In order to meet the users' heterogeneous QoS requirements, a partial HARQ-CC scheme is proposed. Specially, user u transmits a new signal during each transmission; while user v's signals are identical among re-transmissions. In this partial HARQ-CC scheme, a feedback from user v to the BS carries an acknowledgment (ACK) signal if the user decodes its signal successfully by combining the previously received signals. Otherwise, it stores the received signal and sends a negative acknowledgment (NACK) binary signal to the BS for repeating request. Then it shall perform re-transmission until the maximum allowed number of re-transmissions is reached.

At the BS, the superposition codeword of user u and user v for the t-th transmission can be expressed as $\sqrt{p_{u,t}}s_{u,t} + \sqrt{p_{v,t}}s_{v,t}$, where $s_{j,t}, j = u, v$, is the unit-power signal of user j; $p_{j,t}$ denotes the transmission power with $p_{u,t}+p_{v,t} \leq p_{\max}$, where $p_{\max}$ is the maximum transmission power of the BS. Then, the observed signal at user j can be expressed as

$$y_{j,t} = g_{jt}(\sqrt{p_{u,t}}s_{u,t} + \sqrt{p_{v,t}}s_{v,t}) + n_{j,t}, \tag{1}$$

where $n_{j,t}$ is additive white Gaussian noise (AWGN) with variance σ_0^2; $g_{j,t} = h_{j,t}/(1+d_j^{\zeta})^{1/2}$ denotes the channel coefficient of user j during the t-th transmission, where d_j is the distance between user j and the BS, $h_{j,t}$ is the time-correlated Rayleigh fading channel coefficient, and ζ is the path loss exponent. It is assumed that the CSI, $g_{j,0} = h_{j,0}/(1+d_j^{\zeta})^{1/2}$, is periodically fed back to the BS each frame (T time slots), where $h_{j,0} \sim \mathcal{CN}(0,1)$. Due to the time-varying nature of the wireless channels, the superposition codeword will experience the channel coefficients, $g_{j,t}$, which is different form the CSI feedback, $g_{j,0}$. The small fading coefficient, $h_{j,t}$, of $g_{j,t}$ in the frame is given by

$$h_{j,t} = \tau_j^t h_{j,0} + \sqrt{1-\tau_j^{2t}}\omega_{j,t}, \tag{2}$$

where $\omega_{j,t} \sim \mathcal{CN}(0,1)$ is the fading term, which is independent of $h_{j,0}$; $\omega_{j,t}$ and $\omega_{j,\tilde{t}}$ are independent from each other for any $t \neq \tilde{t}$; $\tau_j^t = E[h_{j,0}h_{j,t}]$ denotes the time correlation factor, and it is given by a Bessel function in Jakes' scattering model [10], such as

$$\tau_j = J_0(2\pi f_c \varpi \nu_j / c),$$

where $J_0(\cdot)$ denotes the zero-th order Bessel function of the first kind; f_c is the carrier frequency; ϖ is the time duration between two sampling instances; ν_j is the mobile speed and c is the speed of light.

In order to perform SIC at the receiver, the power allocation has to be firstly decided at the BS based on the CSI feedback. Here, two types of imperfect CSI are considered, i.e., the outdated and statistical CSI. For the first type, the channel gains are sorted as $|g_{u,0}|^2 \leq |g_{v,0}|^2$. For the second case, only the statistical channel gains are sorted as $E[|g_{u,0}|^2] \leq E[|g_{v,0}|^2]$, i.e., $d_u \geq d_v$. According to the NOMA scheme, the powers of these two users should satisfy $p_{u,t} > p_{v,t}$ for user fairness.

The received signal-to-interference-plus-noise-ratio (SINR) at user v to decode user u's signal is

$$\gamma_{v\to u}^t = \frac{p_{u,t}|g_{v,t}|^2}{p_{v,t}|g_{v,t}|^2 + \sigma_0^2}, \tag{3}$$

and the received signal-to-noise-ratio (SNR) to decode its own signal is

$$\gamma_{v\to v}^t = \frac{p_{v,t}|g_{v,t}|^2}{\sigma_0^2}. \tag{4}$$

At user u, the signals of user v are considered as noise during each transmission round and the received SINR can be expressed as

$$\gamma_{u\to u}^t = \frac{p_{u,t}|g_{u,t}|^2}{p_{v,t}|g_{u,t}|^2 + \sigma_0^2}. \tag{5}$$

With partial HARQ-CC, the signal of user u is first decoded for each transmission, then user v combines its received signals with maximum ratio combination (MRC) to do joint decoding. The outage probability of user v after T transmissions is formulated as

$$P_{v,T}^{\text{out}} = 1 - \Pr\Bigg(\log_2(1+\gamma_{v\to u}^1) > \hat{R}_u, ..., \log_2(1+\gamma_{v\to u}^T) > \hat{R}_u, \frac{1}{T}\log_2\left(1+\sum_{t=1}^{T}\gamma_{v\to v}^t\right) > \hat{R}_v\Bigg), \tag{6}$$

where $\hat{R}_j$ is the target rate of user j.

However, user u transmission its own signal without re-transmission in partial HARQ-CC. The outage probability of this user is given by

$$P_{u,t}^{\text{out}} = \Pr(\log_2(1+\gamma_{u\to u}^t) < \hat{R}_u). \tag{7}$$

III. Performance Analysis

In this section, we first derive a closed-form expression of the outage probability for the NOMA without re-transmission ($T = 1$) over time-correlated Rayleigh fading channels. With the developed analytical results, the diversity order and the condition on the superiority of NOMA to OMA are investigated. Secondly, the approximated expression of outage probability for NOMA with HARQ-CC is derived. In order to obtain insight, a high SNR approximation of outage probability is further provided.

A. Outage Probability of NOMA without Re-transmission

To reveal the impact of the outdated CSI, we derive the outage probability of NOMA without re-transmission in the following proposition.

Proposition 1: The outage probabilities of user u and user v in NOMA with outdated CSI are expressed as

$$P_{u,t}^{\text{out},o} = 1 - \exp\left(-\frac{2r_u\sigma_0^2}{(2-\tau_u^{2t})(p_{u,t}-p_{v,t}r_u)\lambda_u}\right), \tag{8}$$

and

$$P_{v,t}^{\text{out},o} = 1 - 2\exp\left(-\frac{\varphi\sigma_0^2}{\lambda_v}\right) + \exp\left(-\frac{2\varphi\sigma_0^2}{(2-\tau_v^{2t})\lambda_v}\right), \tag{9}$$

respectively, where $\varphi = \max\left\{\frac{r_u}{p_{u,t}-p_{v,t}r_u}, \frac{r_v}{p_{v,t}}\right\}$ and $\lambda_j = \frac{1}{1+d_{j0}^{\zeta}}$.

Proof: See Appendix A. ■

By defining $\alpha_{u,t} = \frac{p_{u,t}}{p_{u,t}+p_{v,t}}, \alpha_{v,t} = \frac{p_{v,t}}{p_{u,t}+p_{v,t}}$, we have $\frac{p_{u,t}}{\sigma_0^2} = \alpha_{u,t}\frac{p_{u,t}+p_{v,t}}{\sigma_0^2} \triangleq \alpha_{u,t}\rho$ and $\frac{p_{v,t}}{\sigma_0^2} \triangleq \alpha_{v,t}\rho$, where ρ denotes the total transmission SNR at the BS. With the developed outage probability in (8), the diversity order of user u can be formulated as

$$d_u = -\lim_{\rho\to\infty}\frac{\log\left(1-\exp\left(-\frac{2r_u}{(2-\tau_u^{2t})(\alpha_{u,t}-\alpha_{v,t}r_u)\rho\lambda_u}\right)\right)}{\log(\rho)}$$
$$\overset{\hat{\rho}\triangleq\frac{1}{\rho}}{=}\lim_{\hat{\rho}\to 0}\frac{\log\left(\frac{2r_u\hat{\rho}}{(2-\tau_u^{2t})(\alpha_{u,t}-\alpha_{v,t}r_u)\lambda_u}\right)}{\log(\hat{\rho})} = 1.$$

Similarly, the diversity order of user v can be given by

$$d_v = \lim_{\hat{\rho}\to 0}\frac{\log\left(1-2\exp\left(-\frac{\hat{\varphi}\hat{\rho}}{\lambda_v}\right)+\exp\left(-\frac{2\hat{\varphi}\hat{\rho}}{(2-\tau_v^{2t})\lambda_v}\right)\right)}{\log(\hat{\rho})},$$

where $\hat{\varphi} = \max\left\{\frac{r_u}{\alpha_{u,t}-\alpha_{v,t}r_u}, \frac{r_v}{\alpha_{v,t}}\right\}$. With the aid of L' Hospital's Rule, $d_v = 1$ for $0 \le \tau_v < 1$, and $d_v = 2$ for $\tau_v = 1$.

For the case that NOMA with statistical CSI, the cumulative distribution function (CDF) of $|h_{j,t}|^2$ conditioned on $|h_{j,0}|^2$ can be given by

$$F_{|h_{j,t}|^2||h_{j,0}|^2}(x) = \int_0^x \frac{\exp\left(-\frac{\tau_j^{2t}|h_{j,0}|^2+y}{1-\tau_j^{2t}}\right)}{1-\tau_j^{2t}} I_0\left(\frac{2\sqrt{\tau_j^{2t}|h_{j,0}|^2 y}}{1-\tau_j^{2t}}\right)dy, \quad (10)$$

where $I_0(x) = \sum_{l=0}^{\infty}\frac{(x/2)^{2l}}{l!\Gamma(l+1)}$ is the zero-order modified Bessel function of the first kind. Then, the individual conditional outage probability can be given by

$$P_{u||h_{u,0}|^2}^{\text{out},s} = F_{|h_{u,t}|^2||h_{u,0}|^2}\left(\frac{r_u}{(p_{u,t}-p_{v,t}r_u)\lambda_u}\right), \quad (11)$$

$$P_{v||h_{v,0}|^2}^{\text{out},s} = F_{|h_{v,t}|^2||h_{v,0}|^2}(\varphi). \quad (12)$$

Taking expectation of $|h_{j,0}|^2$ and $|h_{v,0}|^2$ in (11)-(12), the individual outage probability can be obtained and the diversity order of each user is equal to one, where the proof is similar to [3]. It is important to point out that NOMA with these types of CSI cannot obtain full diversity order due to the inaccurate CSI. Therefore, NOMA with statistical CSI is a more realistic scheme.

B. Conditions Analysis on the Superiority of NOMA to OMA

For comparison, a conventional OMA scheme, e.g., TDMA, is introduced here. Similar to the developed outage probability in NOMA, the outage probability of user u in OMA with outdated and statistical CSI over time-correlated fading channels is expressed as

$$\hat{\mathbb{P}}_u^{\text{out}} = 1-\exp\left(-\frac{2(2^{2\hat{R}_u}-1)}{(2-\tau_u^{2t})\rho\lambda_u}\right). \quad (13)$$

For user v, the outage probabilities of OMA with outdated and statistical CSI are given by

$$\hat{P}_v^{\text{out},s} = 1-2\exp\left(-\frac{2^{2\hat{R}_v}-1}{\rho\lambda_v}\right)+\exp\left(-\frac{2(2^{2\hat{R}_v}-1)}{(2-\tau_v^{2t})\rho\lambda_v}\right), \quad (14)$$

and

$$\hat{P}_v^{\text{out},s} = 1-\exp\left(-\frac{2^{2\hat{R}_v}-1}{\rho\lambda_v}\right), \quad (15)$$

respectively. With the derived outage probabilities of NOMA and OMA, a condition on the superiority of NOMA to OMA will be shown in the following proposition.

Proposition 2: For a given power allocation scheme, e.g., $p_{u,t}, p_{v,t}$, the target rates of user u and user v should be selected as follows:

$$\hat{R}_u \in \mathcal{D} \text{ and } \hat{R}_v \in \bar{\mathcal{D}}, \quad (16)$$

where $\mathcal{D}$ denotes the region $[0, \log_2(\frac{p_{u,t}}{p_{v,t}}))$, and $\bar{\mathcal{D}} = \mathcal{R}^+ - \mathcal{D}$.

Proof: 1) From (8) and (13), the performance of NOMA with outdated and statistical CSI is better than that of OMA if and only if the following condition is satisfied

$$\frac{2^{\hat{R}_u}-1}{\alpha_{u,t}-\alpha_{v,t}(2^{\hat{R}_u}-1)} < 2^{2\hat{R}_u}-1$$
$$\Leftrightarrow 0 < 1-\alpha_{v,t}2^{\hat{R}_u}-\alpha_{v,t}$$
$$\Leftrightarrow \log_2\left(\frac{\alpha_{u,t}}{\alpha_{v,t}}\right) > \hat{R}_u. \quad (17)$$

Recall that $\alpha_{u,t}-\alpha_{v,t}r_u > 0$, we then have $\log_2\left(\frac{1}{\alpha_{v,t}}\right) > \hat{R}_u$. Since $\log_2(\frac{1}{\alpha_{v,t}}) > \log_2(\frac{\alpha_{u,t}}{\alpha_{v,t}})$, the target rate of user u should satisfy $\hat{R}_u \in \left(0, \log_2\left(\frac{\alpha_{u,t}}{\alpha_{v,t}}\right)\right)$.

2) Form (9) and (14), the outage probability of user v at high SNR for NOMA and OMA with outdated CSI is approximated as

$$\hat{P}_{v,t}^{\text{out},o} \approx \frac{2\hat{\varphi}(1-\tau_v^{2t})}{(2-\tau_v^{2t})\rho\lambda_v}, \quad (18)$$

and

$$\mathbb{P}_{v,t}^{\text{out},o} \approx \frac{2(2^{2\hat{R}_v}-1)(1-\tau_v^{2t})}{(2-\tau_v^{2t})\rho\lambda_v}, \quad (19)$$

respectively. Then, the outage performance of user v in NOMA is superior to OMA if and only if the following condition is satisfied

$$\hat{P}_{v,t}^{\text{out}} < \hat{\mathbb{P}}_{v,t}^{\text{out}} \Leftrightarrow \hat{\varphi} < 2^{2\hat{R}_v}-1. \quad (20)$$

Moreover, based on the values of $\hat{\varphi}$, we need to consider the following two cases: $\hat{\varphi} = \frac{2^{\hat{R}_u}-1}{1-\alpha_{v,t}2^{\hat{R}_u}}$ if $\frac{2^{\hat{R}_u}-1}{1-\alpha_{v,t}2^{\hat{R}_u}} \ge \frac{2^{\hat{R}_v}-1}{\alpha_{v,t}}$. Otherwise, $\hat{\varphi} = \frac{2^{\hat{R}_u}-1}{\alpha_{v,t}}$. Define $\Lambda(\hat{R}_u) = \frac{2^{\hat{R}_u}-1}{1-\alpha_{v,t}2^{\hat{R}_u}}$, we then have $\frac{\partial\Lambda}{\partial\hat{R}_u} > 0$. In this case, $\Lambda(\hat{R}_u)$ increases in $\hat{R}_u > 0$. Then $\frac{2^{\hat{R}_v}}{\alpha_{v,t}} \le \lim_{\hat{R}_u\to 0^+}\lambda(\hat{R}_u) = 0$, i.e., $\hat{R}_v = 0$ in the first case, which contradicts $\hat{R}_v > 0$. Therefore, the condition in (20) can be alternatively written as $\hat{R}_v > \log_2\left(\frac{\alpha_{u,t}}{\alpha_{v,t}}\right)$.

Similarly, the same result can be obtained in case that NOMA and OMA with statistical CSI. ■

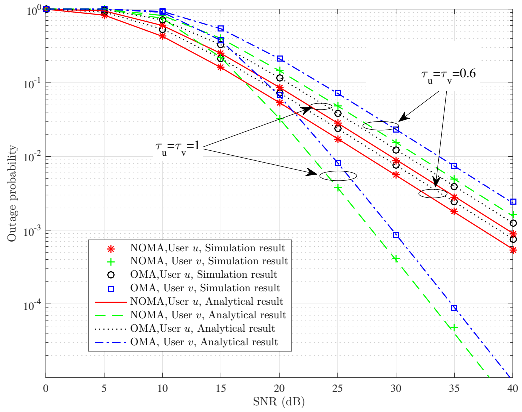

Fig. 1. SNR vs outage probability, where $\hat{R}_u = 0.585$ bps/Hz, $\hat{R}_v = 2$ bps/Hz, $\zeta = 2$.

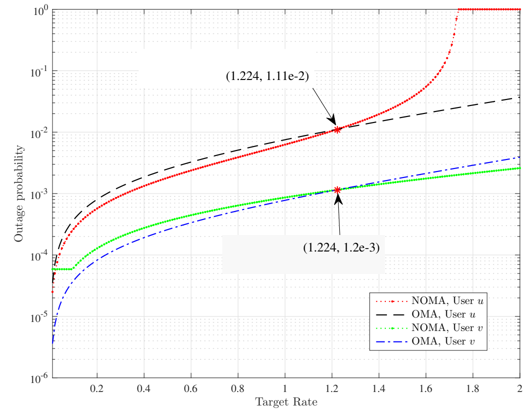

Fig. 2. Target rate vs outage probability at SNR=35 dB, where $\zeta = 2, \tau_j = 0.862$.

C. Outage Probability of NOMA with Partial HARQ-CC

By defining $Y = \sum_{t=1}^{T} \gamma_{v\to v}^{t}$ and $\mathbb{E} = \{\log_2(1+\gamma_{v\to u}^{1}) > \hat{R}_u, ..., \log_2(1+\gamma_{v\to u}^{T}) > \hat{R}_u\}$, the outage probability of user v in (6) can be reformulated as

$$\begin{aligned} P_{v,T}^{\text{out}} &= \Pr(\mathbb{E}, Y < 2^{T\hat{R}_v} - 1) \\ &= \Pr(Y < 2^{T\hat{R}_v} - 1|\mathbb{E}) \Pr(\mathbb{E}) \\ &\overset{(a)}{=} \Pr(Y < 2^{T\hat{R}_v} - 1|\mathbb{E}) \prod_{t=1}^{T} \Pr(\log_2(1+\gamma_{v\to u}^{t}) > \hat{R}_u), \end{aligned}$$

where step (a) uses the fact that the SINRs for decoding the signals of user u in one frame are independent. Notice that to guarantee a high communication reliability, the tolerable outage probabilities of users are generally small numbers, e.g., $\varepsilon_j = 10^{-2}$, or even smaller [11]. Also note that user v has a better channel condition than user u with a high probability due to its smaller path loss (closer to the BS). Thus the outage probability of SIC procedure to decode x_{ut} at user v will be even smaller than ε_u, i.e. $\Pr(\log_2(1+\gamma_{v\to u}^{t}) > \hat{R}_u) \approx 1$. Hence, the outage probability of user v after T transmissions can be approximated as

$$P_{v,T}^{\text{out}} \approx \Pr(Y < 2^{T\hat{R}_v} - 1). \tag{21}$$

The closed-form expressions of outage probability of user v with HARQ-CC is given by the following proposition.

Proposition 3: The outage probability of user v for HARQ-CC aided NOMA with statistical CSI is approximated as

$$\begin{aligned} P_{v,T}^{\text{out}} \approx \sum_{k=1}^{L} \frac{(-1)^{\frac{L}{2}+k}\hat{w}_k}{k} \Bigg[\prod_{t=1}^{T} \left(1 + \frac{p_{v,t}\lambda_v(1-\tau_v^{2t})k\ln 2}{\hat{r}_v\sigma_0^2}\right) \\ \left(1 + \sum_{t=1}^{T} \frac{p_{v,t}\lambda_v\tau_v^{2t}k\ln 2}{\hat{r}_v\sigma_0^2 + p_{v,t}\lambda_v(1-\tau_v^{2t})k\ln 2}\right)\Bigg]^{-1}, \end{aligned} \tag{22}$$

where L is a parameter for Gaver-Stehfest procedure, and ω_k is defined as

$$\hat{w}_k = \sum_{l=\lfloor \frac{k+1}{2} \rfloor}^{\min\{k,l\}} \frac{l^{L/2+1}}{(L/2)!} \binom{L/2}{l} \binom{2l}{l} \binom{l}{k-l},$$

with $\lfloor z \rfloor$ denoting the largest integer in an integer not exceeding z.

Proof: See Appendix B. ∎

Moreover, the following corollary provides a high SNR approximation of outage probability for HARQ-CC.

Corollary 1: At high SNR, the outage probability of user v for HARQ-CC aided NOMA woth statistical CSI is approximated as

$$P_{v,T}^{out} \approx \frac{r_v^T}{T! \prod_{t=1}^{T}(\alpha_{v,t}\rho\lambda_v(1-\tau_v^{2t}))} \left(1 + \sum_{t=1}^{T} \frac{\tau_v^{2t}}{1-\tau_v^{2t}}\right)^{-1}.$$

Proof: Because of the space limitation, the detailed proof are relegated to the full version of this paper [11]. ∎

From Corollary 1, we can observe that the diversity order of user v with HARQ-CC is T.

IV. Numerical Results

Simulations are presented in this section to verify the accuracy of the developed analytical results. In all simulations, we set $f_c = 2.6\text{GHz}, \varpi = 1\text{ms}, c = 3 \times 10^8\text{m/s}, d_{u,0} = 2m, d_{v,0} = 1m, L = 10$ and $\alpha_{u,t} = 0.7, \alpha_{v,t} = 0.3, t = 1, 2, 3$.

In Fig. 1, we depict the individual outage probability versus SNR for time-correlated fading channels. The speeds of two users are set as $\nu_{u0} = \nu_{v0} = 88.4$km/h, which corresponds to $\tau_j = 0.6$. Moreover, the time-correlated coefficient $\tau_j = 1$ if user j is stationary. Fig. 1 demonstrates that the derived results match the simulation results. We can observe from the figure that the diversity order of user v is two for $\tau_j = 1$, while the diversity order is one for $\tau_j = 0.6$. This is because instantaneous perfect CSI is known at the transmitter for $\tau_j = 1$. It can be also seen that NOMA and OMA achieve the same diversity gain, but the NOMA scheme offers a superior performance in terms of the outage probability.

Fig. 2 shows that the outage performance of user u in NOMA is always better than that of OMA when the target rate of user $\hat{R}_u$ in $(0, 1.2224)$. However, the outage performance of user u in NOMA becomes worse when the target outside this region. Particularly, the outage probability of user u will be one when the target rate is large enough. On the other hand, we set $\hat{R}_u = 0.2$ bps/Hz for observing the performance of user

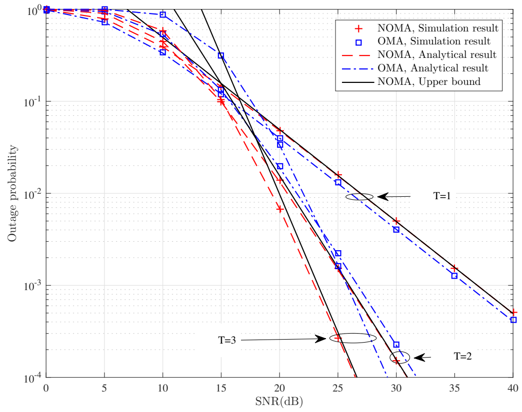

Fig. 3. Outage probability of user u with the aid of HARQ-CC, where $\hat{R}_u = 0.2$ bps/Hz, $\hat{R}_v = 0.8$ bps/Hz, $\tau_v = 0.862, \zeta = 3$.

v. As can be observed from Fig. 2, the performance of user v in NOMA is better than that of OMA when the target rate is larger than 1.2224 bps/Hz. As shown in Fig. 3, the outage performance of user v in NOMA can be enhanced by adopting HARQ-CC. Specifically, the performance gap between NOMA and OMA becomes large when the number of re-transmissions increases. Moreover, the simulations also confirm the accuracy of the developed results in Proposition 3 and the upper bound in Corollary 1.

V. Conclusions

The performance of NOMA over time-correlated fading channels has been investigated in this paper. We have derived the closed-form expression of outage probability for NOMA with two types of imperfect CSI and presented a condition for the target rate selection with outage requirement. It has been shown that the NOMA scheme outperforms the conventional OMA if the condition is satisfied. Otherwise, a partial HARQ-CC scheme was adopted to improve the reliability of transmission. Moreover, the approximated expression and the upper bound of outage probabilities have been derived for the proposed partial HARQ-CC.

Appendix A
The Proof of Proposition 1

Define the unordered channel gain of $|h_{j,0}|^2$ as $|\bar{h}_{j,0}|^2$, which follows exponential distribution with rate parameter 1, then the CDF of the unordered channel gain, $\bar{g}_{j0} = \frac{\bar{h}_{j,0}}{1+d_j^{\zeta}}$, can be given by

$$F_{|\bar{g}_{j,0}|^2}(x_{j0}) = 1 - \exp\left(-\frac{x_{j0}}{\lambda_j}\right). \tag{23}$$

And the probability density function (PDF) can be expressed as folows

$$f_{|\bar{g}_{j,0}|^2}(x_{j0}) = \frac{1}{\lambda_j}\exp\left(-\frac{x_{j0}}{\lambda_j}\right). \tag{24}$$

At BS, the channel gains are sorted as $|g_{u,0}|^2 \leq |g_{v,0}|^2$ to decide the decoding order of SIC. Based on order statistics, the PDFs of ordered channel gains, $|g_{u,0}|^2$ and $|g_{v,0}|^2$, can be expressed as follows

$$\begin{aligned} f_{|g_{u,0}|^2}(x_{u0}) &= 2[1 - F_{|\bar{g}_{u,0}|^2}(x_{u0})] f_{|\bar{g}_{u,0}|^2}(x_{u0}) \\ &= \frac{2}{\lambda_u}\exp\left(-\frac{2x_{u0}}{\lambda_u}\right), \end{aligned} \tag{25}$$

and

$$\begin{aligned} f_{|g_{v,0}|^2}(x_{v0}) &= 2F_{|g_{v,0}|^2}(x_{v0}) f_{|g_{v,0}|^2}(x_{v0}) \\ &= \frac{2}{\lambda_v}\left(1 - \exp\left(-\frac{x_{v0}}{\lambda_v}\right)\right)\exp\left(-\frac{x_{v0}}{\lambda_v}\right). \end{aligned} \tag{26}$$

Due to the change of channel, the ordered channel gains are outdated for the t-th transmission of user j. Based on $g_{j,0}$, the real received channel gain is $|\hat{g}_{j,t}|^2$ for t-th transmission and its PDF can be given by

$$\begin{aligned} f_{|\hat{g}_{j,t}|^2}(x_{jt}) &= \int_0^{\infty} f_{|\hat{g}_{j,t}|^2, |g_{j,0}|^2}(x_{jt}, x_{j0}) dx_{j0} \\ &= \int_0^{\infty} f_{|\hat{g}_{j,t}|^2 | |g_{j,0}|^2}(x_{jt}|x_{j0}) f_{|g_{j,0}|^2}(x_{j0}) dx_{j0} \\ &\overset{(a)}{=} \int_0^{\infty} f_{|g_{j,t}|^2 | |\bar{g}_{j,0}|^2}(x_{jt}|x_{j0}) f_{|g_{j,0}|^2}(x_{j0}) dx_{j0} \\ &= \int_0^{\infty} \frac{f_{|g_{j,t}|^2, |\bar{g}_{j,0}|^2}(x_{jt}, x_{j0})}{f_{|\bar{g}_{j,0}|^2}(x_{j0})} f_{|g_{j,0}|^2}(x_{j0}) dx_{j0}, \end{aligned} \tag{27}$$

where $f_{|g_{j,t}|^2}(x_{jt})$ is the PDF of channel gain $|g_{j,t}|^2$, which is predicted from (2), and (a) uses the fact that $f_{|\hat{g}_{j,t}|^2 | |g_{j,0}|^2}(x_{jt}|x_{j0}) = f_{|g_{j,t}|^2 | |\bar{g}_{j,0}|^2}(x_{jt}|x_{j0})$ [12].

Note that the PDF of $|h_{jt}|^2$ conditioned $|h_{j0}|^2$ can be obtained from (10), then the joint PDF of $|h_{j,t}|^2$ and $|h_{j,0}|^2$ can be given by

$$\begin{aligned} f_{|h_{j,t}|^2, |h_{j,0}|^2}(x_{jt}, x_{j0}) &= f_{|h_{j,t}|^2 | |h_{j,0}|^2}(x_{jt}|x_{j0}) f_{|h_{j,0}|^2}(x_{j0}) \\ &= \frac{\exp\left(-\frac{x_{jt}+x_{j0}}{1-\tau_j^{2t}}\right)}{1-\tau_j^{2t}} I_0\left(\frac{2\sqrt{\tau_j^{2t} x_{jt} x_{j0}}}{1-\tau_j^{2t}}\right). \end{aligned} \tag{28}$$

Therefore, the joint PDF of $|g_{j,t}|^2$ and $|\bar{g}_{j,0}|^2$ can be given by

$$\begin{aligned} & f_{|g_{j,t}|^2, |\bar{g}_{j,0}|^2}(x_{jt}, x_{j0}) \\ &= \frac{\exp\left(-\frac{x_{jt}+x_{j0}}{(1-\tau_j^{2t})\lambda_j}\right)}{(1-\tau_j^{2t})\lambda_j^2} I_0\left(\frac{2\sqrt{\tau_j^{2t} x_{jt} x_{j0}}}{(1-\tau_j^{2t})\lambda_j}\right). \end{aligned} \tag{29}$$

From (25), (26), (27) and (29), the PDFs of $|\hat{g}_{u,t}|^2$ and $|\hat{g}_{v,t}|^2$ can be expressed as follows

$$\begin{aligned} f_{|\hat{g}_{u,t}|^2}(x_{ut}) &= \frac{2}{(1-\tau_u^{2t})\lambda_u^2} \int_0^{\infty} I_0\left(\frac{2\sqrt{\tau_u^{2t} x_{ut} x_{u0}}}{(1-\tau_u^{2t})\lambda_u}\right) \\ & \exp\left(-\frac{x_{ut} + (2-\tau_u^{2t})x_{u0}}{(1-\tau_u^{2t})\lambda_u}\right) dx_{u0} \\ &= \frac{2}{(2-\tau_u^{2t})\lambda_u}\exp\left(-\frac{2x_{ut}}{(2-\tau_u^{2t})\lambda_u}\right), \end{aligned}$$

and

$$f_{|\hat{g}_{v,t}|^2}(x_{vt}) = \frac{2}{(1-\tau_v^{2t})\lambda_v^2}\int_0^\infty \exp\biggl(-\frac{x_{vt}+x_{v0}}{(1-\tau_v^{2t})\lambda_v}\biggr)$$
$$I_0\biggl(\frac{2\sqrt{\tau_v^{2t}x_{vt}x_{v0}}}{(1-\tau_v^{2t})\lambda_v}\biggr)\left(1-\exp\left(-\frac{x_{v0}}{\lambda_v}\right)\right)dx_{v0}$$
$$=\frac{2}{\lambda_v}\exp\left(-\frac{x_{vt}}{\lambda_v}\right)-\frac{2}{(2-\tau_v^{2t})\lambda_v}\exp\left(-\frac{2x_{vt}}{(2-\tau_v^{2t})\lambda_v}\right),$$

respectively, where we use the fact that $\exp(x)=\sum_{l=0}^{\infty}\frac{x^l}{l!}$. Then the CDFs of $|\hat{g}_{u,t}|^2$ and $|\hat{g}_{v,t}|^2$ can be given by

$$F_{|\hat{g}_{u,t}|^2}(x_{ut}) = 1-\exp\left(-\frac{2x_{ut}}{(2-\tau_u^{2t})\lambda_u}\right), \tag{30}$$

and

$$F_{|\hat{g}_{v,t}|^{2t}}(x_{vt}) = 1-2e^{-\frac{x_{vt}}{\lambda_v}}+\exp\left(-\frac{2x_{vt}}{(2-\tau_v^{2t})\lambda_v}\right). \tag{31}$$

The outage probability of user u can be given by

$$\begin{aligned} P_{u,t}^{\text{out}} &= \Pr\left\{\frac{p_{u,t}|\hat{g}_{u,t}|^2}{p_{v,t}|\hat{g}_{u,t}|^2+\sigma_0^2}<r_u\right\} \\ &= F_{|\hat{g}_{u,t}|^2}\left(\frac{r_u\sigma_0^2}{p_{u,t}-p_{v,t}r_u}\right) \\ &= 1-\exp\left(-\frac{2r_u\sigma_0^2}{(2-\tau_u^{2t})(p_{u,t}-p_{v,t}r_u)\lambda_u}\right). \end{aligned} \tag{32}$$

And the outage probability of user v can be given by

$$\begin{aligned} P_{v,t}^{\text{out}} &= 1-\Pr\left\{\frac{p_{u,t}|\hat{g}_{v,t}|^2}{p_{v,t}|\hat{g}_{v,t}|^2+\sigma_0^2}>r_u, \frac{p_{v,t}|\hat{g}_{v,t}|^2}{\sigma_0^2}>r_v\right\} \\ &= F_{|\hat{g}_{vt}|^2}(\varphi). \end{aligned} \tag{33}$$

Submitting (31) in (33) completes the proof.

Appendix B
The Proof of Proposition 3

From (9), the MGF can be expressed as

$$\mathcal{M}_{|h_{v,t}|^2||h_{v,0}|^2}(s) = \frac{1}{1-(1-\tau_v^{2t})s}\exp\left(\frac{\tau_v^{2t}|h_{v,0}|^2 s}{1-(1-\tau_v^{2t})s}\right).$$

Let $Y||h_{v,0}|^2 = \sum_{t=1}^{T}|h_{v,t}|^2||h_{v,0}|^2$, then the CDF of $Y||h_{v,0}|^2$ is

$$F_{Y||h_{v,0}|^2}(y) = \mathcal{L}^{-1}\biggl(\mathcal{L}\biggl(\int_0^y f_{Y||h_{v,0}|^2}(t)dt\biggr)\biggr)$$
$$=\mathcal{L}^{-1}\left(\frac{1}{s}\prod_{t=1}^{T}\mathcal{M}_{|h_{v,t}|^2||h_{v,0}|^2}(-s)\right)$$
$$=\mathcal{L}^{-1}\biggl(\frac{1}{s}\prod_{t=1}^{T}\frac{1}{1+(1-\tau_v^{2t})s}\exp\left(-\frac{\tau_v^{2t}|h_{v,0}|^2 s}{1+(1-\tau_v^{2t})s}\right)\biggr).$$

Using Gaver-Stehfest procedure [13], we have

$$F_{Y||h_{v,0}|^2}(y) \approx \frac{\ln 2}{y}\sum_{k=1}^{L/2}(-1)^{L/2+k}\varpi_k g\biggl(\frac{k\ln 2}{y}\biggr), \tag{34}$$

where $g(x)$ is defined as

$$g(x) = \frac{1}{x}\prod_{t=1}^{T}\biggl(\frac{1}{1+(1-\tau_v^{2t})x}\exp\left(-\frac{\tau_v^{2t}|h_{v,0}|^2 x}{1+(1-\tau_v^{2t})x}\right)\biggr).$$

Then the CDF of Y can be given by

$$\begin{aligned} F_Y(y) &= \int_0^\infty F_{Y||h_{v,0}|^2}(y) f_{|h_{v,0}|^2}(z)dz \\ &\approx \sum_{k=1}^{L/2}\frac{(-1)^{L/2+k}\hat{w}_k}{k}\biggl(\prod_{t=1}^{T}\frac{y}{y+(1-\tau_v^{2t})k\ln 2}\biggr) \\ &\int_0^\infty \exp\left(-\left(1+\sum_{t=1}^{T}\frac{\tau_v^{2t}k\ln 2}{y+(1-\tau_v^{2t})k\ln 2}\right)z\right)dz \\ &= \sum_{k=1}^{L/2}\frac{(-1)^{L/2+k}\hat{w}_k}{k}\biggl(\prod_{t=1}^{T}\frac{y}{y+(1-\tau_v^{2t})k\ln 2}\biggr) \\ &\left(1+\sum_{t=1}^{T}\frac{\tau_v^{2t}k\ln 2}{y+(1-\tau_v^{2t})k\ln 2}\right)^{-1}. \end{aligned} \tag{35}$$

Submitting (35) in (21) completes the proof.

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