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Evaluation of behaviour of hybrid composite cable in the saddle-shaped roof

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ABSTRACT: Hybrid composite cable with increased, in comparison with CFCC, breaking elongation and decreased, in comparison with the steel cables, dead weight could be elaborated for prestressed cable net on the base of carbon and steel. Hybrid composite cable contains three layers: carbon fiber reinforced plastic (CFRP) core, glass fiber reinforced plastic (GFRP) distributional layer and steel wire strands. All layers of hybrid composite cable take up tension stresses, acting in the cable during exploitation. But GFRP has second function – distribution of transversal pressure of steel wire strands at CFRP core. The dependence of external pressure per unit of the surface area of the GFRP (due to the pressure of steel wire strands) of hybrid composite cable on the axial force and angle of steel wire strands twisting was estimated by the engineering method of calculations. Tangential and radial stresses for GFRP and CFRP components of hybrid composite cable were obtained. It was shown, that the angle of steel wire strands twisting should not exceed 20 degrees due to the ultimate strengths of GFRP and CFRP components in radial and tangential directions.

Opportunity to decrease the displacements of composite saddle-shaped cable roof by the using of cable trusses made of hybrid composite cable, as a supporting contour structure, was investigated.

1 INTRODUCTION

Hybrid composite cable with increased, in comparison with carbon fiber composite cable CFCC, breaking elongation and decreased, in comparison with the steel cables, dead weight could be elaborated for prestressed cable roofs on the base of carbon and steel. Hybrid composite cable contains three layers: carbon fiber reinforced plastic (CFRP) core, glass fiber reinforced plastic (GFRP) distributional layer and steel wire strands. All layers of hybrid composite cable take up tension stresses, acting in the cable during exploitation. But GFRP has second function – distribution of transversal pressure of steel wire strands at CFRP core. This pressure has significant value and should be taken into account during the cable design.

Perpendicular to the direction of axial force action pressure causes radial and tangential stresses in GFRP and CFRP.

Volume fractions of steel and carbon were evaluated basing on the assumption, that in an emergency, when the strain of carbon fiber exceeds the ultimate value and these fibers are disrupted, strands of steel wire must be able to take up significantly decreased tension stresses. The decrease of the tension forces, acting in the cable, is joined with the growing of deflection after excluding from the work of GFRP and CFRP components (Figure 1, stages 2 and 3).

Saddle-shaped cable roof is an example of the structures, where hybrid composite cable can be used. The saddle shaped cable roofs with the compliant supporting contour are structures where nearly all load-bearing elements are tensioned. It means that the modern high strength structural materials could be used in the full scale for the saddle shaped cable roofs supported by the tensioned cables. At the same time, the most significant disadvantage of the saddle-shaped cable roofs supported by tensioned cables is the increased compliance.

Basing on the just obtained results and the above mentioned information we can suppose, that the best method to decrease the displacements of the saddle shaped cable roof is to use the cable

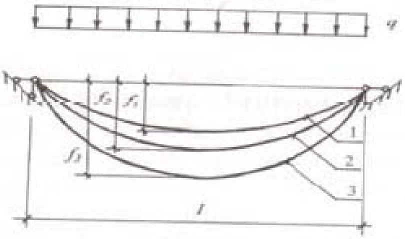

Figure 1. Scheme of hybrid composite cable work: 1 – steel wire, GFRP and CFRP work commonly; 2 – GFRP is excluded from the work; 3 – GFRP with CFRP are excluded from the work and steel wire works alone; q – design vertical load, acting at the cable; f_1 – deflection of the cable, which corresponds to the stage, when steel wire, GFRP and CFRP work commonly; f_2 – deflection, which corresponds to the stage, when GFRP is excluded from the work and steel wire works commonly with CFRP; f_3 – deflection, which is corresponds to the stage, when GFRP and CFRP are excluded from the work and steel wire works alone; l – span of the cable.

trusses made of the materials with the increased moduli of elasticity as structures of the supporting contour.

So, the purpose of this study is to evaluate perpendicular to the direction of axial force action pressure of steel wire strands at GFRP distributional layer and that of GFRP distributional layer at the CFRP core in the hybrid composite cable. Radial and tangential stresses in the GFRP and CFRP also should be determined and compared with compression strengths of the GFRP and CFRP. Effectiveness of cable truss application in supporting contour structure as a method to decrease the displacements of the composite saddle-shaped cable roof also should be evaluated. Rational geometrical characteristics of the cable truss shell be estimated.

2 EVALUATION OF MECHANICAL INTERACTION BETWEEN COMPONENTS IN HYBRID COMPOSITE CABLE

2.1 *Approach to the solution of the problem*

Hybrid composite cable is considered as a system of two cylinders (see Fig.2). Steel wire strands are replaced by the external pressure p_b per unit of external surface area of the GFRP distributional layer.

The GFRP distributional layer is considered as a hollow cylinder inside which another cylinder, i.e., CFRP core is situated. The GFRP distributional layer has constant internal and external radiuses: a and b, respectively. The CFRP core has constant external radius, which is equal to a.

Interaction between the GFRP distributional layer and CFRP core is considered as a pressure p_a at the unit of the surface area of the CFRP core or at the unit of internal surface area of the GFRP distributional layer.

Pressure p_b at the unit of external surface area of the CFRP distributional layer could be determined by the following equation:

$$p_b = -\frac{ntg^2\alpha}{2\pi aR}, \tag{1}$$

where n = part of axial force N, which takes up steel wire strands of the cable; α = angle of steel wire strands twisting; a = radius of the CFRP core; R = radius of the cable.

The equation (1) was obtained for the case, when GFRP distributional layer limits the displacements of the steel wire strands in the radial direction. Pressure p_a per unit of the surface area of the CFRP core and per unit of internal surface area of the GFRP distributional layer could be

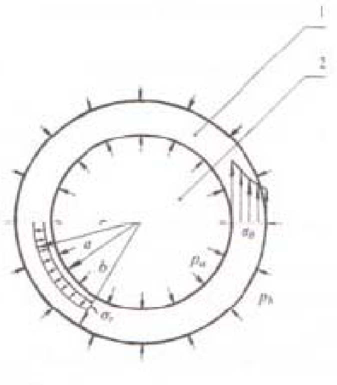

Figure 2. Scheme for determination of pressure at the CFRP core of hybrid composite cable: 1 – GFRP distributional layer; 2 – CFRPcore; σ_r – radial stresses; σ_θ – tangential stresses; p_b – external pressure per unit of the surface area of the GFRP (due to the pressure of steel wire strands); p_a – external pressure per unit of the surface area of the CFRP (due to the pressure of GFRP); a – radius of the CFRP core of the cable and internal radius of GFRP distributional layer; b – external radius of the GFRP distributional layer.

determined by the equation (2). The equation (2) is obtained due to the equal radial deformations of CFRP core and GFRP distributional layer:

$$\frac{1-\nu_{Grz}}{E_{Gr}}\frac{p_a a^2 - p_b b^2}{b^2 - a^2} r + \frac{1+\nu_{Grz}}{E_{Gr}}\frac{a^2 b^2 (p_b - p_a)}{(b^2 - a^2) r} + \delta_{Gr} = \frac{1-\nu_{Crz}}{E_{Cr}} p_a r + \delta_{Cr}, \tag{2}$$

where E_{Gr}, E_{Cr} = modulus of elasticity for GFRP and CFRP, respectively, in the radial directions; r = coordinate of the point, where deformations are determined; ν_{Grz} = Poisson's ratio of GFRP; ν_{Crz} = Poisson's ratio of CFRP; a = radius of the CFRP core of the cable and internal radius of GFRP distributional layer; b = external radius of the GFRP distributional layer; δ_{Gr} = radial deformations of GFRP due to the part of axial force, acting in the GFRP component of the cable; δ_{Cr} = radial deformations of CFRP due to the part of axial force, acting in the CFRP component of the cable.

The left and right parts of the equation (2) are radial deformations of GFRP and CFRP components, respectively, due to the pressures p_b and p_a. Radial deformations of GFRP and CFRP components δ_{Gr} and δ_{Cr} due to the parts of axial force, acting in the components also are taken into account. Values of radial deformations of GFRP and CFRP components δ_{Gr} and δ_{Cr} were determined basing on the consumption that components work in the elastic stage. Radial and tangential stresses act in the GFRP and CFRP due to the pressure p_b. The values of radial and tangential stresses could be determined by the equations, which were obtained for the cylinder with the hole in the center, which is loaded by uniformly distributed by the internal and external surfaces pressures p_a and p_b, respectively.

$$\sigma_{Gr} = \frac{a^2 b^2 (p_b - p_a)}{\left(b^2 - a^2\right)}\frac{1}{r^2} + \frac{p_a a^2 - p_b b^2}{b^2 - a^2}, \tag{3}$$

$$\sigma_{G\theta} = -\frac{a^2 b^2 (p_b - p_a)}{\left(b^2 - a^2\right)}\frac{1}{r^2} + \frac{p_a a^2 - p_b b^2}{b^2 - a^2}, \tag{4}$$

where σ_{Gr} and $\sigma_{G\theta}$ = stresses acting in the GFRP component of hybrid composite cable in the radial and tangential directions.

For determination of radial and tangential stresses acting in the CFRP component of the hybrid composite cable the following equation could be used:

$$\sigma_{Cr} = -p_a = \sigma_{C\theta}, \tag{5}$$

where: σ_{Cr} and $\sigma_{C\theta}$ = stresses acting in the GFRP component of hybrid composite cable in the radial and tangential directions.

Equation (5) was obtained from the equations (3) and (4) when the internal radius of the cylinder (CFRP core) is equal to zero, external radius of the cylinder is equal to a, and external pressure per unit of the surface area of the CFRP (due to the pressure of GFRP) is equal to p_a.

2.2 *Determination of pressures on the components of hybrid composite cables*

The dependence of pressure at the CFRP core of hybrid composite cable on the axial force N and angle of wire twisting α was developed by the example of tension cable of saddle shape cable roof with dimensions in plan 30×30 m.

Cable, which is loaded by the uniformly distributed load, is considered as a scheme for analysis. The cable has rational from the point of view of materials consumption initial deflection $f_1 = 5.7$ m. The uniformly distributed load with intensity $q = 21$ kN/m loads the cable. Mechanical properties of hybrid composite cable components are given in Table 1.

The values of moduli of elasticity correspond to the elastic stages of the materials work. Volume fractions of fibers in GFRP and CFRP are 0.6. The fibers are oriented in the direction of axial force action. Volume fractions of steel wire, GFRP and CFRP are 0.4; 0.2 and 0.4, respectively. Total area of cross sections for hybrid composite cable was equal to 0.00097 m^2.

The value of the axial force N, acting in the cable due to the uniformly distributed load, could be determined by the equations of cable calculation without taking into account elastic elongation of the cable.

The dependence of pressure p_a on the axial force N and angle of steel wire strands twisting α is shown in Figure 3.

Table 1. Mechanical properties of hybrid composite cable components.

Components of hybrid composite cable	E_z, MPa	ν_{zr}	R_{uz}, MPa	E_r, MPa	ν_{rz}	R_{tr}, MPa
Steel wire strand	130000	0.3	1568	–	–	–
GFRP	75000	0.19	1765	9200	0.05	78
CFCC	137000	0.3	1000	8670	0.014	186

In Table 1 R_{uz} are the limits of strengths of components in the direction Z; R_{tr} are the compression strengths of components in the radial direction.

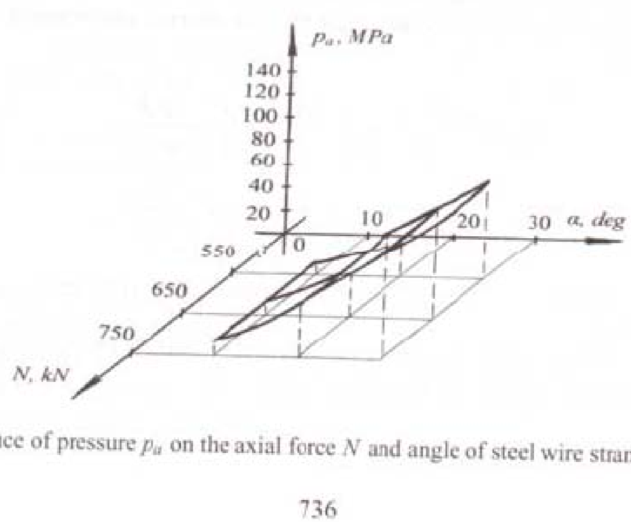

Figure 3. Dependence of pressure p_a on the axial force N and angle of steel wire strands twisting α.

The value of the axial force N, acting in the cable, changes within the limits of 550 to 750 kN. The following values of the axial forces are considered because with the growing of the axial force value exceeding 750 kN, the GFRP component of the cable is excluded from the work.

2.3 *Determination of radial and tangential stresses, acting in the components of hybrid composite cable*

The dependence of maximum radial σ_{Gr} and tangential stresses $\sigma_{G\theta}$, acting in the CFRP component of the hybrid composite cable on the axial force N is shown in Figure 4.

The values of the radial and tangential stresses are equal for the CFRP component of hybrid composite cable.

Comparison of the maximum radial stresses, acting in the CFRP components of the hybrid composite cable with their strengths shows, that the stresses are 1.65 times less, than the strength values, but the maximum value of tangential stresses are 13.79 times less, than the strength value. Maximum values of radial stresses were compared with the compression strength of CFRP in the direction perpendicular to the direction of fiber orientation, which is equal to 186 MPa. Tangential stresses were compared with the compression strength of CFRP in the direction corresponding to the direction of fiber orientation, which is equal to 1558 MPa.

The maximum values of radial σ_{Gr} and tangential $\sigma_{G\theta}$ stresses acting in the GFRP component of hybrid composite cable are given in Table 2 in depending on the angle of steel wire strands twisting α. The maximum values of radial and tangential stresses were obtained when the axial force N, acting in the cable, was equal to 750 kN and $r = a$.

Comparison of the maximum tangential and radial stresses stresses, acting in the GFRP components of the hybrid composite cable with their strengths shows, that the maximum angle of steel wire twisting for the considered case is 20 degrees, when the radial stresses are equal to 45.02 MPa, which are 1.73 times less than the strength value.

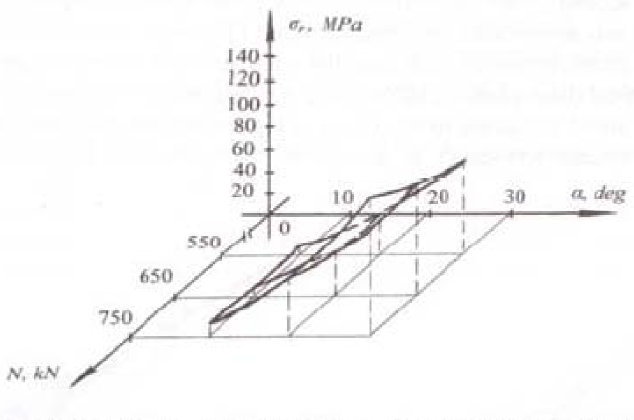

Figure 4. Dependence of the radial σ_{Gr} and tangential $\sigma_{G\theta}$ stresses acting in the CFRP on the pressure p_a and angle of steel wire strands twisting α.

Table 2. Maximum radial and tangential stresses, acting in the GFRP component of hybrid composite cable.

Angle of steel wire strands twisting α, degrees.	σ_{Gr}, MPa	$\sigma_{G\theta}$, MPa
10	10.77	21.29
20	45.02	44.85
30	114.81	93.32

3 DECREASE OF DISPLACEMENTS OF COMPOSITE SADDLE-SHAPED CABLE ROOF

A saddle-shaped cable roof 50 × 50 m in the plan was investigated. The existence of two symmetry planes allows us to regard, as a design scheme, a quarter of the cable net of a saddle-shaped cable roof with a compliant supporting contour in the shape of the cable truss, which is subjected to the prestressing and vertical design load (Figure 5).Three quarters of the cable roof are replaced by the bonds imposed on its one-quarter part. Hybrid composite cables with an elastic modulus of $1.32 \cdot 10^5$ MPa were taken as a material of cable truss elements, since they are most loaded ones. Relatively low elastic modulus of hybrid composite cable is joined with the necessity to use CFRP with the maximum limiting strains (1.6%). Steel cables with an elastic modulus of $1.3 \cdot 10^5$ MPa were assumed as a material for the suspension and stressing cables.

From the viewpoint of material consumption, the saddle shaped cable roof has rational geometrical characteristics: the initial deflection of the contour cables was 8.6 m, the initial deflections of suspension and stressing cables 20 m, and the step in plan of the latter ones was 1.414 m.

The structure was calculated for the basic combination of loads – the dead weight of the structure (0.27 kPa) and the weight of snow (1.12 kPa) – evenly distributed on the horizontal projection of the roof. The design load in the form of point wise forces was applied to the nodes of the cable net. The roof had the following layers: a glass net coated with polymer resin (2 mm), foam plastic, reinforced with a glass net (120 mm), and saddle-shaped plywood sheets (6 mm).

The cable net was prestressed by applying tension forces to the suspension and stressing cables, such that the residual tension forces in the stressing cables were equal to 20% of their initial values under the vertical design load.

The relations between the initial deflection of the top chord of the cable truss, distance between the nodes of the cable truss and the volume of the material of the cable net per unit of the covered area (relative volume) and maximum vertical displacement of the cable net were determined in the form of second power polynomial functions using the method of experimental design. The coefficients of the second power polynomial functions were found from the results of a numerical experiment, which was joined with the determination of forces in net cables, which are necessary to select the cable cross-section and calculate the relative volume of the material of the cable net and maximum vertical displacements of the cable net. The numerical experiment was conducted with the values of initial deflection of top chord of the cable truss, changing from 2.15 to 6.45 m and values of the distance between the nodes of cable truss, changing from 2.5 to7.5 m.

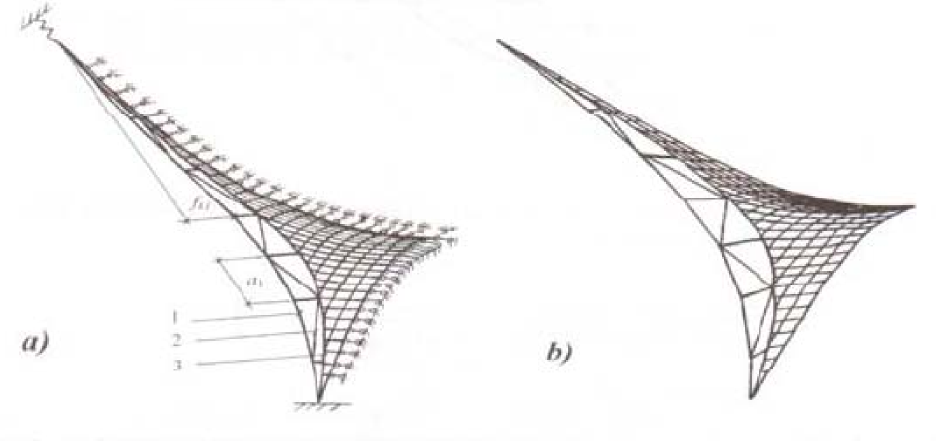

Figure 5. *a)* Quarter of cable net, supported by compliant supporting contour: 1 – top cable of supporting contour, 2 – tie-bar, 3 – bottom cable of supporting contour, f_{k1} – initial deflection of top chord of compliant supporting contour, a_l – distance between the support points of tie-bars. *b)* Shape of prestressed cable net after vertical design load application.

The area, covered by the roof was found with regard to the initial deflections of top chord of the cable truss. It means that the area, covered by the roof includes areas, which are covered by the cable trusses.

Using a computer program "ANSYS/ED 5.3" for WINDOWS the numerical experiment was carried out. The program enables to calculate values of the tension forces acting in the cables of the net and maximum vertical displacements of the cable net. In calculating a cable net, the program uses the Newton-Raphson iteration method, which consists of the division of the applied vertical design load into several parts in an ascending order. The cable net was modeled by finite elements of LINK10 type, with three degrees of freedom for each node. Each finite element was divided into two parts of the same length.

The judicious values of the basic geometrical characteristics of the cable truss were found from the system of equations:

$$\begin{cases} \zeta_0 + \zeta_1 a_1 + \zeta_2 f_{k1} = 0 \\ \chi_0 + \chi_1 a_1 + \chi_2 f_{k1} = 0 \end{cases} \tag{6}$$

where f_{k1} = initial deflections of top chord of the cable truss, a_l = distances between the nodes of the cable truss.

The first and second equation of the system was obtained by taking of partial derivations from the second power polynomial functions by the initial deflection of top chord of the cable truss and distance between the nodes of the cable truss respectively. Coefficients of the equations were equal to: ξ_0, ξ_1 and ξ_0 are equal to −0.1637, 0.5257 and 0.4682 respectively; χ_0, χ_1 and χ_2 are equal to 0.0705, 0.2710 and 0.5257 respectively.

The values of the initial deflection of top chord of the cable truss and distance between the nodes of the cable truss are equal to 3.13 and 7 m respectively.

Maximum vertical displacements of the cable roof for all combinations of the main geometrical characteristics of the cable truss were determined as a maximum difference in the vertical coordinate of the cable net nodes before and after application of design vertical load.

It was stated, that the minimum values of vertical displacements of cable net were obtained, when the initial deflection of top chord of cable truss were equal to 3.13 m and distance between the nodes of the cable truss was equal to 7 m.

Application of hybrid composite cable as a material of supporting contour instead of steel enables to decrease by 1.3% maximum vertical displacements of the cable net.

The using of the cable truss as a structure of support contour enables to decrease by 8% the maximum vertical displacements of the cable net in the case, when the tension cables of the net are hybrid composite cables, but suspension and stressing cables of the net are made of steel.

4 CONCLUSIONS

The dependence of external pressure per unit of the surface area of the GFRP (due to the pressure of steel wire strands) p_b of hybrid composite cable on the axial force N and angle of steel wire strands twisting α was obtained.

It was shown, that increasing of angle of wire twisting α from 10 to 30 degrees causes growing of external pressure per unit of the area of CFCC by 14.61 times when the axial force increases from the 550 to 750 kN.

Tangential and radial stresses for GFRP and CFCC components of hybrid composite cable were obtained. It was shown, that the maximum angle of steel wire strands twisting α is equal to 20 degrees for the considered hybrid composite cable.

It was shown, that maximum radial stresses σ_{Gr} acting in the GFRP component of hybrid composite cable, when the angle of the steel wire twisting α was equal to 20 degrees, and the axial force N was equal to 750 kN, was 1,73 times less than the strengths of GFRP.

Opportunity to decrease the displacements of composite saddle-shaped cable roof by the using of cable trusses as a supporting contour structure was investigated.

It was shown by the numerical experiment, that the rational initial deflections of top chord of the cable truss and distance between the nodes of the cable truss for the cable roof with dimensions in plan 50×50 m are equal to 3.13 and 7 m, respectively.

It was shown, that the using of cable truss as a structure of supporting contour enables to decrease by 8% the maximum vertical displacements of the cable net in the case, when the tension cables of the net are hybrid composite cables but suspension and stressing cables of the net are made of steel.

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