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# Dollar/Euro Exchange Rate: A Monthly Econometric Model for Forecasting


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## Abstract

*The intent of this paper is the construction of an econometric model able to produce reliable and reasonable forecasts for the Dollar/Euro Real Exchange Rate.*
*In order to achieve this aim, a decision must first be made regarding the geographical aggregation versus disaggregation of the data. Hence we analyse whether an area-wide or multi-country model performs better by evaluating the forecasting performance of the two alternative approaches. The arguments that can be set out in favour of either alternative are presented. We consider the problems arising from the non-stationarity of financial variables. By using the well-known cointegration analysis we analyse the long-term relationships among selected real and financial variables and the Dollar/Euro exchange rate. A vector ECM model in which the relevant economic variables are not necessarily of the same order of integration is proposed.*
*An important source of non-stationarity could be the presence of structural breaks.*
*Some relevant economic, political and institutional changes occurred in the Euro Area between January 1990 and December 1999 (the sample period) which could be modelled by structural breaks (e.g. Maastricht Treaty – February 1992, EMS crisis – September 1992, etc). We therefore test the constancy of the models' parameters over the sample period to verify the effectiveness of the deterministic components of the model and the co-breaking concept.*

## 1. Introduction

***Motivation for US$/€ real exchange rate model***

An important motivation in favour of the real, rather than the nominal exchange rate, is the failure (on empirical grounds) of the purchasing power parity (PPP), which states the long-run equilibrium between the exchange rates and the price levels.
Suppose $S_t$ to be the exchange rate US$/€ (price of one unit of Euro in term of US$) and $P_t$ the one country's price level, then the PPP relationship is:

(1) $$S_t = P_{\$t}/P_{€t}$$

or more generally:

(2) $$S_t = Q_t\, P_{\$t}/P_{€t}$$

where $Q_t$ is the real exchange rate US$/€ supposed constant $\forall t$.

An increment of the US inflation rate (versus that of the Euro Area) is followed by an increase of $S_t$, that is a depreciation of US$. The assumption of $Q_t$ to be constant implies that the nominal exchange rate obeys (2) when monetary shocks occur. May not be $Q_t$ constant, as in the case of real shocks (e.g. oil shocks, productivity gaps between the two areas, etc.), then obviously the PPP relationship is no longer valid.
From (2) we obtain

(3) $$Q_t = S_t\, P_{€t} / P_{\$t}$$

and using the log transform:

(4) $$q_t = s_t + p_{€t} - p_{\$t}$$

Here, an increase in $q_t$ means a depreciation of the real US dollar followed by a depreciation of the nominal US dollar or a decrease of the US and Euro inflation rate differentials.
The *uncovered interest parity condition* (UIP) states the long-run equilibrium between the money market and the foreign exchange market, that is:

(5) $$E_t\Delta s_{t+k} = i_{\$t} - i_{€t}$$

in real terms, we subtract from both sides the inflation differential:

$$E_t\Delta s_{t+k} - (E_t\Delta p_{\$t+k} - E_t\Delta p_{€t+k}) = (i_{\$t} - i_{€t}) - (E_t\Delta p_{\$t+k} - E_t\Delta p_{€t+k})$$

$$E_t\Delta s_{t+k} - E_t\Delta p_{\$t+k} + E_t\Delta p_{€t+k} = (i_{\$t} - E_t\Delta p_{\$t+k}) - (i_{€t} - E_t\Delta p_{€t+k})$$

Using (4), we obtain:

$$E_t\Delta q_{t+k} = r_{\$t} - r_{€t}$$

where:

$$r_{\$t} = i_{\$t} - E_t \Delta p_{\$t+k}$$

$$E_t q_{t+k} - q_t = r_{\$t} - r_{€t}$$

(6) $$q_t = E_t q_{t+k} - (r_{\$t} - r_{€t})$$

In formula (6), we indicate the unknown $E_t q_{t+k}$ as $\underline{q}_t$ , which is called *Fundamentals Exclusive of the Real Interest Differential* (*FERID*) and is driven by fundamentals, such as productivity variables (e.g. the ratio of Tradable to Non-tradable Goods), which should be able to capture the so-called Balassa–Samuelson effect, commodity shocks (such as the Real Price of Oil and relative Terms of Trade) and budget policy (such as Fiscal Budget Surplus or Deficit and Net Foreign Assets).
The difference $(r_{\$t} - r_{€t})$ in formula (6) is usually known as the *Real Interest Differential* (*RID*) and it is modelled in this paper as the Real Long-term Interest Rate Differential (*RRL*); therefore (6) it can lead to the **Foreign exchange market** relationship written in the subsequent section.

## 2. The complete economic model

In order to specify a structural model we endogenize the long-term interest rate differential and the differential between US and Euro GDP annual growth rate.
We therefore consider the following three markets (all variables are considered log-transformed).

### *a) The Foreign Exchange Market*

The real foreign exchange rate's equilibrium behaviour, given the previous considerations, is therefore affected in our model by the time path of several fundamental variables (such as foreign trade efficiency, commodity shocks and budget policy) as well as by the real interest rates differential. We model the real foreign exchange rate's equilibrium behaviour in the line of the recent works of MacDonald (1997) and MacDonald and Marsh (1999).
The foreign trade efficiency is modelled, in our theoretical framework, as the differential between US and Euro ratio of consumer price index to the production price index (noted *LTNT*). This variable should be able to capture the Balassa-Samuelson effect, probably the best-known source of systematic changes in the relative price of traded to non-traded goods across countries. The Balassa-Samuelson theory states that the nominal exchange rate moves to ensure the relative price of traded goods is constant over time. Productivity differences in the production of traded goods across countries, however, usually introduce a bias into the overall real exchange rate, since productivity advances are preferably concentrated in the traded goods sector rather than the non-traded one. If, as usual, all (tradable and non-tradable) finished products' prices are strictly linked to wages, wages are linked to productivity and linked across tradable and non-tradable industries as well, then the price of tradable goods will rise less rapidly in the country with a higher productivity in the tradable sector. This will cause an increase in the foreign demand for tradable goods produced in such a country (less expensive) and therefore to an appreciation of the real exchange rate (a decrease of $q_t$). The sign we expect for *LTNT* is therefore negative.
The fiscal budget, both in terms of direct expenditure and in terms of net foreign assets (national savings), also affects the equilibrium behaviour of the real exchange rate.
In our model we use two variables to describe these effects: *FBAL*, which is the differential between US and Euro ratio of government debt's annual rate of growth to GDP rate of growth, and *NFA* which is the ratio of US to Euro ratio of net foreign asset to GDP. A tight fiscal policy in United States implies, *ceteris paribus,* a decrease of *FBAL* or an increase of *NFA*.
The effect of fiscal policy on the real exchange rate usually leads to the following question: "*Will a positive fiscal budget strengthen or weaken the external value of a currency?*"
Unfortunately, there is no one single answer.
On the one hand, in fact, in the traditional Mundell–Fleming two country model, a tight fiscal policy, which increases the aggregate national savings, would lower the domestic interest rate and generate a permanent real exchange rate depreciation (an increase of $q_t$).
On the other hand, however, considering only the pure effect of fiscal policy in terms of an increase in national savings is somehow misleading. This is just a partial view, since lower interest rates will induce also net funds outflows towards countries paying higher

interest rates. In this case models which account also for the stock implications of the initial fiscal tightening are portfolio balance models. In this class of models, the long-run is defined as a point at which any interest earnings on net foreign assets are offset by a corresponding trade imbalance. Hence, if the fiscal tightening is perceived as permanent by the markets, this will induce a permanent increase in net foreign assets and therefore a permanent appreciation of the long-run equilibrium exchange rate (a decrease of $q_t$).

The last source of shocks affecting the real equilibrium exchange rate is that referring to shocks in the commodity markets.

In our theoretical framework they are modelled by means of two variables: the differential between domestic and foreign ratio of export unit value to the import unit value (*LTOT* in our notation), and the real price of oil (*ROIL* in our notation).

Changes in the terms of trade usually induce a shock to one country's foreign trade structure, in the sense that this will affect both the foreign demand (increase/decrease) and the domestic production structure (more or less foreign trade driven).

Changes in the real price of oil can also have an effect on the relative price of traded goods, usually through their effect on the above-described terms of trade. In comparing a country which is self-sufficient in oil resources with one which needs to import oil, the latter, *ceteris paribus*, will experience a depreciation of its currency vis-à-vis that of the former as the price of oil rises. More generally, countries that have at least some oil (and/or other commodities) resources could find their currencies appreciating relative to countries that are net importers of oil (and/or other commodities).

The comparison of US and Euro areas, both prevailingly importers of oil, leaves the sign of *ROIL* uncertain.

Taking into account these considerations, we model the long-run equilibrium real exchange rate as follows:

$$Q = h(LTNT, FBAL, NFA, LTOT, ROIL, RRL)$$

where $Q$ indicates the US Dollar/Euro real exchange rate and *RRL* (see end of section 1) the 10-year real interest rate differential.

### *b)* ***The Money Market***

We modelled the equation for the long-term real interest rates differential as follows:

$$RRL = g(MG, Y)$$

where *MG* denotes the differential between the annual growth rate of the US and Euro real money supply and *Y* denotes the differential between the annual growth rate of the US and Euro GDP.

The money market's equilibrium equation usually describes the real money supply as a function of both the real "policy" interest rate and the output growth,

$$M/P = L(r, Y)$$

and therefore, according to that interpretation, money growth would be endogenous, while real policy interest rate (in our model approximated by the 10-year real interest rate) would be exogenous.

We can assume, however, that the total amount of money supply is determined by the two countries' central banks. If this is the case, given *M* (total amount of money supply), *P* (the level of prices, in which we are not interested, since we are modelling the real not the nominal exchange rate), and *Y*, the only variable determined by the money market is the real interest rate. This will then, in turn, play its influence on the real exchange rate through the above-mentioned real interest rate differential.

In our model, we make the assumption that central banks fix the money growth target and therefore money growth can be considered as exogenous, while the markets fix the equilibrium interest rate.

The economic theory tells us what follows about dynamics. An easy monetary policy, if perceived as permanent and not just a spot increase in the monetary base, usually induces a decrease in the long-term interest rate. In fact, once liquidity has been injected in the system, banks experience the need to invest this new and a large amount of liquidity and will be willing to do this even in correspondence of lower interest rates.

As to the output, instead, an increase in output levels induces a rise in the volume of transactions and therefore in the demand for money, which will resolve in an increase in the level of interest rates.

c) ***The Goods Market***

In order to take into account both domestic effects (national savings and budget policy) and foreign trade effects (Balassa-Samuelson effect, and commodity market shocks) the dynamic equilibrium of the goods market has been formalised in the following way:

$$Y = f(RRL, LTNT, NFA, FBAL, LTOT, ROIL)$$

where the impact of monetary conditions on gross domestic product growth has been taken into account as well in terms of the long-term interest rates differential.

As to the ratio of tradable to non-tradable goods prices, we argue that an increase in productivity denotes an improved ability to face competition across markets. This will resolve in an increase in the foreign demand of the country's products and therefore to an increase in the production and finally in the output.

According to the classical economic theory, the impact of a tight monetary policy on the real gross domestic product growth is negative, in the sense that higher interest rates will discourage investments and, therefore, result in a lower economic growth.

Passing to the analysis of the fiscal policy on output, we observe that an easy fiscal policy (increase of *FBAL* or decrease of *NFA*), if directed to investments, in the first step should increase the total output, while in the long-run, this fact could be perceived as an obstacle to growth (because of the tight policies motivated from debt repayments) and therefore having a negative impact on the latter.

Finally, passing to the analysis of the impact of commodity markets on output, it is useful to take into account the same considerations described above in term of the effects on the real exchange rate. Comparing a country which is self-sufficient in oil resources with one which needs to import oil an increase in the cost of oil leads to an increase in the output growth of the former. To take into account a wide concept of commodities, we also consider the terms of trade.

## 3. Variables Definitions

For both models described in this work (aggregated and disaggregated approach), we took into consideration monthly data from January 1990 to December 1999, with the last twelve observations (from January to December 1999) used to produce ex-post forecasts. Therefore, we test the forecasting ability of the model, both in terms of evaluating the proximity of forecasted data to the observed ones (Root Mean Square Error, Mean Error, Mean Absolute Error, Theil's U) and in terms of the model's ability of the model to capture signs of the changes in the real Dollar/Euro exchange rate (percentage of signs correctly forecasted).

The real Dollar/Euro exchange rate (*Q*) used for this analysis is the logarithm of the synthetic[4], nominal Dollar/Euro Exchange rate minus the differential between the logarithms of the Euro Area Consumer Price Index (base 1995 = 100) and the US Consumer Price Index (base 1995 = 100).

In order to take into account the well known Balassa-Samuelson effect, we have built a proxy of the ratio of traded to non-traded prices as the ratio of Consumer Price Index to Producer Price Index and we have considered the differential between domestic (United States) and foreign logarithms of these ratios (*LTNT*)[5].

The fiscal policy effects are adequately captured, in our opinion, by taking into consideration the differential between US and foreign ratios of annual real public debt growth to annual real gross domestic product growth.

The *NFA* variable is computed as the ratio of domestic and foreign ratios of total real net foreign assets to the real gross domestic product (in billions of dollars). It captures the fundamental dynamics of funds flows and the effect of fiscal policies on the exchange rate as well as other factors more closely associated with private sector savings, such as demographics.

Two variables have been used to model the impact of the dynamics of commodity prices on both the gross domestic product growth (*Y*) and the real exchange rate. The first variables are the terms of trade (*LTOT*), that is, constructed as the ratio of US export unit value to import unit value as a proportion of the equivalent effective foreign ratio, expressed in logarithms. The second variable is the real price of oil (*ROIL*), expressed in US Dollars per barrel and defined as the nominal price of crude oil (Brent) to the US producer price index.

Money markets have been taken into consideration by means of two variables. The real money (*MG*) supplied to the economic system by the central banks of the countries involved in our analysis, is represented by the differential between domestic and foreign annual real M3 growth (deflated by subtracting from the nominal growth rate the annual domestic inflation rate). This variable, in our opinion, is able to capture the differences between the European Central Bank and the Federal Reserve with regards to the total amount of credit allowed to the system. The second variable used to model money markets is the real long term interest rates differential (*RRL*), computed as the differential between the US and European 10-year real interest rate. The real interest rate both for the Euro Area and the US has been computed as the ratio of nominal 10-

[4] The synthetic Dollar/Euro nominal exchange rate is that produced by Warburg Dillon Read.

[5] In the case of presence of seasonal patterns in the time series, they have been removed by means of the usual ARIMA techniques.

year interest rate to a centred 13-month average of the annual inflation rate[6]. All the variables are synthetically presented in Table A1 in Appendix A.

## 4. The Area-Wide Model

### 4.1. Motivations for an Aggregated Approach to Euro Modelling

The arguments in favour of the specification of an area-wide model come from the considerations that an area-wide model would be more parsimonious than the multi-country alternative. This consideration can not be undervalued because the larger the degrees of freedom, the more meaningful become the computed statistics, often based on asymptotic assumption. Not less important is the possibility to specify the model taking into account the different structure behaviour of the economic variables for specific sub-sample periods. Furthermore, under a single monetary policy, some macroeconomic variables are the same across the Euro-Area with better simplifications in the specification of the model. Finally, policy makers can decide to monitor some disagregate variables in any case if they play a role of leading indicators.

### 4.2. The Econometric Approach

#### 4.2.1. Cointegration analysis in presence of structural breaks

The recent econometric literature has given a strong relevance to structural breaks [see Clements and Hendry (1999)]. Some results regarding structural breaks in the context of univariate autoregressive time series with a unit root are well known. A time series given by stationary fluctuations around a broken constant level is better described by a random walk than a stationary time series [see Perron (1989, 1990) and Rappoport and Reichlin (1989)].

Special issues of the *Journal of Business & Economics Statistics*, volume 10, 1990 and the *Journal of Econometrics*, volume 70, 1996 have discussed the parameter stability in econometric models assuming known break points. Testing hypotheses for known break points in connection with cointegration testing has been suggested by Inoue (1999), and breaks in the cointegration parameter by Kuo (1998), Seo (1998) and Hansen and Johansen (1999).

The importance of cointegration analysis in the presence of structural breaks relies on the undesired results when these breaks are ignored. In fact, when the series are trend stationary and the trend is a broken trend, if the structural breaks are not considered, the cointegration hypothesis may be rejected. Furthermore, the forecasts using VAR might be better than a VECM which does not consider structural breaks. On the contrary, if cointegration analysis with structural breaks is performed, VECM forecasts better than VAR model as usual.

[6] The lack of data for the period between January 2000 and June 2000 has been faced by the computation of six forecasts for the inflation rate of each country by means of ARIMA class models.

*Basic idea and approach*

The idea here is to analyse cointegration in a Gaussian vector autoregressive model with a broken linear trend with *known* break points.
A comparison between stationary and non-stationary with broken deterministic trend is given in the following figures.

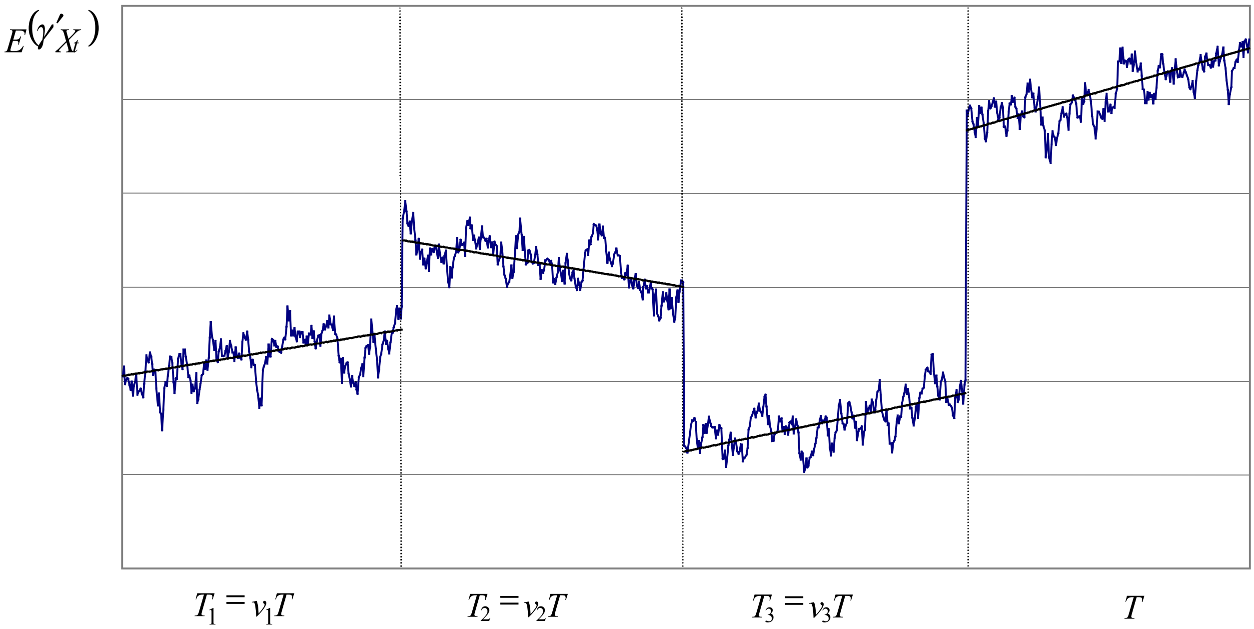


**Fig. 1** – *Stationary process with broken deterministic trend*

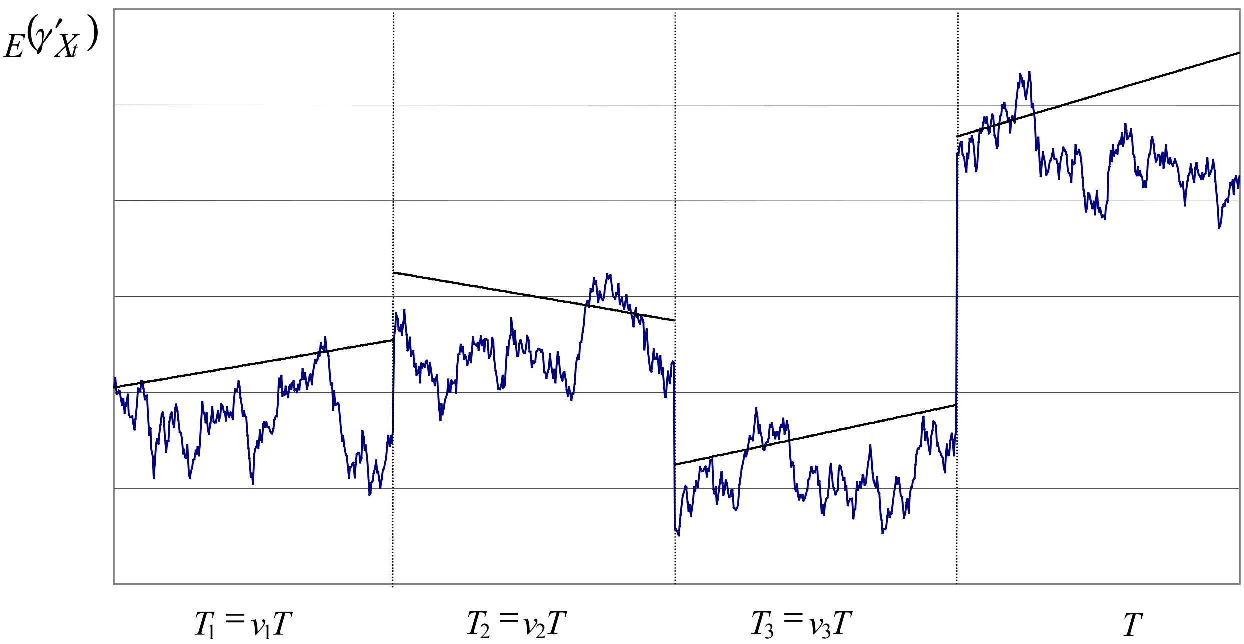


**Fig. 2** – *Non-stationary process with broken deterministic trend*

where $\mathbf{X}_t$ is a $p$-dimensional vector, $\boldsymbol{\gamma}'\mathbf{X}_t$ is a linear combination of the $p$-dimensional vector and $\nu_j = T_j/T$ are the relative break points such that $0 = \nu_0 < \nu_1 < \cdots < \nu_h = 1$.
The Fig. 1 represents a stationary process with broken deterministic trend. As we can see, the process appears non-stationary over the whole sample period, nevertheless it looks stationary around the trend line. In Fig.2 it is drawn a stochastic process which is generated by simulating a random walk process around a broken *true* trend line. It can be noticed the systematic deviation of the process in the sub-periods from their trend line. Both Fig. 1 and Fig. 2 illuminate the possible misinterpretation of the behaviour of stochastic processes in presence of a broken trend. In fact, if none of the true trend lines had been drawn on the figures, the stationary or non-stationary behaviour then would have been uneasily distinguishable looking at the graphs.
The cointegration in the presence of structural breaks is a slight generalisation of the likelihood-based cointegration analysis in vector autoregressive models suggested by Johansen (1988, 1996) and it is based on a very recent work of Johansen, Mosconi and Nielsen (2000)
There are only few conceptual differences and the major issue for the practitioner is that new asymptotic tables are needed.

### 4.2.2. The VECM model with structural breaks

Let $\mathbf{X}_t$, $t = 1,...,T$ the observed time series and divided the sample period $T$ into sub-samples according to the position of $h$ pre-specified break points and denote the lengths of this sub-samples by $(T_0, \cdots, T_1)$, $(T_1+1, \cdots, T_2)$, $\cdots$, $(T_{h-1}+1, \cdots, T_h)$, with $T_0 = 1$ and $T_h = T$.
We assume that $\mathbf{X}_t$ is a Gaussian VAR of order $k$ in each sub-sample with the same parameters with the exception of the constant and the trend, that is the deterministic (non-stochastic) components of the multivariate process. Therefore the model can be written:

$$(7) \qquad \Delta\mathbf{X}_t = \begin{pmatrix}\mathbf{\Pi} & \boldsymbol{\pi}_j\end{pmatrix}\begin{pmatrix}\mathbf{X}_{t-1} \\ t\end{pmatrix} + \boldsymbol{\mu}_j + \sum_{i=1}^{k-1}\mathbf{\Gamma}_i\Delta\mathbf{X}_{t-i} + \boldsymbol{\varepsilon}_t \qquad \begin{cases} j = 1, \cdots, h \\ T_{j-1} + k < t < T_j \end{cases}$$

where $\mathbf{\Pi}$, $\boldsymbol{\pi}_j$, $\boldsymbol{\mu}_j$ and $\mathbf{\Gamma}_j$ are $p$-dimensional matrices or vectors.

We consider the hypothesis in which

$$(8) \quad rank\begin{pmatrix}\Pi & \pi_1 & \ldots & \pi_h\end{pmatrix} \leq r \quad \text{or} \quad \begin{pmatrix}\Pi & \pi_1 & \ldots & \pi_h\end{pmatrix} = \boldsymbol{\alpha}\begin{pmatrix}\boldsymbol{\beta} \\ \boldsymbol{\gamma}_1 \\ \vdots \\ \boldsymbol{\gamma}_h\end{pmatrix}'$$

where the parameters vary freely so that $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are of dimension $(p \times r)$ and $\boldsymbol{\gamma}_j$ is of dimension $(1 \times r)$. This hypothesis indicates that in each sub-sample the deterministic component is linear both for non-stationary and cointegrating relations.

Other hypotheses can be made, e.g. of no linear trend but a broken constant level or more generally that common trends have a broken linear trend while the cointegrating relation has a broken constant level.
These last hypotheses are less attractive mainly for the reason that the asymptotic analysis is heavily burdened with nuisance parameters as demonstrated by Nielsen and Rahbek (2000).
The model (7) involves $h$ model equations and under the hypothesis (8) can be re-written in more compact way for $t=k+1, \cdots, T$ as:

$$(9) \qquad \Delta \mathbf{X}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}' \quad \boldsymbol{\gamma}')\begin{pmatrix} \mathbf{X}_{t-1} \\ t\mathbf{E}_t \end{pmatrix} + \boldsymbol{\mu}\mathbf{E}_t + \sum_{i=1}^{k-1} \boldsymbol{\Gamma}\Delta\mathbf{X}_{t-i} + \sum_{i=1}^{k}\sum_{j=2}^{q} \boldsymbol{\kappa}_{j,i} D_{j,t-i} + \boldsymbol{\varepsilon}_t$$

with

$$D_{j,t} = \begin{cases} 1 & \text{for } t = T_{j-1} \\ 0 & \text{otherwise} \end{cases} \quad j = 1,\ldots,h$$

which is an indicator function for the *i-th* observation in the *j-th* period
and $\boldsymbol{E}_t$ is a $h$-dimensional vector whose $j$-elements is:

$$E_{j,t} = \sum_{i=k+1}^{T_j - T_{j-1}} D_{j,t-i} = \begin{cases} 1 & \text{for } T_{j-1} + k + 1 \le t \le T_j, \\ 0 & \text{otherwise} \end{cases} \quad j = 1,\ldots,h$$

The dummy parameters $\kappa_{j,i}$, are $p$-vectors and the observations $\mathbf{X}_1, \ldots, \mathbf{X}_k$ are held fixed as initial observations. Note that the dummy variables $D_{j,t}, \ldots, D_{j,t-k}$ correspond to the observations $X_{Tj+1}, \ldots, X_{Tj+k}$ which are held fixed above and have the role to exclude such observations from the analysis.

### 4.2.3. Test for rank

The cointegration rank can be tested by modifying the procedures suggested by Johansen (1996).
The statistical analysis is unchanged and Johansen, Mosconi and Nielsen (2000) show that the asymptotic results are related but different. The trace-test is defined by:

$$(10) \qquad LR(r/p) = -T \sum_{i=r+1}^{p} \log\left(1 - \hat{\lambda}_i\right)$$

where $\lambda_i$, $i = 1, \cdots, p$ is the squared sample canonical correlations of the appropriate regression residuals.
The asymptotic distribution of the trace-test is a function of Brownian motions which has some important features:

a) it only depends on the $(p\text{-}r)$ dimensions, i.e. the number of non-stationary relations and (depends) on $\Delta\nu_i = \nu_i - \nu_{i-1}$ of the relative break points $\nu_j = T_j/T$ , but does not depend on the parameters of the model (9) (asymptotic similarity of the test). This is important because it means that the asymptotic distribution of the test is not affected by nuisance parameters;

b) it depends on the relative length of the sample periods $\Delta\nu_i$, not on their ordering. For instance, in case of one break point, the asymptotic distribution is the same if $T_1 = T/4$ as if $T_1 = 3T/4$ value;

c) denoting the asymptotic distribution by $DF_h(p\text{-}r, \Delta\nu_1, \cdots, \Delta\nu_h)$, then:

$$\lim_{\Delta\nu_j \to 0} DF_{h+1}(p-r,\ \Delta\nu_1, \cdots, \Delta\nu_{h+1}) =$$
$$= DF_h(p-r,\ \Delta\nu_1, \cdots, \Delta\nu_{j-1},\ \Delta\nu_{j+1}, \cdots, \Delta\nu_{h+1}) + \chi^2_{p\text{-}r}$$

where the $DF_h$ and the $\chi^2$ distributions are independent.
The additional $\chi^2$ term arises because the dimension of the vector $(\mathbf{X}'_{t-1},\ t\mathbf{E}'_t)$ is preserved although the dimension of the relative sample length vanishes, and hence the dimension of the restrictions imposed by the rank hypothesis is unaltered.
On the other hand, if the dummies with the vanishing sample length are taken out of the statistical analysis, the additional $\chi^2$-distributed element disappears.
Exact analytic expressions for the asymptotic distributions are not known and the quantiles have to be determined by simulation.
In order to avoid the simulations for any possible set $(\nu_1, \cdots, \nu_h)$, the moments of these distributions have been approximated $\Gamma$ distributions. In fact, it can be shown that the right tails of $DF_h(p\text{-}r, \Delta\nu_1, \cdots, \Delta\nu_h)$ and $\Gamma$ distribution are almost identical. The latter have parameters given by the first two moments and it suffices to report adequate approximations to the asymptotic mean and variance by a large number of simulations.
For example, if $h \le 3$ and $(p\text{-}r) \le 10$, then:

mean = $\exp\{3.06 + 0.456n + 1.47a + 0.993b - 0.0269n^2 - 0.0363na - 0.0195nb - 4.21a^2 - 2.35b^2 + 0.00084n^2 + 6.01a^3 - 1.33a^2b + 2.04b^3 - 2.05n^{-1} - 0.304an^{-1} + 1.06bn^{-1} + 9.35a^2n^{-1} + 3.82abn^{-1} + 2.12b^2\ n^{-1} - 22.8a\ 3\ n^{-1} - 7.15ab^2\ n^{-1} - 4.95b^3n^{-1} + 0.681n^{-2} - 0.828bn^{-2} - 5.53a^2\ n^{-2} + 13.1a\ 3\ n^{-2} + 1.5b^3n^{-2}\} - (2-h)n$

variance = $\exp\{3.97 + 0.314n + 1.79a + 0.256b - 0.00898n^2 - 0.0688na - 4.08a^2 + 4.75a^3 + 2.04b^3 - 2.47n^{-1} + 1.62an^{-1} + 3.13bn^{-1} - 4.52a^2n^{-1} - 1.21abn^{-1} - 5.87b^2n^{-1} + 4.89b\ 3\ n^{-1} + 0.874n^{-2} - 0.865bn^{-2}\} - 2(2-h)n$

where $a = min(\nu_1 - 0,\ \nu_2 - \nu_1,\ 1 - \nu_2)$ and $b$ is the second minimum of the three lengths.

## 4.3 Empirical Results of Area-Wide Model

### 4.3.1. Cointegration analysis with structural breaks

The analysis has been performed by using MALCOLM 2.4 (Mosconi, 1998).
We consider the vector

$$\mathbf{X}_t = [Q_t, RRL_t, Y_t, LTNT_t, FBAL_t, LTOT_t, ROIL_t, MG_t],$$

where the variables are defined as before (Table A1).
Notice that the information set for the area-wide model does not include the net foreign asset (NFA) variable. This exclusion has two main reasons. First, aggregate data for the net foreign assets of the Euro area do not exist and they are not correctly aggregable using the individual European country variables if the information on bilateral net foreign assets among them are not available. Second, as already mentioned, both NFA and FBAL could interpret the effects of fiscal policy with an antithetic behaviour. Therefore, we can suppose that the information bearing on the fiscal policy, held in NFA, could be partially captured by FBAL variable. Once more, in our opinion, FBAL is more representative of the fiscal policy interventions and it allows us to consider the country risk.
As mentioned in the previous section we use monthly data, ranging from January 1990 to December 1999 ($T$ = 120). We keep the last 12 months (from January 1999 to December 1999) as the forecasting period.
The time series seem to be trending, therefore we use the model that considers a linear trend both for the I(1) and I(0) components.
We introduce a break in coincidence with the crisis of the European Monetary System (September 1992) so that $T_1$ = 33 and $v_1$ = 0.28. This choice seems to be strengthened by an evaluation of Graphs A1 to A4. Trends in the first period are not the same as in the second. The different behaviour appears more relevant for the short-term real interest differential (Graph A4) in which the downward peak looks like a good indicator of the break point.
The first step of our analysis consists in the estimation of a $p$-dimensional Vector AutoRegressive model, where $p$ = 8.
In order to overcome the residuals cross-correlation, given the high VAR dimension, we choose $k$ = 3, the maximum number of lags considered.
We report in Table A2 the results of the Jarque-Bera normality test of the VAR model's residuals. The normality hypothesis at system level is not accepted because of some problems with the skewness in the $LTOT_t$ equation and kurtosis in the $Y_t$, $LTNT_t$ and $MG_t$ equations.
With regards to the kurtosis, we can note that all the residuals based on misspecification tests should be modified to take into account the fact that the first $k$ residuals of each period are set to zero by the presence of dummies $D_{j,t}$.
The trace-test (Table A3) shows that there is evidence of four cointegrating vectors. However, we have considered only three cointegration relationships because the failure of the residual's normality test can affect the meaning of trace-test results. Furthermore, our goal is to verify if the long-run relationships suggested by our theoretical framework, enable us to obtain reasonable and reliable forecasts of the Dollar/Euro real

exchange rate. However, the presence of four cointegrating vectors could be a further chance to improve the forecast capability of our model in future researches.

### 4.3.2 Long-run Dynamics

Following the proposed theoretical model, we suggest these long-run relationships:

(11)

$$
\begin{aligned}
Q_t &= -\beta_{12} RRL_t - \beta_{14} LTNT_t - \beta_{15} FBAL_t \pm \beta_{16} LTOT_t \pm \beta_{17} ROIL_t + ecm_{1t} \\
RRS_t &= +\beta_{23} Y_t - \beta_{29} MG_t + ecm_{2t} \\
Y_t &= -\beta_{32} RRL_t + \beta_{34} LTNT_t - \beta_{35} FBAL_t \pm \beta_{36} LTOT_t \pm \beta_{37} ROIL_t + ecm_{3t},
\end{aligned}
$$

where:

- $\beta_{ij}$ is the $j$-th coefficient of the $i$-th column of the cointegration matrix $\boldsymbol{\beta}$, supposing $\beta_{ij}>0$, $\forall i$, $i = 1,\ldots,r$ and $j = 1,\ldots,p$;
- $ecm_{it}$ is the stationary error correction component of the $i$-th equation.

We have verified that the constraints suggested by the economic model (11) are able to identify the cointegration space, in other words that the three linear combinations of the variables proposed by the system (11) are linearly independent.
Tables A4 to A6 present the coefficients of the estimated cointegration relationships.
All the coefficients of the variables are significantly different from zero. Furthermore, the presence of different deterministic trends in the two sub-samples is accepted for all the equations. Only in the cointegrating relationship for the real growth rate differential ($Y_t$) the coefficients of both the deterministic trends seems to be quite similar. Graphs A5 to A7 show the error correction components (*ecm*) of the system (11): we can see that, especially in the second sub-sample, these residuals appear stationary around the deterministic trend.
The null hypothesis of the over-identification test[7], which places the proposed restrictions on the $\boldsymbol{\beta}$ matrix, is not rejected at the 95% confidence level (p-values = 0.18961).
Hence, from the cointegration analysis we have obtained the long-run relationships that can be used both for modelling the dynamic equations via the Error Correction Mechanism and for interpreting the expected interactions suggested by the economic theory.
In the equation of the real exchange rate, all the signs for the coefficients are in accordance with our expectations, but the positive sign for *ROIL* is not consistent with the positive sign in the long-run equation of the GDP's growth rates differential. We can justify the second as a consequence of the stronger dependence to the oil prices of the Euro-Area rather than the United States.
In the real interest rates differential equation, the coefficient for *MG* is negative and for *Y* is positive in accordance with the economic theory.

[7] The mentioned test verifies if the restricted VAR model can be accepted with respect to the unrestricted model on the basis of the comparison between the maximum likelihood functions.

Finally, in the third equation, the coefficients for all variables except the long-term real interest rate differential (*RRL)* have the expected sign.
This last unsatisfactory sign of the real interest rates differential could be justified as follows: as the first long-run equation suggests, an increase of the differential between real interest rates allows for an appreciation of the US dollar. Consequently, if the appreciation is more relevant for the commodity import than goods import, then this increase could be interpreted as a signal of the expansion of real output (an increase of $Y_t$).

### 4.3.3 Short-term Dynamics

In the previous section we have determined the long-run patterns of the Dollar-Euro exchange rate, the differential between the US and European 10-year real interest rates and the differential in the GDP growth rates. Therefore, we can analyse the short-term dynamics for these variables. To do that we estimate, over the period from January 1990 to December 1998, a conditional Vector Error Correction Model in the form:

$$\mathbf{\Delta X}_t = \boldsymbol{\alpha}\,\mathbf{ecm}_{t-1} + \boldsymbol{\mu}\,\mathbf{E}_t + \sum_{i=1}^{k-1}\mathbf{\Gamma}_i\mathbf{\Delta X}_{t-i} + \sum_{i=1}^{k}\sum_{j=2}^{h}\boldsymbol{\kappa}_{j,i}\mathbf{D}_{j,t-i} + \boldsymbol{\varepsilon}_t$$

where

$$\mathbf{ecm}_{t-1} = \begin{pmatrix}\boldsymbol{\beta}' & \boldsymbol{\gamma}'\end{pmatrix}\begin{pmatrix}\mathbf{X}_{t-1} \\ t\mathbf{E}_t\end{pmatrix},$$

where $h = 2$ indicates the number of sub-samples and, coherently with the specification of the VAR model, $k$, the number of maximum lags, is equal to three.
$E_t$ and $D_{j,t}$, are the dummy variables described in the section 4.2.2.
The estimated short-run equations are presented in Tables A7 to A9 where the first differences of the endogenous variables $Q_t$, $RRL_t$ and $Y_t$ are considered (*DQ*, *DRRL* and *DY*). The predetermined variables are: the dummy variables (*E2*, the first, second and third lags of *D2*); the first lag of endogenous variables; lags from zero to two of exogenous variables' first differences (*DLTNT*, *DLTOT*, *DROIL*, *DFBAL*, *DMG* and *DNFA*); the first lag of the stationary components derived from the cointegration analysis (*ECM(Q)*, *ECM(RRL)*, *ECM(Y)*).
The dynamic behaviour of fitted and observed first differences of the endogenous variables are reported in Graphs A8 to A10.
The estimated model presents satisfactory properties from a statistical point of view. Table A10 shows that the normality hypothesis of model residuals can not be rejected for all the equations and for the entire system. Furthermore, there is no evidence of autoregressive conditional heteroschedasticity (ARCH) effects, and statistically and economically significant autocorrelation structures, according to the standard Bartlett's bands test with the exception of the real growth differential changes (Graphs A11 to A13). In this last case the Graphs exhibit a slight indication of seasonal component in the residuals. The seasonality effect could be removed by using seasonal deterministic

components in the model. We do not follow this procedure for two reasons. First, all the variables in the model are seasonally adjusted in the usual way, then the remaining seasonality effect would be removed changing the seasonally adjustment techniques. Second, the use of the seasonal deterministic components in the model change all the cointegration test statistics and their computation is not so easy in the presence of structural breaks.
Finally, the model's residuals are not cross-correlated (Table A11).

### 4.3.4. Forecasting Performance

As we said, the model was estimated over the sample period from January 1990 to December 1998 and then the parameters are maintained fixed to produce out-of-sample forecasts for the period from January to December 1999. The out-of-sample forecasts are constructed using the actual observations of the exogenous variables to compute our one-step ahead forecasts.
We present the results obtained for monthly percentage changes of the Dollar/Euro real exchange rate in Table A12 and in Graph A14, while in Table A13 and in Graph A15 the results for the level variable.
The forecasting performances of the Dollar/Euro exchange rate are reasonably satisfactory: the monthly changes forecasts showed in Table A12 have the indicator of correct signs at the percentage of 50 % and the Theil's U statistic of 0.4839 reveals the good prevailing of econometric model forecasts with respect to the random walk forecasts. The monthly levels slightly worsen the Theil's U statistic, which rises up to 0.6105.
The graphs are drawn without the forecast intervals for a matter of better readability. However, the computations of test statistics lead to the acceptance of the hypothesis that none of the forecast values are significantly different from the actual values, at the usual significance levels.

## 5. The Multi-Country Model

### 5.1 Motivations for a Disaggregated Approach to Euro Modelling

The fundamental reasons of a disaggregated approach to model the Euro lie in the fact that, despite the creation of the European Monetary Union on January, 1st 1999, throughout the 1990s and at present (last year and the early months of 2000) fiscal and economic policies in the eleven countries members of the EMU have sometimes been substantially different. This fact has obviously largely influenced, still influences, and will probably continue to influence the behaviour of the countries macroeconomic and financial fundamentals (e.g. different growth rates, different shapes of the term structure of interest rates, different levels and growth rates of fiscal and commercial imbalances, different impacts from the increases in commodity prices, different inflation rates, etc.).
Thus, on the one hand we observe substantial differences in the economic structure of the eleven countries belonging to the EMU Area, and on the other, all these differences are somewhat "restricted" to respect a common trend in the monetary interest rate and currency market. Further restrictions induced by the aggregate approach can create distortion in policy making. For example, it may happen that a policy maker who deliberately chooses to restrict his information set to area-wide aggregated data, overlooking the information conveyed by national variables, will achieve sub-optimal results [Angelini, Del Giovane, Siviero and Terlizzese (2000)].
As a matter of fact, from an econometric point of view, it is particularly interesting to study the relative importance the different countries have in determining the level of the new European currency versus the US Dollar (by definition the "World currency") and of the money markets' (short-term) interest rates.
We dealt with this issue by making reference to the theoretical model outlined in the previous sections and extended it to take into account the above-mentioned differences among countries belonging to the Euro Area. Because of the lack of data regarding some of the economic fundamentals and financial indicators considered in this paper, we restricted our analysis to three countries: Germany, France (representing Core Europe) and Italy, whose credit spreads vis-à-vis the United States and the Core Europe countries should be able to explain (in our opinion) some sudden fluctuations of both the European interest rates and the Euro currency.

### 5.2 The Econometric Approach[8]

The theoretical partial economic equilibrium model, whose properties and fundamentals have been outlined in sections 1 and 2, has been specified to take into account the increased amount of both endogenous and exogenous variables, and information about the presence of unit roots in the autoregressive representations of the time series. It took (as to the long-run equilibrium) the following econometric structure:

[8] All the tables and graphs regarding estimation, tests and forecasts from system (12) are included in appendix B.

(12)

$$
\begin{aligned}
Q_t \;=\;& \beta_0 + \beta_1\, LTNT\_USBD_t + \beta_2\, LTNT\_USFR_t + \beta_3\, LTNT\_USIT_t + \beta_4\, FBAL\_USBD_t + \\
& \beta_5\, FBAL\_USFR_t + \beta_6\, FBAL\_USIT_t + \beta_7\, NFA\_USBD_t + \beta_8\, NFA\_USFR_t + \\
& \beta_9\, NFA\_USIT_t + \beta_{10}\, LTOT\_USBD_t + \beta_{11}\, LTOT\_USFR_t + \beta_{12}\, LTOT\_USIT_t + \\
& \beta_{13}\, ROIL_t + \beta_{14}\, RRL_t + ecm_{1,t}
\end{aligned}
$$

$$
\begin{aligned}
RRL_t \;=\;& \delta_0 + \delta_1\, MG\_USBD_t + \delta_2\, MG\_USFR_t + \delta_3\, MG\_USIT_t + \delta_4\, Y\_USBD_t + \delta_5\, Y\_USFR_t + \\
& \delta_6\, Y\_USIT_t + ecm_{2,t}
\end{aligned}
$$

$$
\begin{aligned}
Y\_USBD_t \;=\;& \gamma_0 + \gamma_1\, NFA\_USBD_t + \gamma_2\, FBAL\_USBD_t + \gamma_3\, LTOT\_USBD_t + \gamma_4\, LTNT\_USBD_t + \\
& \gamma_5\, ROIL_t + \gamma_6\, RRL_t + ecm_{3,t}
\end{aligned}
$$

$$
\begin{aligned}
Y\_USFR_t \;=\;& \phi_0 + \phi_1\, NFA\_USFR_t + \phi_2\, FBAL\_USFR_t + \phi_3\, LTOT\_USFR_t + \phi_4\, LTNT\_USFR_t + \\
& \phi_5\, ROIL_t + \phi_6\, RRL_t + ecm_{4,t}
\end{aligned}
$$

$$
\begin{aligned}
Y\_USIT_t \;=\;& \theta_0 + \theta_1\, NFA\_USIT_t + \theta_2\, FBAL\_USIT_t + \theta_3\, LTOT\_USIT_t + \theta_4\, LTNT\_USIT_t + \\
& \theta_5\, ROIL_t + \theta_6\, RRL_t + ecm_{5,t}
\end{aligned}
$$

where $ecm_t = (ecm_{1,t}, ecm_{2,t}, ecm_{3,t}, ecm_{4,t}, ecm_{5,t})'$, as in the case of the area-wide model, is the vector stationary error correction component of the system.
The suffixes *_USBD*, *_USFR* and *_USIT* in the different tables indicate, respectively, the differential between US and Germany, US and France, US and Italy (in the case of *NFA* they indicate the ratio of the US *NFA* variable to that of the corresponding European country).
The econometric analysis of system (12) has been undertaken according to the framework of analysis outlined by Johansen's consolidated works (Johansen, 1996).
The first step of our analysis has been the estimation of the vector autoregression including all the 21 variables considered in our model and constant term. Furthermore, we carry out the evaluation of the number of cointegration relations among the set of variables by means of the usual tests on the rank of the matrix $\Pi_{t-k}$. The high number of variables forced us to undertake this step considering just one, two and three lags for the above-mentioned vector autoregression.
Results reported in Table B1 show that according to the test results there is evidence of six cointegrating vectors for lag 3.
This result is much more than satisfactory from our point of view, that is, the number of cointegrating vectors is higher than the number of relationships in system (12) and therefore we can conclude that our hypothesis of a long-run dynamic equilibrium among both the endogenous and exogenous variables considered in our model is statistically

accepted. Thus, our hypothesis concerning the impact of regional (or, if the reader prefers, country) data on both the Dollar/Euro dynamics and the long-term real interest rates dynamics is also accepted.
Furthermore, the evidence of six cointegrating vectors would give us, to some extent, a further degree of freedom, intended as the chance to exploit a further long-run dynamic relationship among variables to increase the ability of our model to explain past dynamics and forecast future ones. This chance has not been exploited at this stage, since we wanted to test the capability of our theoretical model to supply us with reasonable and reliable forecasts concerning the Dollar/Euro exchange rate basically linking it to interest rates and gross domestic product growth dynamics. A further step to complete our model, for example, could be that of rendering endogenous the dynamic pattern of a world commodity market variable, such as the real price of oil (used as a proxy of the world demand for energy goods), which in fact is statistically significant in two out of three cointegration regressions concerning the *GDP* growth differentials.

### 5.3 Long-run Dynamics

As previously stated, the system was estimated over the sample period from January 1990 to December 1998 and then tested over the remaining twelve months (from January to December 1999).
The results, reported in Tables B2 to B6, are in our opinion quite satisfactory, both in terms of signs of the coefficients and in terms of the ability of the estimated equations to explain the equilibrium dynamics of the exchange and interest rates.
As to the analysis of the exchange rate equation (Table B2 and Graph B1), we highlight its ability to explain more than 80% of the variability of the Dollar/Euro real exchange rate, and the strong role played by the financial and macroeconomic fundamentals of the three European countries considered in our sample: Germany, Italy and France.
Economic theory tells us that the overall coefficient of fiscal balances should be positive, and this is the case also for our analysis (in fact, roughly, the sum of the three coefficients is positive), but we also observe the strange coefficient regarding the differential between the US and Italian fiscal balances. In our opinion that is because US fiscal imbalances are somehow perceived by financial markets as more growth-aimed than Italian ones, which are considered more debt-servicing driven.
The flows-of-funds effect enters our model in two different forms: the first is the negative coefficient of the real interest rate differential, perfectly consistent with economic theory, and the second is the overall positive coefficient on net foreign assets, which tells us that an increase in net overseas assets detained by residents usually induces a currency depreciation, since these financial resources are not used to invest in the domestic economy and therefore increase one country's aggregate level of both effective and potential wealth. The negative coefficient of the US-Germany ratio is explained by the fact that the time series of German net foreign assets is constantly positive, while the correspondent time series for the United States is constantly negative.
The motivations for a disaggregated approach to model the US Dollar/Euro real exchange rate find empirical confirmation if we consider the real long-term interest rates differential (Table B3 and Graph B2).

In this case, a common, indirect policy variable (*RRL*) is explained by different monetary and fiscal/growth policies undertaken by the different European countries. Not unexpectedly (at least to some extent), the Core Europe's monetary and fiscal policy plays a central role in the determination of the credit spread of Euro denominated investments vis-à-vis those in US dollars. By looking more closely at the coefficients, we confirm our preliminary impression of the existence of two different European economies: the Core Europe economy, whose monetary and fiscal policies are coincident in sign (even if not in magnitude) with those of the United States, and the Italian economy, which is characterised by periods of massive injections of liquidity in the system.

As to the equations regarding the output differentials, we intend to stress the satisfactory results obtained in terms of coefficients' signs.

In the case of US-Germany real growth differential (Table B4 and Graph B3), as expected for the reasons outlined commenting the equation of the real exchange rate, an increase in net foreign assets ratio induces a decrease in output differential. The same decreasing effect is produced by an increase in the tradable to non tradable prices ratio, via foreign trade.

The disappointing sign regarding the coefficient of real oil price, instead, may be explained by noting that, in our sample, the highest prices of oil were observed in correspondence of both the latest economic recession in the United States and the beginning of restructuring processes in Germany's eastern Länder. These joint events forced the differential between the United States and Germany to lie in negative territory for two years in the period between January 1990 and January 1992.

In the case of the US-France differential (Table B5 and Graph B4) the negative coefficient of the fiscal balance differential may be explained if we consider the fact that US growth has become more and more "private sector driven" and that the federal government in the last years has undertaken a substantial debt reduction policy aimed at a "zero debt goal" within the next twenty years. The other signs, however, are perfectly in line with what is suggested by economic theory.

Finally, in the case of the US-Italy gross domestic product growth differential (Table B6 and Graph B5), the reason for a positive coefficient for the real interest rates differential lies (in our opinion) in the fact that from the mid 1990s Italy has experienced the convergence (apart from the usual credit spread versus Germany and France) of both short- and long-term interest rates to the Core European level and that, in spite of this fact, the ability to produce an acceleration in the rate of growth has remained at the planning stage.

As the reader will surely note, the signs of the real interest rates coefficients in all the equations concerning the growth differentials are positive.

This fact seems to be inconsistent with the generally accepted economic theory, postulating a negative impact of the interest rates on output. However, the evidence that the transmission of the monetary policy to the real sector occurs usually with a time lag, whose length depends on the structure of the economic system, is now widely accepted (according to several academic and central bank studies)

In our model, instead, the relationship between real interest rates and output is contemporaneous, and by-passed through the equations concerning the money market and the forex market.

As outlined in the section dedicated to the comment of the results obtained for the area-wide model, our first dynamic equilibrium equation suggests that an increase in the real

interest rate differential allows the US dollar to appreciate. If this happens, and if the above-mentioned appreciation is more relevant for the commodity import sector than for the goods import sector, then the appreciation of the US dollar versus the Euro by means of an increase in the interest rate differential might be interpreted as the signal of the contemporaneous expansion of the real output (an increase of *Y*).
Considered overall, the dynamic equilibrium equations are satisfactory even from the statistical point of view. Their $R^2$ coefficients, in fact, are usually well above 60% (almost 80% for the real exchange rate, almost 73% for the US-Germany and 80% for the US-France growth differentials, almost 60% for the real interest rate differential, while only almost 50% of the US-Italy growth differential's variability is explained by our corresponding equilibrium equation), while residuals show some degree of correlation, especially as far as the growth differentials equations are concerned, as the reader can see by looking at Table B7.

### 5.4 Short-run Dynamics

Having analysed in the last section the equilibrium relationships among variables, we now move to the analysis of the short-term dynamics jointly determining the behaviour of the real exchange rate, the real interest rate differential and the growth differentials (Tables B8 to B12 and Graphs B6 to B10).
To do that, over the period from January 1990 to December 1998 (leaving the last 12 observations in our sample to test the forecasting ability of the model) we estimated a Vector Error Correction Model (VECM) involving the first simple differences (monthly percentage changes) of the endogenous variables considered in our analytic framework (noted as *DQ*, *DRRL* and *DY*)[9], the first simple differences of the exogenous variables of our system (noted as *DLTNT*, *DLTOT*, *DROIL*, *DFBAL*, *DMG* and *DNFA*) and, finally, the first lag of the residuals from the long-run equilibrium equations (noted as *ECM(Q)*, *ECM(DRRL)* and *ECM(Y)*). The lag for the system estimation was set equal to one.
In the case of the real exchange rate dynamics (see Table B8 and Graph B6), once again the attention must be drawn to the different impact on the real exchange rate of the disequilibrium in US-Germany and US-Italy growth differentials, which can be interpreted as a further proof of the different contribution of the sub-areas not only to the behavioural (long-run) equilibrium of the US Dollar/Euro exchange, but also to the short-term (perhaps also speculative) dynamics.
The estimated model presents very satisfactory properties under the statistical point of view. Analysis of Table B13 shows that the model isolates non cross-correlated residuals, with the exception of the US-Germany and US-Italy growth differentials. Analysis of Table B14 together with Graphs B11 to B15 shows that residuals are to be considered as normally distributed, and in four cases out of five without autoregressive conditional heteroschedasticity (ARCH) effects and without statistically and economically significant autocorrelation structures according to the standard Bartlett's bands test.
The only exceptions to the optimality are the residuals coming from the equation of the real exchange rate, which seem to confirm the time varying volatility usually observed when referring to financial variables such as interest and/or exchange rates. In a further step of this work, in fact, it could be useful to correct the behaviour of the exchange rate

[9] Suffixes have the same meaning as for the long-run equilibrium equations (see section 5.2).

by means of a conditional variance model, probably obtaining even more precise forecasts.

## 5.5 Forecasting Performance

When talking about forecasts, it seems to be obvious to some extent to evaluate a model's forecasting performance in comparison to some other model. Ever since the well-known seminal paper by Meese and Rogoff (1983) containing their famous criticism of structural econometric models, the benchmark by which a fundamental-based econometric exchange rate model is assessed is through comparison to the simple random walk, that is the most simple way to explain asset prices' behaviour.

As other authors have done, and indeed continue to do, we also accept the challenge to compare the forecasts supplied by our model to those supplied by the random walk.

The model was therefore, as mentioned above, estimated over the sample period from January 1990 to December 1998 and then the parameters maintained fixed to produce out-of-sample forecasts for the period from January to December 1999. The out-of-sample forecasts are constructed as "perfect foresight forecasts", in the sense that we use the actual observations of the exogenous variables to compute our one-step ahead forecast.

The results obtained for monthly percentage changes and levels of the real Dollar/Euro exchange rate, both from a graphical point of view (Graphs B16 and B17) and a numerical one (Tables B15 and B16), seem to lean particularly toward the use, in a forecasting optic, of the model described in past sections.

Graphs B16 and B17 show how the multi-country model avoids the problem of "*structurally*" overestimating, or underestimating the Dollar/Euro exchange rate changes or levels, while Tables B15 and B16 prove that, on average, the *absolute forecasting error* of the model is well below 2% and that the same is true for the *root mean square error*.

Particularly interesting are also the indications coming from the *percentage of signs correctly forecasted*: in this case the model is able to capture the direction changes of the exchange rate induced by market activity in the majority of cases: ten out of twelve (83%). Directional forecasts are particularly satisfactory even for the other variables in our system: the model correctly forecasts the sign of nine out of twelve monthly changes in the case of the US-Germany growth differential (75%), eight in the case of the US-France growth differential (67%) and, surprisingly, eleven in the case of the US-Italy growth differential (92%), while it is slightly less precise in the case of the long-term interest rate differential for which only six monthly changes (50%) have been correctly predicted.

Finally, the Theil's U statistic shows us that, in both the cases, the estimated structural econometric model performs better than the random walk. The efficiency gain in the case of real exchange rate changes is around 63%, while it decreases to 54 % in the case of the exchange rate levels.

These results are extremely important in our opinion, since they prove that, contrarily to the original statement by Meese and Rogoff, there is room for further research aimed at explaining the dynamic behaviour of exchange rates by means of structural econometric models, which could give further ground for fundamentally or financially based analyses.

## 6. Conclusions

The main results of the paper are the specification and estimation of two econometric models on Dollar/Euro Real Exchange Rate in VECM form. The aim to produce reliable and reasonable forecasts is pursued by comparing an area-wide to a multi-country model. For both models, the main structural feature is given by making endogenous the long-term interest rate differential and the differential between US and Euro GDP annual growth rate in addition to the exchange rate. In this way all the three relevant markets, i.e. the foreign exchange market, the money market and the goods market are modelled jointly.

As far as the area-wide model is concerned, the admissibility of the VECM form is achieved by using cointegration analysis in the presence of structural breaks. We introduce a break inside the sample period (between January 1990 and December 1999) in coincidence with the crisis of the European Monetary System (September 1992). The presence of different deterministic trends in the two sub-sample periods is accepted and three long-run relationships are obtained. The estimates of the coefficients in the long-run relationships are consistent with the interactions suggested by economic theory.

The forecasting performances of Dollar/Euro exchange rate are to some extent satisfactory: the Theil's U statistic shows an efficiency gain in the forecasting performances with respect to the competing random walk model of 39% and of 50% for the correct signs.

The motivations for the multi-country approach rely on the fact that, despite the single European monetary policy, the fiscal and economic policies in the eleven member countries of the EMU may be substantially different. This aspect is reflected in the structure of the model by expanding the differential between the US and each European country GDP annual growth rate. Because of the lack of data concerning economic fundamentals, we restricted our analysis to three countries, Germany and France representing Core Europe, and Italy.

Due to the degrees of freedom insufficiency, the cointegration analysis among the disaggregated variables is carried out without the assumption of a structural break. The results obtained are consistent with the hypothesis of five long-run relationships, so that the impact of each European country on both the Dollar/Euro dynamics and the long-term real interest rates dynamics is accepted. In this case the signs of the coefficients and the ability of the estimated equations to explain the equilibrium dynamics of the exchange and interest rates are quite satisfactory. Also, the short-run dynamic model presents good properties from a statistical point of view. A comparison with the area-wide short-run dynamic model points out a light presence of ARCH effects in the residuals coming from the equation of the real exchange rate.

The main good result of the multi-country estimated model is indeed in terms of forecasting performance: the Theil's U statistic shows an efficiency gain in the forecasting performances with respect to the competing random walk model of 54% and of 83% for the correct signs.

If both the area-wide and the multi-country models perform well in this respect, the latter nevertheless exhibits a lesser superiority than the former. The comparison between the respective Theil's U statistic shows that the multi-country forecasts out-performs the area-wide ones by 25% and the percentage of correct signs is much higher. The Table C1 and the Graph C1 are a good synthesis of this comparison.

As a conclusive consideration, we can state that the wider information exploited by the multi-country model with respect to the area-wide model is effective in having better forecasts. In turn, the insertion of a structural break in the area-wide model substantially improves the diagnostic statistics and the forecasting performances with respect to the model without its presence.


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# *Appendix A*

***Results Regarding the Area-Wide Model***

## *Variables Description*

| **Variables** | **Description** |
|---|---|
| *Q* | Real Dollar/Euro exchange rate (logarithm) |
| *RRL* | Differential between US and Euro 10-years real interest rate |
| *Y* | Differential between US and Euro annual real GDP growth rates (logarithms) |
| *LTNT* | Differential between US and Euro ratio of consumer price index to producer price index (logarithms) |
| *FBAL* | Differential between US and Euro ratio of annual real public debt growth and the GPD growth |
| *NFA* | Ratio of US to Euro ratio of net foreign asset to annual real GDP |
| *LTOT* | Ratio of US to Euro ratio of export unit value to import unit value (logarithms) |
| *ROIL* | Real price of oil expressed in US Dollars per barrel |
| *MG* | Differential between US and Euro annual real M3 growth rates (logarithms) |

***Table A1***

## *Normality Test of VAR Model's Residuals*

| Equation | Skewness | Kurtosis | Sk. + Kur |
|---|---|---|---|
| *Q* | 0.4480 | 0.2910 | 0.4300 |
| *RRL* | 0.3350 | 0.3050 | 0.3710 |
| *Y* | 0.9320 | 0.0120 | 0.0420 |
| *LTNT* | 0.8750 | 0.0310 | 0.0970 |
| *FBAL* | 0.0570 | 0.2120 | 0.0750 |
| *LTOT* | 0.0010 | 0.3190 | 0.0030 |
| *ROIL* | 0.3260 | 0.4780 | 0.4800 |
| *MG* | 0.8770 | 0.0100 | 0.0370 |
| *System* | 0.0360 | 0.0260 | 0.0060 |

***Table A2:*** *Jarque - Bera Normality Test - p-values*

## *Cointegration Analysis*

Number of Lags Considered: 3

| Ho : rank = r | Test Statistic<br>- T $\Sigma$ log(.) | 95% | p-value |
|---|---|---|---|
| r = 0 | 303.9600 | 223.2700 | 0.0000 |
| r ≤ 1 | 233.5400 | 181.8600 | 0.0000 |
| r ≤ 2 | 172.5500 | 144.6100 | 0.0004 |
| r ≤ 3 | 119.5000 | 111.3900 | 0.0128 |
| r ≤ 4 | 75.4000 | 82.2000 | 0.1467 |
| r ≤ 5 | 45.7500 | 57.0600 | 0.3316 |
| r ≤ 6 | 23.7200 | 35.7500 | 0.5055 |
| r ≤ 7 | 6.8100 | 18.0800 | 0.7677 |

***Table A3:*** *Cointegration Analysis of System*

### *Equation 1 for Q*

| *Variable* | *Coefficient* | *Std. Error* | *t-value* |
|---|---|---|---|
| *RRL* | -4.7978 | 0.2989 | -16.0515 |
| *LTNT* | -2.1057 | 0.2281 | -9.2315 |
| *FBAL* | -2.0330 | 1.1169 | -1.8202 |
| *LTOT* | 0.6540 | 0.1178 | 5.5518 |
| *ROIL* | 0.1318 | 0.0367 | 3.5913 |
| *t*E1* | 0.0255 | 0.0039 | 6.5254 |
| *t*E2* | 0.0134 | 0.0014 | 9.6800 |

***Table A4***

### *Equation 2 for RRL*

| *Variable* | *Coefficient* | *Std. Error* | *t-value* |
|---|---|---|---|
| *Y* | 6.0033 | 0.5337 | 11.2485 |
| *MG* | -8.5467 | 0.5620 | -15.2077 |
| *t*E1* | 0.0169 | 0.0029 | 5.7944 |
| *t*E2* | 0.0174 | 0.0004 | 44.8581 |

***Table A5***

## *Equation 3 for Y*

| *Variable* | *Coefficient* | *Std. Error* | *t-value* |
|---|---|---|---|
| *RRL* | 4.1181 | 0.2546 | 16.1748 |
| *LTNT* | 1.5180 | 0.1942 | 7.8167 |
| *FBAL* | -1.4335 | 0.9608 | -1.4920 |
| *LTOT* | -0.4262 | 0.1037 | -4.1099 |
| *ROIL* | 0.1370 | 0.0319 | 4.2947 |
| *t*E1* | -0.0131 | 0.0030 | -4.3202 |
| *t*E2* | -0.0132 | 0.0012 | -11.3529 |

***Table A6***

## *Short-Run Dynamics*

### *Equation 1 for DQ*

| *Variable* | *Coefficient* | *Std.Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *DRRL_1* | -0.2312 | 0.0813 | -2.8460 | 0.0055 |
| *DLTNT* | -0.1213 | 0.0608 | -1.9950 | 0.0490 |
| *DLTNT_2* | -0.1106 | 0.0591 | -1.8710 | 0.0645 |
| *DLTOT_1* | -0.0480 | 0.0206 | -2.3290 | 0.0220 |
| *DLTOT_2* | -0.0414 | 0.0203 | -2.0420 | 0.0440 |
| *DMG* | -0.4110 | 0.0556 | -7.3930 | 0.0000 |
| *ECM(Q)_1* | -0.1102 | 0.0359 | -3.0660 | 0.0028 |
| *ECM(RRL)_1* | 0.0461 | 0.0152 | 3.0290 | 0.0032 |
| *ECM(Y)_1* | -0.1094 | 0.0377 | -2.9020 | 0.0046 |
| *E2* | 0.0347 | 0.0134 | 2.5810 | 0.0114 |
| *D2(t-1)* | -0.0610 | 0.0215 | -2.8380 | 0.0056 |
| *D2(t-2)* | 0.0175 | 0.0226 | 0.7730 | 0.4415 |
| *D2(t-3)* | 0.0198 | 0.0225 | 0.8800 | 0.3811 |
| *Constant* | -0.0546 | 0.0241 | -2.2690 | 0.0256 |

$\sigma$ =0.0200956

***Table A7***

### *Equation 2 for DRRL*

| *Variable* | *Coefficient* | *Std.Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *DQ_2* | -0.2062 | 0.0799 | -2.5800 | 0.0114 |
| *DRRL_2* | -0.2596 | 0.0845 | -3.0730 | 0.0028 |
| *DLTOT_1* | -0.0648 | 0.0223 | -2.9030 | 0.0046 |
| *DMG* | 0.1159 | 0.0541 | 2.1430 | 0.0347 |
| *ECM(Q)_1* | 0.0763 | 0.0360 | 2.1210 | 0.0366 |
| *ECM(RRL)_1* | 0.0290 | 0.0149 | 1.9460 | 0.0546 |
| *ECM(Y)_1* | 0.1340 | 0.0383 | 3.5010 | 0.0007 |
| *E2* | -0.0236 | 0.0133 | -1.7740 | 0.0794 |
| *D2(t-1)* | 0.1008 | 0.0224 | 4.4910 | 0.0000 |
| *D2(t-2)* | -0.0213 | 0.0237 | -0.8990 | 0.3708 |
| *D2(t-3)* | -0.0298 | 0.0245 | -1.2160 | 0.2269 |
| *Constant* | 0.0621 | 0.0243 | 2.5530 | 0.0123 |

$\sigma$ = 0.0207249

***Table A8***

***Equation 3 for DY***

| *Variable* | *Coefficient* | *Std.Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *DLTNT_1* | -0.0575 | 0.0162 | -3.5620 | 0.0006 |
| *DFBAL* | -0.2734 | 0.0760 | -3.5980 | 0.0005 |
| *DFBAL_1* | -0.1680 | 0.0752 | -2.2360 | 0.0278 |
| *DLTOT_2* | 0.0124 | 0.0052 | 2.3670 | 0.0200 |
| *DROIL* | 0.0156 | 0.0050 | 3.1050 | 0.0025 |
| *DMG* | 0.9985 | 0.0135 | 74.1630 | 0.0000 |
| *E2* | -0.0030 | 0.0012 | -2.5690 | 0.0118 |
| *D2(t-1)* | 0.0087 | 0.0056 | 1.5560 | 0.1232 |
| *D2(t-2)* | -0.0032 | 0.0057 | -0.5650 | 0.5736 |
| *D2(t-3)* | -0.0083 | 0.0058 | -1.4340 | 0.1549 |
| *Constant* | 0.0013 | 0.0010 | 1.3280 | 0.1873 |

$\sigma$ = 0.00533911

***Table A9***

# *Tests on Model's Residuals*

***Normality, Autocorrelation and ARCH effect Tests for Residuals***

| *Equation* | *Residuals - Normality Test* ($\chi^2$ *Test)* | *Residuals - ARCH Test* *(F-form Test)* | *Residuals - Autocorrelated* *(Bartlett's Test - Significant Lags)* |
|---|---|---|---|
| *DQ* | 0.64931 [0.7228] | 1.46560 [0.2299] | Lag 6 - Lag 12 |
| *DRRL* | 4.72790 [0.0940] | 0.20512 [0.6519] | Lag 20 |
| *DY* | 4.14850 [0.1257] | 0.41592 [0.5210] | Lag 12 - Lag 14 - Lag 17 - Lag 24 |

*Normality Test for the System* : $\chi^2(6)$ = 8.5581 [0.2000]

*Values in parenthesis indicate the significance level; * = significant at 5% level; ** = significant at 1% level.*

***Table A10***

***Correlation of Residuals***

| | ***DQ*** | ***DRRL*** | ***DY*** |
|---|---|---|---|
| ***DQ*** | 1.0000 | | |
| ***DRRL*** | -0.3164 | 1.0000 | |
| ***DY*** | -0.3438 | -0.0828 | 1.0000 |

***Table A11***

# *Forecasting Performances*

***Forecasting Performance (1) - Monthly Changes of Real Euro/Dollar Exchange Rate***

| *Date* | *Actual* | *Forecasted* | *Error* | *Square Error* | *Absolute Error* | *Actual Change Sign* | *Forecasted Change Sign* |
|---|---|---|---|---|---|---|---|
| *12/98* | 0.0162 | | | | | | |
| *01/99* | -0.0360 | -0.0149 | 0.0211 | 0.0004 | 0.0004 | (–) | (–) |
| *02/99* | -0.0320 | -0.0313 | 0.0007 | 0.0000 | 0.0000 | (–) | (–) |
| *03/99* | -0.0159 | 0.0078 | 0.0237 | 0.0006 | 0.0006 | (–) | (+) |
| *04/99* | -0.0236 | -0.0174 | 0.0062 | 0.0000 | 0.0000 | (–) | (–) |
| *05/99* | -0.0141 | 0.0052 | 0.0193 | 0.0004 | 0.0004 | (–) | (+) |
| *06/99* | -0.0104 | 0.0045 | 0.0150 | 0.0002 | 0.0002 | (–) | (+) |
| *07/99* | 0.0366 | 0.0290 | -0.0076 | 0.0001 | 0.0001 | (+) | (+) |
| *08/99* | -0.0141 | 0.0031 | 0.0172 | 0.0003 | 0.0003 | (–) | (+) |
| *09/99* | 0.0044 | -0.0076 | -0.0120 | 0.0001 | 0.0001 | (+) | (–) |
| *10/99* | -0.0124 | -0.0122 | 0.0002 | 0.0000 | 0.0000 | (–) | (–) |
| *11/99* | -0.0427 | -0.0210 | 0.0218 | 0.0005 | 0.0005 | (–) | (–) |
| *12/99* | -0.0012 | 0.0023 | 0.0035 | 0.0000 | 0.0000 | (–) | (+) |

| | |
|---|---|
| Mean Error: | 0.0091 |
| Root Mean Square Error: | 0.0148 |
| Mean Absolute Error: | 0.0124 |
| Theil's U Statistic: | 0.4839 |
| Signs Correctly Forecasted: | 50.00% |

***Table A12***

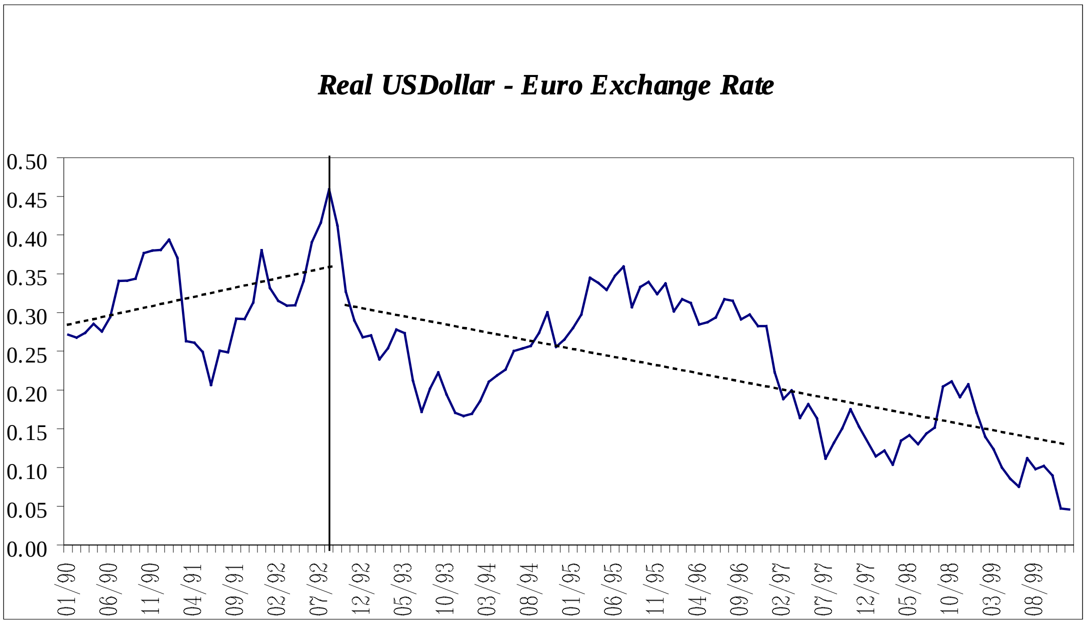
***Forecasting Performance (2) - Monthly Levels of Real Euro/Dollar Exchange Rate***

| *Date* | *Actual* | *Forecasted* | *Error* | *Square Error* | *Absolute Error* | *Actual Change Sign* | *Forecasted Change Sign* |
|---|---|---|---|---|---|---|---|
| *12/98* | 1.2305 | | | | | | |
| *01/99* | 1.1870 | 1.2123 | 0.0253 | 0.0006 | 0.0006 | (–) | (–) |
| *02/99* | 1.1496 | 1.1505 | 0.0009 | 0.0000 | 0.0000 | (–) | (–) |
| *03/99* | 1.1314 | 1.1586 | 0.0272 | 0.0007 | 0.0007 | (–) | (+) |
| *04/99* | 1.1051 | 1.1120 | 0.0069 | 0.0000 | 0.0000 | (–) | (–) |
| *05/99* | 1.0896 | 1.1108 | 0.0213 | 0.0005 | 0.0005 | (–) | (+) |
| *06/99* | 1.0783 | 1.0945 | 0.0163 | 0.0003 | 0.0003 | (–) | (+) |
| *07/99* | 1.1185 | 1.1100 | -0.0085 | 0.0001 | 0.0001 | (+) | (+) |
| *08/99* | 1.1028 | 1.1219 | 0.0191 | 0.0004 | 0.0004 | (–) | (+) |
| *09/99* | 1.1077 | 1.0945 | -0.0132 | 0.0002 | 0.0002 | (+) | (–) |
| *10/99* | 1.0940 | 1.0943 | 0.0002 | 0.0000 | 0.0000 | (–) | (–) |
| *11/99* | 1.0483 | 1.0713 | 0.0231 | 0.0005 | 0.0005 | (–) | (–) |
| *12/99* | 1.0470 | 1.0507 | 0.0037 | 0.0000 | 0.0000 | (–) | (+) |

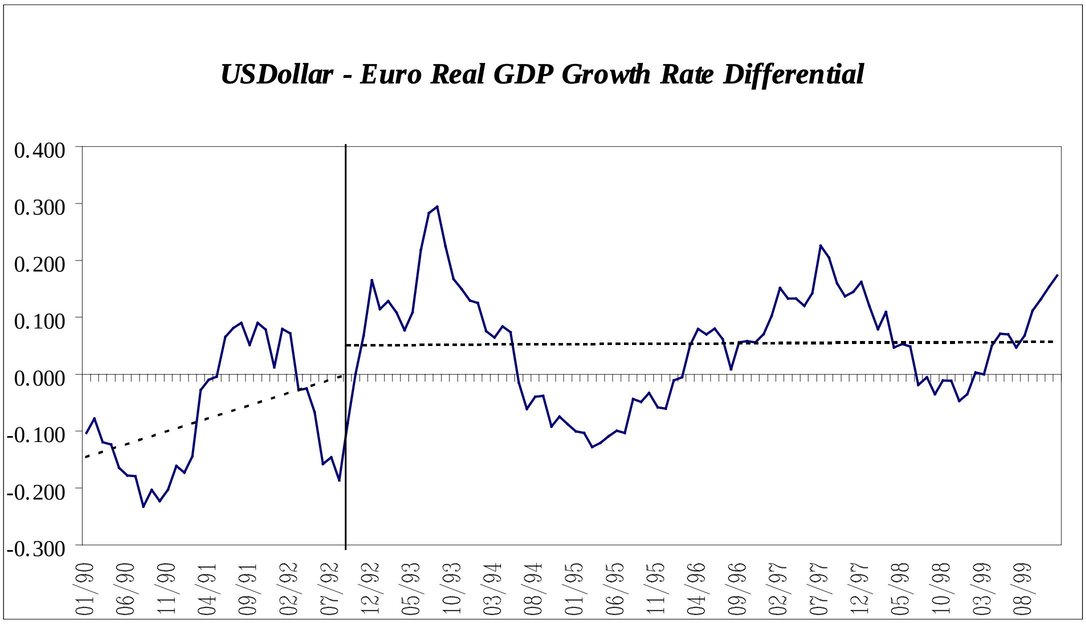
| | |
|---|---|
| Mean Error: | 0.0102 |
| Root Mean Square Error: | 0.0166 |
| Mean Absolute Error: | 0.0138 |
| Theil's U Statistic: | 0.6105 |
| Signs Correctly Forecasted: | 50.00% |

***Table A13***

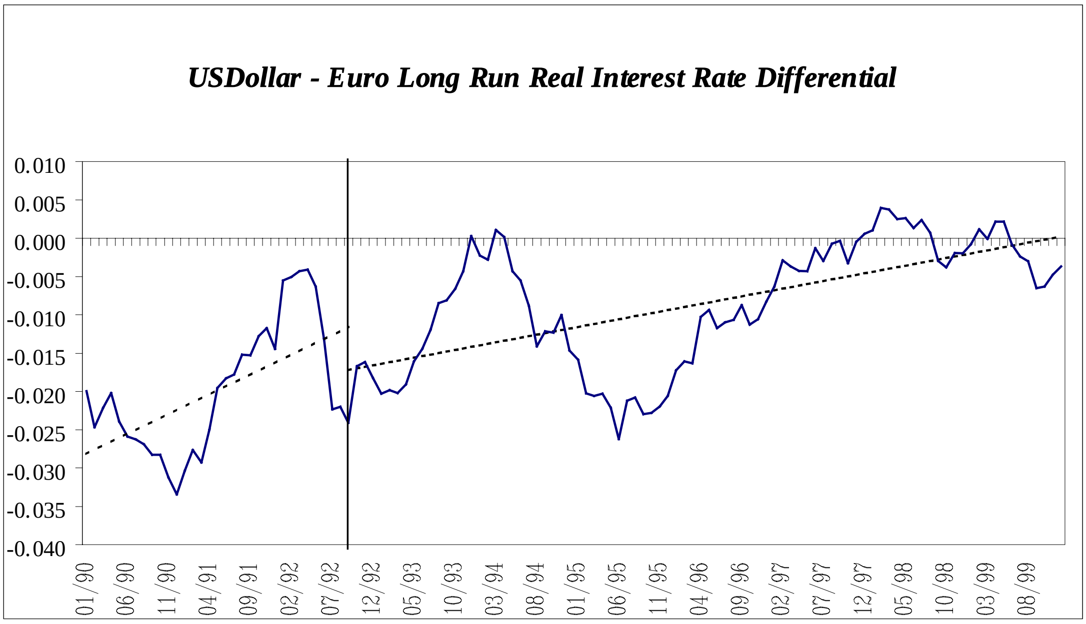
# *Time Series Graphs*



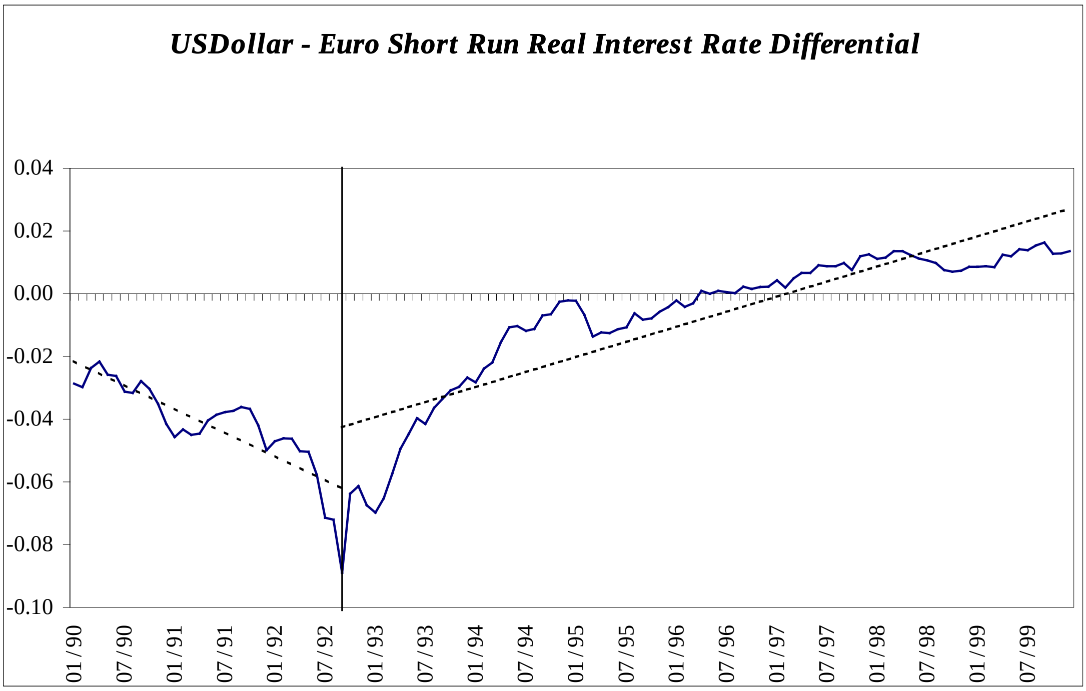

***Graph A1***




***Graph A2***

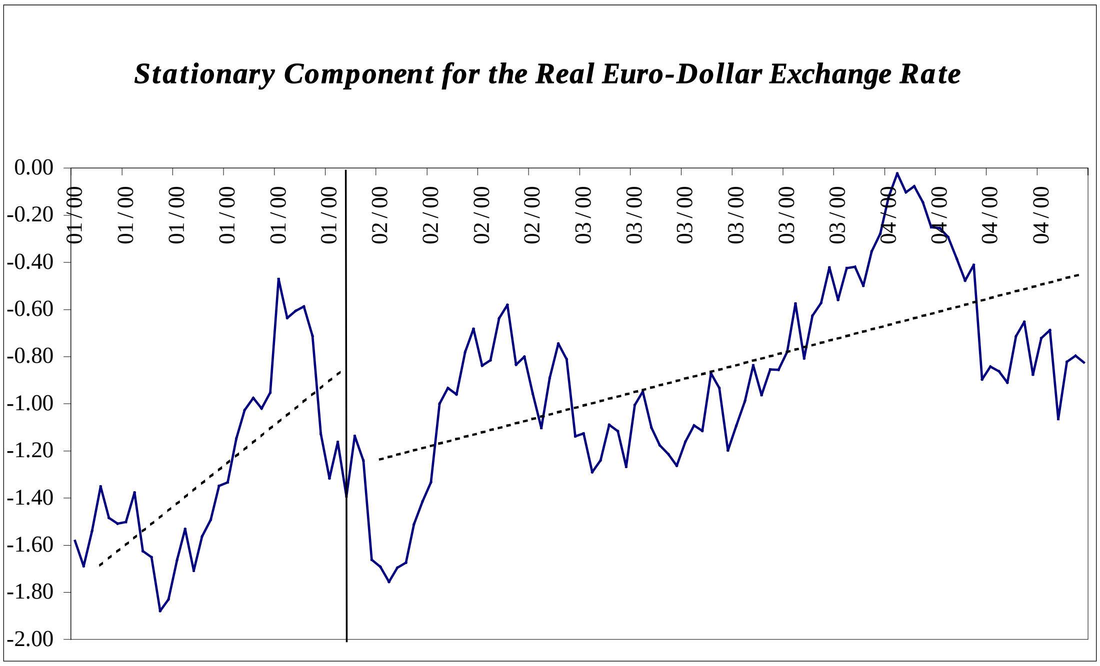



*Graph A3*

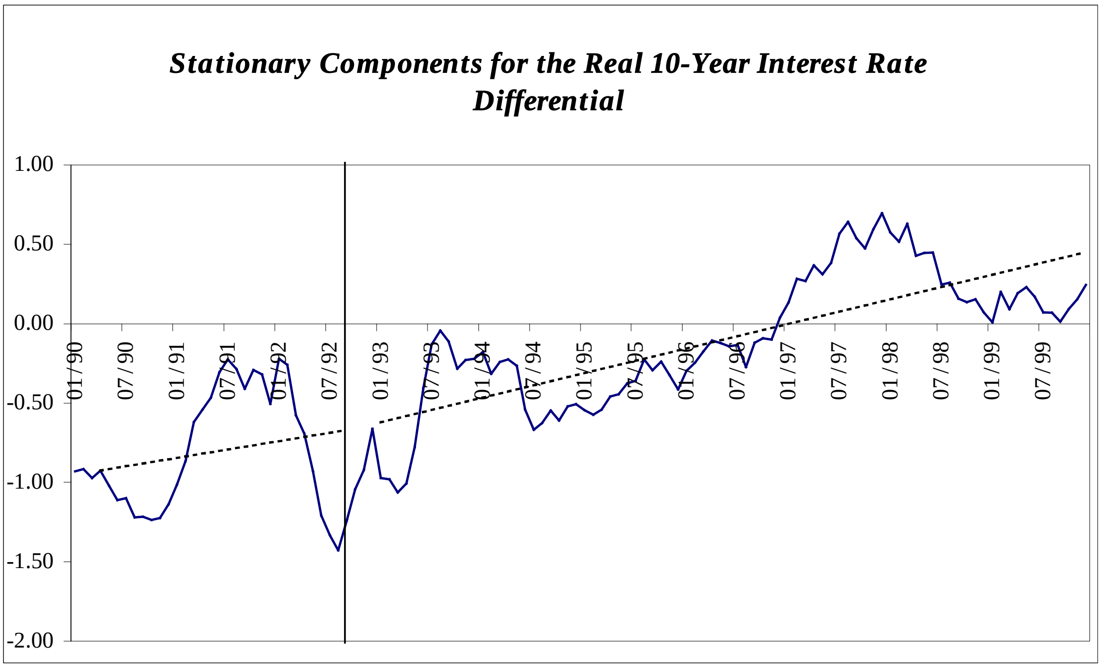



*Graph A4*

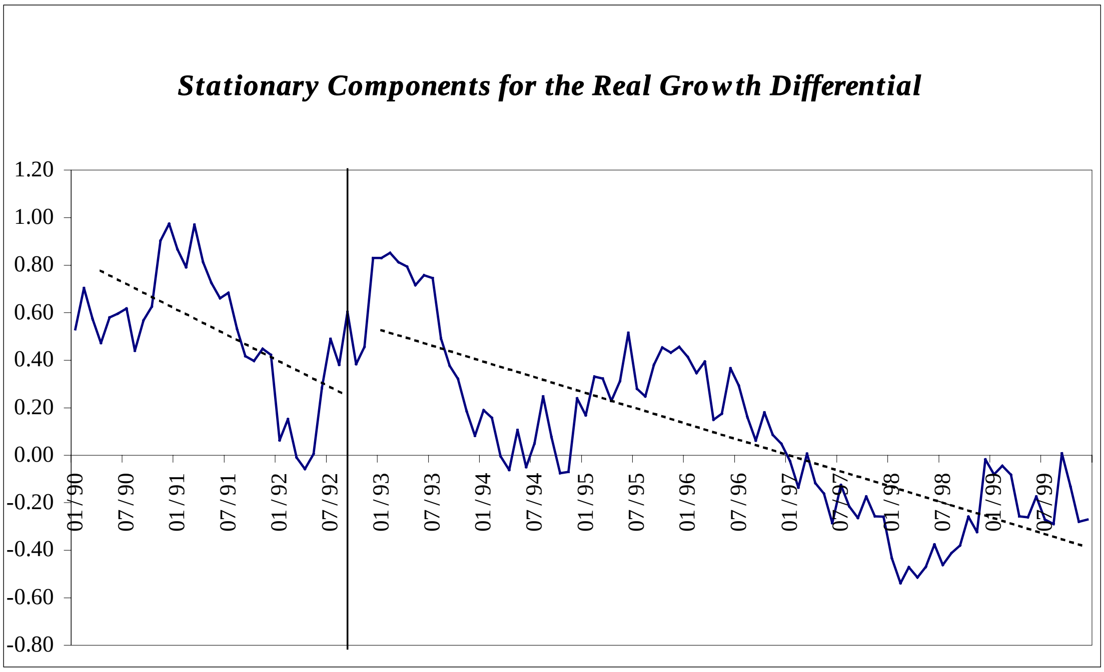
# *Stationary Components*




***Graph A5***

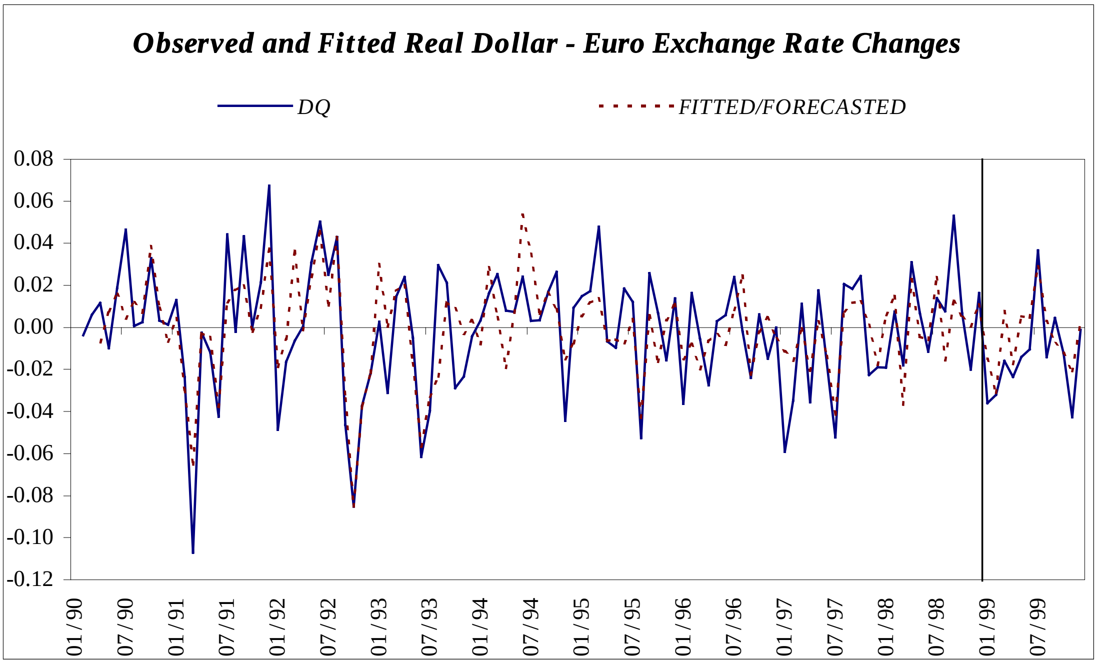



***Graph A6***

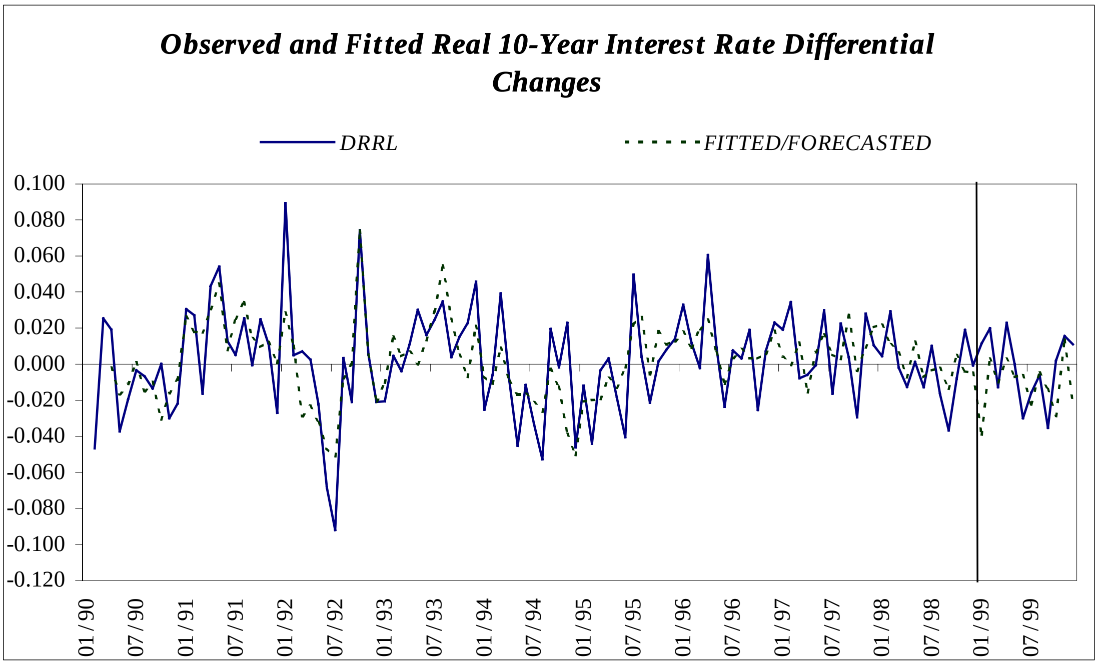

Stationary Components for the Real Growth Differential
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
01/90
07/90
01/91
07/91
01/92
07/92
01/93
07/93
01/94
07/94
01/95
07/95
01/96
07/96
01/97
07/97
01/98
07/98
01/99
07/99

***Graph A7***

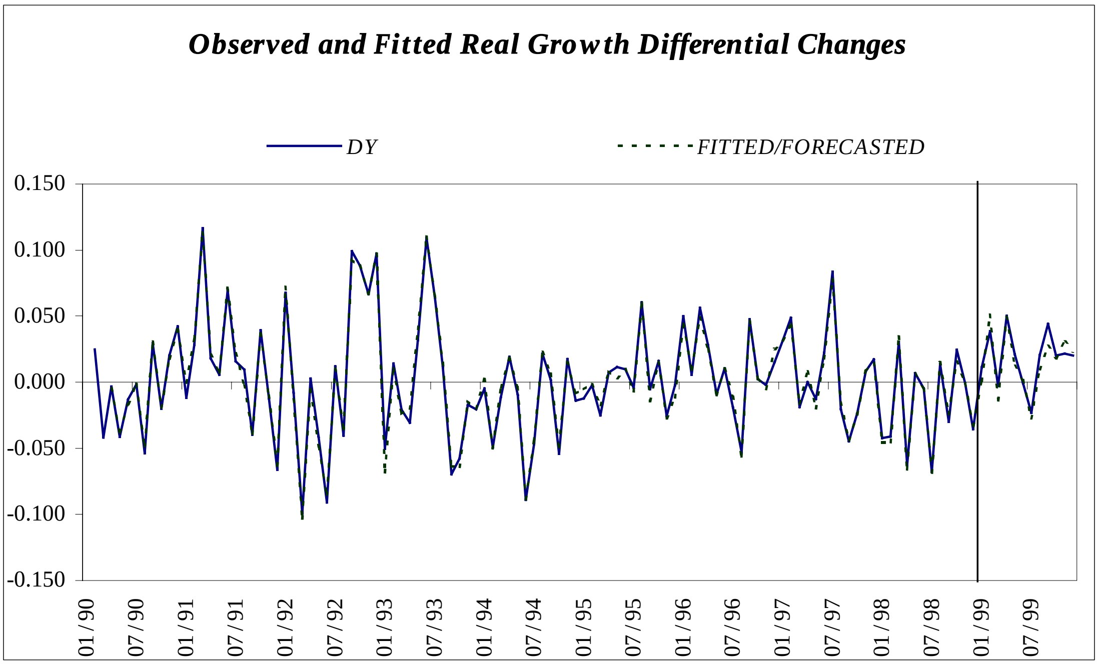
## *Short Term Dynamics*




***Graph A8***

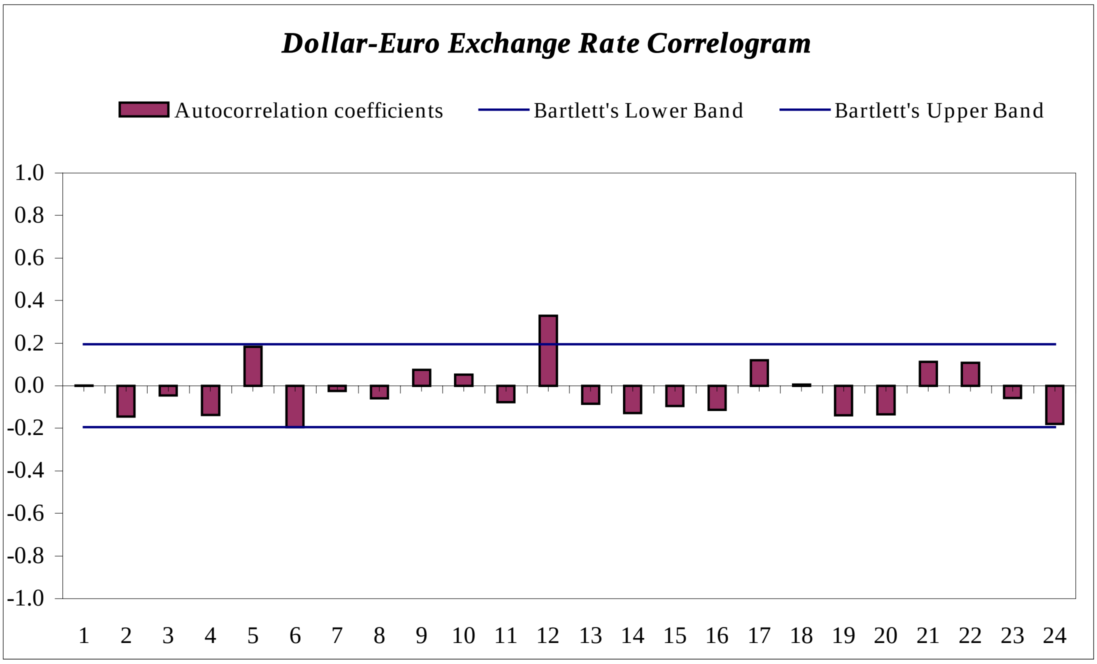



***Graph A9***

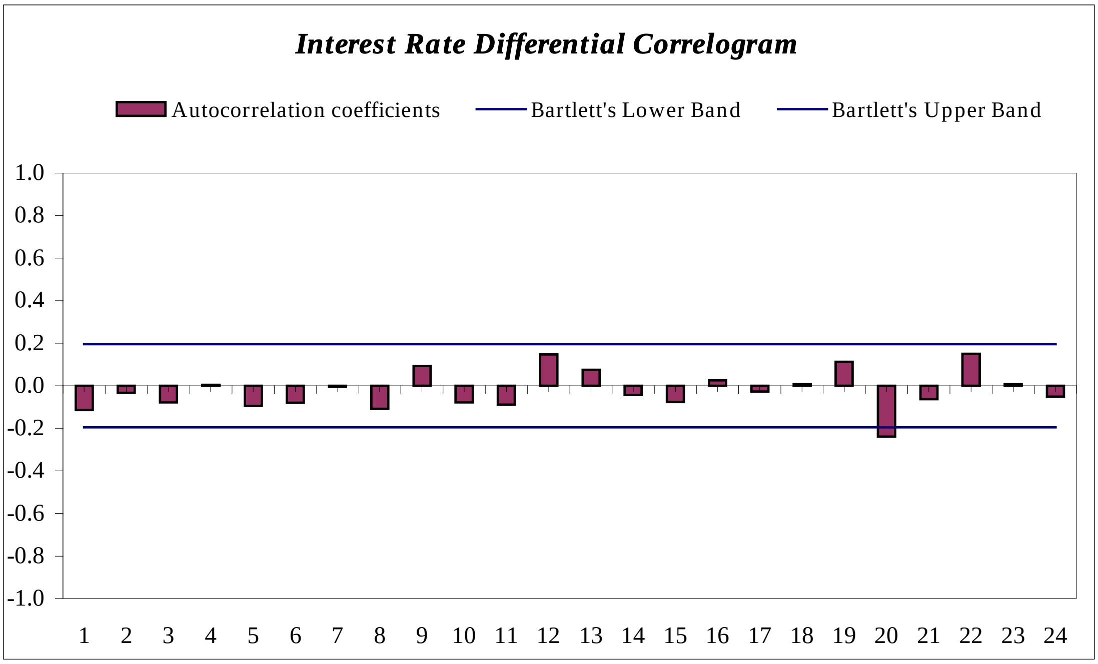

Observed and Fitted Real Growth Differential Changes
DY
FITTED/FORECASTED
0.150
0.100
0.050
0.000
-0.050
-0.100
-0.150
01/90
07/90
01/91
07/91
01/92
07/92
01/93
07/93
01/94
07/94
01/95
07/95
01/96
07/96
01/97
07/97
01/98
07/98
01/99
07/99

***Graph A10***

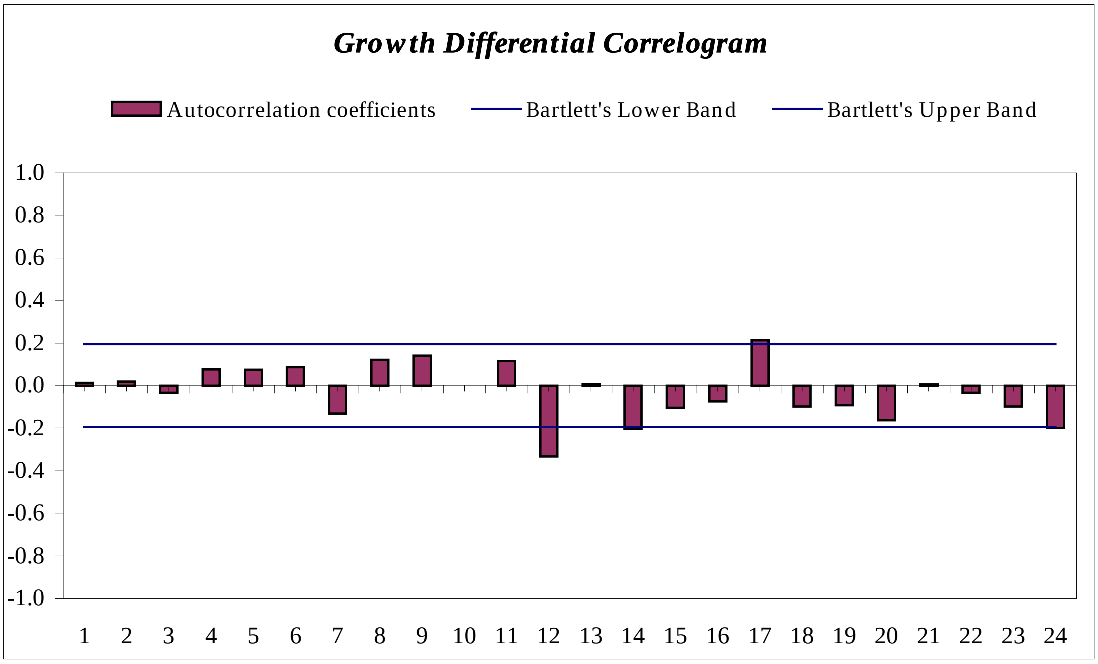
# *Residuals Autocorrelations*




***Graph A11***

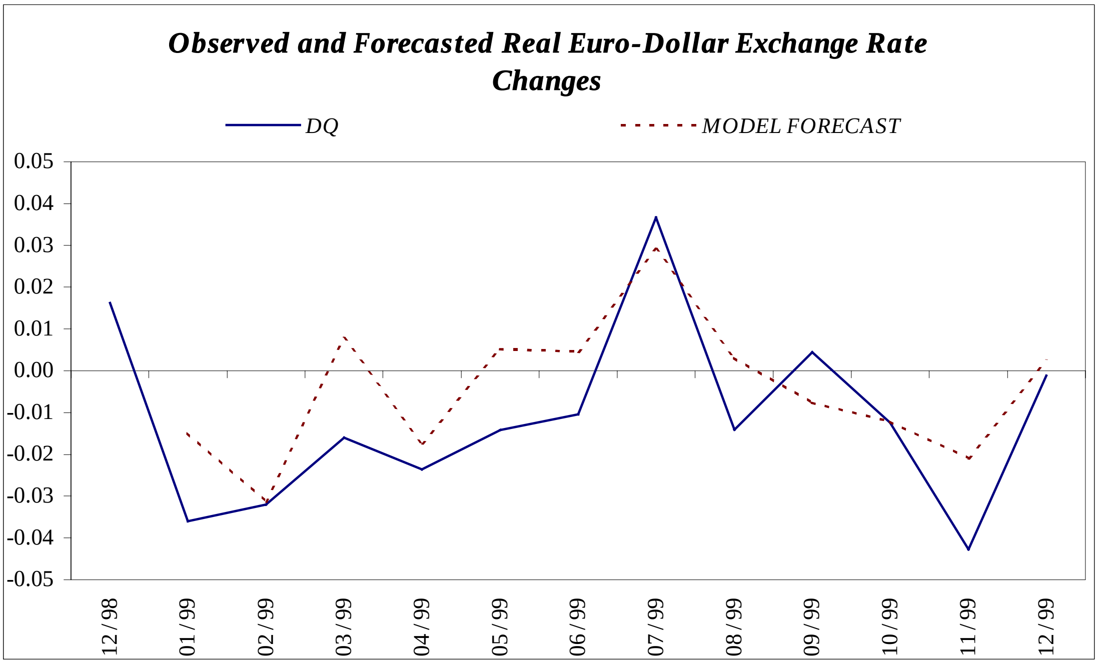



***Graph A12***

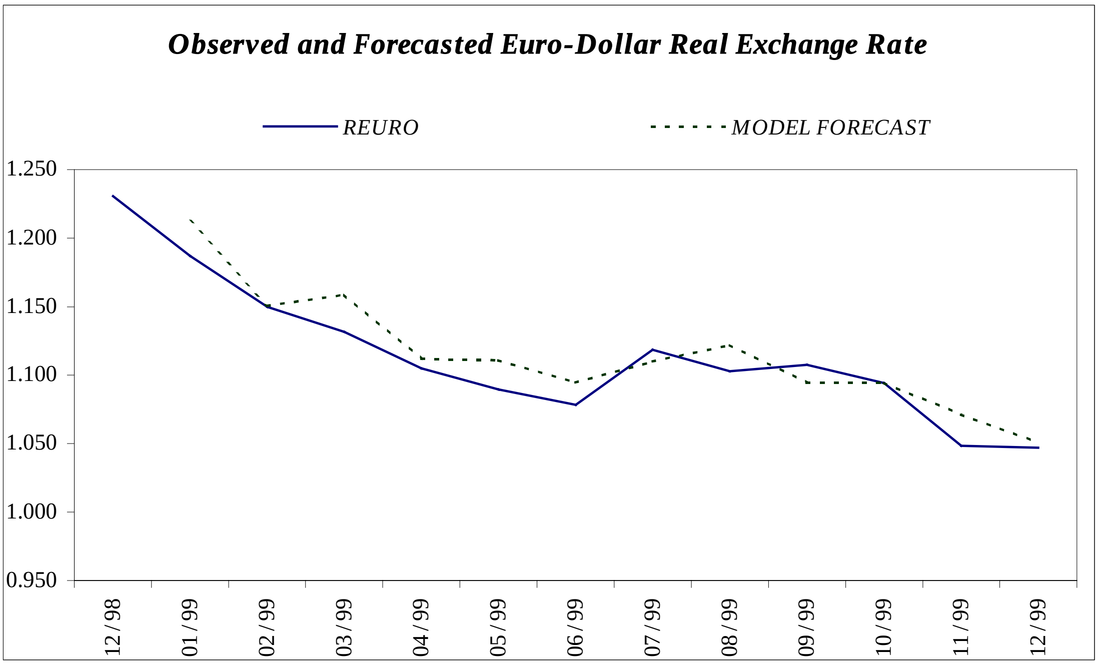

Growth Differential Correlogram
Autocorrelation coefficients
Bartlett's Lower Band
Bartlett's Upper Band
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

***Graph A13***

## *Forecasting Performance*




***Graph A14***




***Graph A15***

# *Appendix B*

***Results Regarding the Multi-Country System (12)***

## *Cointegration Analysis of System (12)*

Number of Lags Considered: 3

| Ho : rank = r | Test Statistic - T log(1 - m) | 95% | Test Statistic - T S log(.) | 95% |
|---|---|---|---|---|
| r > 1 | 0.00 | 3.80 | 0.00 | 3.80 |
| r > 2 | 11.34 | 14.10 | 11.34 | 15.40 |
| r > 3 | 12.72 | 21.00 | 24.07 | 29.70 |
| r > 4 | 18.51 | 27.10 | 42.58 | 47.20 |
| r > 5 | 24.99 | 33.50 | 67.57 | 68.50 |
| r > 6 | 28.64 | 39.40 | 96.21* | 94.20 |
| r > 7 | 38.12 | 45.30 | 134.30* | 124.20 |
| r > 8 | 46.53 | 51.40 | 180.9** | 156.00 |
| r > 9 | 54.11 | 57.10 | 235.0** | 192.90 |
| r > 10 | 66.61* | 62.80 | 301.6** | 233.10 |
| r > 11 | 71.59* | 68.80 | 373.2** | 277.70 |
| r > 12 | 77.28 | | 450.40 | |
| r > 13 | 111.70 | | 562.10 | |
| r > 14 | 115.50 | | 677.60 | |
| r > 15 | 123.70 | | 801.30 | |
| r > 16 | 132.00 | | 933.30 | |
| r > 17 | 153.20 | | 1087.00 | |
| r > 18 | 168.90 | | 1255.00 | |
| r > 19 | 183.30 | | 1439.00 | |
| r > 20 | 202.70 | | 1641.00 | |
| r > 21 | 221.60 | | 1863.00 | |

***Table B1*** *: Cointegration Analysis of System (12)*

***Equation 1 for Q***

| *Variable* | *Coefficient* | *Std. Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *FBAL_USBD* | 0.3110 | 0.0858 | 3.6260 | 0.0004 |
| *FBAL_USFR* | 0.6617 | 0.1048 | 6.3120 | 0.0000 |
| *FBAL_USIT* | -0.5486 | 0.0769 | -7.1330 | 0.0000 |
| *NFA_USBD* | -0.0601 | 0.0227 | -2.6490 | 0.0092 |
| *NFA_USIT* | 0.0280 | 0.0081 | 3.4730 | 0.0007 |
| *LTOT_USBD* | -0.7819 | 0.2096 | -3.7310 | 0.0003 |
| *LTOT_USFR* | 0.4439 | 0.1950 | 2.2770 | 0.0247 |
| *LTOT_USIT* | 0.3680 | 0.1728 | 2.1300 | 0.0354 |
| *LTNT_USBD* | -2.6782 | 0.4078 | -6.5680 | 0.0000 |
| *LTNT_USFR* | 1.3393 | 0.4777 | 2.8040 | 0.0059 |
| *RRL* | -2.4087 | 0.9677 | -2.4890 | 0.0143 |
| *Constant* | 0.2746 | 0.0199 | 13.7880 | 0.0000 |

$\sigma = 0.0373014$

***Table B2***

***Equation 2 for RRL***

| *Variable* | *Coefficient* | *Std. Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *MG_USFR* | 0.0336 | 0.0049 | 6.8740 | 0.0000 |
| *MG_USIT* | -0.0134 | 0.0062 | -2.1760 | 0.0316 |
| *Y_USBD* | 0.1834 | 0.0396 | 4.6250 | 0.0000 |
| *Y_USFR* | -0.3699 | 0.0622 | -5.9500 | 0.0000 |
| *Y_USIT* | 0.3691 | 0.0676 | 5.4590 | 0.0000 |
| *Constant* | -0.0141 | 0.0007 | -19.1020 | 0.0000 |

$\sigma = 0.0207432$

***Table B3***

**_Equation 3 for Y_USBD_**

| *Variable* | *Coefficient* | *Std. Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *NFA_USBD* | -0.0229 | 0.0066 | -3.4620 | 0.0008 |
| *ROIL* | -0.0237 | 0.0066 | -3.5730 | 0.0005 |
| *LTNT_USBD* | -0.6675 | 0.0720 | -9.2750 | 0.0000 |
| *RRL* | 0.8624 | 0.1963 | 4.3940 | 0.0000 |
| *Constant* | 0.0902 | 0.0181 | 4.9740 | 0.0000 |

$\sigma$ = 0.0143976

**_Table B4_**

**_Equation 4 for Y_USFR_**

| *Variable* | *Coefficient* | *Std. Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *FBAL_USFR* | -0.1373 | 0.0141 | -9.7200 | 0.0000 |
| *NFA_USFR* | -0.0076 | 0.0024 | -3.2350 | 0.0016 |
| *ROIL* | 0.0239 | 0.0045 | 5.3350 | 0.0000 |
| *LTNT_USFR* | -0.4161 | 0.0597 | -6.9720 | 0.0000 |
| *RRL* | 0.7551 | 0.1131 | 6.6780 | 0.0000 |
| *Constant* | -0.0585 | 0.0129 | -4.5490 | 0.0000 |

$\sigma$ = 0.00935584

**_Table B5_**

***Equation 5 for Y_USIT***

| *Variable* | *Coefficient* | *Std. Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *FBAL_USIT* | -0.0416 | 0.0140 | -2.9710 | 0.0036 |
| *LTNT_USIT* | -0.1472 | 0.0596 | -2.4720 | 0.0149 |
| *RRL* | 1.9008 | 0.1761 | 10.7940 | 0.0000 |
| *Constant* | 0.0323 | 0.0024 | 13.4330 | 0.0000 |

$\sigma$ = 0.0156231

***Table B6***

***Correlation of Residuals***

| | ***Q*** | ***Y_USBD*** | ***Y_USFR*** | ***Y_USIT*** | ***RRL*** |
|---|---|---|---|---|---|
| ***Q*** | 1.0000 | | | | |
| ***Y_USBD*** | 0.2844 | 1.0000 | | | |
| ***Y_USFR*** | 0.1561 | 0.6511 | 1.0000 | | |
| ***Y_USIT*** | -0.0263 | -0.5827 | 0.7713 | 1.0000 | |
| ***RRL*** | -0.0633 | -0.6184 | -0.6234 | -0.8795 | 1.0000 |

***Table B7***

## *Short-Run Dynamics*

### *Equation 1 for DQ*

| *Variable* | *Coefficient* | *Std.Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *DQ_1* | 0.1302 | 0.0823 | 1.5820 | 0.1171 |
| *DRRL_1* | -1.8498 | 0.6465 | -2.8610 | 0.0052 |
| *DMG_USFR* | 0.2536 | 0.1351 | 1.8760 | 0.0638 |
| *DROIL* | 0.0429 | 0.0172 | 2.4870 | 0.0147 |
| *DLTOT_USFR* | 0.2497 | 0.1026 | 2.4330 | 0.0169 |
| *DFBAL_USIT* | -0.6344 | 0.1354 | -4.6860 | 0.0000 |
| *DFBAL_USIT_1* | 0.1001 | 0.0550 | 1.8190 | 0.0721 |
| *DNFA_USBD* | -0.0246 | 0.0125 | -1.9670 | 0.0521 |
| *DNFA_USIT* | 0.0095 | 0.0037 | 2.5490 | 0.0124 |
| *ECM(Q)_1* | -0.2818 | 0.0498 | -5.6600 | 0.0000 |
| *ECM(Y_USBD)_1* | 0.3995 | 0.1564 | 2.5550 | 0.0123 |
| *ECM(Y_USIT)_1* | -0.4656 | 0.1394 | -3.3400 | 0.0012 |

$\sigma$ = 0.018378

***Table B8***

### *Equation 2 for DY_USBD*

| *Variable* | *Coefficient* | *Std.Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *DYG_USBD_1* | 0.8168 | 0.0534 | 15.3030 | 0.0000 |
| *DLTNT_USBD_1* | 0.1088 | 0.0563 | 1.9340 | 0.0562 |
| *DLTNT_USIT_1* | -0.1129 | 0.0698 | -1.6180 | 0.1091 |
| *DMG_USBD* | 0.0204 | 0.0097 | 2.1080 | 0.0377 |
| *DLTOT_USBD* | 0.0430 | 0.0122 | 3.5300 | 0.0006 |
| *DLTOT_USIT* | -0.0266 | 0.0124 | -2.1530 | 0.0339 |
| *DFBAL_USBD* | -0.0879 | 0.0163 | -5.3890 | 0.0000 |
| *DFBAL_USFR_1* | -0.0338 | 0.0140 | -2.4190 | 0.0175 |
| *DFBAL_USIT* | 0.0660 | 0.0189 | 3.4920 | 0.0007 |
| *DNFA_USBD_1* | 0.0038 | 0.0015 | 2.5890 | 0.0112 |
| *DNFA_USIT* | -0.0009 | 0.0005 | -1.8050 | 0.0743 |
| *ECM(Q)_1* | 0.0193 | 0.0067 | 2.8890 | 0.0048 |
| *ECM(Y_USBD)_1* | -0.0928 | 0.0170 | -5.4510 | 0.0000 |
| *ECM(RRL)_1* | 0.1910 | 0.0476 | 4.0110 | 0.0001 |

$\sigma$ = 0.00253141

***Table B9***

### *Equation 3 for DGDP_USFR*

| *Variable* | *Coefficient* | *Std.Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *DY_USFR_1* | 0.6021 | 0.0690 | 8.7250 | 0.0000 |
| *DY_USIT_1* | 0.1579 | 0.0554 | 2.8520 | 0.0054 |
| *DRRL_1* | -0.1685 | 0.0688 | -2.4490 | 0.0162 |
| *DLTNT_USIT_1* | 0.0976 | 0.0562 | 1.7370 | 0.0857 |
| *DMG_USFR* | 0.0186 | 0.0057 | 3.2750 | 0.0015 |
| *DMG_USIT* | -0.0275 | 0.0072 | -3.8350 | 0.0002 |
| *DMG_USIT_1* | -0.0131 | 0.0058 | -2.2800 | 0.0249 |
| *DROIL* | 0.0060 | 0.0020 | 3.0120 | 0.0033 |
| *DLTOT_USBD_1* | -0.0232 | 0.0109 | -2.1300 | 0.0359 |
| *DLTOT_USIT_1* | 0.0212 | 0.0109 | 1.9420 | 0.0552 |
| *DFBAL_USFR* | -0.0517 | 0.0117 | -4.4230 | 0.0000 |
| *DFBAL_USIT_1* | 0.0320 | 0.0058 | 5.4940 | 0.0000 |
| *DNFA_USFR* | 0.0022 | 0.0009 | 2.6170 | 0.0103 |
| *DNFA_USFR_1* | 0.0018 | 0.0008 | 2.3070 | 0.0233 |
| *DNFA_USIT_1* | 0.0008 | 0.0004 | 1.9240 | 0.0574 |
| *ECM(Q)_1* | 0.0175 | 0.0054 | 3.2180 | 0.0018 |
| *ECM(Y_USFR)_1* | -0.1186 | 0.0212 | -5.6000 | 0.0000 |
| *ECM(RRL)_1* | 0.1114 | 0.0386 | 2.8850 | 0.0049 |

$\sigma$ = 0.00198212

***Table B10***

### *Equation 4 for DGDP_USIT*

| *Variable* | *Coefficient* | *Std.Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *DY_USIT_1* | 0.5964 | 0.0632 | 9.4410 | 0.0000 |
| *DLTNT_USIT* | -0.1336 | 0.0642 | -2.0810 | 0.0402 |
| *DMG_USFR* | 0.0816 | 0.0179 | 4.5700 | 0.0000 |
| *DFBAL_USBD_1* | -0.0331 | 0.0190 | -1.7440 | 0.0845 |
| *DFBAL_USIT* | -0.0878 | 0.0179 | -4.8920 | 0.0000 |
| *DFBAL_USIT_1* | 0.0317 | 0.0189 | 1.6750 | 0.0973 |
| *DNFA_USBD* | 0.0040 | 0.0016 | 2.6020 | 0.0108 |
| *ECM(Y_USIT)_1* | -0.0363 | 0.0125 | -2.8970 | 0.0047 |

$\sigma$ = 0.00268984

***Table B11***

### *Equation 5 for DRRL*

| *Variable* | *Coefficient* | *Std.Error* | *t-value* | *t-prob* |
|---|---|---|---|---|
| *DQ_1* | -0.0182 | 0.0112 | -1.6310 | 0.1063 |
| *DY_USIT_1* | -0.1417 | 0.0721 | -1.9640 | 0.0525 |
| *DMG_USFR* | 0.0462 | 0.0186 | 2.4770 | 0.0150 |
| *DMG_USFR_1* | 0.0426 | 0.0194 | 2.1980 | 0.0304 |
| *DLTOT_USBD_1* | -0.0222 | 0.0137 | -1.6210 | 0.1083 |
| *DFBAL_USIT* | -0.0403 | 0.0187 | -2.1580 | 0.0335 |
| *DFBAL_USIT_1* | -0.0518 | 0.0206 | -2.5180 | 0.0135 |
| *DNFA_USIT_1* | 0.0009 | 0.0005 | 1.8540 | 0.0670 |
| *ECM(Y_USBD)_1* | 0.0645 | 0.0234 | 2.7510 | 0.0071 |
| *ECM(Y_USFR)_1* | -0.1716 | 0.0505 | -3.3970 | 0.0010 |
| *ECM(Y_USIT)_1* | 0.0774 | 0.0266 | 2.9050 | 0.0046 |
| *ECM(RRL)_1* | -0.2889 | 0.0585 | -4.9360 | 0.0000 |

$\sigma =$ 0.00242131

***Table B12***

## *Tests on Model's Residuals*

***Correlation of Residuals***

| | DQ | DY_USBD | DY_USFR | DY_USIT | DRRL |
|---|---|---|---|---|---|
| **DQ** | 1.0000 | | | | |
| **DY_USBD** | 0.0340 | 1.0000 | | | |
| **DY_USFR** | -0.2064 | 0.5232 | 1.0000 | | |
| **DY_USIT** | -0.2640 | 0.5597 | 0.4695 | 1.0000 | |
| **DRRL** | -0.3716 | 0.2121 | 0.0743 | 0.1865 | 1.0000 |

***Table B13***

***Normality, Autocorrelation and ARCH effect Tests for Residuals***

| *Equation* | *Residuals - Normality Test ($\chi^2$ Test)* | *Residuals - ARCH Test (F-form Test)* | *Residuals - Autocorrelated (Bartlett's Test - Significant Lags)* |
|---|---|---|---|
| *DQ* | 0.82775 [0.6611] | 4.0463 [0.0487] ** | Lag 22 |
| *DRRL* | 2.87490 [0.2375] | 0.2389 [0.6268] | Lag 10 - Lag 18 |
| *DY_USBD* | 0.77820 [0.6777] | 2.9221 [0.0925] | Lag 22 |
| *DY_USFR* | 1.86100 [0.3944] | 0.1206 [0.7301] | Lag 11 - Lag 23 |
| *DY_USIT* | 0.91831 [0.6318] | 1.3168 [0.2557] | Lag 9 - Lag 24 |

*Normality Test for the System*: $\chi^2(10)$ = 8.6608 [0.5646]

*Values in parenthesis indicate the significance level; * = significant at 5% level; ** = significant at 1% level.*

***Table B14***

# *Forecasting Performances*

***Forecasting Performance (1) - Monthly Changes of Real Euro/Dollar Exchange Rate***

| *Date* | *Actual* | *Forecasted* | *Error* | *Square Error* | *Absolute Error* | *Actual Change Sign* | *Forecasted Change Sign* |
|---|---|---|---|---|---|---|---|
| *12/98* | 0.0162 | | | | | | |
| *01/99* | -0.0360 | -0.0202 | 0.0159 | 0.0003 | 0.0003 | (−) | (−) |
| *02/99* | -0.0320 | -0.0378 | -0.0058 | 0.0000 | 0.0000 | (−) | (−) |
| *03/99* | -0.0159 | -0.0055 | 0.0105 | 0.0001 | 0.0001 | (−) | (−) |
| *04/99* | -0.0236 | -0.0105 | 0.0131 | 0.0002 | 0.0002 | (−) | (−) |
| *05/99* | -0.0141 | -0.0087 | 0.0054 | 0.0000 | 0.0000 | (−) | (−) |
| *06/99* | -0.0104 | 0.0031 | 0.0135 | 0.0002 | 0.0002 | (−) | (+) |
| *07/99* | 0.0366 | 0.0424 | 0.0058 | 0.0000 | 0.0000 | (+) | (+) |
| *08/99* | -0.0141 | -0.0111 | 0.0030 | 0.0000 | 0.0000 | (−) | (−) |
| *09/99* | 0.0044 | -0.0114 | -0.0158 | 0.0003 | 0.0003 | (+) | (−) |
| *10/99* | -0.0124 | -0.0034 | 0.0090 | 0.0001 | 0.0001 | (−) | (−) |
| *11/99* | -0.0427 | -0.0248 | 0.0179 | 0.0003 | 0.0003 | (−) | (−) |
| *12/99* | -0.0012 | -0.0082 | -0.0071 | 0.0000 | 0.0000 | (−) | (−) |

| | |
|---|---|
| Mean Error: | 0.0054 |
| Root Mean Square Error: | 0.0113 |
| Mean Absolute Error: | 0.0102 |
| Theil's U Statistic: | 0.3681 |
| Signs Correctly Forecasted: | 83.33% |

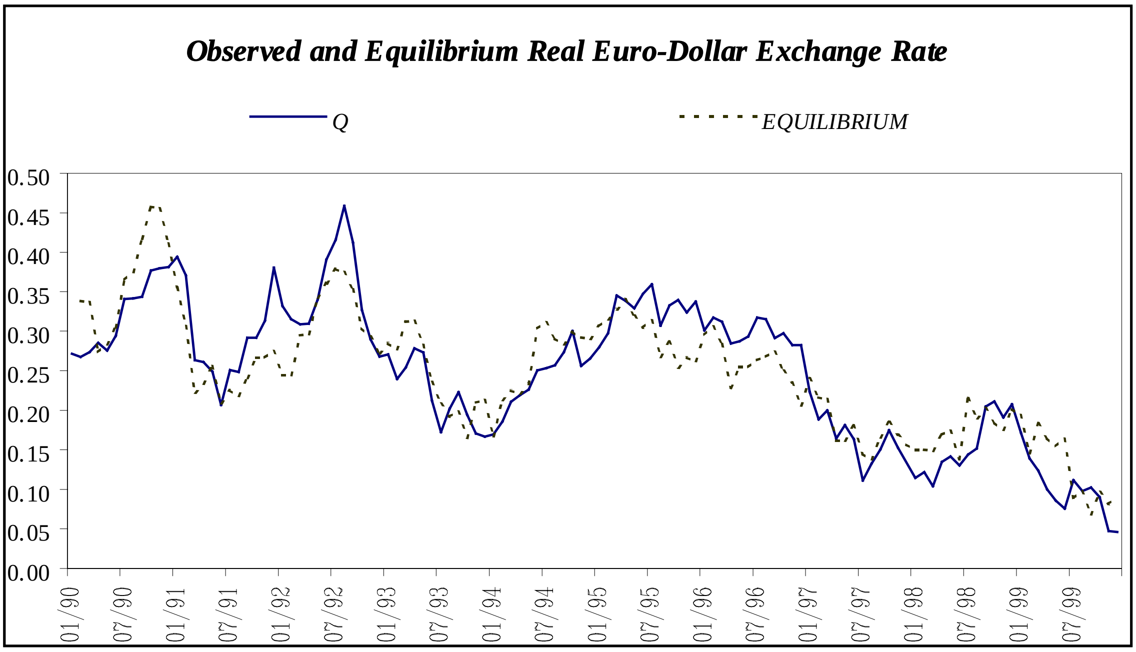
***Table B15***

***Forecasting Performance (2) - Monthly Levels of Real Euro/Dollar Exchange Rate***

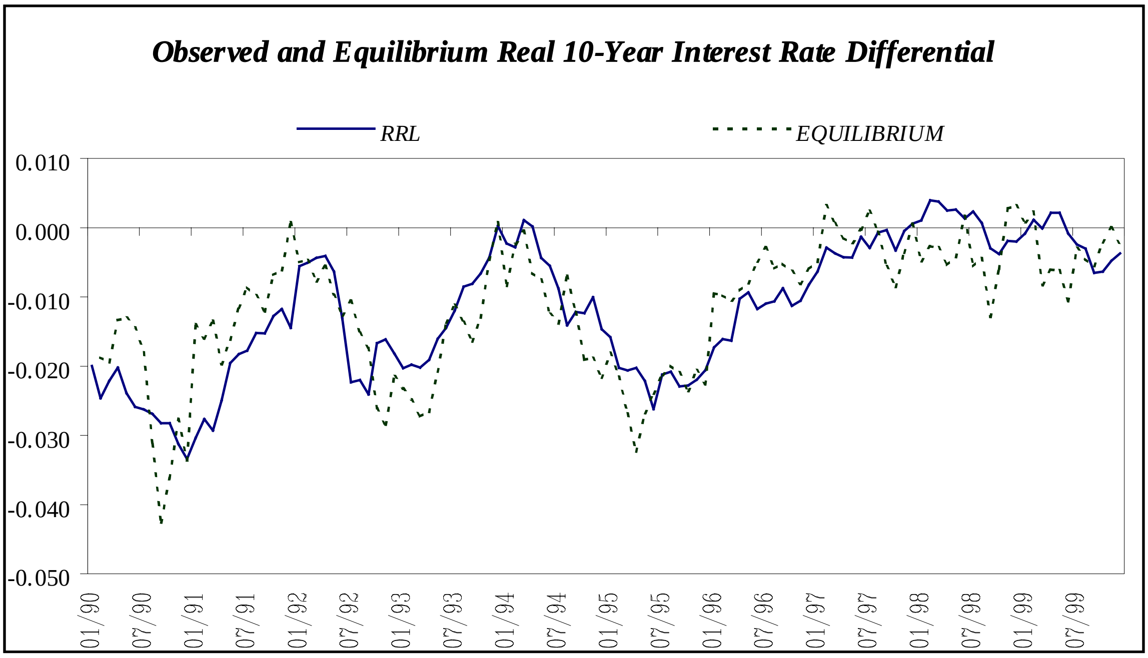
| *Date* | *Actual* | *Forecasted* | *Error* | *Square Error* | *Absolute Error* | *Actual Change Sign* | *Forecasted Change Sign* |
|---|---|---|---|---|---|---|---|
| *12/98* | 1.2305 | | | | | | |
| *01/99* | 1.1870 | 1.2060 | 0.0190 | 0.0004 | 0.0004 | (–) | (–) |
| *02/99* | 1.1496 | 1.1429 | -0.0067 | 0.0000 | 0.0000 | (–) | (–) |
| *03/99* | 1.1314 | 1.1434 | 0.0119 | 0.0001 | 0.0001 | (–) | (–) |
| *04/99* | 1.1051 | 1.1196 | 0.0145 | 0.0002 | 0.0002 | (–) | (–) |
| *05/99* | 1.0896 | 1.0955 | 0.0059 | 0.0000 | 0.0000 | (–) | (–) |
| *06/99* | 1.0783 | 1.0929 | 0.0147 | 0.0002 | 0.0002 | (–) | (+) |
| *07/99* | 1.1185 | 1.1250 | 0.0065 | 0.0000 | 0.0000 | (+) | (+) |
| *08/99* | 1.1028 | 1.1061 | 0.0033 | 0.0000 | 0.0000 | (–) | (–) |
| *09/99* | 1.1077 | 1.0903 | -0.0174 | 0.0003 | 0.0003 | (+) | (–) |
| *10/99* | 1.0940 | 1.1039 | 0.0099 | 0.0001 | 0.0001 | (–) | (–) |
| *11/99* | 1.0483 | 1.0672 | 0.0189 | 0.0004 | 0.0004 | (–) | (–) |
| *12/99* | 1.0470 | 1.0397 | -0.0074 | 0.0001 | 0.0001 | (–) | (–) |

| | |
|---|---|
| Mean Error: | 0.0061 |
| Root Mean Square Error: | 0.0125 |
| Mean Absolute Error: | 0.0113 |
| Theil's U Statistic: | 0.4599 |
| Signs Correctly Forecasted: | 83.33% |

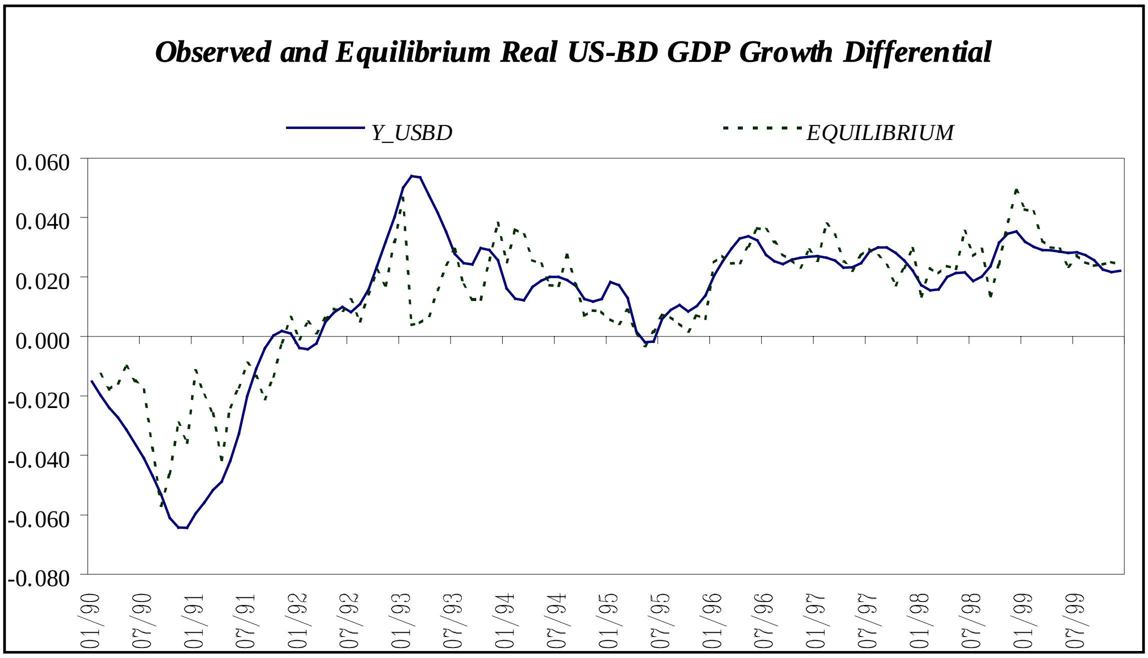
***Table B16***



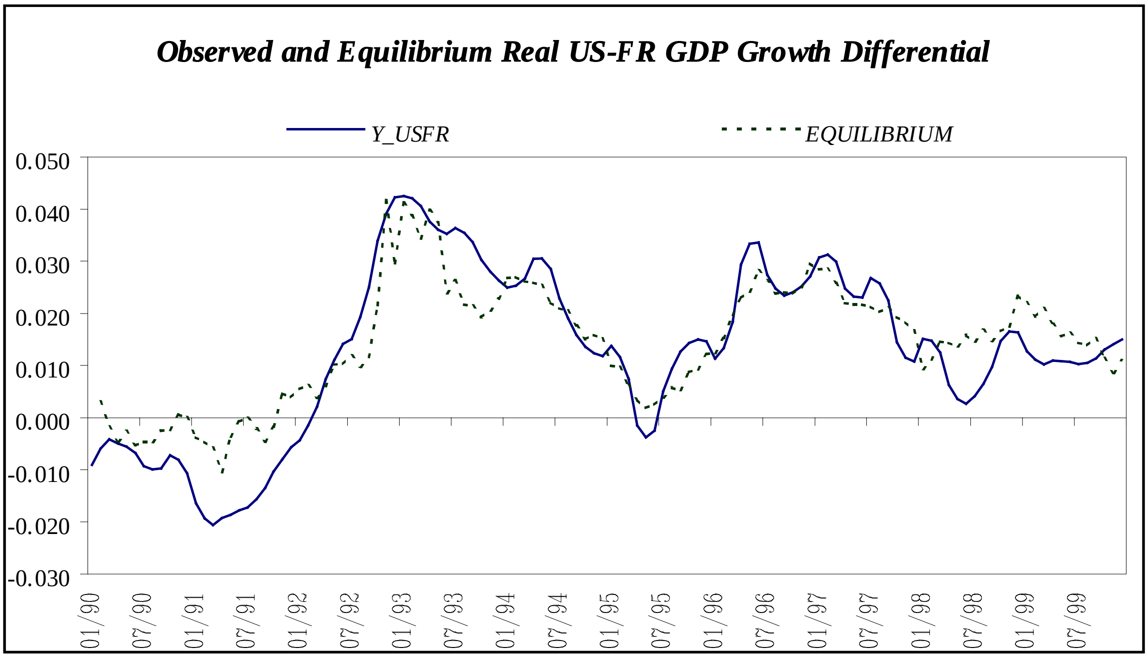

***Graph B1***



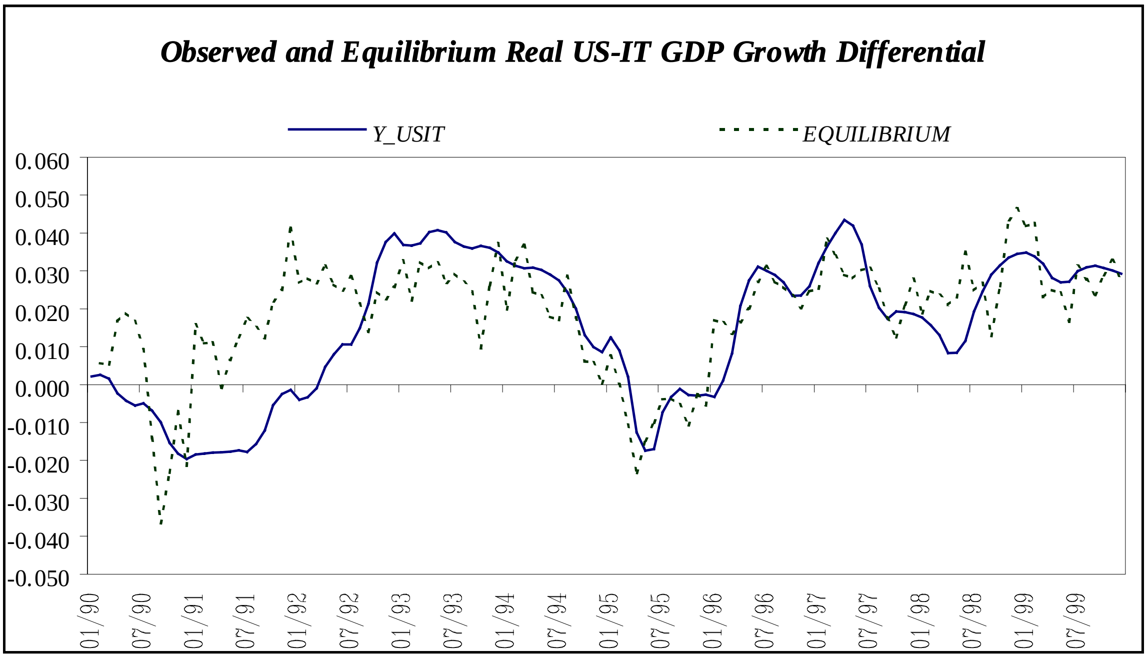

***Graph B2***



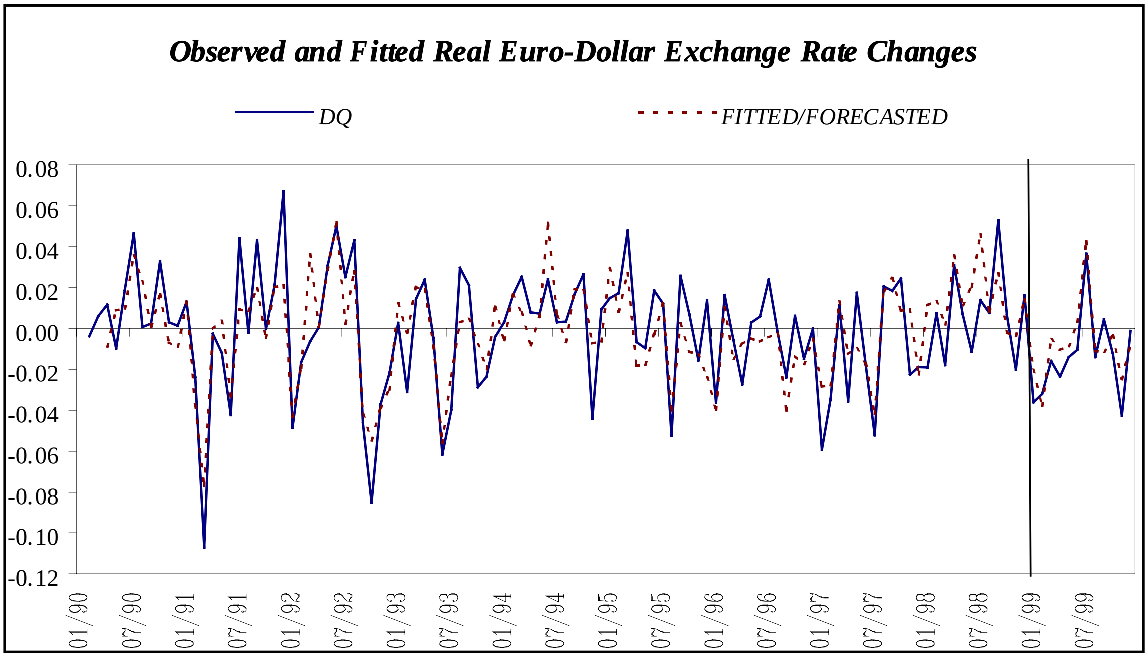

***Graph B3***



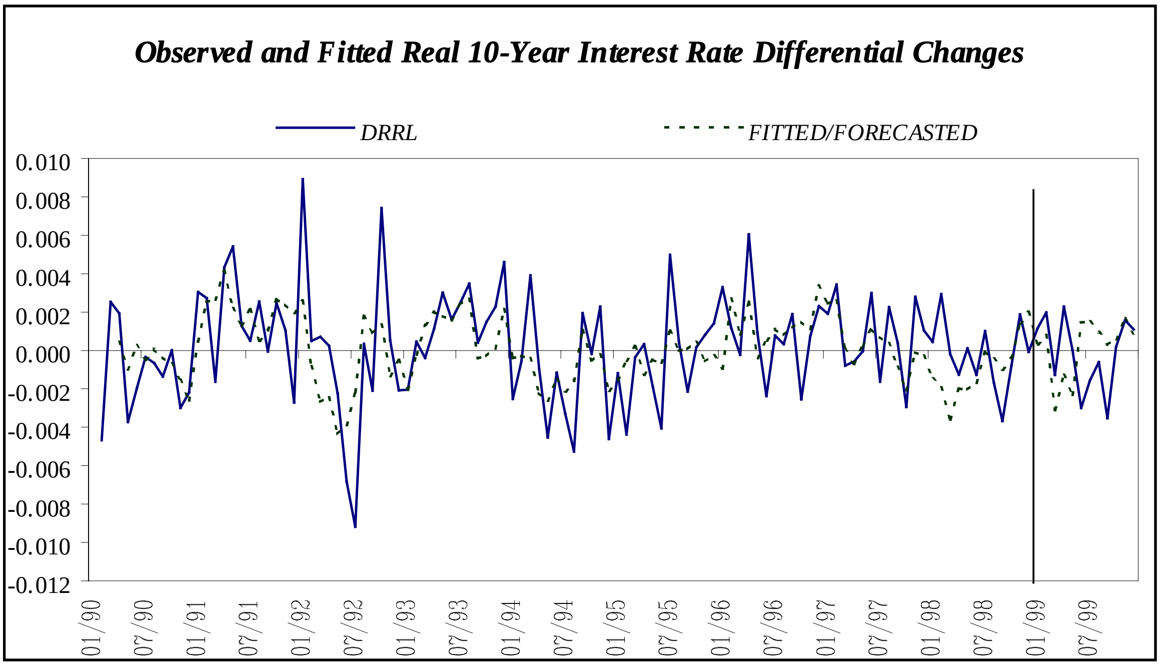

***Graph B4***



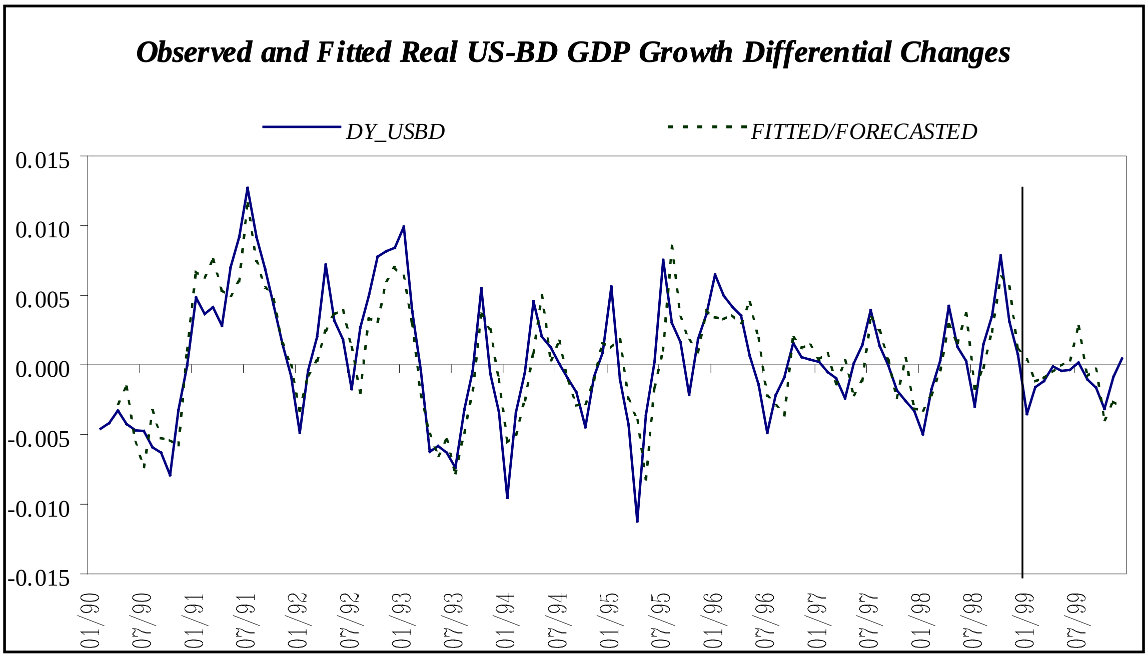

***Graph B5***



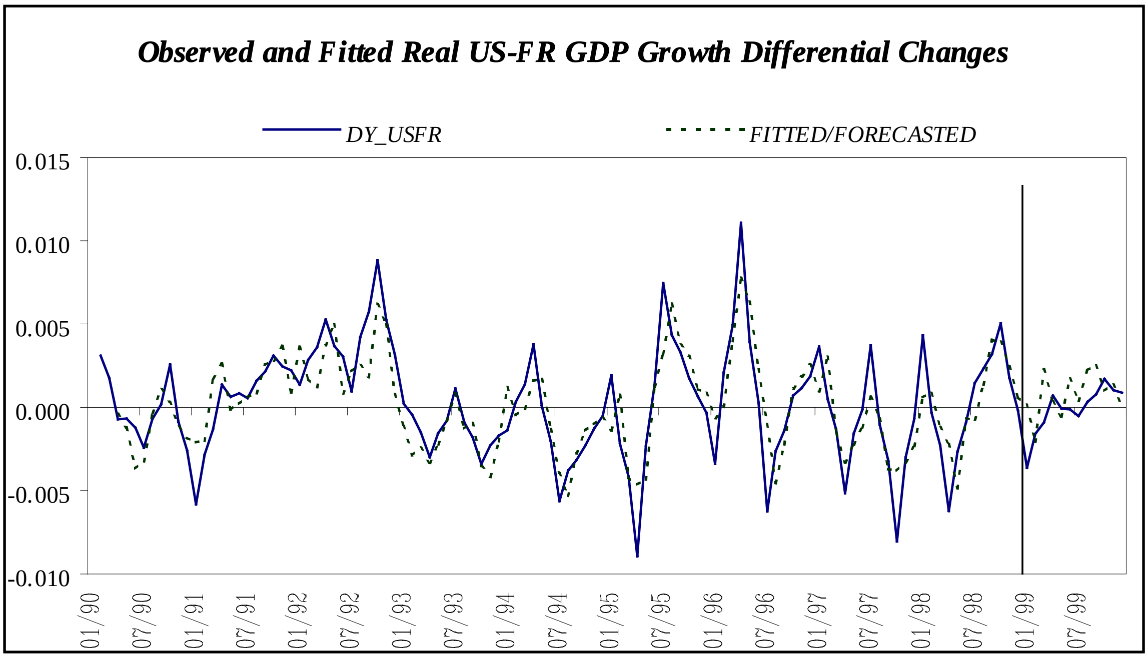

***Graph B6***



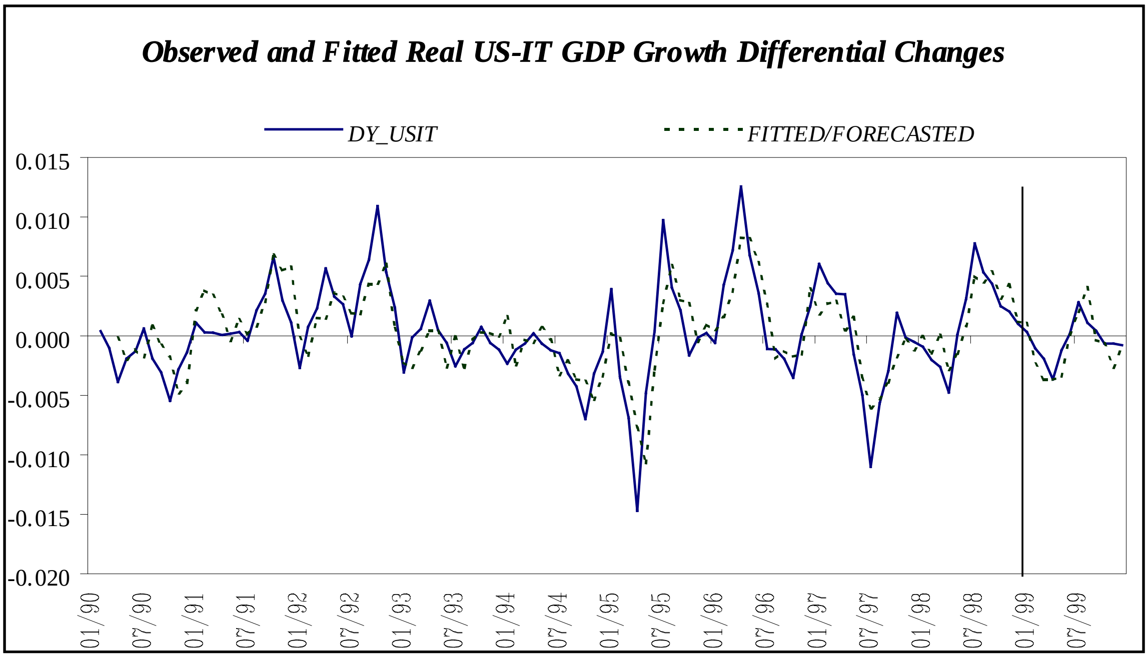

***Graph B7***




***Graph B8***

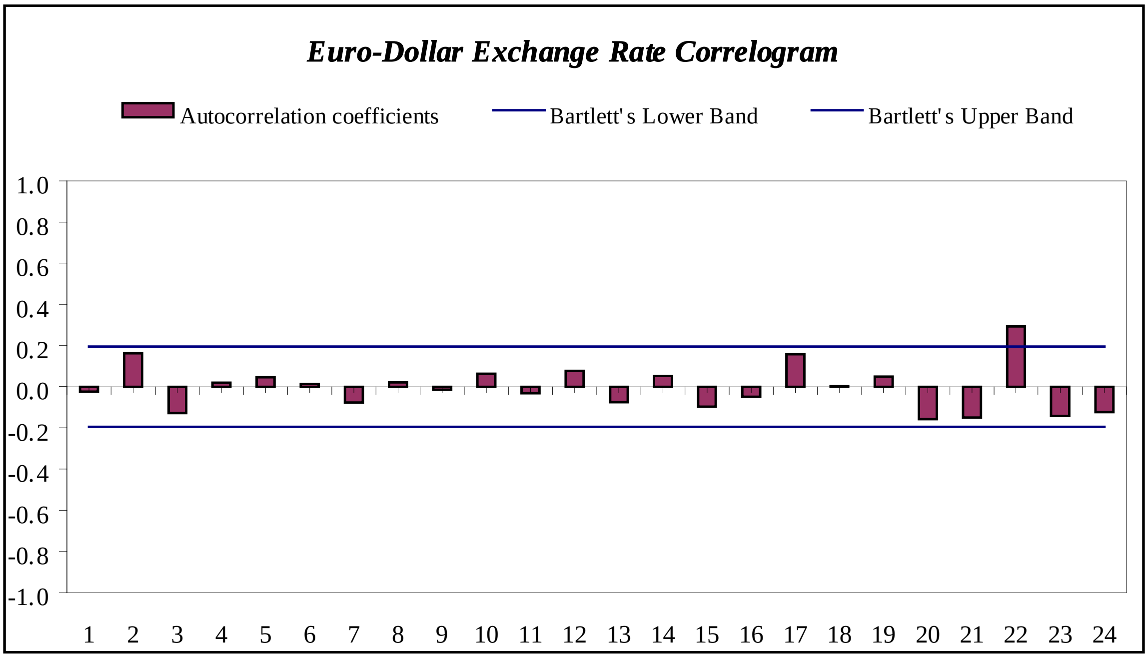



***Graph B9***

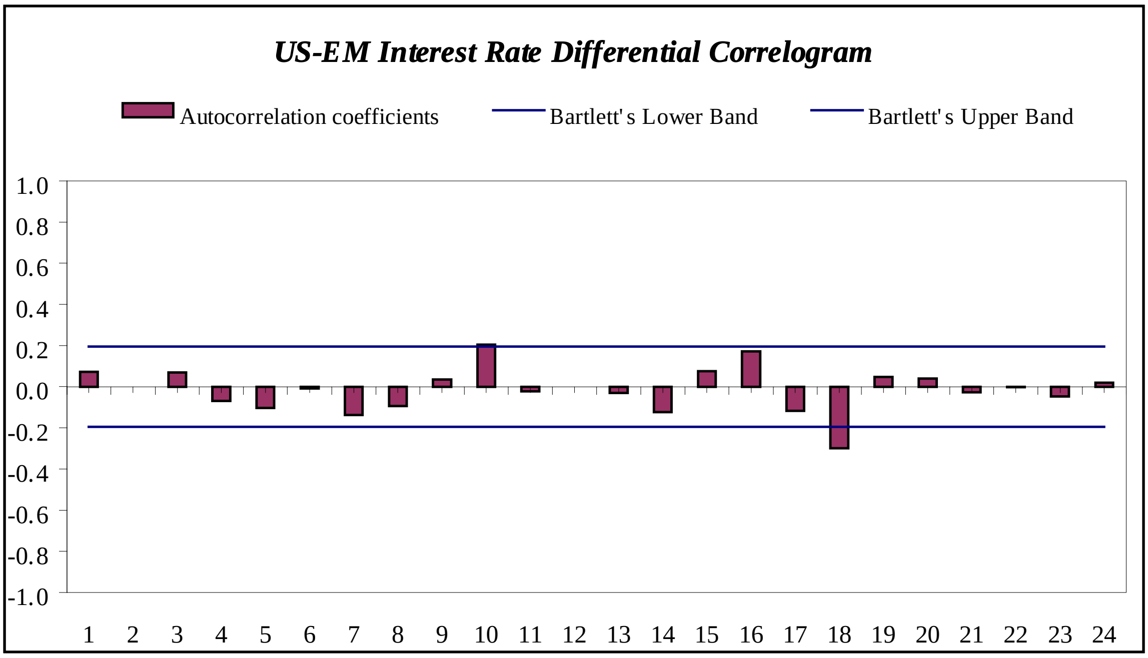



***Graph B10***

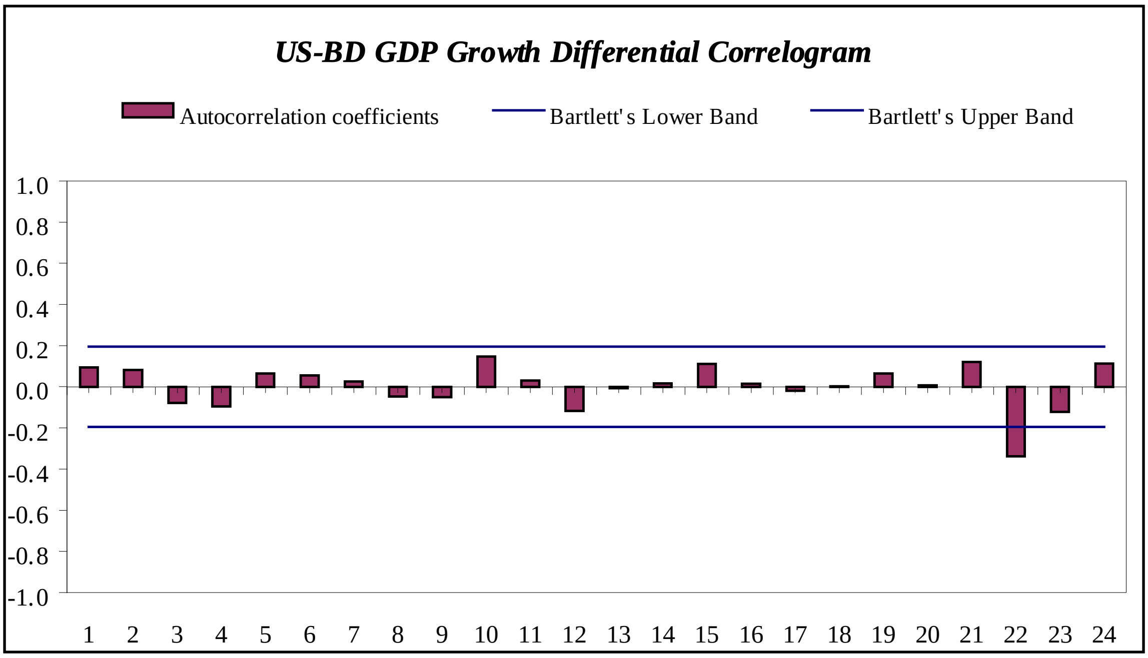
# *Residuals Autocorrelations*



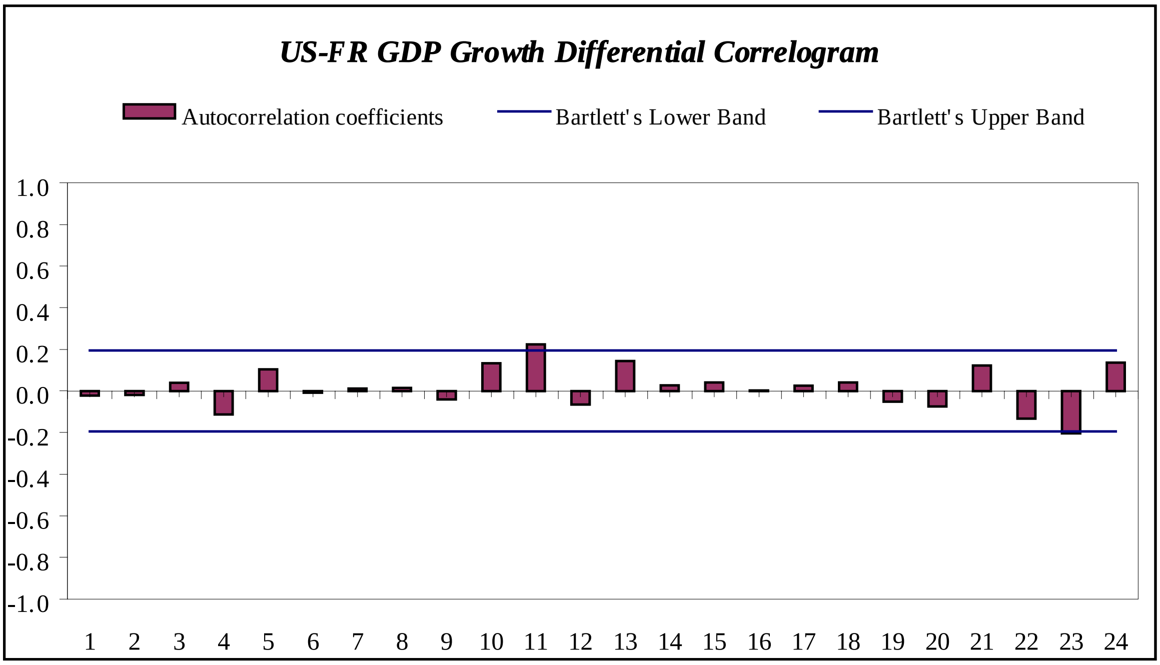

***Graph B11***



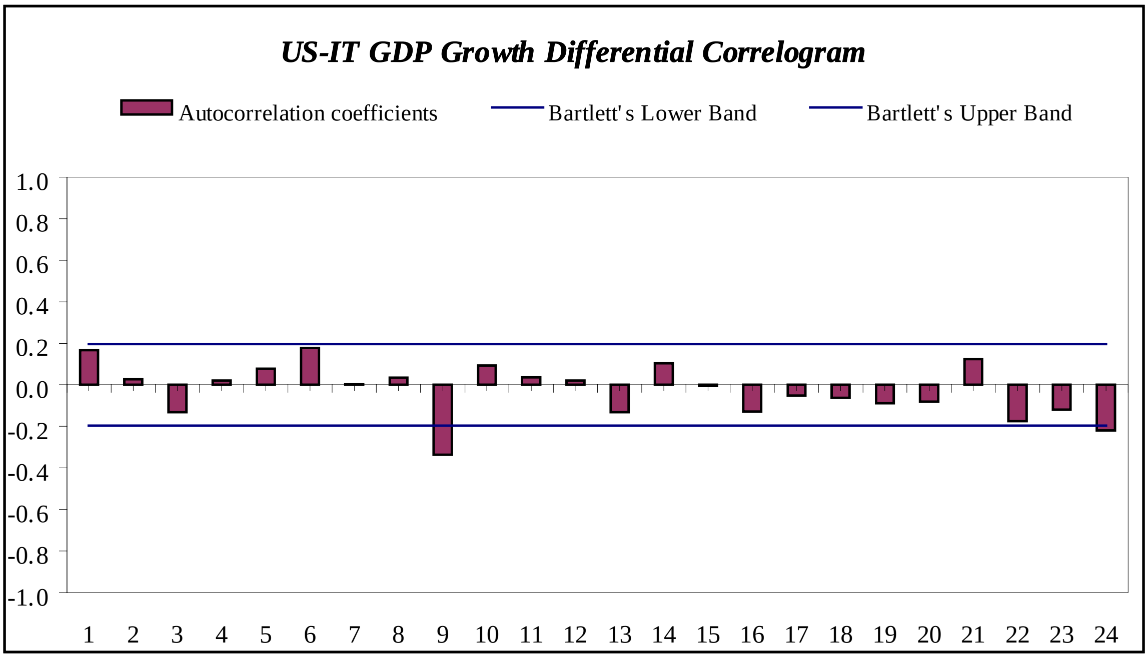

***Graph B12***




***Graph B13***

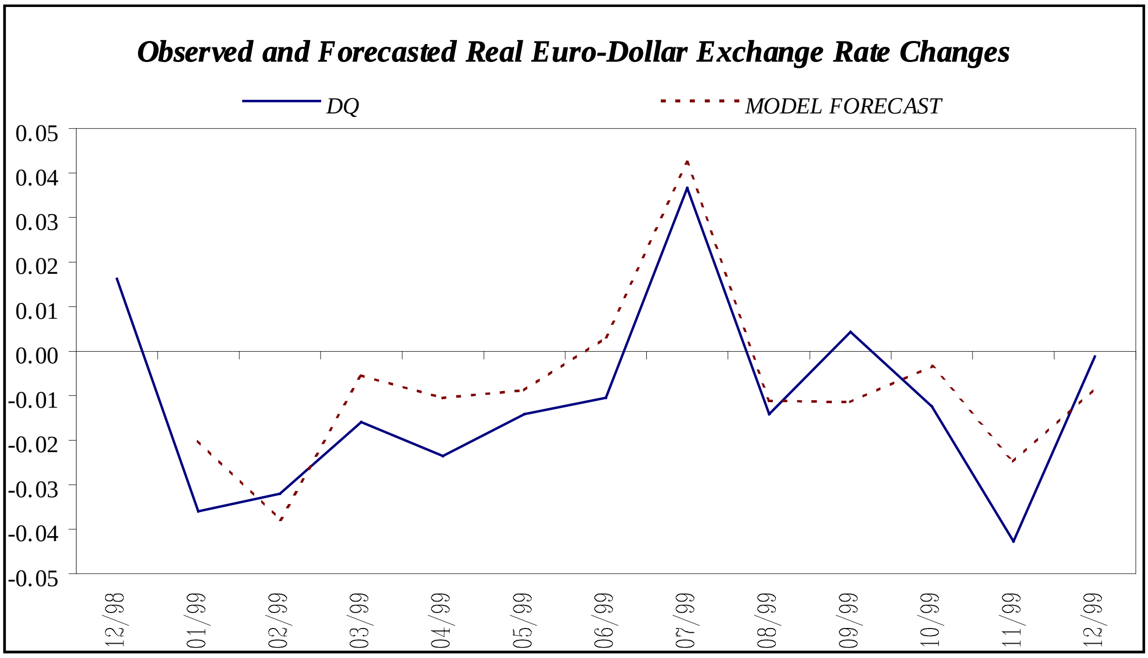



***Graph B14***

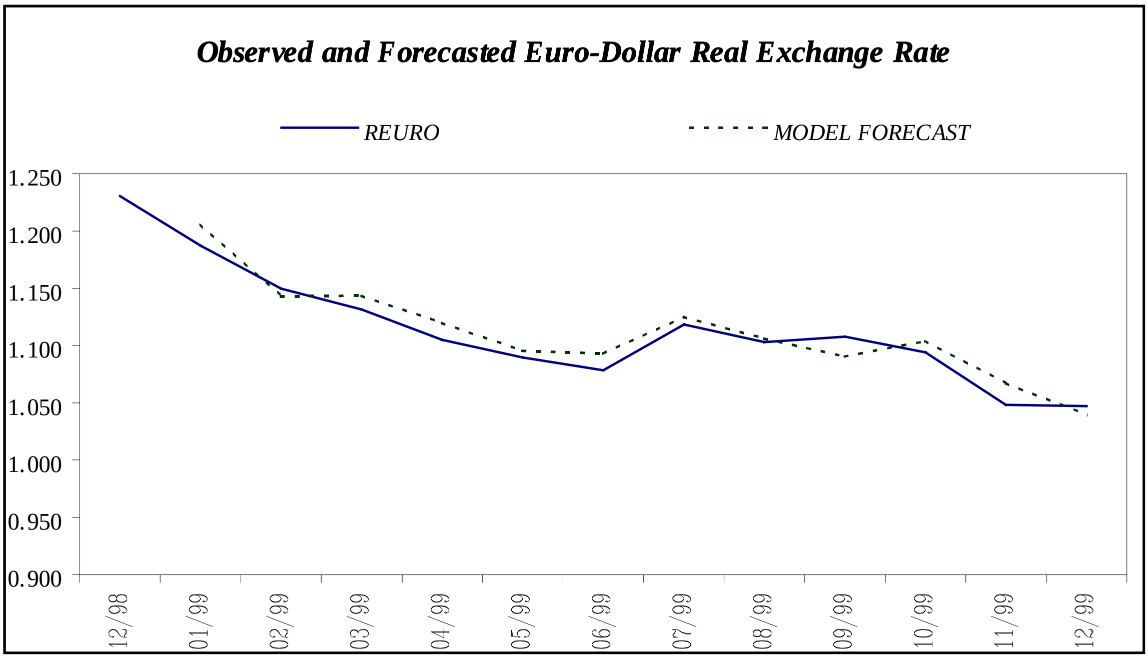

US-IT GDP Growth Differential Correlogram
Autocorrelation coefficients
Bartlett's Lower Band
Bartlett's Upper Band
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

***Graph B15***

## *Forecasting Performance*




***Graph B16***




***Graph B17***

# *Appendix C*

***Forecasts Comparison***

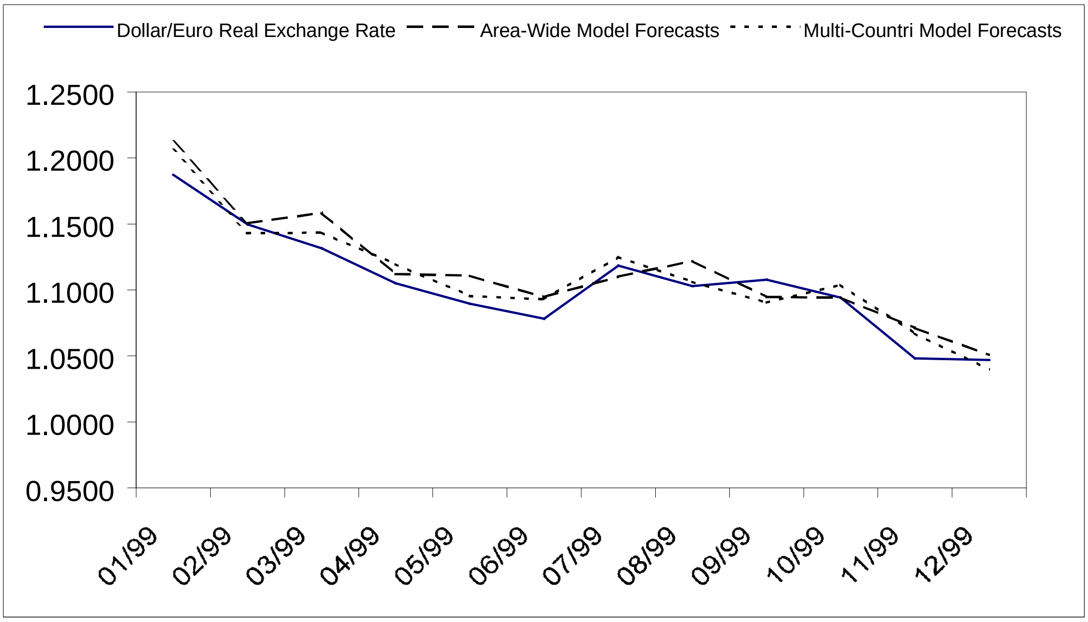
***Numerical Comparison of Area -Wide and Multi Country Models Forecasts***

***(Dollar/Euro Real Exchange Rate Level)***

| *Model* | *ME* | *RMSE* | *MAE* | *Theil's U Statistic* | *Signs Correctly Forecasted* |
|---|---|---|---|---|---|
| ***Multi Country Model*** | 0.0061 | 0.0125 | 0.0113 | 0.4599 [54.01%] | 83.33% |
| ***Area-Wide Model*** | 0.0102 | 0.0166 | 0.0138 | 0.6105 [38.95%] | 50.00% |
| ***Random Walk*** | 0.0153 | 0.0272 | 0.0228 | | |

***Table C1***

***Graphical Comparison of Area-Wide and Multy-Country Models Forecasts***




***Graph C1***