Impedance of nonlinear current or voltage dependent devices

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Abstract— many real devices such as batteries are nonlinear and their impedance values depend on either current or voltage. However, impedance analysis is based on the assumption of a linear system. If certain conditions such as sufficiently small excitation amplitude to ensure quasi-linearity and measurements at several operating points are fulfilled, also the impedance of a nonlinear system can be interpreted. Equations to calculate the large signal impedance from small signal impedance measurements of current, voltage and mixed dependent devices are derived and analyzed.

Keywords-component; nonlinear, large signal impedance, small signal impedance, negative resistance, negative capacitance, inductive semicircle, inductive behaviour

I. INTRODUCTION

Impedance spectroscopy is a powerful and widely used tool for the parameterization of simulation models. From a single impedance spectrum of a linear device, both the structure of its equivalent circuit and the parameter values can be extracted. For nonlinear devices, the same information can be obtained, but measurements at several operating points are necessary to obtain the complete behavior. Besides the dependency on the operating point, which means the bias current or voltage, the impedance of a nonlinear device also depends on the excitation amplitude. In order to guarantee quasi-linear conditions and thus usable spectra, the amplitude has to be kept small enough [1]. Furthermore it has to be considered that – in contrast to the linear case – the measured (small signal) impedance of a nonlinear device is not equal to the large signal impedance which is needed for a time domain simulation model.

There are several publications dealing with nonlinear circuits; already in the 1950s, 1960s and 1970s, researchers focused on the mathematical description of polarized electrodes ([2][3][4]), nonlinear capacitors ([5]) and nonlinear networks. ([6][7][8]). Those researchers had to face the problem that they could not employ their theory on a grand scale because of the limitations in computing power. The focus of these works was to calculate current and voltage behaviour of a circuit with given nonlinearities, while the focus in this work is to determine the nonlinearities from current and voltage measurements. More recent publications deal with this problem by the usage of impedance spectroscopy, but they typically only determine the small signal impedance in the frequency domain, e.g. [9][10][11].

In a previous paper by the authors, a procedure was developed to determine the large signal impedance of a voltage dependent device from its small signal impedance [12]. Starting from the differential equation of the current-voltage relationship, the equation is linearized after an AC and DC perturbation. The resulting AC part is transformed into the frequency domain giving differential equations for the small signal impedance elements. In this paper, the method is applied to current and mixed dependency.

The simulations presented in the following show a flipping of semicircles that is caused by the nonlinearities. In practice, similar spectra are measured for different systems. Especially if a semicircle switches from being capacitive to being inductive, this could be interpreted as a change of the equivalent circuit from a parallel connection of resistance and capacitance (capacitive semicircle) to a series connection of resistance and inductance (inductive semicircle). At first sight, this makes sense physically because then only positive equivalent circuit elements are used. However, we will show in this paper that it is not necessary to change the equivalent circuit if negative small signal resistances and capacitances are accepted.

II. IMPEDANCE OF CURRENT AND VOLTAGE DEPENDENT DEVICES

Many electrochemical processes can be described as a current or voltage dependent RC parallel circuit. Fig. 1 shows the equivalent circuit of a nonlinear RC circuit that is considered here.

The voltage-dependent large signal capacitance $C_{lsi}$ and resistance $R_{lsi}$ of a RC circuit can be calculated from equations 1 and 2 [12].

![Figure 1. RC circuit](image-url)

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III. SIMULATIONS OF CURRENT AND VOLTAGE DEPENDENT DEVICES

Fig. 2 shows the given voltage dependencies of the large signal parameters for the resistor and the capacitance and the resulting simulated impedance spectra of the RC circuit. It can be seen that for higher voltages the real part of the small signal impedance becomes negative although the large signal parameters are always positive in the considered voltage range. This behaviour can be explained by considering the equation to calculate the small signal conductance from the large signal conductance $G_{ssi} = 1/R_{ssi}$, which is the inverse of equation 2:

$$C_{ssi}(U_{DC}) = \text{Im}\left\{\frac{Y_{ssi}(U_{DC})}{\omega}\right\} = C_{ssi}(U_{DC})$$ (1)

$$R_{ssi}(U_{DC}) = \frac{U_{DC}}{\int \frac{Y_{ssi}(u)}{G_{ssi}(u)} du + c} = \frac{U_{DC}}{\int U_{DC} G_{ssi}(u) du + c}$$ (2)

where $Y_{ssi}$ is the measured small signal admittance, $\omega = 2\pi f$ is the frequency and $U_{DC}$ is the bias voltage of the operating point. The small signal capacitance $C_{ssi}$ is used as an abbreviation for $\text{Im}\{Y_{ssi}/\omega\}$ and the small signal conductance $G_{ssi}$ is used as an abbreviation for $\text{Re}\{Y_{ssi}\}$. Using the same procedure for a current dependent RC circuit gives similar equations:

$$C_{ssi}(I_{DC}) = \frac{C_{ssi}(I_{DC}) \cdot R_{ssi}(I_{DC})}{R_{ssi}(I_{DC})}$$ (3)

$$R_{ssi}(I_{DC}) = \int \frac{R_{ssi}(i)}{I_{DC}} di + c$$ (4)

where $R_{ssi} = 1/G_{ssi}$ is the small signal resistance of the circuit.
It can be seen that the small signal conductance depends on the derivative of the large signal conductance and if the first term of equation 5 becomes sufficiently negative, the small signal impedance becomes negative. This relationship is illustrated in Fig. 3: For voltages larger than 5 V, the slope of the large signal conductance is negative and above about 7 V, also the small signal conductance is negative.

Fig. 4 shows the simulated impedance spectra (left hand picture) for a given current dependency of $R$ and $C$ (right hand picture). Again both large signal parameters are always positive in the considered current range, but for high currents, both the real and the imaginary part become negative. Similarly to voltage dependency, this behaviour can be explained using the relationships between small and large signal impedance (inverse of equations 3 and 4):

$$G_{ssi}(U_{DC}) = \frac{dG_{li}(u)}{du} \cdot U_{DC} + G_{ss}(U_{DC})$$

(5)

$$C_{ssi}(I_{DC}) = \frac{C_{li}(I_{DC}) \cdot R_{li}(I_{DC})}{R_{ss}(I_{DC})}$$

(6)

$$R_{ssi}(I_{DC}) = \frac{dR_{li}(i)}{di} \cdot I_{DC} + R_{ss}(I_{DC})$$

(7)

Equation 7 shows that also the current dependent small signal resistance depends on the derivative of the large signal resistance and if it is sufficiently negative, the small signal resistance becomes negative. Since also the small signal capacitance (equation 6) depends on the small signal resistance, it becomes negative for the same current range. This is illustrated in Fig. 5.

IV. MIXED DEPENDENCY

Also the equations of mixed dependencies, e.g. $R(u)$ with $C(i)$ and $R(i)$ with $C(u)$, can be derived applying the same procedure. As in the cases of pure current or voltage dependency, the large signal resistance can be calculated independently from the capacitance according to equation 2 for voltage dependent resistance and equation 4 for current dependent resistance. However, the calculation of the large signal capacitance can depend on the large and small signal...
resistance. The equations for the large signal capacitance for all four cases of current or voltage dependencies are given in Table 1. It can be seen that the capacitance equation depends on the dependency of the resistance. If the resistance is voltage dependent, the large signal capacitance is equal to the small signal capacitance and if the resistance is current dependent, the large signal capacitance depends on the large and small signal resistance as well.

The equations were verified by simulated impedance spectroscopy of the four dependencies of RC circuits: A sinusoidal current with a fixed frequency is applied to the parallel connection of resistance and capacitance and the impedance is calculated from the FFT of current and voltage. This is repeated for a set of frequencies for each direct current offset. Fig. 6 shows the Simulink model used for simulation. For each of the four combinations, the large signal resistance is calculated according to equation 2 for voltage dependent resistance and equation 4 for current dependent resistance. Afterwards, the large signal capacitance is calculated with both equations 1 and 3 and the resulting curve is compared to the given characteristic. Fig. 7 shows the resulting curves for the two possibilities of mixed current and voltage dependency, verifying that the equations in Table 1 are correct.

V. CONCLUSION

Based on the procedure introduced in [12] to derive equations for the large signal impedance as a function of the small signal impedance, the equations for an RC parallel circuit with pure voltage, pure current or mixed voltage and current dependency of the elements have been derived and verified by simulations. It turned out that the equation to calculate the large signal capacitance depends on whether the resistance is current or voltage dependent – independent from the kind of dependency of the capacitance. These equations apply to any current or voltage dependent system. Most systems can be modelled both as current and voltage dependent, but typically one dependency is more convenient or makes more sense because of physical or chemical interpretation. According to the decision to model either current or voltage or mixed dependency, the corresponding equations have to be used for parameter extraction from measurement.

Simulations of RC circuits current and voltage dependent elements have shown that for suitable resistance characteristics, the resulting impedance spectra (small signal impedance) show apparently negative resistance semicircles for voltage dependency and apparently negative resistance and capacitance semicircles for current dependency, although both the large

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<th>Table 1. Capacitance Equations for All Four Combinations of Current and Voltage Dependent Resistance and Capacitance</th>
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<tr>
<td><strong>R(u)</strong></td>
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<td><strong>C(u)</strong></td>
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<td><strong>C(i)</strong></td>
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Figure 7. Simulink model for simulated impedance spectroscopy of parallel connection of current and voltage dependent resistance and capacitance

Figure 6. Comparison of possible equations to calculate the large signal capacitance from simulations of mixed dependent RC circuits. Left hand figure: \( R(u) \) with \( C(i) \), right hand figure: \( R(i) \) with \( C(u) \).
signal resistance and capacitance are positive at all times in the performed simulations. This behaviour can be explained mathematically from the corresponding equations for the small signal impedance. The inductive behaviour in the case of current dependent resistance can also be modelled with positive small signal resistance and inductance. However, if such behaviour is observed in measured spectra, it does not necessarily mean that the large signal impedance is also inductive, but a detailed analysis is needed. Before searching for a physical or chemical explanation for inductive behaviour, the large signal impedance should be calculated to find out if it is inductive as well. In the case of voltage dependent resistance, a negative small signal resistance can be observed. Negative large signal resistances are of course impossible for passive circuits, so if such behaviour is measured, it is most probably caused by the nonlinearities of the circuit or otherwise, active elements are present.

REFERENCES


