VALIDATION OF NUMERICAL SCHEMES AND TURBULENCE MODELS COMBINATIONS FOR TRANSIENT FLOW AROUND AIRFOIL

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ABSTRACT: In the present study, combinations of turbulence models and numerical schemes are evaluated in terms of accuracy and computational cost for the prediction of transient flow at fixed points around the non-symmetrical ONERA-A airfoil for pre- and stall conditions using a finite volume method. The computed results were validated by experimental data. The purpose is to investigate whether TVD discretization schemes, developed initially for compressible flows, can help computation of incompressible flows when accuracy and low computational cost are important considerations, as in the case of aeroelastic calculations. The TVD discretization schemes are applied to the convection terms of the momentum equations and they are compared to central difference, upwind, hybrid and QUICK discretization schemes for several turbulence models. From the parametric studies, it comes out that TVD schemes can be a promising tool for aeroelastic calculations.

Keywords: CFD, TVD discretization schemes, transition, turbulence models, airfoil separation/stall

1. INTRODUCTION

The flow around wind turbine blades is usually turbulent with the whole or part of the blade very often operating under stall conditions. This may be due either to sudden wind changes or to design needs to control the turbine power output. In such conditions, the aerodynamic behaviour of the blade is important because of the appearance of dynamic stall flutter, for which numerical predictions become difficult owing to inadequate physical modelling (transition, turbulence) combined with the strong non-linear behaviour of the flow. Numerous researchers have examined numerically and experimentally airfoil stall and its implication for the dynamics of wind turbine blades. For example, Ekaterinaris and Platzer (1997) reviewed the most common numerical schemes and turbulence and transition models, and compared the main computational results with the measurements. Chaviaropoulos (1999) discussed the effects of the aerodynamic loading on blade dynamics by studying the flap lead-lag aeroelastic stability of wind turbine blades. Voutsinas and Riziotis (1999) developed a viscous-inviscid interaction model for the simulation of dynamic stall on airfoils. Sarkar and Venkatraman (2005) used a discrete vortex method to simulate a heaving airfoil undergoing non-sinusoidal motions.

The performance of numerical schemes and turbulence models has also been studied for wing sections of aircraft. For example, Kral (1998) evaluated turbulence models for compressible flows in aircraft applications, such as the Baldwin-Lomax and Thomas algebraic models, the Baldwin-Barth and Spalart-Almaras one-equation models, five low Re models and the Menter SST blended k-ε/k-ω model. A zonal, upwind, implicit-factored algorithm was employed to solve both the mean turbulent flow, using a cell-centred, finite-volume method, and a generalized Roe’s upwind flux-difference splitted with an optional TVD operator. Zingg et al. (2000) compared the accuracy of several discretization schemes for subsonic and transonic airfoil flows. Their discretization for the inviscid fluxes included: a) a second-order accurate central differences with third-order matrix numerical dissipation, b) a second-order convective upstream split scheme, c) a third-order upwind-biased differencing with Roe’s flux-splitting, and d) a fourth-order central differences with third-order matrix numerical dissipation. Turbulence was modelled using the Baldwin-Lomax model and a far-field circulation correction at the outer boundary. Baxevanou and Vlachos (2004) have...
carried out a study similar to the present work, for the symmetrical NACA0015 airfoil using a finite-volume CFD model equipped with various numerical schemes and turbulence models. Li (2007) calculated pre- and post-stall aerodynamic characteristics for the symmetrical NACA64A006 airfoil applying a detached-Eddy simulation (DES) method. Although more accurate turbulence modelling techniques (DNS, LES, etc) are available today, one- and two-equation turbulence models based on the Boussinesq approximation are still a necessary choice for practical aeroelastic calculations (and unsteady calculations in general) owing to their lower computational cost.

TVD discretization schemes were initially designed for compressible flows and they have been used successfully for the prediction of compressible flow around airfoils (Rezgui, Cinnella and Lerat, 2001). In the present work, their efficiency in predicting incompressible flows was investigated. Various turbulence models and interpolation schemes were combined and assessed in terms of their ability to calculate the steady-state flow around an ONERA-A non-symmetrical airfoil, before and after separation, with forced transition from laminar to turbulent flow. The combinations were compared with the research findings released by ONERA (Piccin and Cassoudesalle, 1987) on the basis of their accuracy and rates of convergence. The intention is to identify the combinations that are more promising for wind turbine aeroelastic calculations. Although the present study deals with the ONERA airfoil only, it may indicate the expected behaviour for other non-symmetrical airfoils with similar characteristics.

2. GOVERNING EQUATIONS

2.1 Flow governing equations

The governing Navier-Stokes equations of the present incompressible flow are:

- Continuity
  \[ \frac{\partial U_j}{\partial x_j} = 0 \]  
  (1)

- Momentum
  \[ \rho \frac{\partial U_i}{\partial t} + \rho U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu + \mu_t \right) \frac{\partial U_i}{\partial x_j} \]  
  (2)

where the turbulent viscosity, \( \mu_t \), is calculated from the specific turbulence model.

2.2 Turbulence models

The effect of turbulence is implemented via the following turbulence models:

a) High Re k-\( \varepsilon \) model (standard)
   
   It is the most widely used model but it cannot predict flow separation accurately since it does not integrate properly up to the wall nor does it account for the effect of adverse pressure gradient on turbulence (Ferziger and Peric, 1996).

b) High Re k-\( \omega \) model (with wall functions)
   
   This model also does not integrate up to the wall, but it accounts for adverse pressure gradient effects (Wilcox, 1994) and thus, it is more suitable for airfoil flow predictions.

c) High Re k-\( \omega \) model (with wall conditions)
   
   This model integrates up to the wall, providing a specific value of \( \omega_w \) (Eq. 3) on the boundary (Wilcox, 1994) and it has been demonstrated as suitable for separated flows.

d) Low Re k-\( \omega \) model
   
   This model also integrates up to the wall and adjusts the constants in the region close to it (Wilcox, 1994). It requires a finer grid, but it is less demanding than the low Re k-\( \varepsilon \) model.

\[ \omega_w = \frac{u^2}{\nu} S_r \]  
(3)

where \( S_r = (50 / k^+_r)^2 \) for \( k^+_r < 25 \) or \( = 100 / k^+_r \) for \( k^+_r \geq 25 \), \( k^+_r = u_c k_r / \nu \).

2.3 Transition modelling

In the present study, the combination of turbulence and numerical results are evaluated for flow around a non-symmetrical airfoil with forced transition from laminar to turbulent flow at specific points. In order to simulate appropriately the transition, a forced transition model is adopted and incorporated in the CFD code. In this model, a “flag” which takes the value of zero for laminar and one for turbulent flow is adopted. This way the turbulence equations are solved for the entire field but their influence in the flow equations is taken into account only after the transition point, which is known in the present case (Howard, Alam and Sandham, 1999). By adopting a flag and a very low level of
free stream turbulence (0.5%), the problem of zero k and ε values for laminar calculations is overcome.

3. NUMERICAL DETAILS

3.1 Numerical model

For the present study, the CFD code CAFFA (Ferziger and Peric, 1996) was modified appropriately and extended further to include more numerical schemes and turbulence models (Baxevanou, 2004). It solves the Navier-Stokes equations for 2D incompressible flow, using finite volumes and structured, collocated, curvilinear, body-fitted grids. It uses the algorithm SIMPLE and the SIP method to solve the resulting systems of algebraic equations. Turbulence effects were implemented using the standard k-ε high Re model, the high Re k-ω model with wall functions or integration up to the wall (Wilcox, 1994), and the k-ω low Re model. Central differencing was used for the diffusion terms, and the seven different schemes for the convection terms are: a) upwind interpolation, b) central differencing, c) the hybrid scheme, d) the QUICK scheme (Hayase, Humphrey and Grief, 1992; Rahman, Miettinen and Siikonen, 1996), e) the Harten-Yee upwind TVD with five different limiters, f) the Roe-Sweby upwind TVD with three different limiters, and g) the Davis-Yee symmetric TVD with three different limiters. The first four schemes will not be discussed here since they are well documented in the literature. In addition, the CAFFA code was modified so that it can consider laminar flow up to a prescribed point on the airfoil surface and the turbulent flow beyond it and as a result, the physical experiment used for the validation could be better simulated. The 2D code allows a large number of turbulence models-discretization schemes combinations to be examined with an acceptable computational cost. So the present study could serve as a guide for a future 3D study since it identifies the inappropriate combinations and therefore, helps save on computational effort.

The measurements used for validation were in the form of aerodynamic coefficients or distribution of coefficients around the airfoil, not time-loops of aerodynamic properties. This means that they are time-averaged values in the regions of separation where the flow is inherently unsteady. So the measured and computed data have to be regarded as average values (Weber and Platzer, 2000).

3.2 Interpolation schemes

Hybrid numerical schemes should not be used for highly convective flows in regions of large angles to the grid and in separation zones because they lead to large numerical diffusion. The central difference and QUICK schemes work very well in regions with significant diffusion, but produce oscillations in regions where convection is dominant. In order to overcome these difficulties, a series of total variation diminishing (TVD) schemes were introduced into the model. In general, they are second-order accurate but may become first-order accurate in regions of numerical oscillations and thus securing convergence at the expense of reduced accuracy (Hirsch, 1990; Peyret, 1996). In the TVD scheme, the value of the variable at the surface of the computational cell is calculated from:

\[
\phi_e = (1 - f_c)\phi_p + f_c\phi_E - \frac{\phi_{\mid i+1,e}}{F_e} \quad (4)
\]

Three schemes were used for the calculation of the flux vector limiters (TVD limiters), as described below.

a) Harten-Yee upwind TVD scheme

Flux vector limiter:

\[
\phi_{i+\frac{1}{2}} = \frac{1}{2} \psi \left( \alpha_{i+\frac{1}{2}} \right) \left( G_i + G_{i+1} \right) - \frac{1}{2} \psi \left( \alpha_{i+\frac{1}{2}} + \beta_{i+\frac{1}{2}} \right) \delta_{i+\frac{1}{2}} \quad (5)
\]

\[
\delta_{i+\frac{1}{2}} = \begin{cases} 
X_{i+\frac{1}{2}} & \text{first row} \\
\phi_{i+1} - \phi_i & \text{otherwise}
\end{cases} \quad (6)
\]

\[
\beta_{i+\frac{1}{2}} = \frac{1}{2} \psi \left( \alpha_{i+\frac{1}{2}} \right) \begin{cases} 
G_{i+1} - G_i & \text{if } \delta_{i+\frac{1}{2}} \neq 0 \\
0 & \text{if } \delta_{i+\frac{1}{2}} = 0
\end{cases} \quad (7)
\]

The limiters proposed for the specific scheme are the following:

(i) if \( G_i = \text{mod} \left( \delta_{i-\frac{1}{2}} + \delta_{i+\frac{1}{2}} \right) \neq 0 \),

\[
G_{i+1} = 0 \quad (8a)
\]

\[
G_{i+1} = \text{mod} \left( \delta_{i+\frac{1}{2}} - \delta_{i+\frac{3}{2}} \right) \quad (8b)
\]

where \( \delta_{i-\frac{1}{2}} = (\phi_i - \phi_{i-1}) \) and \( \delta_{i+\frac{1}{2}} = (\phi_{i+2} - \phi_{i+1}) \).
(ii) \( G_i = \frac{\delta_i \cdot \delta_{i+\frac{1}{2}} + \delta_{i+\frac{1}{2}} \cdot \delta_i}{\delta_i + \delta_{i+\frac{1}{2}}} \) if \( \delta_i \cdot \delta_{i+\frac{1}{2}} \neq 0 \), otherwise \( G_i = 0 \) \hfill (9a)

\[
G_{i+1} = \frac{\delta_{i+\frac{1}{2}} \cdot \delta_i + \delta_i \cdot \delta_{i+\frac{1}{2}}}{\delta_i + \delta_{i+\frac{1}{2}}} \text{ if } \delta_{i+\frac{1}{2}} \cdot \delta_i \neq 0 , \text{ otherwise } G_{i+1} = 0 \hfill (9b)
\]

(iii) \( G_j = \frac{\delta_{i+\frac{1}{2}} \left( \delta_i^2 + \omega \right) + \delta_i \left( \delta_{i+\frac{1}{2}}^2 + \omega \right)}{2} + 2\omega \) \hfill (10a)

\[
G_{i+1} = \frac{\delta_{i+\frac{1}{2}} \left( \delta_i^2 + \omega \right) + \delta_i \left( \delta_{i+\frac{1}{2}}^2 + \omega \right)}{2} + 2\omega \hfill (10b)
\]

where \( 10^{-7} \leq \omega \leq 10^{-5} \).

(iv) \( G_i = \min \left( 2\delta_{i+\frac{1}{2}}, 2\delta_i, 2\delta_{i+\frac{1}{2}} \right) \)

\[
G_{i+1} = \min \left( 2\delta_{i+\frac{1}{2}}, 2\delta_i, 2\delta_{i+\frac{1}{2}} \right) \text{ if } \delta_{i+\frac{1}{2}} \neq 0 , \text{ otherwise } G_{i+1} = 0 \hfill (11b)
\]

(v) \( G_i = \frac{\left| \delta_{i+\frac{1}{2}} \right|}{\delta_{i+\frac{1}{2}}} \max \left( 0, \min \left( 2\delta_i, \frac{\left| \delta_{i+\frac{1}{2}} \right|}{\delta_{i+\frac{1}{2}}} \cdot \delta_{i+\frac{1}{2}} \right) \right) \)

\[
G_{i+1} = \frac{\left| \delta_{i+\frac{1}{2}} \right|}{\delta_{i+\frac{1}{2}}} \max \left( 0, \min \left( 2\delta_i, \frac{\left| \delta_{i+\frac{1}{2}} \right|}{\delta_{i+\frac{1}{2}}} \cdot \delta_{i+\frac{1}{2}} \right) \right) \text{ if } \delta_{i+\frac{1}{2}} \neq 0 , \text{ otherwise } G_{i+1} = 0 \hfill (12b)
\]

\[\phi_i^{n+1} = \alpha_i^{n+1} \left( 1 - G_i \right) \delta_i^{n+1} \]

b) Roe-Sweby upwind TVD scheme

\[\phi_{i+\frac{1}{2}} = \alpha_{i+\frac{1}{2}} \left( 1 - G_{i+\frac{1}{2}} \right) \delta_{i+\frac{1}{2}} \]

with the following limiters:

(i) \( G_i = \min \left[ 1, r \right] = \max \left[ 0, \min \left( 1, r \right) \right] \)

(ii) \( G_i = r + \frac{\left| r \right|}{1 + r} \)

(iii) \( G_i = \frac{1}{3} \max \left[ 0, \min \left( 4, r \right) \right] + \frac{2}{3} \max \left[ 0, \min \left( 1, 4r \right) \right] \)

\[
\phi_{i+\frac{1}{2}} = -\left( \psi \left( \alpha_{i+\frac{1}{2}} \right) \left( \delta_{i+\frac{1}{2}} - G_{i+\frac{1}{2}} \right) \right) \]

with the following limiters:

(i) \( G_{i+\frac{1}{2}} = \min \left[ 2\delta_{i+\frac{1}{2}}, 2\delta_i, 2\delta_{i+\frac{1}{2}} \right] \)

(ii) \( G_{i+\frac{1}{2}} = \min \left[ \delta_{i+\frac{1}{2}}, \delta_i, \delta_{i+\frac{1}{2}} \right] \)

(iii) \( G_{i+\frac{1}{2}} = \min \left[ \delta_{i+\frac{1}{2}}, \delta_{i+\frac{1}{2}}, \delta_{i+\frac{1}{2}} \right] \)

3.3 Computational grids

The cases of air flow around the ONERA-A airfoil (Piccin and Cassoudesalle, 1987) at \( \text{Re} = 2.07, 3.13, 5.25 \) and 7.54 \(( \times 10^6 \) for angles of attack of 8° and
were considered as reference cases for the present simulations. Piccin and Cassoudesalle provided measurements of distributions of $C_p$ and $C_f$, and the values of $C_L$ and $C_D$ for angles of attack from $3^\circ$ to $20^\circ$. Two different computational grids were employed. The first was used for turbulence models with wall functions and the second for those integrating up to the wall. The grids were C-type with boundaries at 20 chords from the airfoil (Weber and Platzer, 2000), constructed carefully in order to handle the extreme cases of minimum or maximum friction coefficient. Wall boundary conditions were applied on the airfoil surface. On the C outer part of the grid, on the boundary cells where applied specific values of the calculated variables, while at the normal outer part a zero pressure gradient normal to the boundary was assumed. The experimental results for $Re = 2.05 \times 10^6$ gave a maximum friction coefficient $C_f = 1.53 \times 10^{-2}$ at the leading edge for an angle of attack of $14.5^\circ$, and $1.86 \times 10^{-2}$ at the trailing edge for $18^\circ$. Based on these values, the grid requirements related to the wall distance ($y^+$) were selected as follows:

**Grid-1**: A computational grid size of $451 \times 61$ (=27511 cells) with 371 computational cells on the airfoil surface was used for the standard high $Re$ $k-\varepsilon$ turbulence model and the high $Re$ $k-\omega$ model with wall functions. The grid was constructed with the first computational point at a distance of $10 < y^+ < 40$ from the airfoil surface (Sankar, Sayer and Fraser, 1997).

**Grid-2**: A computational grid size of $450 \times 91$ (=40950 cells) with 390 computational cells on the airfoil surface was used for the $k-\omega$ models that integrate up to the wall (high $Re$ $k-\omega$ model with wall conditions and low $Re$ $k-\omega$ model). In accordance with the study of Wilcox (1993), the grid was constructed with the first computational point at a non-dimensional distance of $y^+ < 2.5$.

For this grid, a special problem arises at the leading edge where the flow is laminar and the requirements for the distance of the first computational point from the wall are closer to those of wall functions grid configuration. In the area of the expected transition, the final grid is a combination of Grid-1 and Grid-2, which are smoothed appropriately. The smoothing area covers 12% of the chord length on the pressure side and the 5% on the suction side. Thus, numerical oscillations due to high gradients in the transition points are avoided.

In all the simulations, an upwind interpolation scheme was used for the convective terms of the turbulence transport equations and a central difference scheme for the diffusion terms. The results will be compared on the basis of the most appropriate interpolation scheme (for the convective terms of the momentum equations) for each turbulence model.

4. AVAILABLE EXPERIMENTAL DATA

Detailed experimental data for the cases considered here were provided by Piccin and Cassoudesalle (1987). The flow around the ONERA-A airfoil has been systematically measured at the F1 and the F2 wind tunnels of ONERA/FAUGA. The experiments were conducted in a wind tunnel with a free-stream Mach number was low enough ($Ma=0.15$) to avoid compressibility effects and thus, the incompressible flow assumption made here is valid. The flow was tripped on both the suction and pressure sides of the airfoil and the transition point on the suction side was fixed at a distance of 15% of the chord from the leading edge while on the pressure side, at 30%, i.e., before natural transition would have appeared. The airfoil geometry and grid are given in Fig. 1. The maximum thickness to chord ratio is 0.16.

![Fig. 1 Geometry and grid of airfoil ONERA-A.](image)

5. RESULTS AND DISCUSSION

5.1 Initial choice of interpolation scheme

The four turbulence models and the fifteen combinations of numerical schemes and limiters were compared in terms of their accuracy and
convergence against the experiments of Piccin and Cassoudesalle (1987). For the validation of the interpolation scheme, four indicative cases were examined corresponding to \( \text{Re}=2.07 \times 10^6 \) and \( 5.25 \times 10^6 \) and angles of attack of 8° and 13°. Thus, for each turbulence model, sixty simulations were performed in order to account for all interpolation schemes for the velocities.

a) High Re k-\( \varepsilon \) model (standard)

With respect to the distribution of \( C_p \) and the values of \( C_L \) and \( C_D \), the simulations with the upwind and hybrid schemes and the Harten-Yee upwind TVD scheme with the limiter (ii) were the worst for the two Reynolds numbers and angles of attack used. The simulations with the QUICK scheme were the slowest to converge (10 times more iterations than the other schemes) giving modest results in terms of accuracy. For \( \text{Re}=5.25 \times 10^6 \), the simulations with the Harten-Yee upwind TVD scheme with limiter (iii) and central differences did not converge.

For the rest of the schemes, the simulations with the Davis-Yee symmetric TVD scheme with limiters (ii) and (iii) are the fastest in converging. The simulations using the Roe-Sweby scheme with the minmod limiter and the Harten-Yee upwind scheme with limiter (i) gave the best predictions for \( C_D \) and \( C_L \). Overall, the best behaviour (high accuracy with fast convergence) was obtained from the Roe-Sweby scheme combined with the minmod limiter. Thus, for the rest of this work, this scheme is considered as the best choice for the discretization of the convective terms of the momentum equations when the k-\( \varepsilon \) high Reynolds turbulent model is used.

b) High Re k-\( \omega \) model with wall functions

With respect to the prediction of \( C_p \), \( C_L \) and \( C_D \), the simulations with the upwind and hybrid schemes and the Harten-Yee upwind TVD scheme with the limiter (ii) were the worst, for all \( \text{Re} \) and angles of attack. The simulations with the QUICK scheme, the Roe-Sweby scheme with the minmod limiter, the Harten-Yee Upwind TVD scheme with limiters (i) and (iii), the Davis-Yee scheme with limiters (i) and (iii), and central differences did not converge in most cases.

The best prediction of \( C_D \) was obtained from the Roe-Sweby scheme combined with the Chakravarthy and Osher limiter, and that of \( C_L \) was from the Harten-Yee upwind scheme with limiter (v) and the Davis-Yee scheme with limiter (iv). Overall, the best behaviour was obtained by using the Roe-Sweby scheme with the Chakravarthy and Osher limiter or the Harten-Yee scheme with limiter (v). The next best overall prediction was obtained from the Harten-Yee scheme with limiter (iv). The last three schemes presented similar convergence behaviour, affordable for unsteady calculations. For the rest of this work, the last scheme will be used for the discretization of the convective terms of the momentum equations when the k-\( \omega \) high Re turbulence model with integration up to the wall (wall conditions) is used. Considering that this turbulence model can give results close to those of the Low Re k-\( \omega \) model, this choice allows us to investigate the effect of a discretization scheme which is not the most accurate on the overall behaviour.

c) High Re k-\( \omega \) model with integration up to the wall

With respect to the distribution of \( C_p \) and the values of \( C_L \) and \( C_D \), the simulations with the upwind and hybrid schemes and the Harten-Yee upwind TVD scheme with limiter (ii) were the worst for all Reynolds numbers and angles of attack. The simulations with the QUICK scheme, the Roe-Sweby scheme with the minmod limiter, the Harten-Yee Upwind TVD scheme with limiters (i) and (iii), the Davis-Yee scheme with limiters (i) and (iii), and central differences did not converge in most cases.

The best prediction of \( C_D \) was obtained from the Harten-Yee upwind scheme with limiter (v), which will be used for the discretization of the convective terms of the momentum equations when using the k-\( \omega \) high Reynolds turbulent model with wall function for the rest of this study.

d) Low Re k-\( \omega \) model

With respect to the distribution of \( C_p \) and the values of \( C_L \) and \( C_D \), the simulations with the upwind and
hybrid schemes and the Hatren-Yee upwind TVD scheme with limiter (ii) were the worst for all Re and angles of attack. The simulations with the QUICK, the Roe-Sweby scheme with the minmod limiter, the Harten-Yee upwind TVD scheme with limiters (i) and (iii), the Davis-Yee scheme with limiters (i) and (iii), and central differences did not converge in most cases. The best prediction of $C_D$ is obtained from the Roe-Sweby scheme combined with the Chakravarthy and Osher limiters and of that of $C_L$ was from the Harten-Yee Upwind scheme with limiter (iv). Overall, the best performance was obtained by using the Roe-Sweby scheme with the Chakravarthy and Osher limiters and the Harten-Yee scheme with limiter (iv) since they are characterised by fair convergence rates. For the rest of this work, the Roe-Sweby scheme with the Chakravarthy and Osher limiters will be used for the discretization of the convective terms of the momentum equations with the k-$\omega$ low Re model. The matrix of simulations described above are summarized in Table 1. The calculated values of $C_L$ and $C_D$ for all turbulence models and the chosen interpolation schemes are summarized in Table 2 for Re=$2.07 \times 10^6$ and an angle of attack of 8.1°.

### Table 1 Summary of simulations for the initial choice of discretization scheme.

<table>
<thead>
<tr>
<th>Discretization schemes</th>
<th>Turbulence models</th>
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<tbody>
<tr>
<td></td>
<td>High Re k-\varepsilon (standard)</td>
</tr>
<tr>
<td></td>
<td>High Re k-\omega With wall functions</td>
</tr>
<tr>
<td></td>
<td>High Re k-\omega With wall conditions</td>
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<tr>
<td></td>
<td>Low Re k-\omega</td>
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<tr>
<td>Upwind</td>
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<td>Hybrid</td>
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<tr>
<td>Central Differences</td>
<td></td>
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<tr>
<td>QUICK</td>
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</tr>
<tr>
<td>Harten-Yee Upwind TVD</td>
<td></td>
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<tr>
<td>minmod limiter, limiters (ii) to (v)</td>
<td></td>
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<tr>
<td>Roe-Sweby Upwind TVD</td>
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<tr>
<td>minmod limiter, Van Leer limiter,</td>
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<tr>
<td>Chakravarthy-Osher limiter</td>
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<tr>
<td>Davis-Yee Symmetric TVD</td>
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<tr>
<td>limiters (i) to (iii)</td>
<td></td>
</tr>
</tbody>
</table>

$\alpha=8^\circ, 13^\circ \ & \ Re=2.07, 5.25 \times 10^6$

(for all combinations of discretization schemes and turbulence models)

### Table 2 Experimental versus calculated $C_L$, $C_D$ for Re=$2.07 \times 10^6$ and $\alpha=8^\circ$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$C_L$ (calculated)</th>
<th>$C_L$ (exper.)</th>
<th>Diff. (%)</th>
<th>$C_D$ (calculated)</th>
<th>$C_D$ (exper.)</th>
<th>Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Re k-\varepsilon model –</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Roe-Sweby scheme &amp; minmod limiter</td>
<td>1.110839</td>
<td>1.09</td>
<td>1.91</td>
<td>0.015085</td>
<td>0.0139</td>
<td>8.53</td>
</tr>
<tr>
<td>High Re k-\omega model with wall functions –</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harten-Yee &amp; limiter (v)</td>
<td>1.088732</td>
<td>1.09</td>
<td>0.12</td>
<td>0.016897</td>
<td>0.0139</td>
<td>21.56</td>
</tr>
<tr>
<td>High Re k-\omega model with wall functions –</td>
<td></td>
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<td>Roe-Sweby scheme &amp; Chakravarthy-Osher limiter</td>
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<td>0.017446</td>
<td>0.0139</td>
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<td>0.67</td>
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<tr>
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<td>0.68</td>
<td>0.015203</td>
<td>0.0139</td>
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<tr>
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<td>0.014412</td>
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5.2 Grid independence tests

After determining the best interpolation scheme for each turbulence model with a sufficiently large grid, grid independence tests were carried out in order to confirm it. For the turbulence models with wall functions, two more grids were used, one coarser and one finer than the “normal” (451x61) grid described earlier. The coarser grid had 321x61 (=19581 cells) with 241 computational cells on the airfoil surface and the finer 561x61 (=34221 cells) with 481 cells on the surface. Two more grids were also used for the turbulence models that integrate up to the wall, one coarser and one finer than the normal. The coarser grid had 320x91 (=29120 cells) with 260 computational cells on the airfoil surface and the finer 560x91 (=50960 cells) with 500 cells on airfoil surface. In all cases, the normal distance to the airfoil surface was selected to satisfy the y' requirement. The results of the test cases are summarized in Table 3. They correspond to higher values of percentage deviations between the normal and finer grids for two reference angles of attack (8° and 13°) and two Reynolds numbers (2.07x10^6 and 5.25x10^6). From the test results it comes out that the “normal” grids can be used safely for the calculations.

<table>
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<tr>
<th>Case</th>
<th>dC_L [%]</th>
<th>dC_D [%]</th>
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<tr>
<td>High Re k-ε model</td>
<td>0.06</td>
<td>0.66</td>
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<td>High Re k-ω model with wall functions</td>
<td>0.17</td>
<td>1.55</td>
</tr>
<tr>
<td>High Re k-ω model with wall conditions</td>
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<td>0.51</td>
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<tr>
<td>Low Re k-ω model</td>
<td>0.09</td>
<td>1.36</td>
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</table>

Table 3 Grid independence tests.

Fig. 2 Distribution of C_L for Re=2.07x10^6, 3.13x10^6, 5.25x10^6 and 7.54x10^6.
5.3 Comparison of turbulence models

In Fig. 2(a–d), the predicted values of $C_L$ versus angle of attack are shown for $Re=2.07, 3.13, 5.25$ and $7.54 \times 10^6$, respectively, for the four turbulence models used with the chosen interpolation schemes for each model. The distribution of $C_L$ shows that for low angles of attack, all the models behave well. When separation begins, the high Re k-$\varepsilon$ model fails to predict the maximum value of $C_L$ while the other models behave better and produce similar results with the low Re k-$\omega$ giving the closest-to-experiment estimation of the separation point for all Re examined.

In Fig. 3(a–d), the distribution of $C_D$ versus angle of attack is shown for the same Reynolds numbers as above. These distributions show that the best prediction of $C_D$ is obtained from the low Re k-$\omega$ model while the high Re k-$\varepsilon$ model gives equally good estimations at low angles of attack. The two high Reynolds k-$\omega$ models give worse but similar results, indicating that a less accurate interpolation scheme can cancel the effect of a more accurate turbulence model.

In Fig. 4(a–d), the distribution of $C_f$ versus x/c position on the upper airfoil surface is shown, as calculated using the four turbulence models for an angle of attack of $8^\circ$ for the above four Reynolds numbers. Close to the point of triggering turbulence, the best predictions are obtained from the models that use wall functions, while close to the trailing edge, the models that integrate up to the wall give better results.

Fig. 5(a–d) gives the $C_p$ distribution around the airfoil for the same angle of attack. It can be observed that the four turbulence models behave similarly.

![Fig. 3 Distribution of $C_D$ for $Re=2.07 \times 10^6, 3.13 \times 10^6, 5.25 \times 10^6$ and $7.54 \times 10^6$.](image-url)
Fig. 4    Distribution of $C_f$ on the airfoil surface for $\alpha=8^\circ$ and $Re=2.05 \times 10^6$, $3.13 \times 10^6$, $5.25 \times 10^6$ and $7.54 \times 10^6$.

Fig. 5    Distribution of $C_p$ for $\alpha=8^\circ$ and $Re=2.05 \times 10^6$, $3.13 \times 10^6$, $5.25 \times 10^6$ and $7.54 \times 10^6$. 
Fig. 6 Distribution of $C_F$ on the upper surface for $\alpha = 16^\circ$ and $Re = 2.05 \times 10^6$, $3.13 \times 10^6$, $5.25 \times 10^6$ and $7.54 \times 10^6$.

Fig. 7 Distribution of $C_p$ for $\alpha = 16^\circ$ and $Re = 2.05 \times 10^6$, $3.13 \times 10^6$, $5.25 \times 10^6$ and $7.54 \times 10^6$. 

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In Fig. 6(a–d) the distribution of $C_f$ versus $x/c$ position on the upper airfoil surface is shown, as calculated with the same turbulence models and Reynolds numbers and an angle of attack of 16°. For this angle of attack, when separation begins, the models that integrate up to the wall give better predictions on the whole airfoil surface. Similarly, in Fig. 7(a–d) where the $C_p$ distribution around the airfoil is given, it is observed that the $k-\omega$ model gives better predictions than the $k-\varepsilon$ model does. It should be noted, as far as convergence is concerned, that low Re $k-\omega$ model gives slower convergence and incurs a high computational cost as a result of a denser grid requirement, making the model less attractive for unsteady calculations.

5.4 Unsteady calculations

Fig. 8(a & b) gives the distributions of $C_L$ and $C_D$ versus angle of attack for $Re=2.07 \times 10^6$ calculated with the high Re $k-\omega$ turbulence model with wall condition and the Harten-Yee Upwind TVD scheme equipped with limiter (iv) for the ONERA-A airfoil undergoing a pitching motion. The unsteady calculations are carried out for two time steps $dt=0.5$ and $dt=1$. The angle of attack increases from 8° to 18° and 20° by an increment of $\frac{d\alpha}{dt}=0.01^\circ$. In order to create a flow field to be used as initial condition, an unsteady flow around the airfoil is simulated, keeping the angle of attack fixed at 8° for a time period long enough to achieve steady state conditions. This appears at $t=50$ giving $C_L=1.09$ and $C_D=0.0156$. From the distribution it comes out that the difference between the two time steps are negligible and so we can conclude that the calculations are time step independent. Plotted in the same figures are the steady state results with the same combination of turbulence model and discretization scheme as well as the experimental results. From the comparison it comes out that our initial effort to carry out steady-state calculations in order to simulate the experimental conditions renders better the flow physics.

6. CONCLUSIONS

In the present work, the flow around the non-symmetrical ONERA-A airfoil with forced transition at fixed points was predicted, for angles of attack corresponding to pre- and stall conditions using several combinations of turbulence models and numerical schemes in order to evaluate their performance in terms of accuracy and computational cost. A first general conclusion is that the TVD discretization schemes, although initially designed for compressible flows, can be a useful tool for incompressible flows too, especially when fast convergence must be combined with accuracy as it happens to aeroelastic calculations. From the results presented in this work, it appears that the most appropriate interpolation scheme for the high Re $k-\varepsilon$ model is the Roe-Sweby upwind TVD scheme with the minmod limiter. For the symmetric NACA0012 airfoil (Baxevanou and Vlachos, 2004), the same TVD scheme with the Chakravarthy and Osher limiter was the best. For that case the minmod limiter follows to the classification rank. In the present study, the position ranks of the specific limiters were inversed, which means that both worth to be considered in such kind of simulations.
For the high Reynolds k-ω model with wall functions, the best choice is the Harten-Yee upwind TVD scheme with limiter (v). For the high Reynolds k-ω model with wall conditions, the best choice is the Harten-Yee Upwind TVD scheme with limiter (v) and the Roe-Sweby upwind TVD scheme with the Chakravarthy and Osher limiter. Finally for the low Reynolds k-ω model, the best choice is the Roe-Sweby upwind TVD scheme with the Chakravarthy and Osher limiter. The conclusions concerning the k-ω models are similar to those for the symmetrical airfoil NACA0012 (Baxevanou and Vlachos, 2004).

For low angles of attack, all the turbulence models give good predictions for C_L. For high angles of attack, and after separation begins, all the k-ω models give better prediction of the maximum value of C_L, with the low Re k-ω model being closer to experiment. However, none of the models is able to predict the separation point and the C_L behaviour accurately.

For low angles of attack, the high Re k-ε model and the low Re k-ω model give the best predictions of C_D. The latter model also gives good predictions for high angles of attack. The two high Re k-ω models give poor predictions of C_D, with the failure of the model that integrates up to wall attributed to the interpolation scheme.

For low angles of attack, all the models give good prediction of the distribution of C_P, while for high angles of attack, the prediction of the high Re k-ε model begins to deviate from the experiment.

Noticeable differences appear in the prediction of the distribution of C_P, where the superiority of k-ω models, especially of those integrating up to the wall, is obvious mainly at high angles of attack.

**LIST OF SYMBOLS**

**Latin letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>C_D</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>C_f</td>
<td>friction coefficient</td>
</tr>
<tr>
<td>C_L</td>
<td>lift coefficient</td>
</tr>
<tr>
<td>C_M</td>
<td>pitching coefficient</td>
</tr>
<tr>
<td>C_p</td>
<td>pressure coefficient</td>
</tr>
<tr>
<td>F_i</td>
<td>mass flux in the i surface</td>
</tr>
<tr>
<td>f_i</td>
<td>geometrical coefficient (distance of cell center to i surface/distance of cell to the neighbouring cell center in the i-direction)</td>
</tr>
<tr>
<td>G_i</td>
<td>TVD limiter</td>
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<tr>
<td>k</td>
<td>turbulence kinetic energy</td>
</tr>
<tr>
<td>k_R</td>
<td>wall roughness</td>
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<td>Ma</td>
<td>Mach number</td>
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<tr>
<td>P</td>
<td>average pressure</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>U_i</td>
<td>average velocity in i-direction</td>
</tr>
<tr>
<td>X_{i+1}^-</td>
<td>RHS eigenvalue of the RANS system</td>
</tr>
<tr>
<td>x_i</td>
<td>length in i-direction</td>
</tr>
<tr>
<td>y^+</td>
<td>non-dimensional distance (y^+ = y_u*/\nu)</td>
</tr>
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</table>

where u* = friction velocity and \( \nu = \text{kinematic viscosity} \)

**Greek letters**

<table>
<thead>
<tr>
<th>Symbol</th>
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<td>\alpha_{i+\frac{1}{2}^-}</td>
<td>eigenvalue of the RANS system Jacobian</td>
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<td>\psi</td>
<td>correction determined by the entropy condition</td>
</tr>
<tr>
<td>\omega</td>
<td>specific dissipation rate</td>
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</table>

**REFERENCES**


